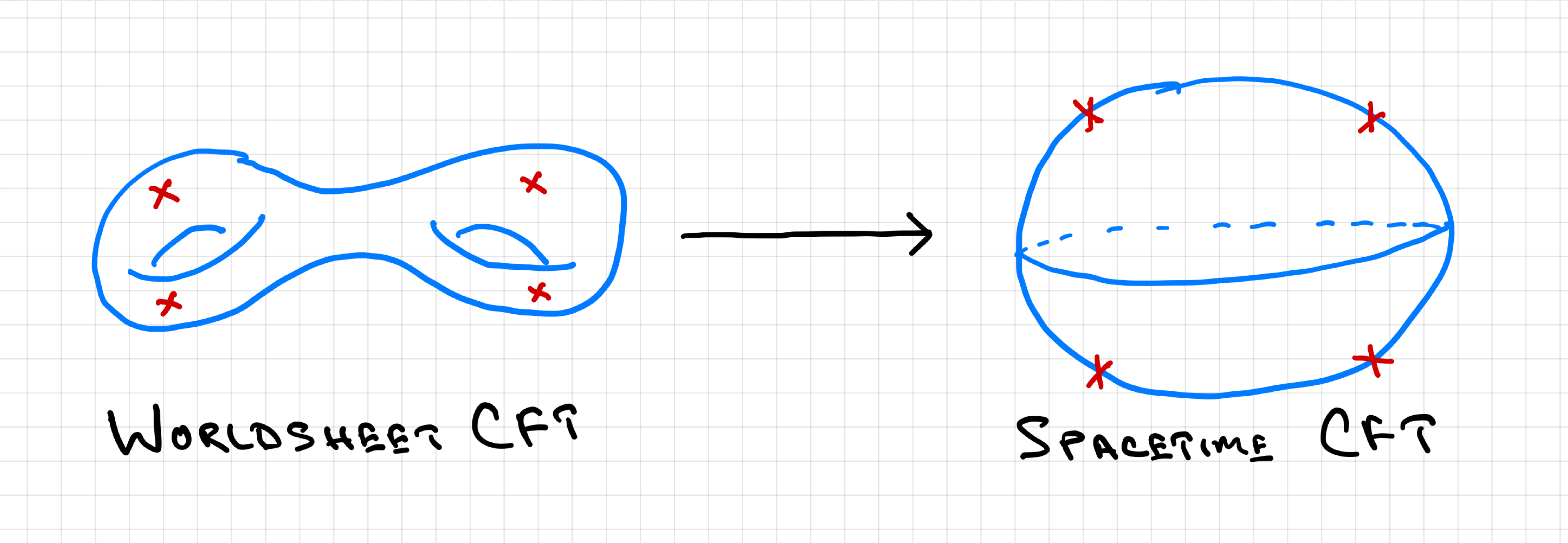

Deriving the AdS_3/CFT_2 Correspondence

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Based on: arXiv:1911.00378 (w/ L. Eberhardt & M. Gaberdiel);

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Derivation vs. Verification

- What does it mean to derive the AdS/CFT correspondence?
- Top down construction of dual pairs via Maldacena's near horizon limit.
- Match (BPS/integrable) spectrum, compute correlators, Wilson lines, EE…
- Verification of equality of both sides appears miraculous/mysterious.
- Also does not help in delineating the scope of gauge-string duality - how and to what extent, can we systematically come up with new dual pairs?

Can We Tautologise AdS/CFT?

- Thus derivation of AdS/CFT is not mathematical fastidiousness.
- Rather a physics necessity to lay bare the inner workings of the duality.
- Perhaps ultimately a geometrisation of quantum information (tensor networks).
- Can aim at a less ambitious but concrete goal: equivalence of worldsheet CFT description of certain AdS_{d+1} backgrounds with (large N) boundary CFT_d.
- Dictionary that relates two (separately) mathematically well defined entities - can we make the equality manifest?

From Worldsheet CFT to Spacetime CFT

- A large part of the dictionary is the equality of (euclidean) correlators.

$$\int_{\mathcal{M}_{g,n}} \langle \mathcal{V}^{w_1}_{h_1}(x_1;z_1)\mathcal{V}^{w_2}_{h_2}(x_2;z_2)\dots\mathcal{V}^{w_n}_{h_n}(x_n;z_n)\rangle_{\Sigma_{g,n}} = \langle \mathcal{O}^{(w_1)}_{h_1}(x_1)\mathcal{O}^{(w_2)}_{h_2}(x_2)\dots\mathcal{O}^{(w_n)}_{h_n}(x_n)\rangle_{\mathrm{S}^d}\Big|_g$$

- Assumes a matching of spectrum: on shell vertex operators of physical states in the worldsheet CFT $\leftrightarrow$ single trace operators in spacetime CFT

addl. labels

spacetime position

$$\mathcal{V}^w_h(x;z) \leftrightarrow \mathcal{O}^{(w)}_h(x)$$

worldsheet position

spacetime operator dim.

Can we transform the LHS correlator into the RHS correlator (or vice versa)?

A Testing Ground

- Need an example where this program can be carried through.

- PROPOSAL [Eberhardt-Gaberdiel-RG ('18)]: String theory on $AdS_3 \times S^3 \times T^4$ with ($k = 1$) unit of NS-NS flux (in a perturbative expansion) = Large N limit of $(T^4)^N / S_N$ - Symmetric orbifold 2d CFT.

