New Holographic CFTs: Finding the Needle in the Haystack

Alexandre Belin

Based on:

2002.07819 with N. Benjamin, A. Castro, S. Harrison, C. Keller
+ earlier work with A. Castro, J. Gomes, C. Keller, B. Mühlmann

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What is the space of consistent,
UV-complete theories of quantum gravity?

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For gravity with $\Lambda < 0$, AdS/CFT has “solved” this question.

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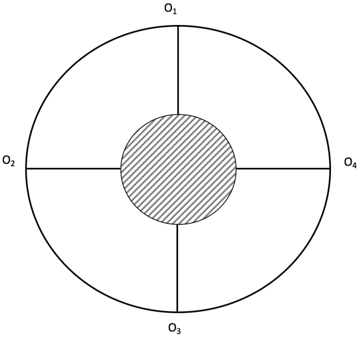
$< O_1 \, O_2 \, O_3 \, O_4 > =$



The CFT $\Longrightarrow$ UV-complete and non-perturbative definition of Q.G. in AdS

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But we currently have no complete classification of all CFTs. $\Longrightarrow$ the conformal bootstrap

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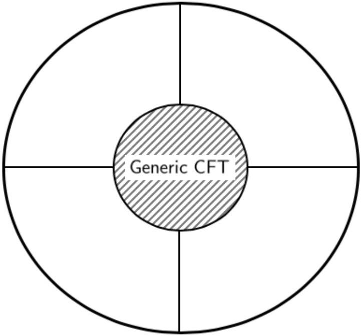
For a generic CFT:



$\not\sim$ semi-classical General Relativity

Organizing the search

A 2-step process:

- S1: What are the conditions such that a CFT has a gravity dual described by semi-classical G.R. at low energies? CFTs that satisfy these conditions are called holographic CFTs.

  [Hartman Strings '18; Zhiboedov Strings '19]

- S2: Can we construct (ideally all) such CFTs?

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[Hartman Strings '18; Zhiboedov Strings '19]

- S2: Can we construct (ideally all) such CFTs?

Main goal of the talk

Present evidence for a new infinite family of holographic CFTs!

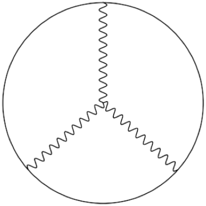
Outline

1. Introduction
2. Review of holographic conditions
3. Symmetric Product Orbifolds
4. A new infinite family of holographic CFTs

Review of holographic conditions

1. Gravity is semi-classical / large N

$$\frac{1}{N} = \frac{\langle TTT \rangle}{\langle TT \rangle^{3/2}} =$$



$$\sim \sqrt{G_N}$$

In this talk, I will focus on AdS$_3$/CFT$_2$:

$$\boxed{c = \frac{3\ell_{\text{AdS}}}{2G_N}}$$

[Brown, Henneaux]

2. Bulk EFT is local / large gap

$$\boxed{\Delta_{s>2}^{\text{S.T.}} \geq \Delta_{\text{gap}} \gg 1}$$

[Heemskerk, Penedones, Polchinski, Sully]

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3. $S_{\text{CFT}} = S_{\text{BH}}$ / sparse spectrum

$$\boxed{\rho(\Delta) \leq e^{2\pi\Delta}\,, \qquad \Delta \leq \tfrac{c}{12}}$$

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[Hartman, Keller, Stoica]

4. Bulk locality v2 / very sparse spectrum

$$\boxed{\rho(\Delta) \sim e^{\Delta^{\gamma}} \,, \quad \gamma < 1}$$

[AB, Freivogel, Jefferson, Kabir]

Bulk locality v2

If the bulk low energy EFT is local:

$$F^{\text{bulk fields}} \sim T^D \,, \qquad AdS_{d+1} \times M_{D-(d+1)}$$

$$\Longleftrightarrow$$

$$\rho(\Delta) \sim e^{\Delta^\gamma} \,, \qquad \gamma = \frac{D-1}{D}$$

It is currently unknown how large gap and very sparse spectrum interplay.

