Anomalies of SCFTs and Geometric Engineering in M-theory

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- Geometric Engineering of QFTs is a powerful tool for exploring Strongly Coupled Systems
- The Landscape of SCFTs can be explored by studying the low-energy dynamics of various brane systems
- Reduction of SCFTs on compact manifolds, X – Lower D SCFT defined by X
- Typical SCFT is strongly coupled and may not admit Lagrangian descriptions [Gaiotto '09; Gaiotto, Moore, Neitzke '09]
- Many of such SCFTs can admit an arbitrarily large flavor symmetry – For example: Compactification of 6D SCFTs on punctured Riemann surfaces
- Physical observables of SCFTs from the geometric definitions

Compute 't Hooft anomalies of SCFTs from geometric setup

't Hooft Anomalies

- 't Hooft Anomalies: Gauge anomalies for global symmetry G – Poincaré, flavor, discrete, higher form [Gaiotto, Kapustin, Seiberg, Willett, '12]
- Quantum Anomalies: Partition function not invariant under gauge transformation in presence of background gauge fields

$$Z_{QFT}[A'] = e^{i\alpha(A,\epsilon)} Z_{QFT}[A]$$

A' & A are background gauge fields of G related by gauge transformation ϵ

- There is a **'t Hooft anomaly when**
  - $\alpha(A,\epsilon)$ cannot be removed by local counterterms
  - $\alpha(A,\epsilon)$ vanish in the limit $A \to 0$

- The anomaly measures obstruction to gauging the symmetry G
  - **Non-renormalized under RG flows** – must be reproduced by any effective description
  - Constrain IR phases of quantum systems
  - Yield central charges in supersymmetric theories

't Hooft anomalies provide a measure for degrees of freedom for QFTs – Defining data for non-Lagrangian theories

Anomaly Polynomials

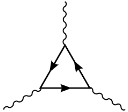
- Anomalies for **continuous** global symmetries (Most of the talk)
- The anomaly for a QFT on W_d is given by an integral of a local density

$$\alpha(A,\epsilon) = \delta_\epsilon \mathcal{W}_{QFT}[A] = 2\pi \int_{W_d} I_d^{(1)}$$

- Wess-Zumino consistency conditions imply descent relations for anomaly

$$dI_d^{(1)} = \delta I_{d+1}^{(0)}, \qquad dI_{d+1}^{(0)} = I_{d+2}$$

  - $I_{d+1}^{(0)}$ is a Chern-Simons form in W_{d+1} with boundary W_d
  - I_{d+2} is a gauge invariant form in W_{d+2} with boundary W_{d+1}
- I_{d+2} is a polynomial in curvatures of the background fields whose coefficients encode the 't Hooft anomaly of the global symmetry – **Anomaly Polynomial**
- Example in 4d: a_{IJK} and a_I are anomaly coefficients from triangle diagram



$$I_6 = a_{IJK} F^I \wedge F^J \wedge F^K + a_I F^I \wedge \mathrm{tr}(R \wedge R),$$

Reducing anomalies

- Lower D CFTs emerge in the compactification of Higher D CFTs on X_p
- Anomaly polynomials is integrated down as anomaly matching

$$I_{d-p} = \int_{X_p} I_d$$

- Successful approach in the reduction of 6d CFTs to 4d CFTs [Benini, Tachikawa, Wecht '09; IB, Beem, Bobev, Wecht '12; IB, Hanany, Maruyoshi, Razamat, Tachikawa, Zafrir '17; Kim, Razamat, Vafa, Zafrir '17; Lawrie, Martelli, Schäfer-Nameki '18;$\cdots$] or to 2d CFTs [Harvey, Minasian, Moore '98; Alday, Benini, Tachikawa '09; Benini, Bobev '13; Benini, Bobev, Crichigno '16; Lawrie, Schäfer-Nameki '18, Weigand '17; Morteza Hosseini, Hristov, Tachikawa, Zaffaroni '20;$\cdots$]
- However procedure can miss new symmetries that result from the compactification – Leads to wrong central charges in lower D theory
- Cannot account for defects added on X_p such as punctures on Riemann surfaces in 6d to 4d reductions
- A complete anomaly computation requires to consider string theory setups that realize these RG flows

