Domain walls and integrability in N=4 SYM

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Based on:

- A. Gimenez-Grau, C.K., M. Volk & M. Wilhelm, arXiv:1912.02468[hep-th],
- JHEP 04 (2020) 132
- M. de Leeuw, T. Gombor, C.K., G. Linardopoulos & B. Pozsgay ArXiv:1912.09338[hep-th], JHEP 01, (2020) 176
- C.K., D. Müller & K. Zarembo, ArXiv:2005.01392[hep-th], to appear in JHEP

Strings 2020

Cape Town, South Africa

June 29th, 2020

AdS/CFT

$\mathcal{N} = 4$ SYM in 4D $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$

- Conformal symmetry
- Supersymmetry
- Planar integrability

AdS/dCFT

$\mathcal{N} = 4$ SYM in 4D with 3D domain wall $\longleftrightarrow$ IIB strings on $AdS_5 \times S^5$ with probe brane

- Conformal symmetry partially broken
- Supersymmetry partially or completely broken

In the planar limit, $N \to \infty$:

Motivation

- Insights on the interplay between conformal symmetry, supersymmetry and integrability
- Exact results for novel types of observables such as one-point functions
- Positive tests of AdS/dCFT dictionary for set-ups with and without supersymmetry
- Interesting connections to statistical physics: matrix product states and quantum quenhes.
- Possible cross-fertilization with the boundary conformal bootstrap program.

Plan of the talk

I. Motivation

II. The defect set-ups and their parameters

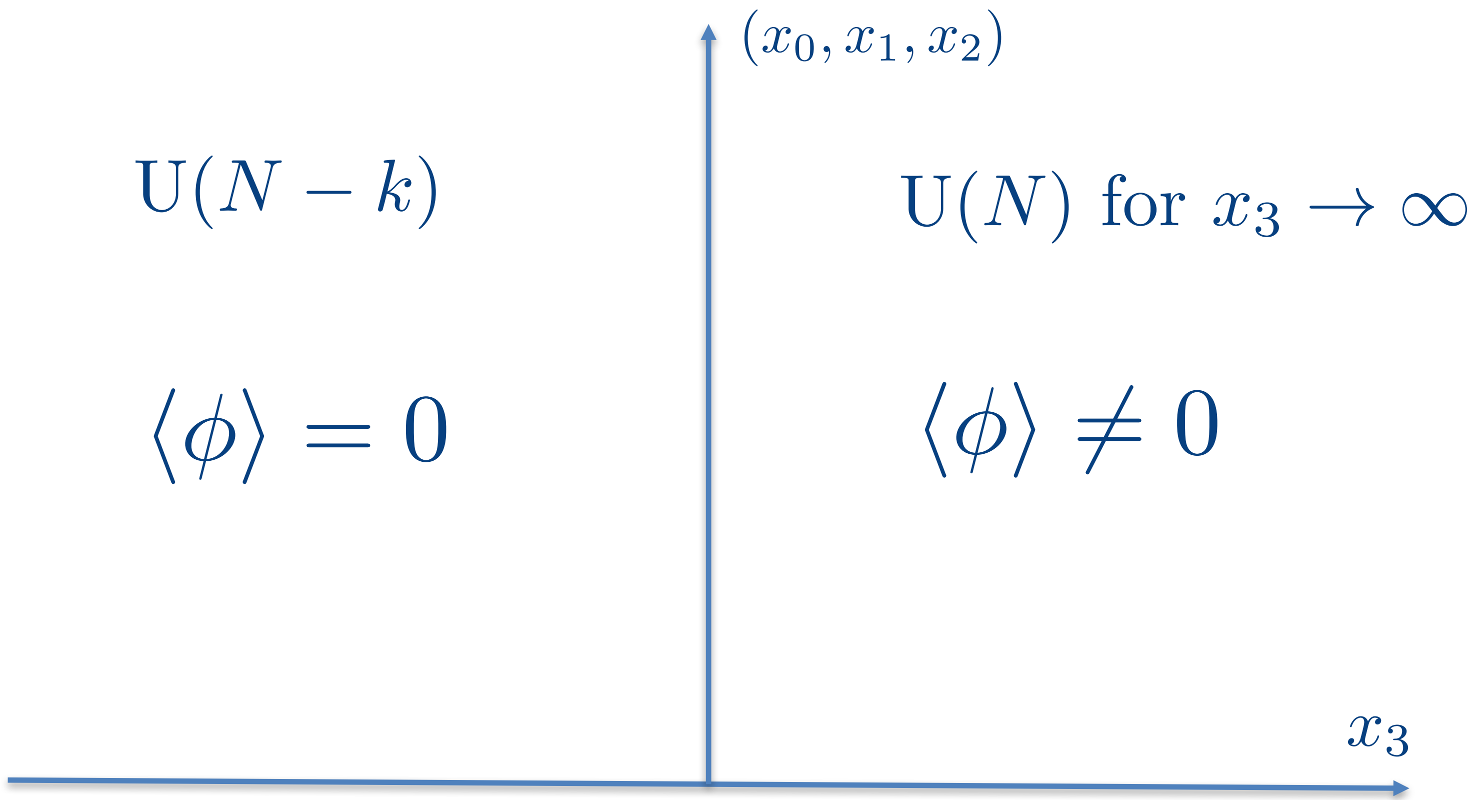
III. Integrability properties and exact results for one-point functions