Symmetric Product Orbifolds

Start with a “seed” CFT $\mathcal{C}$, then take N copies.

$$\boxed{\mathcal{C}_N \equiv \frac{\mathcal{C}^{\otimes N}}{S_N}}$$

$\Longrightarrow$ huge landscape of large c CFTs!

$\mathcal{C}_N$ completely determined from $\mathcal{C}$.

Generalization: permutation orbifolds for $G_N \subseteq S_N$.

[AB, Keller, Maloney; Haehl, Rangamani]

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For all choices of the seed CFT, we have for $\mathcal{C}_N$:

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Large N ✓

Sparse spectrum ✓

$\rho(\Delta) = e^{2\pi\Delta}$ [Keller]

Very sparse spectrum ✗

Large gap ✗

$\Delta_{\text{gap}} = \mathcal{O}(1)$

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Game Over?

The D1D5 CFT

The canonical example of $\mathrm{AdS}_3/\mathrm{CFT}_2$:
symmetric orbifold with $\mathcal{C} =$ NLSM on T^4 or K3.

The D1D5 CFT

The canonical example of AdS$_3$/CFT$_2$:
symmetric orbifold with $\mathcal{C}$ = NLSM on T^4 or K3.

There is a conformal manifold:
$S_{CFT} + \lambda \int d^2 x O(x)$ is also a CFT.

Orbifold description is the free/tensionless point

[Talk by Rajesh yesterday]

	$\lambda = 0$	$\lambda \gg 1$
$\rho(\Delta)$	$e^{2\pi\Delta}$	$e^{\Delta^{5/6}}$
Δ_{gap}	$\mathcal{O}(1)$	$\sim \lambda^{1/4} \gg 1$

Can we find new examples for other $\mathcal{C}$?

$\Longrightarrow$ Focus on CFT$_2$ with $\mathcal{N} = (2, 2)$ SUSY.

I will argue for a new infinite family!

First, revisit holographic conditions for symmetric orbifolds.

Diagnostics for Symmetric Orbifolds

Recall large N, sparse spectrum ✓

Missing large gap + sparse → very sparse

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Missing large gap $+$ sparse $\rightarrow$ very sparse

For large gap, we require $\exists$ exactly marginal:

$$\delta S_{\mathrm{CFT}} = N^{1-\frac{K}{2}} \lambda \int d^2 z O^K(z, \bar{z})$$

- $(h, \bar{h})_O = (1,1)$ + protected
- K: K-trace operator
- $O \in$ twisted sector

Exact marginality can be obtained thanks to SUSY.

Take chiral primary with $h = \bar{h} = \frac{1}{2}$,

$$G^{-}_{-1/2}\bar{G}^{-}_{-1/2}\,|h = \bar{h} = \tfrac{1}{2}, Q = \bar{Q} = 1\rangle$$

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The lightest chiral primary in twist-L sector:

$$h^{\text{min}}_{\text{c.p.}} = \tfrac{c}{12}\,(L-1)$$

$\Longrightarrow \boxed{c_{\text{seed}} \leq 6}$

Very sparse spectrum

What is the growth $\rho(\Delta)$ at strong coupling?

Again use supersymmetry:

$$\rho(\Delta) \geq \rho^{\text{BPS}}(\Delta)$$

If $\rho^{\text{BPS}} \sim e^{\Delta^{\gamma}}$, good start!

We will study ρ^{BPS} through the elliptic genus:

$$Z_{\text{EG}}(\tau, z) = Tr_{RR}(-1)^F q^{L_0-\frac{c}{24}} y^{J_0} \bar{q}^{\bar{L}_0-\frac{c}{24}}$$

Advantages of the elliptic genus

- It is protected on the conformal manifold.

$\Longrightarrow$ Window into the putative strong coupling regime

[Benjamin, Cheng, Kachru, Moore, Paquette; Benjamin, Kachru, Keller, Paquette]

- $Z_{\text{E.G.}}$ is (up to unwrapping) a weak Jacobi form.