Outline

Outline

Setup with M5-branes

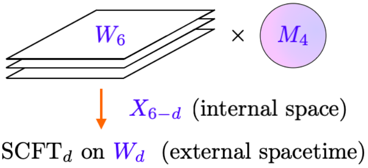
- Consider a stack of N M5-branes in M-theory
  - Flat branes: $(2,0)$ A_{N-1} SCFTs in 6D
  - Probing $\mathbb{C}^2/\Gamma$ singularity: $(1,0)$ SCFTs in 6D
  - Wrapped on a surface X: SCFTs in 4D, SCFTs in 2D
  - $\cdots$
- The 4-form flux of M-theory admits a singular magnetic source and the M-theory background has an internal boundary

$$dG_4 = N\delta_{W_6}, \qquad M_{11} = \mathbb{R}^+ \times M_{10}$$

- M_{10} is the boundary of a tubular neighborhood of the source:

$$M_{10-d} \hookrightarrow M_{10} \rightarrow W_d, \quad M_4 \hookrightarrow M_{10-d} \rightarrow X_{6-d}$$

- M_{10-d}: defines the SCFT in M-theory, can have orbifold fixed points
- M_4: The angular directions that surround the branes
- M_4 fibration fixed by topological twist



- Reducing M-theory on M_{10-d} can lead to interesting gauge symmetry, G
- Components of G: the isometry group of M_{10-d}, massless fluctuations of the C_3 potential – Expanded on $H^*(M_{10-d}, \mathbb{Z})_{free}$
- G induces a global symmetry G for QFT on W_d
- Due to the singular source of G_4, the classical variation of the M-theory action under diffeomorphisms and the gauge group G is anomalous
- Consistency of the full theory, including the M5-brane sources, must be anomaly free [Callan, Harvey '85]
- **Anomaly Inflow**: The quantum anomalies for the boundary degrees of freedom on the M5-branes must **cancel** the classical bulk anomaly

The bulk supergravity action can be used to obtain the anomalies for SCFTs from M5-branes

Flux Boundary Condition

- The anomalous variation of the M-theory action depends on the boundary condition of G_4 corresponding to the singular source [Freed, Harvey, Minasian, Moore '98]

$$G_4 = 2\pi\rho(r)\bar{G}_4 + \cdots \quad \text{with} \quad \int_{M_4} \bar{G}_4 = N$$

  $\rho(r)$ is bump function that vanishes away from the boundary
- The boundary term $\bar{G}_4$ is a closed and globally defined four-form on M_{10-d}
- $\bar{G}_4$ can be extended to a closed, gauge invariant and globally defined four-form, E_4, on the space M_{10} by gauging the action of the group G

$$\bar{G}_4 \quad \text{on} \quad M_{10-d} \quad \Rightarrow \quad E_4 \quad \text{on} \quad M_{10}$$

On W_d, the gauging corresponds to turning on background fields for the global symmetry

Anomalous Variation of M-theory [IB, Bonetti, Minasian, Nardoni '18, '19]

- The variation of the M-theory action localizes on the boundary

$$\frac{\delta S_M}{2\pi} = \int_{M_{10}} \mathcal{I}_{10}^{(1)}, \qquad d\mathcal{I}_{10}^{(1)} = \delta\mathcal{I}_{11}^{(0)}, \quad d\mathcal{I}_{11}^{(0)} = \mathcal{I}_{12}$$

- The 12-form anomaly polynomial is completely characterized by E_4 and the M-theory action

$$\mathcal{I}_{12} = -\frac{1}{6} E_4 \wedge E_4 \wedge E_4 - E_4 \wedge X_8$$

the 8-form, $X_8 = \frac{1}{192}\left[p_1(TM_{11})^2 - 4p_2(TM_{11})\right] \sim R^4$, decomposed on $M_{11} = \mathbb{R}^+ \times M_{10}$ – Gravitational anomalies

Anomaly inflow statement:

$$I_{d+2}^{\text{inf}} + I_{d+2}^{\text{CFT}} + I_{d+2}^{\text{decoupled}} = 0, \qquad I_{d+2}^{\text{inf}} = \int_{M_{10-d}} \mathcal{I}_{12}$$

I_{d+2}^{inf} captures much more than integrating anomaly polynomials of 6d SCFTs

- M_{10} and boundary condition for G_4 are

$$M_{10} = W_6 \times S^4, \qquad \bar{G}_4 = N\, d\Omega_4$$

M_4: Round 4-sphere and the induced global symmetry is $SO(5)$ – the R-symmetry of the $(2,0)$ SCFT

