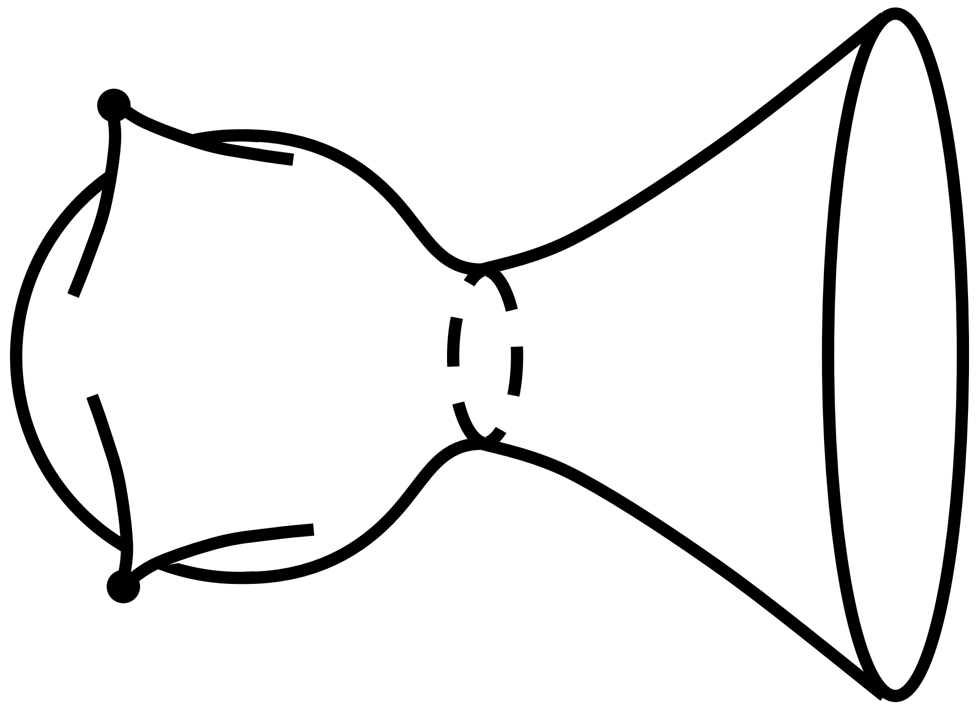
JT gravity with defects and 3D gravity



Based on [2006.11317] with Joaquin Turiaci

See also [2006.13414] Witten

Henry Maxfield, University of California, Santa Barbara

Strings 2020

JT gravity wit … ts
and 3D gravit

Based on [2006.11317] with **Joaquin Turiaci**

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Henry Maxfield, University of California, Santa Barbara

Strings 2020

The gravitational path integral

How much can a semiclassical theory tell us?

Two simple case studies

The gravitational path integral

How much can a semiclassical theory tell us?

Two simple case studies:

3D pure gravity:

Construct partition function from sum over saddle points *[MaloneyWitten][KellerMaloney]*

A problem: negative density of states near extremal BTZ *[BenjaminOoguriShaoWang]*

Aim 1: a solution within pure gravity

The gravitational path integral

How much can a semiclassical theory tell us?

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Aim 1: a solution within pure gravity