IV. Positive tests of AdS/dCFT dictionary with and without susy

V. Summary & Open problems

The defect set-up

$\mathcal{N} = 4$ SYM



Classical Fields (simplest case)

Assume only x_3 -dependence and $x_3 > 0$, $A_\mu^{\rm cl} = 0, \qquad \Psi_A^{\rm cl} = 0$

Classical e.o.m.: $$\frac{d^2\phi_i^{\rm cl}}{dx_3^2} = \left[\phi_j^{\rm cl}, \left[\phi_j^{\rm cl}, \phi_i^{\rm cl}\right]\right].$$

(x_3 is distance to defect)

Solution: $$\phi_i^{\rm cl} = \frac{1}{x_3}\begin{pmatrix} (t_i)_{k\times k} & 0 \\ 0 & 0 \end{pmatrix}, \ i = 1, 2, 3$$

Constable, Myers & Tafjord '99

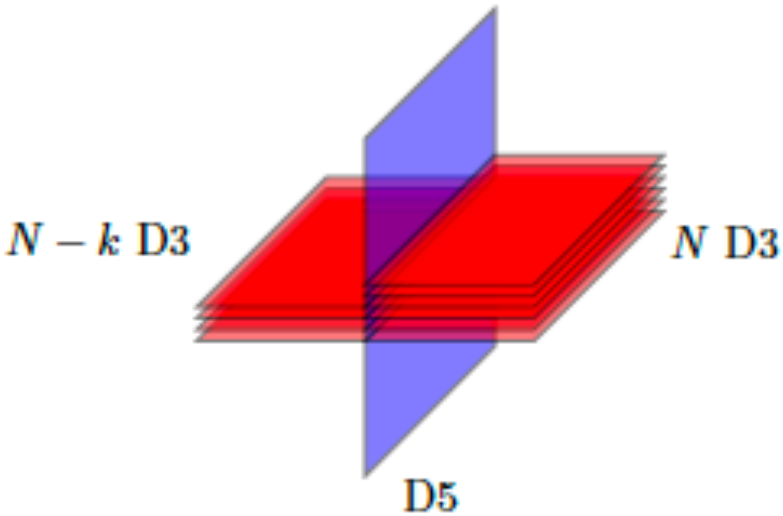
$$\phi_4^{\rm cl} = \phi_5^{\rm cl} = \phi_6^{\rm cl} = 0$$

where t_i, i=1,2,3, constitute a k-dimensional irreducible repr. of SU(2). (Nahm eqns. also fulfilled.)

Set-up ½ BPS (for appropriate choice b.c. for zero-modes, Gaiotto & Witten '08)

AdS/dCFT --- The string theory side

	x^0	x^1	x^2	x^3	x^4	x^5	x^6	x^7	x^8	x^9
D3	$\times$	$\times$	$\times$	$\times$						
D5	$\times$	$\times$	$\times$		$\times$	$\times$	$\times$			



Karch & Randall '01,

Geometry of D5 brane: $AdS_4 \times S^2$

Background gauge field: k units of magnetic flux on S^2

AdS/dCFT set-ups

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Symmetry of vevs	SU(2)	SU(2) × SU(2)	SO(5)
Dim. of rep. / Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
Gauge Groups	$\mathrm{U}(N)$, $\mathrm{U}(N-k)$	$\mathrm{U}(N)$, $\mathrm{U}(N-k_1 k_2)$	$\mathrm{U}(N)$, $\mathrm{U}(N-d)$