$\Longrightarrow$ Finite number of possibilities to explore

Elliptic genus for the orbifolds

Let us define

$$Z_{\mathrm{E.G.}}(\tau, \kappa z) =: \phi(\tau, z) = \sum_{n,l} c(n,l) q^n y^l$$

For ϕ_m the (unwrapped) E.G. of $\mathcal{C}_N$ we have

$$\mathcal{Z}(\tau, z, \rho) = \sum_m \phi_m(\tau, z) p^m = \prod_{n,l,m} \frac{1}{(1 - p^m q^n y^l)^{c(nm,l)}}$$

[Dijkgraaf, Moore, Verlinde, Verlinde]

Information on the growth is in the $c(n,l)$!

A complete classification

We obtained a complete classification for which wJf lead to slow growth.

[AB, Castro, Keller, Mühlmann]

Two possible outcomes:

1. The growth is $\rho^{\text{BPS}}(\Delta) \sim e^{\Delta}$
2. The growth is $\rho^{\text{BPS}}(\Delta) \sim e^{\sqrt{\Delta}}$

$\Longrightarrow$ Simple criterion to distinguish based on $c(n, l)$.

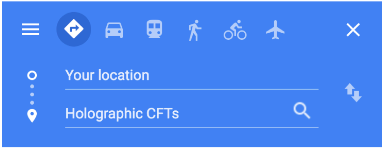
Our criterion gives

$$\boxed{c_{\text{seed}} \leq 6}$$

Same as we found for the existence of marginal operators!

Important: very different conditions! But they seem to give the same outcome.

To sum up, the classification serves as a map



Your location

Holographic CFTs

Originally, we found four new slow-growing functions. [AB, Castro, Gomes, Keller; Alejandra's talk Strings '19]

Now, infinitely more! [AB, Castro, Keller, Mühlmann]

Some are known examples, orbifolds of T^4

[Datta, Eberhardt, Gaberdiel]

The stragegy

- Look for CFTs that have these functions as their E.G.

- Study the spectrum, look for exactly marginal, twisted S.T. operators.

- If both succeed, strong evidence for new holographic CFT!

- Split the search into $1 \leq c_{\text{seed}} < 3$ and $3 \leq c_{\text{seed}} \leq 6$

$1 \leq c_{\text{seed}} < 3$

Full classification of CFTs with $1 \leq c_{\text{seed}} < 3$:

$$\boxed{\text{The } \mathcal{N} = 2 \text{ minimal models}}$$

[Boucher, Friedan, Kent; Di Vecchia, Petersen, Yu, Zheng]

They have

$$c = \tfrac{3k}{k+2}\,, \qquad k = 1, \ldots$$

$\Longrightarrow$ ADE classification: A-series, D-series, E_6,E_7,E_8.

Main Result

For a seed theory given by *any* $\mathcal{N} = 2$ min. model:

1. The E.G. exhibits slow-growth.
2. There is at least one exactly marginal, single-trace, twisted sector operator.

$\Longrightarrow$ New infinite family of holographic CFTs

The spectra are known, can be checked explicitly.

$3 \leq c_{\text{seed}} \leq 6$

No full classification, but we can consider some cases:

- Products of minimal models
- Kazama-Suzuki theories

$$\frac{SU(M+1)_k \times SO(2M)_1}{SU(M)_{k+1} \times U(1)_{M(M+1)(M+k+1)}} , \qquad c = \frac{3kM}{k+M+1}$$

[Kazama,Suzuki]

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[Kazama,Suzuki]

We find:

$\boxed{\text{Slow growing E.G.} \Rightarrow \text{There is a marginal operator}}$

The converse isn't true (one counter-example)

Interesting properties of the models

The dimension of the moduli-space grows very fast with k.

There are many exactly marginal multi-trace operators.

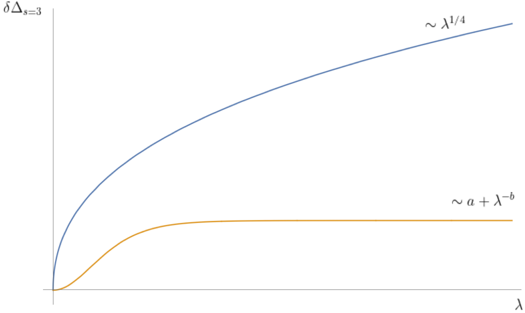
They are very interesting for AdS/CFT:

[Aharony, Berkooz, Silverstein]

$\Longrightarrow$ tunable bulk couplings in the 't Hooft limit.