- Worldsheet theory with NS-NS flux can be quantised - in RNS formalism essentially a WZW model: $\mathfrak{sl}(2,\mathbb{R})_{k+2} \oplus \mathfrak{su}(2)_{k-2} \oplus \mathbb{T}^4 \oplus \text{fermions} \oplus \text{ghosts} \quad (k \geq 2)$.

  [Maldacena-Ooguri ('00)]

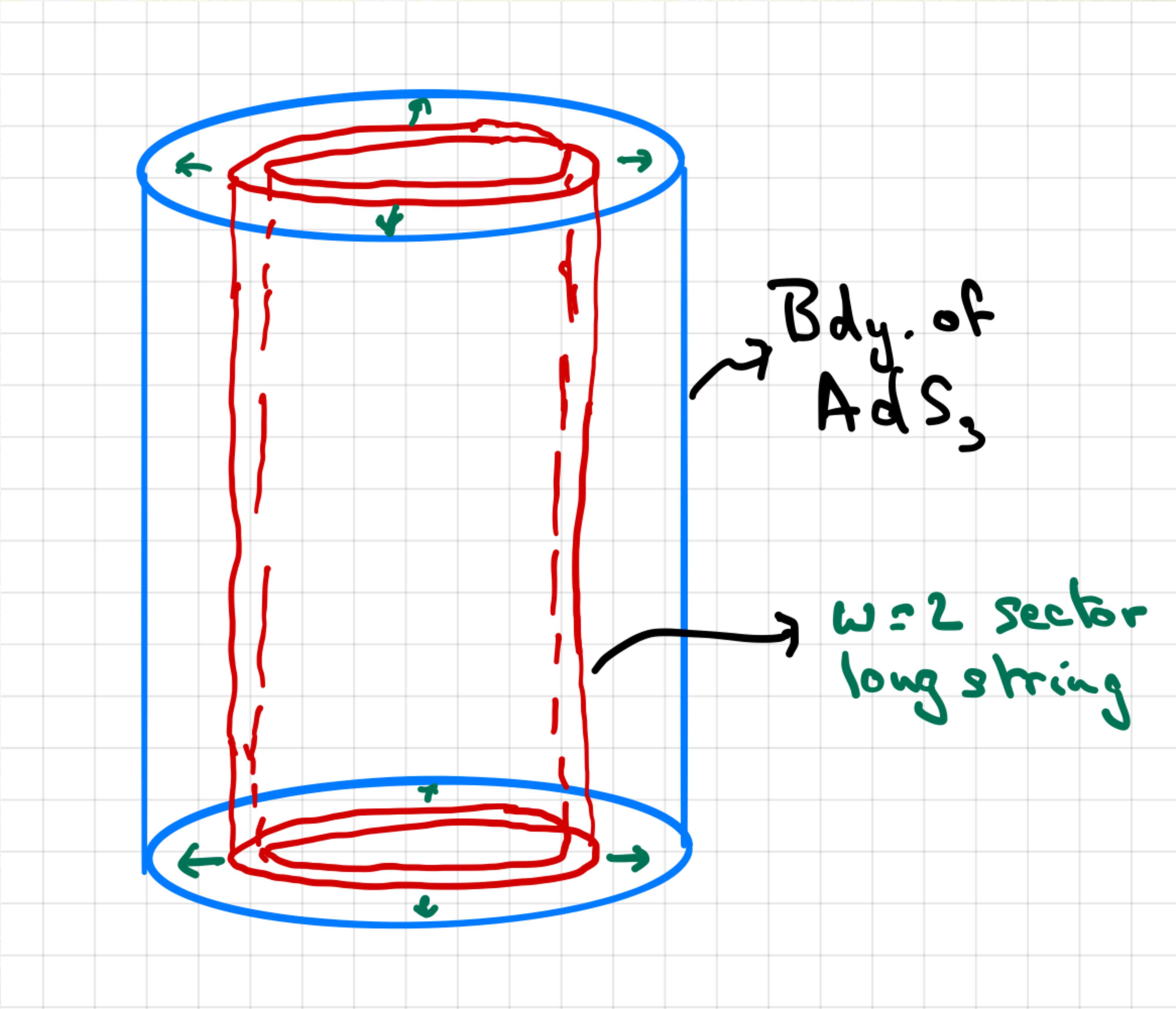
- Spectrum organised in terms of sectors with `spectral flow' (w) - asymptotic winding of string on AdS_3. Also continuum of long strings with mom. p.

A Tractable Case

- For $(k=1)$ - `tensionless limit' - can employ the hybrid formalism of Berkovits-Vafa-Witten based on a $\mathfrak{psu}(1,1|2)_1$WZW model.
- Spectrum truncates to short reps of $\mathfrak{psu}(1,1|2)_1$.
- Only states at the bottom of the long string continuum survives: $j = \frac{1}{2} + i(p=0)$.

  sl(2) qtm no.
- Match of the entire perturbative spectrum with dual CFT (and other checks).

  [Eberhardt-Gaberdiel-R.G. ('18); see Lorenz@Strings 2019]



Correlating Correlators

- Thus we have $\mathcal{V}_h^w(x;z) \leftrightarrow \mathcal{O}_h^{(w)}(x)$; spectral flow sector $(w) \leftrightarrow$ twisted sector (w).