One-point functions in dCFT's

$$\langle \mathcal{O}_\Delta^{\text{bulk}}(x) \rangle = \frac{C}{|x_3|^\Delta}$$

Cardy ´84
McAvity & Osborn '95

Normalization given by: $$\lim_{x_3 \to \infty} \langle \mathcal{O}_\Delta^{\text{bulk}}(y+x)\mathcal{O}_{\Delta'}^{\text{bulk}}(z+x) \rangle = \frac{\delta_{\Delta\Delta'}}{|y-z|^{2\Delta}}$$

Due to vevs scalar operators can have non-zero 1-pt fcts at tree-level

$$\langle \mathcal{O}_\Delta(x) \rangle = \left(\text{Tr}(\phi_{i_1} \ldots \phi_{i_\Delta}) + \ldots\right)\big|_{\phi_i \to \phi_i^{\text{cl}} = \frac{t_i}{x_3}}$$

Wish: A Systematic approach to the computation of 1-pt functions of *conformal* scalar operators using the tools of integrability

An exact formula valid for any operator and for any loop order

One-point functions at tree level

Full scalar sector: $\phi_i, \quad i = 1, \ldots, 6$

Conformal scalar operators=Eigenstates of integrable SO(6) spin chain

Minahan & Zarembo ´02

Eigenstates of length L: $|u_i, v_j^+, v_k^-\rangle_L$

characterized by three sets of rapidities $\{u_i\}_{i=1}^M, \ \{v_j^+\}_{i=1}^{N^+}, \ \{v_j^-\}_{i=1}^{N^-}$

$$\langle \mathcal{O}_\Delta(x)\rangle = (\mathrm{Tr}(\phi_{i_1}\ldots\phi_{i_\Delta}) + \ldots)\,|_{\phi_i\to\phi_i^{\mathrm{cl}}=\frac{t_i}{x_3}} \equiv \frac{C(\{u_i, v_j^+, v_l^-\})}{x_3^\Delta}$$

Matrix Product State associated with the defect:

$$|\mathrm{MPS_k}\rangle = \sum_{\vec{i}} \mathrm{tr}[t_{i_i}\ldots t_{i_L}]|\phi_{i_1}\ldots\phi_{i_L}\rangle,$$

deLeeuw, C.K.
& Zarembo ‘15,

Object to calculate:

$$C_k\left(\{u_i, v_j^+, v_l^-\}\right) = \frac{\langle \mathrm{MPS}_k\,|\{u_i, v_j^+, v_l^-\}\rangle_L}{\langle\{u_i, v_j^+, v_l^-\}|\{u_i, v_j^+, v_l^-\}\rangle^{\frac{1}{2}}}$$

Integrability criterion

When can $\langle \text{MPS}_k \,|\{u_i, v_j^+, v_l^-\}\rangle_L$ be calculated in closed form?

NB: Parameters:

$L = \Delta$,

M, N_+, N_- number of Bethe roots/fields of various types,

k representation label $(SU(2)$ or $SO(5))$

Integrability criterion: $\hat{Q}_{2m+1}|MPS_k\rangle = 0, \quad m \geq 1$

Ghoshal & Zamolodchikov '94

Piroli, Pozsgay Vernier '17

$\hat{Q}_{2m+1}$: Conserved charges which are odd under parity $(p \to -p)$

$|MPS_k\rangle$ only involves excitation pairs with momenta $(+p, -p)$

$|MPS_k\rangle$ boundary state which only allows pure reflection (after Wick rotation)

Also inspired by
Korepin '82, Izgerzin '87, Tsuchiya '98, Pozsgay '13, Brockmann et al '14, Buhl-Mortensen, de Leeuw, CK & Zarembo '15, Foda and Zarembo '15

Integrability of MPS

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Dim. of rep./ Flux	k	k_1, k_2	$d = \frac{(n+1)(n+2)(n+3)}{6}$
$\vert MPS\rangle$	Integrable	Non-integrable	Integrable
Overlaps	Exact formula derived	—	Exact formula derived