$\Longrightarrow$ Interpolate between 't Hooft type bulk, and strongly coupled AdS matter

How could the models not be holographic?



Saturation + miraculous cancellations in the E.G.

If this happens, $\ell_{AdS}/\ell_s \sim \mathcal{O}(1)$. Similar to SUSY SYK in $d = 2$. [Murugan, Stanford, Witten]

A renormalization theorem?

Take a CFT with an 't Hooft limit and an exactly marginal operator which plays the role of a gauge coupling.

Define β:

$$\delta\Delta \sim_{\lambda\to\infty} \lambda^{\beta}$$

Under which conditions do we have $\beta > 0$?

Conclusion and Open Questions

- Evidence for a new infinite family of holographic CFTs: Symmetric Orbifold of $\mathcal{N} = 2$ min. models + deformation.
- The theories have interesting moduli spaces, richer than D1D5.
- Can we find the dual SUGRA backgrounds?
- Can we find top-down derivation with branes?
- A renormalization theorem? Integrability? Localization?

Backup Slides

Some scenarios for the light spectrum

$$\rho_\infty(\Delta) = \lim_{N\to\infty} \rho_N(\Delta)\,, \quad \Delta \text{ fixed}$$

$\rho_\infty(\Delta)$	EFT
$e^{\sqrt{\Delta}}$	Pure gravity in AdS_3
$e^{\Delta^{2/3}}$	Gravity + scalar in AdS_3
$e^{\Delta^{5/6}}$	Pure gravity in $\mathrm{AdS}_3 \times S^3$
$e^{\Delta^{\frac{d-1}{d}}}$	LQFT in $\mathrm{AdS}_3 \times M_{d-3}$
e^{Δ}	String theory in AdS_3
e^{Δ^2}	??

Holographic CFTs

Sparser

The α criterion

The wJf is:

$$\phi_{0,t}(\tau, z) = 0 \cdot y^{-t} + \cdots y^{-b} + \cdots$$

y^{-b} is the most polar term. We have $c = \frac{6b^2}{t}$.

For a term $q^n y^l$ in ϕ, we define

$$\alpha = \max_{j=0,\ldots,b-1} \left(-\frac{t}{b^2} j \left(j - \frac{b\ell}{t} \right) - n \right) \ .$$

If $\alpha < 0$ for all polar terms $\Longrightarrow$ slow growth

Ex. of slow-growing wJF and associated CFTs

t	b	$c = \frac{6b^2}{t}$	CFT Examples
1	1	6	K3 sigma model
2	1	3	$T^2/\mathbb{Z}_2$
3	1	2	D_4
4	1	$\frac{3}{2}$	A_3
4	2	6	T^4/G
6	1	1	A_2
6	2	4	$(A_2)^4$
8	2	3	$(A_3)^2$
9	3	6	$(A_2)^6$
10	2	$\frac{12}{5}$	D_6
12	2	2	A_5
12	3	$\frac{9}{2}$	$(A_3)^3$
15	3	$\frac{18}{5}$	$(A_4)^2$
16	4	6	$(A_3)^4$
18	3	3	$(A_2)^3$, $A_2 \otimes A_5$

Example of BPS spectra to compare to SUGRA

For the first minimal model, we have:

$$\chi_{\infty}^{\mathrm{NS},A_2} = \prod_{n=1}^{\infty} \frac{(1-q^{n-\frac{1}{12}}y^{-1})(1-q^{n+\frac{1}{12}}y)}{(1-q^{\frac{n}{12}}y^{n})(1-q^{\frac{n+1}{12}}y^{n+1})(1-q^{\frac{n}{2}}y^{-6n})^2}\,.$$

To compare with K3:

$$\chi_{\infty}^{\mathrm{NS},K3} = \prod_{n\geq 1} \frac{(1-q^{n})^{20}(1-q^{n-1/2}y)^2(1-q^{n-1/2}y^{-1})^2}{(1-q^{n/2}y^{n})^{24}(1-q^{n/2}y^{-n})^{24}}\,.$$