- GOAL: to derive the relation between correlators rather than verify.

$$\int_{\mathcal{M}_{0,n}} \langle \mathcal{V}_{h_1}^{w_1}(x_1;z_1)\mathcal{V}_{h_2}^{w_2}(x_2;z_2)\dots\mathcal{V}_{h_n}^{w_n}(x_n;z_n)\rangle_{\Sigma_{0,n}} = \langle \mathcal{O}_{h_1}^{(w_1)}(x_1)\mathcal{O}_{h_2}^{(w_2)}(x_2)\dots\mathcal{O}_{h_n}^{(w_n)}(x_n)\rangle_{S^2}\Big|_{g=0}$$

- Restrict to ground states in each sector: $h = \dfrac{w^2-1}{4w}$ - states at the bottom of the would-be continuum (i.e. no torus oscillators excited).

- Also to genus zero (for generalisation to higher genus see Eberhardt ('20)).

Localisation

- Uncover the structural reason why correlators in this worldsheet CFT match with those of the spacetime CFT.

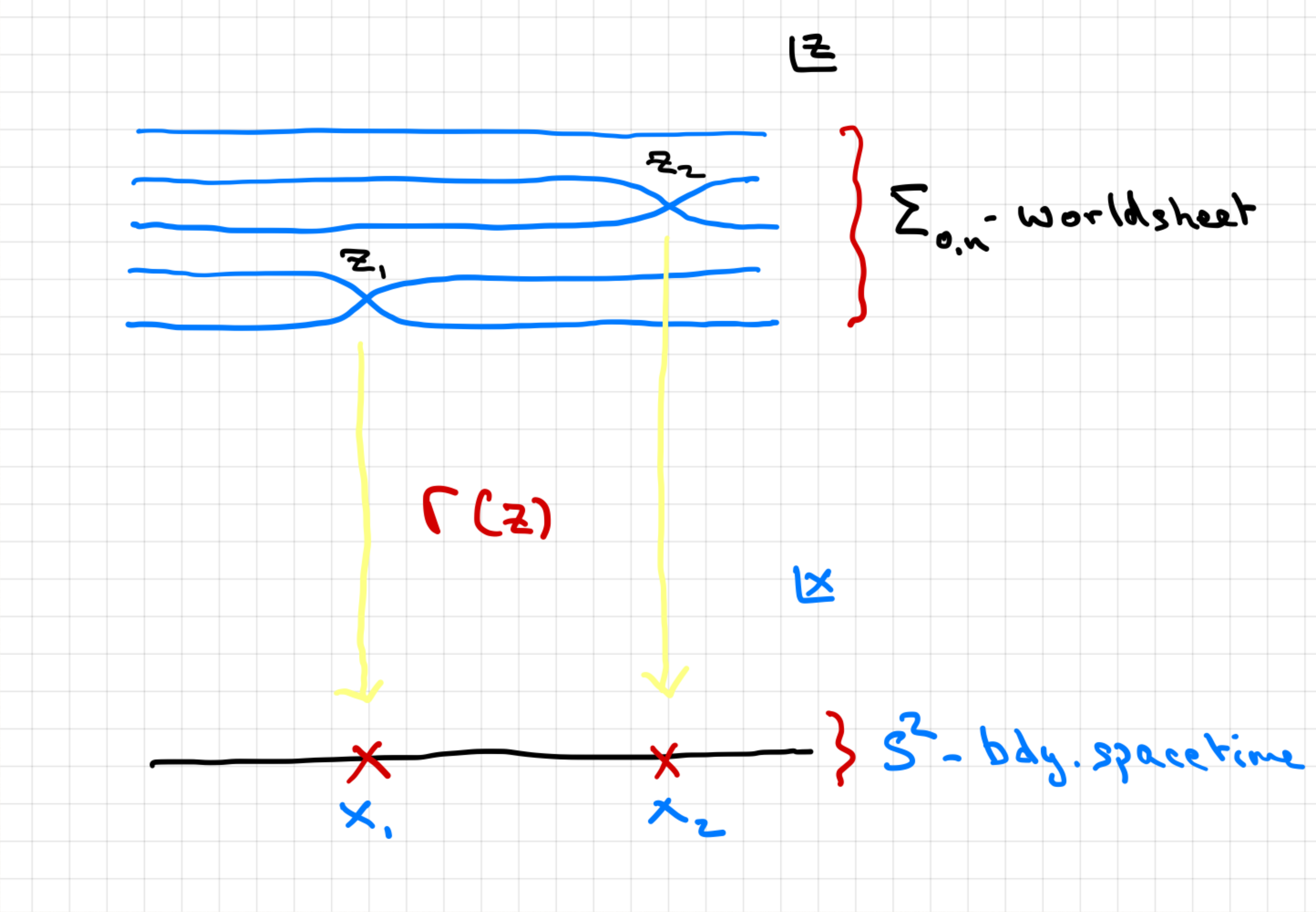
- Localisation on moduli space to holomorphic covering maps: $x = \Gamma(z)$ with specified branching (w_i) at the insertions $x \approx x_i + a_i^\Gamma (z - z_i)^{w_i}; \quad (i = 1 \ldots n)$.

- CLAIM: Worldsheet correlator $\propto \prod_{i=1}^{n-3} \delta(x_i - \Gamma(z_i))$ - discrete set of points.

- Leads to a realization of covering space computation of symmetric product orbifold correlators (and its identification with worldsheet).