- The extension of $\bar{G}_4$: global angular form of the 4-sphere

$$E_4 = \frac{N}{64\pi^2}\epsilon_{a_1\cdots a_5} y^{a_5}\left[Dy^{a_1}\cdots Dy^{a_4} + 2F^{a_1a_2}Dy^{a_3}Dy^{a_4} + F^{a_1a_2}F^{a_3a_4}\right]$$
$$Dy^a = dy^a - A^{ab}y^b, \qquad y^a y^a = 1$$

- Here A^{ab} is the $SO(5)$ connection with field strength F^{ab}
- Integrating $\mathcal{I}_{12}$ on S^4: [Freed, Harvey, Minasian, Moore '98; Harvey, Minasian, Moore '98]

$$I_8^{\text{inf}} + I_8[(2,0)\ \text{SCFT}] + I_8[\text{Free (2,0) tensor}] = 0$$

- The extension E_4 has different components

$$E_4 = \sum_p E_4^p$$

- E_4^p: expansion along a basis of $H^p(M_{10-d}, \mathbb{Z})_{free}$

$$\bar{G}_4 = N^a \Omega_a^4 \quad \rightarrow \quad E_4^4 = N^a \left[\Omega_a^{4,g} + F^I \omega_{a,I}^g + F^I F^J \sigma_{a,IJ} \right]$$

  - $F^I = DA^I$: Background gauge fields for isometry group
  - Closure of E_4:

$$\iota_I \Omega_a^4 + d\omega_{a,I} = 0, \quad \iota_{(I}\omega_{a,J)} + d\sigma_{a,IJ} = 0$$

- The expansion along 2-forms is

$$E_4^2 = F^\alpha \left[\omega_\alpha^{2,g} + F^I \sigma_{\alpha,I} \right], \qquad \iota_I \omega_\alpha^2 + d\sigma_{\alpha,I} = 0$$

Choices for E_4 labeled by G_{isom}-equivariant cohomology of M_{10-d}

Compute anomaly by considering local ansatz for metric and p-forms on M_{10-d} consistent with symmetry and topology

- Impose regularity conditions on E_4
- Regularity conditions related to integrals of internal forms $(\Omega^4, \omega^2, \cdots)$
- The Inflow anomaly depends on background fields and on flux parameters of M_{10-d}

Consistency with SUGRA on $(d+1)$ spacetimes requires tadpole condition on background fields!

$$\int_{M_{10-d}} \left[E_4^2 + 2X_8\right] = 0$$

- Interesting example in 6D:

$$E_4 \;\to\; E_4 + \gamma_4, \qquad d\gamma_4 = 0$$

γ_4 is an external field on W_d

- Extra term fixed by Tadpole condition

$$I_8 \;\to\; I_8 + \frac{1}{4}\gamma_4^2 + \gamma_4(\cdots), \qquad \gamma_4 = -\frac{1}{4N}\left[c_2(G_\Gamma^N) - c_2(G_\Gamma^S)\right]$$

- Anomaly inflow for 6D $(1,0)$ SCFTs from $M5$ branes at orbifolds [Ohmori, Shimizu, Tachikawa, Yonekura, '14]
- Interpreted as a Green-Schwarz term associated to the decoupled center of mass mode of the stack in Ohmori et al.
- Tadpole constraint fixes continuous components of some background fields

$$\mathcal{I}_{12} = -\frac{1}{6} E_4 \wedge E_4 \wedge E_4 - E_4 \wedge X_8$$

- Consider an $AdS_{d+1} \times \mathcal{M}_{10-d}$ solution in M-theory supported by a G_4^{ads} flux
- We can identify $\mathcal{M}_{10-d} = M_{10-d}$ and $G_4^{ads} = \bar{G}_4$
- The 4-form E_4 can be constructed and $\mathcal{I}_{12}$ yields the anomaly for the dual SCFT
- The X_8 term in $\mathcal{I}_{12}$ yields the $\frac{1}{N^2}$ corrections to the anomaly polynomial
- Extremization principles [Intriligator, Wecht '03; Benini, Bobev '15]
- We expect the anomaly to be exact up to $\mathcal{O}(1)$ corrections due to decoupled center-of-mass degrees of freedom