Reflection matrix which fulfills BYB of SO(6) spin chain and has the appropriate symmetries can be found for the two cases with $Q_{2m+1}|MPS\rangle = 0$

Solution symmetric case D3-D5

Result for C_k:

- Exact formula valid for any, L, M, N^+, N^- and k

de Leeuw, C.K & Linardopoulos, '18.

$$C_k^{SO(6)} = \sqrt{\frac{Q_0(0)Q_0(\frac{i}{2})Q_0(\frac{ik}{2})Q_0(\frac{ik}{2})}{\bar{Q}_+(0)\bar{Q}_+(\frac{i}{2})\bar{Q}_-(0)\bar{Q}_-(\frac{i}{2})}} \cdot \mathbb{T}_{k-1}(0) \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\mathbb{T}_k(u) = \sum_{a=-\frac{k}{2}}^{\frac{k}{2}} (u+ia)^L \frac{Q_+(u+ia)Q_-(u+ia)}{Q_0(u+i(a+\frac{1}{2}))Q_0(u+i(a-\frac{1}{2}))}.$$

Q's: Baxter polynomials, G Gaudin matrix:

$$\langle\{u_i, v_j^+, v_l^-\}|\{u_i, v_j^+, v_l^-\}\rangle = \det G = \det G_+ \det G_-,$$

Solution SO(5) symmetric case

Result for C_n:

de Leeuw, C.K & Linardopoulos,'18.

de Leeuw, Gombor C.K & Linardopoulos, Pozsgay '19.

- Exact formula valid for any L, M, N^+, N^- and n

$$\frac{\langle \mathbf{u} | \mathrm{MPS}_n \rangle}{\langle \mathbf{u} | \mathbf{u} \rangle^{1/2}} = \Lambda_n \cdot \sqrt{\frac{Q_0(0)\, Q_0\left(\frac{1}{2}\right)}{\bar{Q}_+(0)\, \bar{Q}_+\left(\frac{1}{2}\right) \bar{Q}_-(0)\, \bar{Q}_-\left(\frac{1}{2}\right)}} \cdot \sqrt{\frac{\det G_+}{\det G_-}}$$

$$\Lambda_n = 2^L \sum_{q=-\frac{n}{2}}^{\frac{n}{2}} q^L \left[\sum_{p=-\frac{n}{2}}^{q} \frac{Q_0(p-\frac{1}{2})}{Q_0(q-\frac{1}{2})} \frac{Q_-(q)Q_-(\frac{n}{2}+1)}{Q_-(p)Q_-(p-1)} \right] \left[\sum_{r=q}^{\frac{n}{2}} \frac{Q_0(r+\frac{1}{2})}{Q_0(q+\frac{1}{2})} \frac{Q_+(q)Q_+(\frac{n}{2}+1)}{Q_+(r)Q_+(r+1)} \right].$$

Both overlap formulas are proven by now.

At one loop

Formula works upon modification by a flux factor (su(2) sector)

Buhl-Mortensen, de Leeuw, Ipsen, C.K, Wilhelm '17

$$C_k = i^L \tilde{T}_{k-1}(0) \sqrt{\frac{Q(\frac{i}{2})Q(0)}{Q^2(\frac{ik}{2})}} \sqrt{\frac{\det G_+}{\det G_-}} \, \mathbb{F}_k$$

$$\mathbb{F}_k = 1 + g^2 \left[\Psi(\tfrac{k+1}{2}) + \gamma_E - \log 2 \right] \Delta^{(1)} + O(g^4) \,,$$

and a replacement in the Bethe equations and the transfer matrix

Beisert & Staudacher '05

$$e^{ip} = \frac{u + \frac{i}{2}}{u - \frac{i}{2}} \longrightarrow \frac{x(u + \frac{i}{2})}{x(u - \frac{i}{2})}, \qquad u(x) = x + \frac{g^2}{x}, \qquad g^2 = \frac{\lambda}{8\pi^2}$$

(plus dressing phase via bootstrap plus wrapping corrections via TBA)

NB: A non-trivial field theory calculation is needed for this statement (involving diagonalizing the mass matrix using fuzzy spherical harmonics).