[Lunin-Mathur('00); Pakman-Rastelli-Razamat('09)]

The Worldsheet as a Covering Space



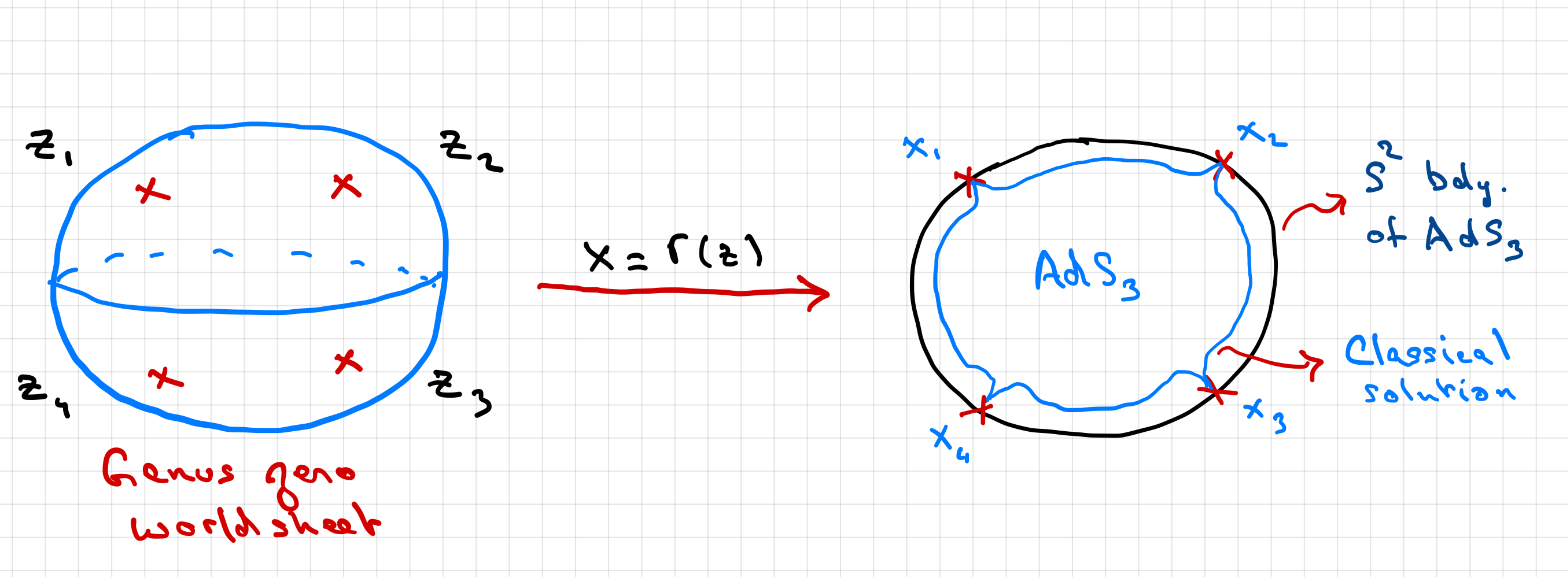
A Two-Pronged Strategy

- Argue (i) that worldsheet correlators localise and (ii) give the right contributions that reproduce the correlators of the spacetime CFT.

- (i) Ward identities for $\mathfrak{sl}(2,\mathbb{R})_{k+2}$ correlators of spectrally flowed vertex operators have the special delta function solution provided: $\sum_{i=1}^{n} j_i = \frac{k}{2}(n-2)+1$ - satisfied for $j_i = \frac{1}{2};\ k=1;\ \forall n$.

- (ii) An exact classical solution of the sigma model corresponding to this covering space map. Gives precisely the contribution to the path integral as in Lunin-Mathur computation of symmetric orbifold correlators.

Semiclassical Picture



Ward Identities

- Ward Identities for $\mathfrak{sl}(2,\mathbb{R})_{k+2}$ are very nontrivial on the spectrally flowed vertex operators.

- Under spectral flow: $\sigma^w(J^\pm)(z) = z^{\mp w} J^\pm(z);\ \sigma^w(J^3)(z) = J^3(z) + \frac{(k+2)w}{2z}\ .$

- Therefore, on the flowed primary vertex operator, the OPE with the currents is

$$J^+(z)V_h^w(0;0) \sim \sum_{p=2}^{w+1} \frac{(J^+_{p-1}V_h^w)(0;0)}{z^p} + \frac{\partial_x V_h^w(0;0)}{z}\ ,$$

$$J^3(z)V_h^w(0;0) \sim \frac{hV_h^w(0;0)}{z}\ ,$$

$$J^-(z)V_h^w(0;0) \sim \mathcal{O}(z^{w-1})\ ,$$

+ve modes with non-zero action

-ve modes which annihilate

Ward Identities (Contd.)