2D Jackiw-Teitelboim gravity: *[SaadShenkerStanford]*

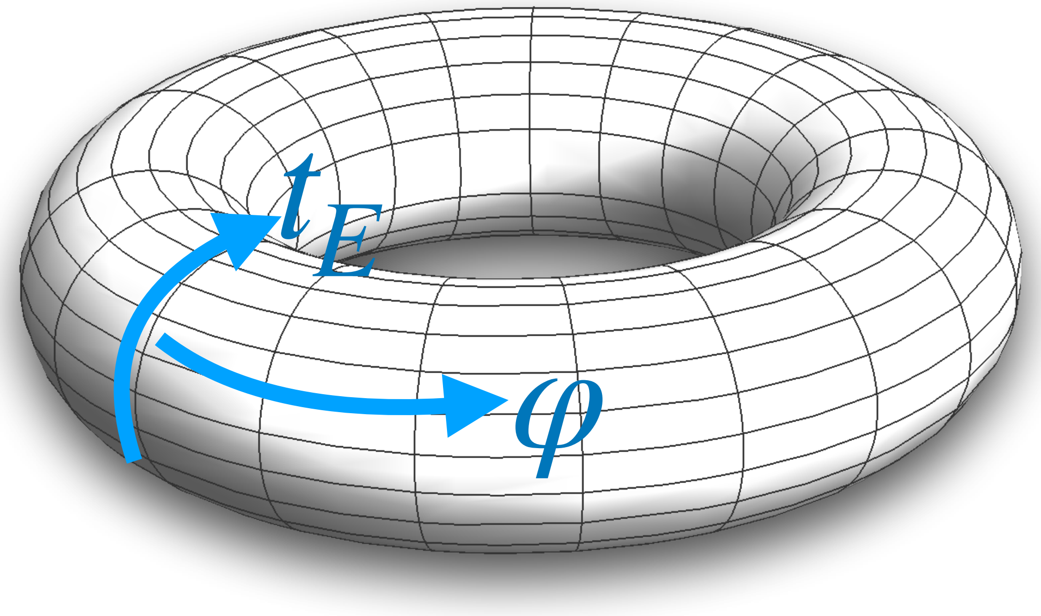
Exact path integral: includes topologies with no classical solutions

A new paradigm for AdS/CFT duality: an ***ensemble*** of dual Hamiltonians

$\text{Prob}\,(H) \propto e^{-L\,\text{Tr}\,V(H)}$

Aim 2: generalise the JT — matrix integral duality

Pure 3D gravity
& Kaluza-Klein instantons



3D pure gravity

[Ghosh,HM,Turiaci]

Near-extremal BTZ black holes

$$I_{\text{EH}} = -\frac{1}{16\pi G_N}\int d^3x\sqrt{g_3}\left(R_3 + \frac{2}{\ell_3^2}\right)$$

Reduce on φ circle

$g_3 = g_2 + \Phi^2(d\varphi + A)^2$

$$I_{\text{AO}} = -\frac{2\pi}{16\pi G_N}\int d^2x\sqrt{g_2}\left(\Phi R_2 - \frac{1}{4}\Phi^3 F^2 + \frac{2}{\ell_3^2}\Phi\right)$$

[Achúcarro,Ortiz]

Ensemble of fixed angular momentum J, integrate out A

$$I = -\frac{2\pi}{16\pi G_N}\int d^2x\sqrt{g_2}\left(\Phi R_2 - \frac{1}{2}(8G_N J)^2\Phi^{-3} + \frac{2}{\ell_3^2}\Phi\right)$$

Near extremal, near horizon, $\Phi = \Phi_0 + 4G_N\phi$
nearly AdS_2

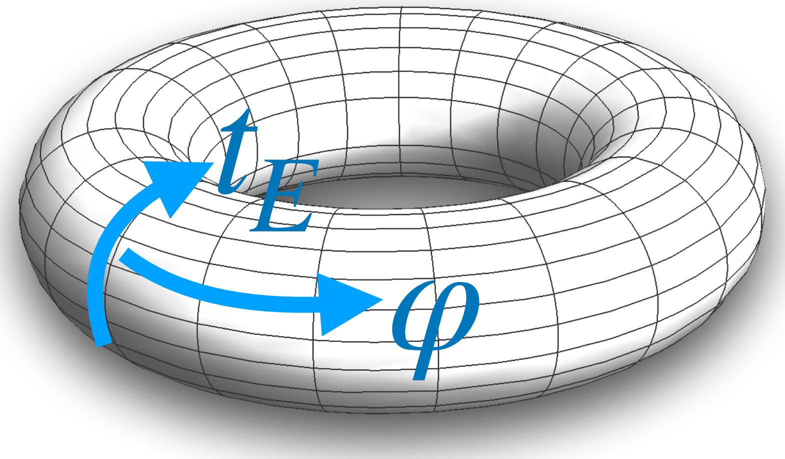
$$I_{\text{JT}} = -S_0\chi - \frac{1}{2}\int d^2x\sqrt{g_2}\,\phi\left(R_2 + \frac{2}{\ell_2^2}\right)$$

Low temperature: well-described by AdS_2 physics (bootstrap!)