Buhl-Mortensen, de Leeuw, Ipsen, C.K, Wilhelm '16

Recently reproduced by a bootstrap argument

Komatsu & Wang '20

$\mathbb{F}_k$: Originates from boundary dressing phase

$\sum_{-\frac{k-1}{2}}^{\frac{k-1}{2}}$ in $T_{k-1}(u)$ originates from sum over boundary bound states

Other sectors & higher loops

Since recently in principle accessible by bootstrap: Komatsu & Wang '20 Gombor & Bajnok '20

Alternatively accessible from the k=1 case: no classical fields, specific b.c. at the defect C.K, Müller, Zarembo '20

For $x_3 > 0$:

$$A_\mu, \Phi_i, \Psi = \overset{\quad k \qquad\quad N-k}{\left[\begin{array}{c|ccc} x & y & y & y \\ \hline y & z & z & z \\ y & z & z & z \\ y & z & z & z \end{array}\right]} \begin{array}{c} k \\ \\ N-k \\ \\ \end{array}$$

For k>1: x and y fields are massive, $m^2 \propto 1/x_3^2$, AdS propagators.
z fields are massless

For k=1:

	$\Phi_{4,5,6}, A_{0,1,2}, c$	$\Phi_{1,2,3}, A_3$
x, y	Dirichlet	Neumann
z	no BCs	no BCs

Prediction of leading order contribution

Propagators for complex scalars: $X = \phi_1 + i\phi_4,$ etc.

$$D_\kappa(x,y) = \frac{1}{4\pi^2}\left(\frac{1}{|x-y|^2} + \frac{\kappa}{|\bar{x}-y|^2}\right), \quad \kappa = \begin{cases} 1 & \text{Neumann} \\ -1 & \text{Dirichlet} \\ 0 & \text{no BCs.} \end{cases}$$

$$\bar{x} = (x_0, x_1, x_2, -x_3)$$

$$\langle X^{1a}(x) X^{b1}(y)\rangle = \frac{g_{\rm YM}^2 \delta^{ab}}{2}\Big(D_1(x,y) - D_{-1}(x,y)\Big) = \frac{g_{\rm YM}^2 \delta^{ab}}{4\pi^2 |\bar{x}-y|^2},$$

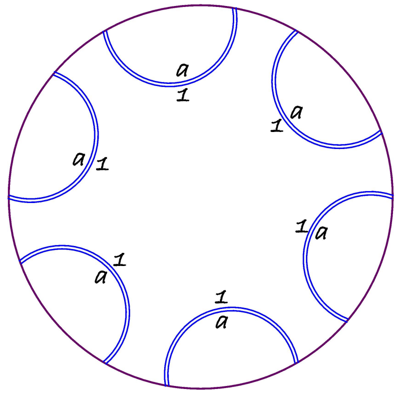
Propagators for fermions in the SU(2|3) sector

$$\langle \Psi_\alpha^{1a}(x) \Psi_\beta^{b1}(y)\rangle = \frac{g_{\rm YM}^2}{8\pi^2}\, \epsilon_{\alpha\beta}\, \delta^{ab} \cdot \frac{\bar{x}_3 - y_3}{|\bar{x}-y|^4}.$$

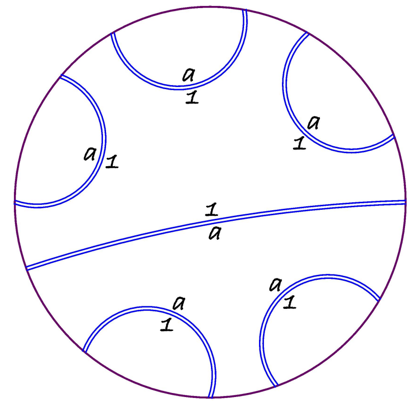
Propagators for self-dual gauge fields

$$\langle f_{\hat\mu}^{1a}(x) f_{\hat\nu}^{b1}(y)\rangle = \frac{g_{\rm YM}^2}{16\pi^2}\, \delta^{ab}\, \frac{\eta_{\hat\mu\hat\nu}}{x_3^4}, \qquad \hat\mu, \hat\nu = 0, 1, 2$$

Feynman diagrams



Leading for large-N



Sub-leading for large-N

$$C_{k=1} = 2\left(\frac{\lambda}{16\pi^2}\right)^{L/2} \frac{\langle \text{VBS}|\mathbf{u}\rangle}{\langle \mathbf{u}|\mathbf{u}\rangle^{1/2}}$$