- So now have (w) new unknowns from the $(J^{+}_{p-1}V^{w}_{h})$. But also have (w) new equations from the regularity of J^{-} OPE upto $\mathcal{O}(z^{w-1})$.
- Apply these OPEs to the correlator $\langle J^{a}(z)\prod_{i=1}^{n} V^{w_i}_{h_i}(x_i;z_i)\rangle$.
- For $\{w_i=0\}$ have the usual $\langle J^{a}(z)\prod_{i=1}^{n} V^{0}_{h_i}(x_i;z_i)\rangle = -\sum_{n=1} \frac{\mathcal{D}^{a}}{z-z_i}\langle\prod_{i=1}^{n} V^{0}_{h_i}(x_i;z_i)\rangle$.

  global (spacetime) conformal generators

- For general $\{w_i\}$ a complicated set of recursion relations for $\langle\prod_{i=1}^{n} V^{w_i}_{h_i}(x_i;z_i)\rangle$ after eliminating unknowns.
- General solution difficult to write down. [See also Hikida-Liu ('20)]

A Special Solution

- Rather remarkably, there exists a special solution which is essentially determined when there exists a covering map: $x = \Gamma(z); \quad x \approx x_i + a_i^\Gamma (z - z_i)^{w_i}$

$$\langle V_{h_1}^{w_1}(0;0) V_{h_2}^{w_2}(1;1) V_{h_3}^{w_3}(\infty;\infty) \prod_{i=4}^{n} V_{h_i}^{w_i}(x_i;z_i)\rangle = \sum_\Gamma \prod_{i=1}^{n} (a_i^\Gamma)^{-h_i} \prod_{i=4}^{n} \delta(x_i - \Gamma(z_i))\, W_\Gamma(z_4,\ldots,z_n)\ .$$

- This solution exists only when $\sum_{i=1}^{n} j_i = \frac{k}{2}(n-2)+1$. True for $j_i = \frac{1}{2};\ k=1;\ \forall n$.
- Delta function localised to a finite set of points on $\mathcal{M}_{0,n}$ where $\Gamma(z)$ exists.
- W_Γ depends only on cross ratios (can constrain further [Dei-Eberhardt-Gaberdiel ('19)]). And $a_i^\Gamma \propto \partial^{w_i}\Gamma(z_i)$ depends on $\{w_i, x_i, z_i\}$. [Note: only holomorphic dependence shown.]

Classical Action

- Now need to figure out the exact contribution to the worldsheet correlator at these discrete points on the moduli space which admit a covering map.

- We look for a classical solution to the AdS_3 sigma model (first order form).

$$S_{\mathrm{AdS}_3} = \frac{k}{4\pi}\int \mathrm{d}^2 z\,\left(4\partial\Phi\bar{\partial}\Phi + \bar{\beta}\partial\bar{\gamma} + \beta\bar{\partial}\gamma - \mathrm{e}^{-2\Phi}\beta\bar{\beta} - k^{-1}R\Phi\right)\,.$$

Radial direction — Boundary direction — [Giveon-Kutasov-Seiberg('98)]

- Might seem a foolhardy thing to do when one is at $k=1$. Highly curved sigma model.

[deBoer-Ooguri-Robbins-Tannenhauser ('98)]

- However, action is also quadratic when $\Phi \to \infty$ i.e. worldsheet at boundary.

Classical Solution

- Ground state in w-spectrally flowed sector given by

$$\gamma(z) = z^w; \quad \Phi(z,\bar{z}) = -\ln\epsilon - \frac{(w-1)}{2}\ln z - \frac{(w-1)}{2}\ln\bar{z}$$

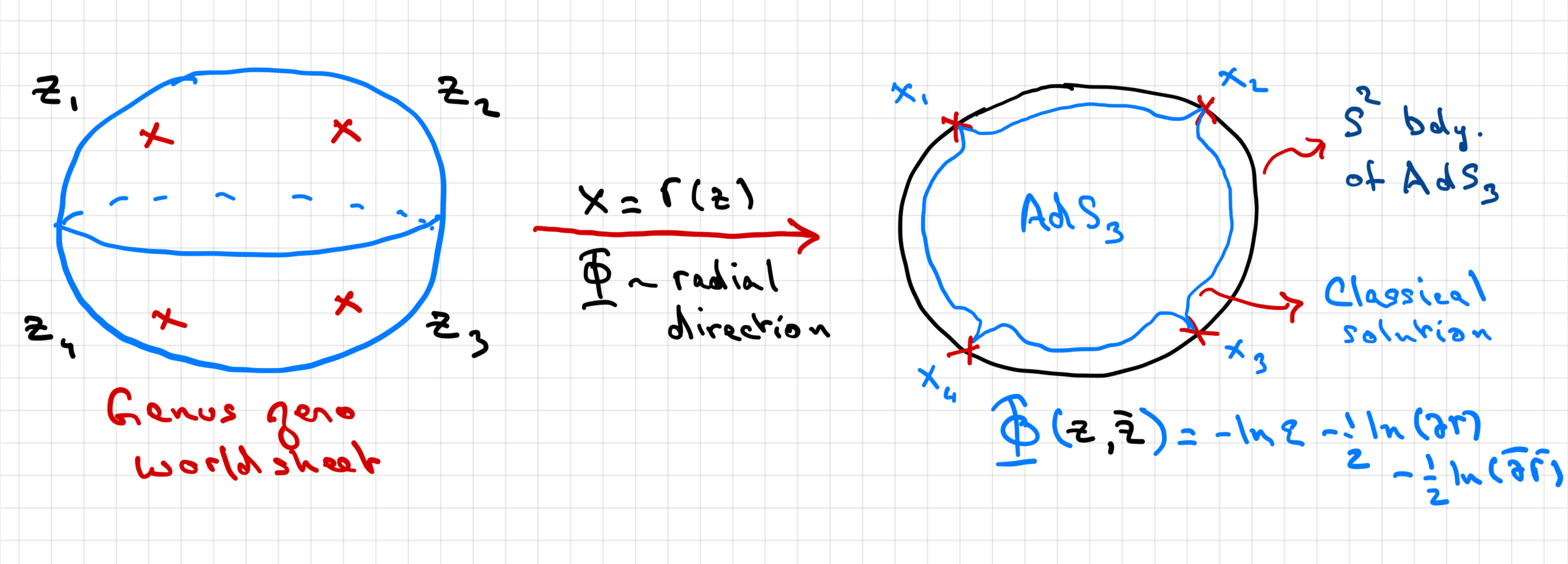
- Obtained by taking a scaling limit ($\epsilon \propto p \to 0$) to the bottom of continuum - worldsheet essentially glued to the boundary.