Outline

Topological Mass in the Bulk [IB, Bonetti, Minasian: To Appear]

- In the reduction of M-theory on M_{10-d}, there can be topological mass terms and part of the gauge symmetry is spontaneously broken
- Example: consider an M_6 with closed p-forms, $(\lambda^1_\alpha, \omega^2_a)$, one expects massless fluctuations for C_3 of the form

$$\delta C_3 = a^a_1 \wedge \omega^2_a + b^\alpha_2 \wedge \lambda^1_\alpha + c_3 + \cdots$$

- (a^a_1, b^α_2, c_3): gauge fields in 5D spacetime for $U(1)$ $(0,1,2)$-form gauge symmetries
- M-theory Chern-Simons can lead to topological mass terms of the 5D theory

$$\mathcal{L} = \frac{1}{2\pi}\Omega_{\alpha\beta}\ b^\alpha_2 \wedge db^\beta_2 + \frac{N_a}{2\pi}\ a^a_1 \wedge dc_3 + \cdots$$

- Gauge symmetry is spontaneously broken – dual continuous global symmetry is not present
- The continuous components for the background fields associated to these symmetries in E_4 are fixed by the tadpole condition

$$\mathcal{L} = \frac{1}{2\pi}\Omega_{\alpha\beta}\ b_2^\alpha \wedge db_2^\beta + \frac{N_a}{2\pi}\ a_1^a \wedge dc_3 + \cdots$$

- In suitable normalization of gauge fields, and due to flux quantization, $(\Omega_{\alpha\beta}, N_a))$ are integrally quantized
- The topological mass terms are BF terms that describe discrete gauge symmetries in the 5D supergravity [Banks, Seiberg '11]
- For $\Omega_{12} = \frac{M}{2\pi}$, and $k = gcd(N_a)$ the discrete gauge symmetries are

$\mathbb{Z}_k$	2-form with	c_3	
$\mathbb{Z}_k$	0-form with	$m_a a_1^a$,	$N_a = k\ m_a$
$\mathbb{Z}_M \times \mathbb{Z}_M$	1-form with	(b_2^1, b_2^2)	

Discrete Symmetry for Field Theory

- The boundary global symmetry dual to the discrete gauge symmetry depends on the choice of boundary condition for the gauge fields [Witten '99]
- Dirichlet boundary conditions **cannot** be imposed on both fields in a BF theory

$$\frac{M}{2\pi} b_1^1 \wedge db_2^2 + \frac{k}{2\pi}\ m_a a_1^a \wedge dc_3$$

- Dirichlet boundary conditions for b_1 fix a source for a $\mathbb{Z}_M$ global 1-form symmetry in the dual theory, Similar for picking Dirichlet BC for $m_a a_1^a$ or for c_3 [Gaiotto, Kapustin, Seiberg, Willett '14; Hofman, Iqbal, '18]
- Mixed boundary conditions between the fields lead to a larger class of possible choices of boundary discrete symmetry [Gaiotto, Kapustin, Seiberg, Willett '14]
- For $M = n_1 n_2$, there is the choice with $\mathbb{Z}_{n_1} \times \mathbb{Z}_{n_2}$ 1-form global symmetry
- Anomaly polynomial terms for fields with Dirchlet BC capture the 't Hooft anomalies for the discrete symmetry [Kapustin, Thorngren '14; Bergman, Tachikawa, Zafrir '20]

$$I_6 = M\ dB_1^1 \wedge dB_2^2 + \mathcal{A}^{\alpha}_{\bullet\bullet} dB_\alpha \wedge F^\bullet \wedge F^\bullet + \cdots$$

Singletons and Decoupled modes [IB, Bonetti, Minasian: To Appear]

- In presence of a boundary, BF theories admit singleton modes [Witten '99; Maldacena, Moore, Seiberg '01]
- Singletons: Pure gauge modes in the bulk and dynamical in the boundary

$$\frac{M}{2\pi} b_p \wedge da_{d-p-1} \qquad \rightarrow \qquad \text{(p-1)-form gauge field singleton}$$