C.K., Müller,
Zarembo '20

$\langle \text{VBS}| = (\langle XX| + \langle YY|)^{\otimes L/2}$, $\quad SU(2)$ sector

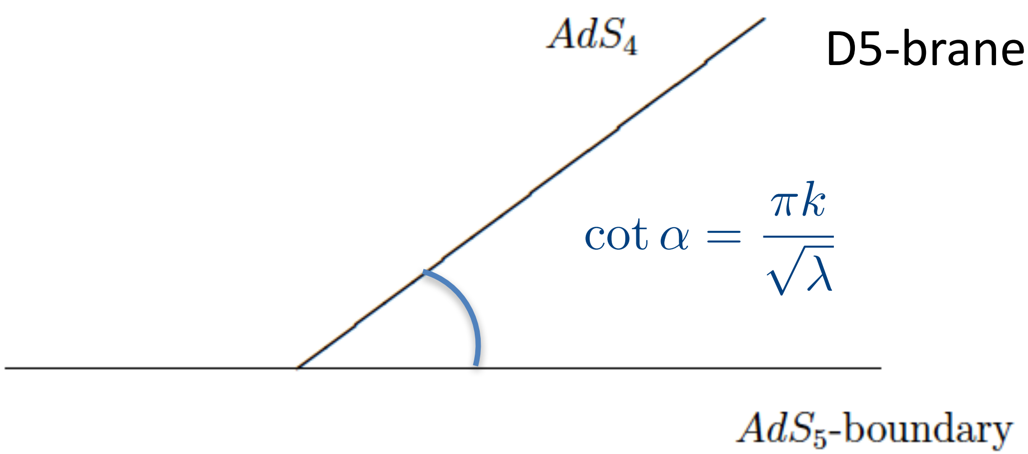
$\langle \text{VBS}| = (\langle XX| + \langle YY| + \langle ZZ| + \langle \uparrow\downarrow| - \langle \downarrow\uparrow|)^{\otimes L/2}$, $\quad SU(2|3)$ sector

$\langle \text{VBS}| = (\sum_{\hat{\mu},\hat{\nu}} \eta^{\hat{\mu}\hat{\nu}} \langle f^{\hat{\mu}} f^{\hat{\nu}}|)^{\otimes L/2}$, $\quad$ self-dual gauge fields

Closed expression of factorized determinant form in all cases.
Result in SU(2) sector agrees with $k \to 1$ limit of dressed formula

The string theory side

D3-D5 probe brane system suggests a new double scaling limit Nagasaki & Yamaguchi '12,



$$\lambda \to \infty, \; k \to \infty, \; \frac{\lambda}{k^2} \text{ finite} \qquad (N \to \infty)$$

One can compare perturbative gauge theory to semi-classical string theory (or sugra).

Similar idea works for the two D3-D7 set-ups C.K., Semenoff & Young '12,

The double scaling parameter

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
Flux/Instanton number	k	k_1, k_2	$d_G = \frac{(n+1)(n+2)(n+3)}{6}$
Double scaling parameter	$\frac{\lambda}{k^2}$	$\frac{\lambda}{k_1^2 + k_2^2}$	$\frac{\lambda}{n^2}$

Results in d.s.l. for *chiral primary* of length L

Match at leading order in d.s.l. both for D3-D5 and D3-D7

Nagasaki & Yamaguchi '12, C.K. , Semenoff, Young '12,

Match to next-to-leading order:

D3-D5 set-up ($\frac{1}{2}$-supersymmetric and integrable)

Buhl-Mortensen, De Leeuw, Ipsen C.K & Wilhelm, '16

$$\frac{\langle \mathrm{Tr}\, Z^L \rangle}{\langle \mathrm{Tr}\, Z^L \rangle|_{\mathrm{tree}}} = 1 + \frac{\lambda}{4\pi^2 k^2} \frac{L(L+1)}{(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 k^2}\right)^2\right)$$

Recently understood how to compute this 1-point function by localization (reproduce gauge as well as sugra results)

Wang '20 Komatsu & Wang '20

Match to next to leading order in d.s.l. for *chiral primary* of length L

D3-D7 set-up with SO(5) symmetry (non-supersymmetric and integrable)

$$\frac{\langle \mathrm{Tr}\, Z^L \rangle}{\langle \mathrm{Tr}\, Z^L \rangle|_{\mathrm{tree}}} = 1 + \frac{\lambda}{4\pi^2 n^2} \frac{L(L+3)}{(L-1)} + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 n^2}\right)^2\right)$$