- Can now find general solution corresponding to the correlator (with right boundary conditions at insertions) - in terms of a covering map:

$$\gamma(z) = \Gamma(z); \quad \Phi(z,\bar{z}) = -\frac{1}{2}\ln(\partial\Gamma) - \frac{1}{2}\ln(\bar{\partial}\bar{\Gamma}) - \ln\epsilon$$

at the boundary

Semiclassical Picture



Semiclassically Exact

- Since these solutions are at the boundary, we can evaluate their contribution to the path integral semiclassically and connect with Ward identity solution.

- Define $\phi_{\rm cl} = -2\Phi + constant = \ln(|\partial\Gamma|^2)$. Then the classical action on-shell is

$$S_{\rm AdS_3} = S_L[\phi_{\rm cl}] = \frac{1}{8\pi}\int {\rm d}^2 z\,\sqrt{g}\left(2\,\partial\phi_{\rm cl}\,\bar\partial\phi_{\rm cl} + R\,\phi_{\rm cl}\right)$$

(Liouville action for the conformal factor.)

- Combining with Ward identity, the worldsheet correlator has the form

$$\left\langle V^{w_1}_{h_1}(0;0)V^{w_2}_{h_2}(1;1)V^{w_3}_{h_3}(\infty;\infty)\prod_{i=4}^{n}V^{w_i}_{h_i}(x_i;z_i)\right\rangle = \sum_{\Gamma}\widetilde{W}_\Gamma e^{-S_L[\phi_{\rm cl}]}\prod_{i=1}^{n}|\partial^{w_i}\Gamma(z_i)|^{-2(h_i-h_i^0)}\prod_{i=4}^{n}\delta^{(2)}(z_i-\Gamma^{-1}(x_i))$$