- SUSY partners from KK singletons
- Singletons dual to Goldstone modes of the spontaneously broken boundary symmetry associated to (b_p, a_{d-p-1}) gauge fields
- Singletons contribute to the inflow anomaly and must be subtracted as part of the decoupled modes

$$I^{inf} + I^{CFT} + I^{decoupled} = 0$$

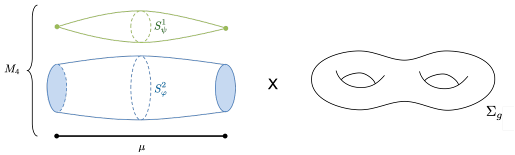
- Singletons account for all decoupling modes in SUSY compactifications of M5-branes on punctured Riemann surfaces! (not including orbifold theories)

The symmetry and topology of M_{10-d} completely fix the anomaly of SCFTs from M5-branes and its compactifications

Outline

Things to do

- Compute the anomalies for $\mathcal{N} = 2$ Class $\mathcal{S}$ of A_N type with arbitrary punctures [IB, Nardoni, '18; IB, Bonetti, Minasian, Nardoni '19]
- The possible choices of E_4 from $M_6 = S^4 \times \Sigma_{g,n}$ is in one-to-one correspondence with the classification from Hitchin equations
- Choices come from different resolutions of punctures on $\Sigma_{g,n}$ in M_6
- This provides an alternate derivation of punctures and the data associated with them from bulk SUGRA
- Explore punctures for $\mathcal{N} = 1$ Class $\mathcal{S}$ [IB, Beem, Bobev, Wecht '12] and from Class $\mathcal{S}_k$ [Gaiotto, Razamat, '15; Hanany, Maruyoshi '15 and $\mathcal{S}_\Gamma$ [Heckmann, Jefferson, Rudelius, Vafa, '16]
- Study Class $\mathcal{S}$ from the D-series (Inflow for 6D SCFT from [Yi, '00]) and E-string theories
- Example – Class $\mathcal{S}_2$



- Consider a stack of N M5-branes on Σ_g and probing a $\mathbb{Z}_2$ orbifold fixed point
- Here $M_6 = M_4 \times \Sigma_g$ and M_4 is $S^4/\mathbb{Z}_2$ with resolution two cycles
- The resolution is supported by threading flux (N^N, N^S) on 4-cycles made from the resolution 2-cycles combined with the Riemann surface
- There are a total of three 4-cycles with three flux parameters (N, N^N, N^S), Associated to them are three closed 2-forms by Poincare duality
- The isometry group is $U(1)_R \times SU(2)_F$ and the naive symmetry from C_3 is $U(1)^3$
- From the 6d $(1,0)$ theory, only $U(1)_N \times U(1)_S$ is visible, the third $U(1)_C$ is an accidental symmetry from the compactification!

A look at class $\mathcal{S}_2$ [IB, Bonetti '19]

- A combination of the three U(1)s is broken by a topological mass – Spontaneous symmetry break of a U(1) global symmetry for the field theory
- The symmetry of low-energy theory is then $U(1)'_N \times U(1)'_S \times U(1)_R \times SU(2)_L$
- The generators of the 2 $U(1)$s visible from the 6d SCFT are shifted as

$$T'_N = T_N - \frac{N^N}{N} T_C, \qquad T'_S = T_S - \frac{N^S}{N} T_C$$

- After obtaining anomaly polynomial, compute large N central charge by a-maximization [Intriligator, Wecht '03]
- Inflow data can be matched with a family of $AdS_5 \times \mathcal{M}_6$ obtained in [Gauntlett, Martelli, Sparks, Waldram '04]

- 5d SUGRA theory admits a rich discrete gauge symmetry! Thus complex network of discrete symmetry in SCFT which is acted upon by $Sp(2g, \mathbb{Z})$

multiplicity	fields	top. mass terms	bulk gauge symm.
$b^2(M_6) = 3$	a_1^a	$\frac{1}{2\pi} N_a\, a_1^a \wedge dc_3$	$U(1)^2$ 0-form symm. $\mathbb{Z}_k$ 0-form symm.
1	c_3		$\mathbb{Z}_k$ 2-form symm.
$b^1(M_6) = 2g$	$b_2^i,\ \tilde{b}_2^i$	$\frac{1}{2\pi} M\, \tilde{b}_2^i \wedge db_2^i$	$(\mathbb{Z}_M \times \mathbb{Z}_M)^g$ 1-form symm.
$b^3(M_6) = 4g$	$a_0^{i\pm},\ \tilde{a}_0^{i\pm}$	—	5D axions