3D pure gravity

Negative density of states?

Boundary torus



3D geometry: “fill in” a cycle

3D pure gravity

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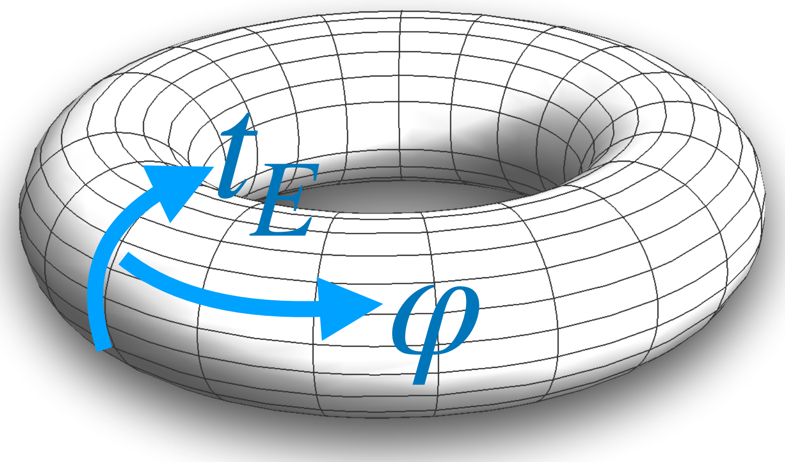
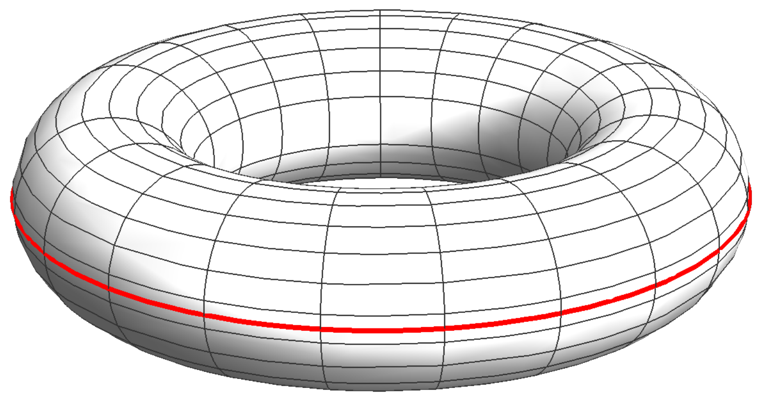


3D geometry: "fill in" a cycle

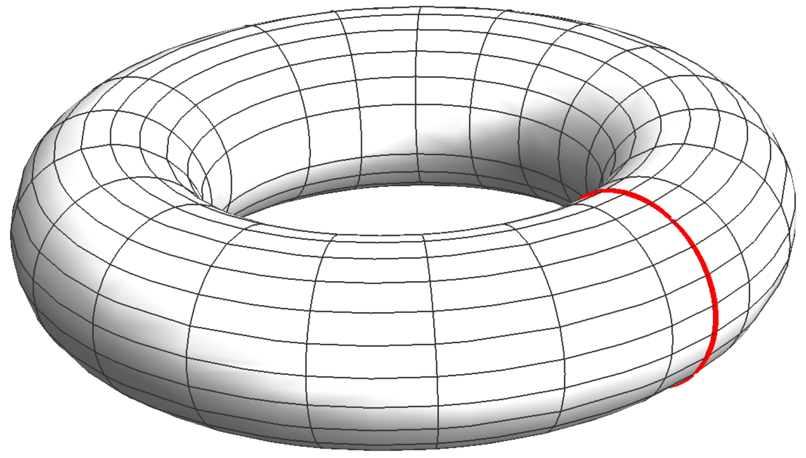
Near extremal: $\rho_J(E) \sim e^{S_0(J)}\sqrt{E - |J|}$