$$\left(h^0 = \frac{w^2-1}{4w}\right)$$

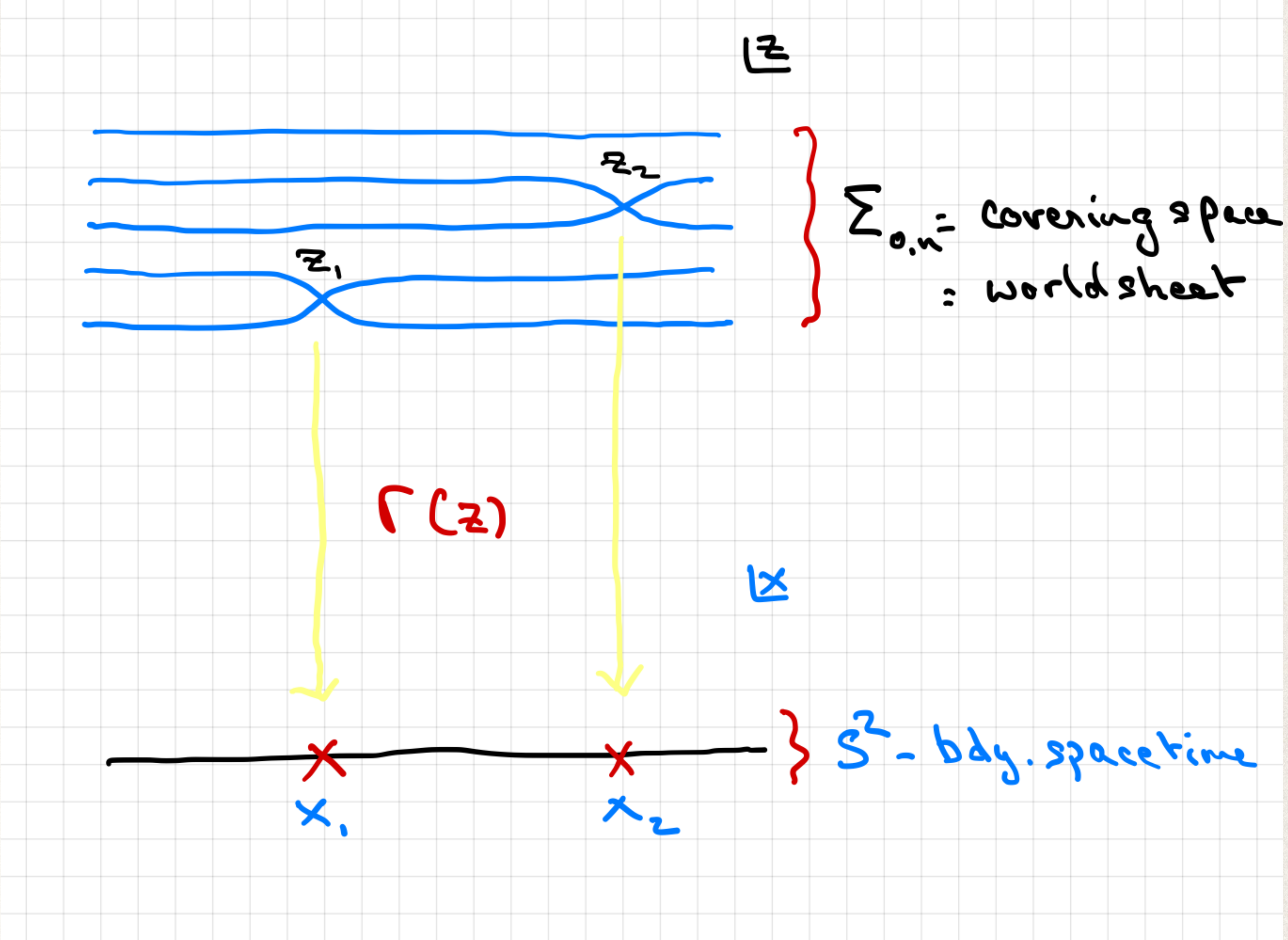
Symmetric Orbifold Correlators

- The worldsheet correlators are now in a form which is manifestly that of the spacetime orbifold CFT correlators.

- Recall Lunin-Mathur ('00) approach to computing symmetric orbifold correlators (e.g. of w-twisted sector ground states).

$$\langle \mathcal{O}_{h_1}^{(w_1)}(x_1)\mathcal{O}_{h_2}^{(w_2)}(x_2)\ldots\mathcal{O}_{h_n}^{(w_n)}(x_n)\rangle_{S^2}\Big|_{g=0}$$

- Lift the correlator to the covering space (each w-twisted sector insertion lifts to a w-sheeted branch cover locally)



- Contribution from each such covering map.
- Twisted sector ground state lifts to vacuum.
- Vacuum path integral on covering space.
- Coord. dependence from conformal factor $\Gamma(z)$.

Making Equality Manifest

- Thus symmetric orbifold CFT correlators (for $c = 6$) take the form $\left(h^0 = \frac{w^2-1}{4w}\right)$

$$\langle \mathcal{O}^{(w_1)}_{h_1}(x_1)\mathcal{O}^{(w_2)}_{h_2}(x_2)\dots\mathcal{O}^{(w_n)}_{h_n}(x_n)\rangle_{S^2}\Big|_{g=0} = \sum_{\Gamma} W_\Gamma e^{-S_L[\phi_{\rm cl}]}\prod_{i=1}^{n}|\partial^{w_i}\Gamma_i|^{-2(h_i-h_i^0)}$$

[See also Dei-Eberhardt ('19)]

- We see that the computation of the integrated worldsheet CFT correlators $\int_{\mathcal{M}_{0,n}}\langle \mathcal{V}^{w_1}_{h_1}(x_1;z_1)\mathcal{V}^{w_2}_{h_2}(x_2;z_2)\dots\mathcal{V}^{w_n}_{h_n}(x_n;z_n)\rangle_{\Sigma_{0,n}}$ reduce to the same Lunin-Mathur path integral.

- With exactly the same Liouville action for the same conformal factor $e^{\phi_{\rm cl}} = |\partial\Gamma|^2$.
($S_L[\phi_{\rm cl}]$ should be understood in an appropriately regularised sense.)

- Worldsheet CFT on the covering space of the spacetime CFT.

[Pakman, Rastelli, Razamat ('09)]

Scattered Remarks

- Delta function localisation of correlators reminiscent of a topological string. Also carries over to higher genus correlators [Eberhardt ('20)].

- The single particle contribution to the torus partition function also showed localisation to holomorphic maps: $Z_{(g=1)} \propto \delta^{(2)}(t - w\tau - m)$ [Eberhardt-Gaberdiel-R.G.('18)].

- Both of these have an analogue for $k \geq 2$. One loop answer as well as 4-pt. correlators have singularities exactly when holomorphic coverings exist.

  [Maldacena-Ooguri ('00-'01)]

- The integration over the continuum (radial momentum) smears out the delta functions into a singularity.

Tensionless Folklore

- Tensionless limit of string theory ought to be a topological string - see a realisation also in reduction in d.o.f. to four bosonic and fermionic oscillators.

  [Gaberdiel-R. G. (in progress)]
- Also a given correlator gets contributions from only up to finite genus.

  [cf. Aharony-Komargodski-Razamat('06)]
- Enhanced Higher Spin symmetries in spacetime CFT [Gaberdiel-R. G ('14)] arising from free worldsheet theory (sigma model free when localised to boundary).
- Gross-Mende like saddle point for worldsheet path integral - from interior of moduli space.

$$\Phi(z,\bar{z}) = -\sum_{i=1}^{n}(w_i - 1)\ln|z - z_i| + 2\sum_{a=1}^{M}\ln|z - u_a| + \text{const.}$$

Radial direction has same Coulomb gas profile.

Poles of $\Gamma(z)$

Drawing Lessons from Drawing Diagrams

- $\Phi(z, \bar{z})$ identified with worldsheet conformal factor. Hence worldsheet curvature

$$\propto \nabla^2 \Phi(z,\bar{z}) \sim \sum_{i=1}^{n} (w_i - 1)\delta^{(2)}(z - z_i) - 2\sum_{a=1}^{M} \delta^{(2)}(z - u_a)$$

(Localised to insertions of vertex operators and poles.)