Grau, C.K, Volk & Wilhelm, '19

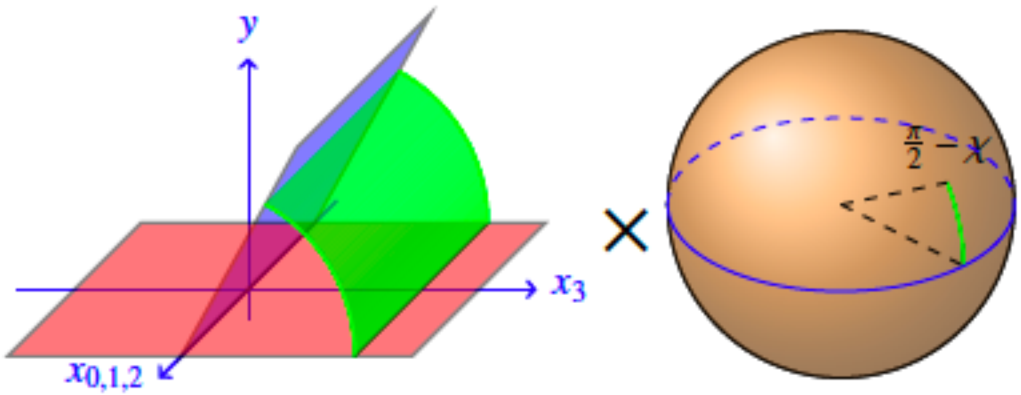
D3-D7 set-up with SO(3)× SO(3) symmetry (non-susy and non-integrable)

$$\frac{\langle \mathrm{Tr}\, Z^L \rangle}{\langle \mathrm{Tr}\, Z^L \rangle|_{\mathrm{tree}}} = 1 + \frac{\lambda}{4\pi^2 (k_1^2 + k_2^2)} \frac{1}{(L-1)\sin(L+2)\phi\, [k_1^2 + k_2^2]^3} \Big[+ 4Lk_1k_2 \left[(k_1)^4 + (k_2)^4 + (k_1k_2)^2(L+1)\right] \cos L\phi + \left[(k_2)^2 - (k_1)^2\right] \left[4(k_1k_2)^2(L^2+L-1) + ((k_1)^4 + (k_2)^4)(L^2+3L-2)\right] \sin L\phi \Big] + \mathcal{O}\left(\left(\frac{\lambda}{4\pi^2 (k_1^2 + k_2^2)}\right)^2\right), \qquad \phi = \arctan\left(\frac{k_1}{k_2}\right)$$

Grau, C.K, Volk & Wilhelm, '18

Non-local observables

Maldacena Wilson line parallel to defect



Match at two leading orders in d.s.l. for D3-D5 as well as D3-D7 (both cases)

Yamaguchi & Nagasaki '12

de Leeuw, Ipsen C.K. Wilhelm '16

Bonansea, Idiab, C.K., Volk '20

Other Wilson loops revealing interesting phases for D3-D5:

- Two anti-parallel lines — Preti, Trancanelli & Vescovi '17
- Circle — Aguilera-Damia, Correa & Giraldo-Rivera '17; Bonansea, Davoli, Griguolo, Seminara '20

Summary

	D3-D5	D3-D7	D3-D7
Supersymmetry	1/2 BPS	None	None
Brane geometry	$AdS_4 \times S^2$	$AdS_4 \times S^2 \times S^2$	$AdS_4 \times S^4$
$\|MPS\rangle$	Integrable	Non-integrable	Integrable
One-point functions	Exact formula derived at tree level and one-loop. Bootstrapped to all orders. Finite size effects via TBA.	—	Exact formula derived at tree level.
Match with string theory: Local obs. (1-pt. fcts) Non-local obs. (Wilson lines)	yes	yes	yes

Future directions

- Understanding the integrability/non-integrability from the string theory side
- Higher loop integrability for D3-D7?
- Complete the bootstrap (info about all sectors, all orders):
  Komatsu & Wang '20 Gombor & Bajnok '20
- Derive the TBA
- Wilson loops by localization?
- Connections to the boundary analytic bootstrap program

Thank you