- There are $4g$ background 1-forms in the anomaly polynomial associated to the axions – Anomaly for background dependent couplings and "(-1)-form symmetry"? [Córdova, Freed, Lam, Seiberg, '19]

Outlook

- Origin of decoupled modes from M_{10-d}

$$I^{inf} + I^{QFT} + I^{decoupled} = 0$$

- Discrete symmetries and higher form symmetries – role of torsion in Cohomology group
- Anomalies related to large gauge transformations and duality groups of QFTs – Global anomalies
- Defects and extended operators – higher form discrete symmetry
- Explore general compactifications of 6D theories in IIB/F-theory (Inflow polynomial in [IB, Bonetti, Minasian, Weck '20]), massive IIA
- Since the analysis relies less on SUSY, we hope to be able to study more general classes of compactifications with punctures and defects

THANK YOU!

Outline

IIB and D3 branes [IB, Bonetti, Minasian, Weck '20]

- One can also consider brane systems in type II string theories
- The polynomials that encode the anomalies are 11-forms, $\mathcal{I}_{11}$ constructed from gauge invariant boundary conditions of various flux
- The anomaly polynomial of IIA is related to the M-theory $\mathcal{I}_{12}$ by a reduction, It is similarly characterized by IIA Chern-Simons terms
- The anomaly polynomial for IIB receives a contribution from the kinetic term of the self-dual five-form flux
- If we consider a stack of D3-branes supported by the five-form flux, F_5

$$F_5 = 2\pi(1+\star)\rho(r)\bar{F}_5 + \cdots \quad \text{on} \quad M_{10} = \mathbb{R}^+ \times W_d \times M_{9-2d}$$

  The boundary term $\bar{F}_5$ on M_{9-2d} can be extended to E_5 on $W_d \times M_{9-2d}$
- The 11-form and the inflow anomaly polynomial are given as

$$\mathcal{I}_{11} = \frac{1}{2} E_5 \wedge dE_5 - E_5 \wedge H_3 \wedge F_3, \qquad I_{2d+2}^{\text{inf}} = \int_{M_{9-2d}} \mathcal{I}_{11}$$

IIB and D3 branes [IB, Bonetti, Minasian, Weck '20]

- The 11-form and the inflow anomaly polynomial are given as

$$\mathcal{I}_{11} = \frac{1}{2} E_5 \wedge dE_5 - E_5 \wedge H_3 \wedge F_3 , \qquad I^{\text{inf}}_{2d+2} = \int_{M_{9-2d}} \mathcal{I}_{11}$$

- For $\mathcal{N} = 4$ SYM, E_5 is the global angular form of the 5-sphere, e_5! Integrating $\mathcal{I}_{11}$ yields the anomaly for the $SO(6)$ R-symmetry group

$$E_5 = N\, e_5 , \qquad dE_5 = -N\, \pi^* \chi\,(SO(6)) ,$$
$$I_6^{inf} = \frac{1}{2} N^2 \chi\,(SO(6)) = \frac{1}{2} N^2 c_3\,(SU(4))$$

- For more general $\mathcal{N} = 1$, E_5 is the volume of SE_5 gauged over the world volume theory! Consistent with holographic analysis by [Benvenuti, Pando Zayas, Tachikawa 06]
- Anomaly of $\mathcal{N} = 4$ SYM on punctured Riemann surface
- This anomaly formula can be used to study compactifications of 4D SCFTs to 2D QFTs

- Generalize type IIB with non-trivial axio-dilaton profile
- Consider an elliptic fibration over the IIB background

$$\mathbb{E}_\tau \hookrightarrow M_{12} \to M_{10}$$

- The anomaly polynomial is

$$\mathcal{I}_{11} = \frac{1}{2} E_5 \wedge dE_5 - E_5 \wedge \pi_* \left[X_8(TM_{12}) + \frac{1}{2}\mathcal{E}_4 \wedge \mathcal{E}_4 \right]$$

- F_3 and H_3 are encoded in $\mathcal{E}_4$, for trivial elliptic fiber

$$\mathcal{E}_4 = F_3 \wedge dx + H_3 \wedge dy$$

- Anomalies of $\mathcal{N} = 4$ with varying coupling, τ_{YM}, can be studied with this generalization [Lawrie, Martelli, Schäfer-Nameki '18]