- Exactly as in `Strebel Gauge': curvature of dual string worldsheet at vertices as well as centres of faces of Feynman diagrams of free Yang-Mills theory.

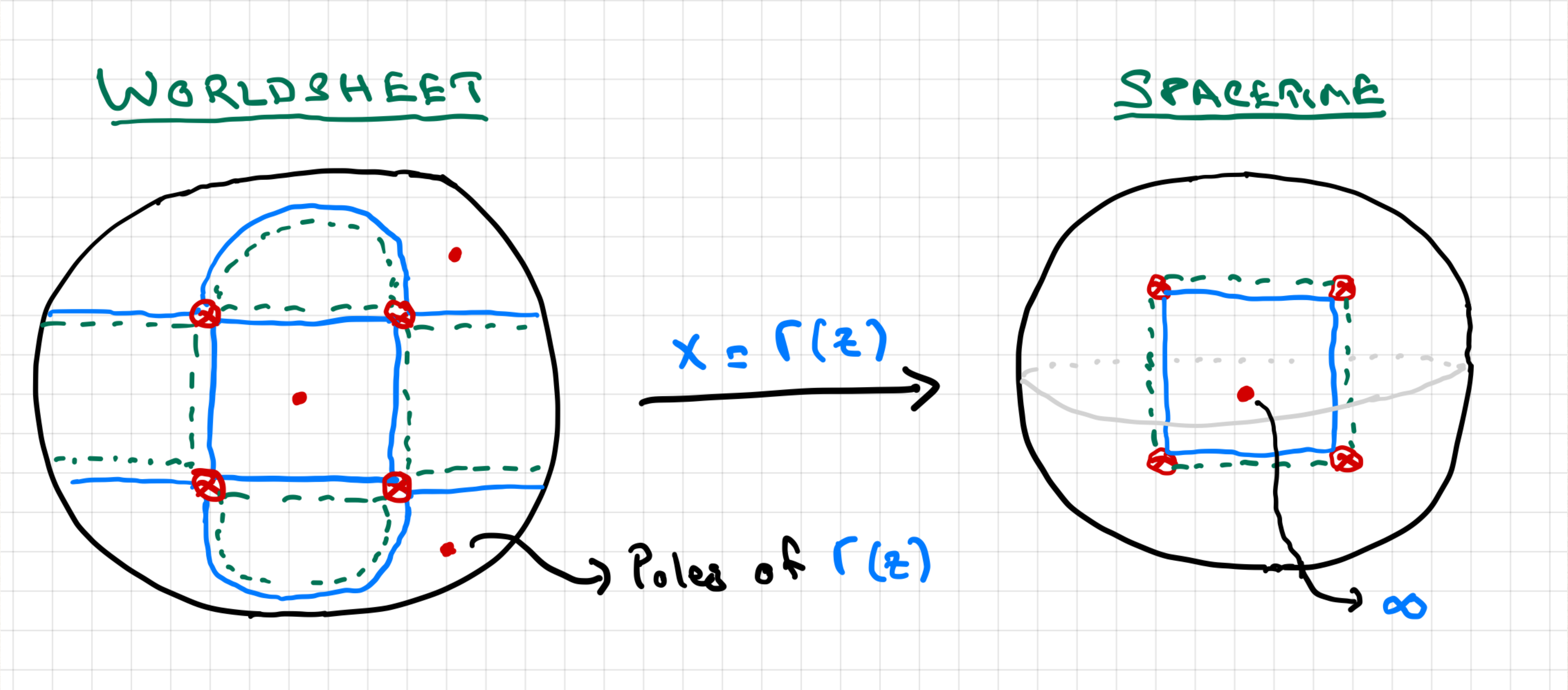
[R.G. ('03-'05), ('11)]

- In fact, can draw Feynman-like diagrams for symm. orbifold correlators with vertices and colour loops. [Pakman, Rastelli, Razamat ('09)]

- Can associate precisely one diagram for each branched cover.

Feynman Coverings?

- Bifundamental edges give rise to a fatgraph on worldsheet.
- Vertices have insertions (and curvature).
- Coloured and dashed faces: former contain preimages of ∞ .
- Colour loops lead to curvature defects when Feynman strips glue together.
- Dashed loops only have information on merging different Riemann sheets.



Analogous (but slightly different from) case of Strebel differentials: poles $\leftrightarrow$ vertices; zeroes $\leftrightarrow$ centres of faces.

Suggestive of associating branched coverings to Feynman diagrams in higher dim as well e.g. $\mathrm{Tr} Z^w$

Wrapping Up

- Argued for the structural equivalence, at the level of correlators, of the tensionless string on $AdS_3 \times S^3 \times T^4$ with the dual symmetric orbifold CFT.

- Localisation of string path integral to a finite # of configurations essential - signifies an underlying novel topological string theory [See Costello-Paquette('20) for a bulk spacetime approach].

- Underlying free field theory description also leads to localisation [Dei-Gaberdiel-R.G.-Knighton].

- Worldsheet is a covering space which wraps the AdS_3 boundary. Good playground to study the interplay of worldsheet CFT and spacetime CFT.

- Interesting to study the BPS sector further [see Li-Troost ('20)], duals for other symmetric orbifolds [Dei-Gaberdiel-Knighton; see also Belin talk]. Perturb away from small radius limit with RR flux.

- Connect with Berkovits' approach - tensionless limit in higher dim. AdS [Gaberdiel-R.G.('20)?].

Thank You!