$$S_0(J) = \frac{\text{Area}}{4G_N} \propto \sqrt{\frac{J}{G_N}}$$



AdS_3

Vacuum



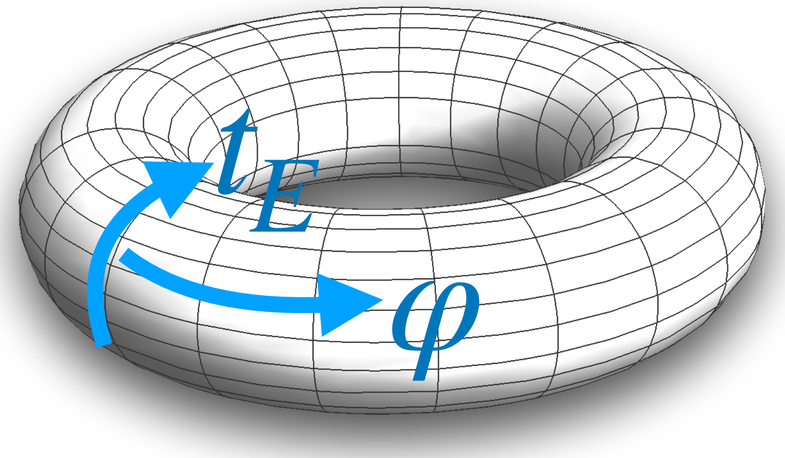
BTZ black hole

$E > |J|$: extremality bound for rotating BTZ

3D pure gravity

Negative density of states?

Boundary torus



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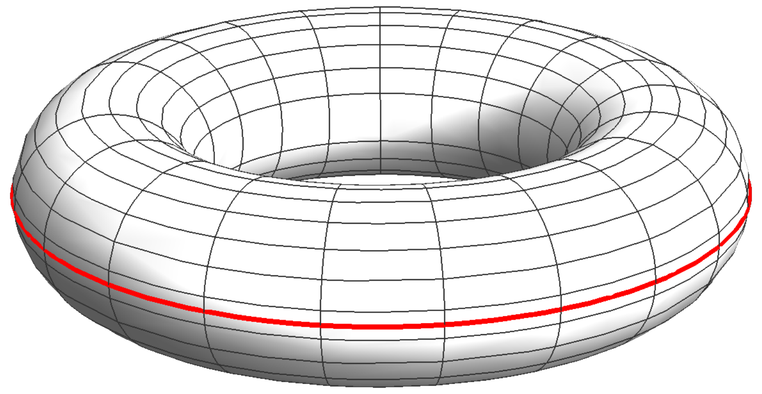

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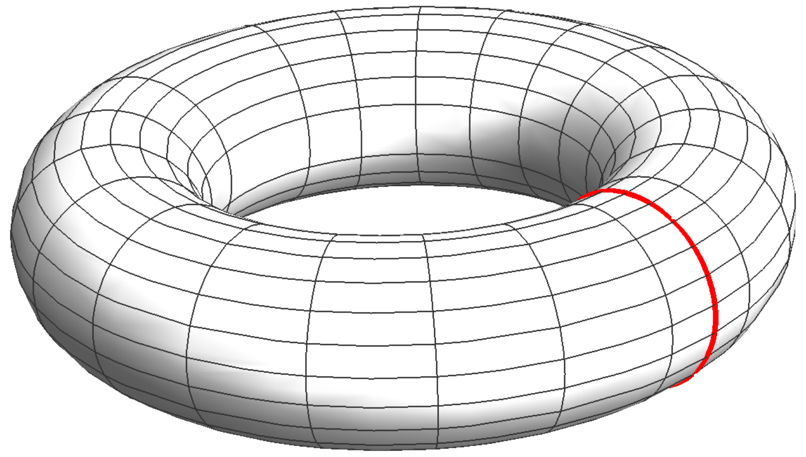
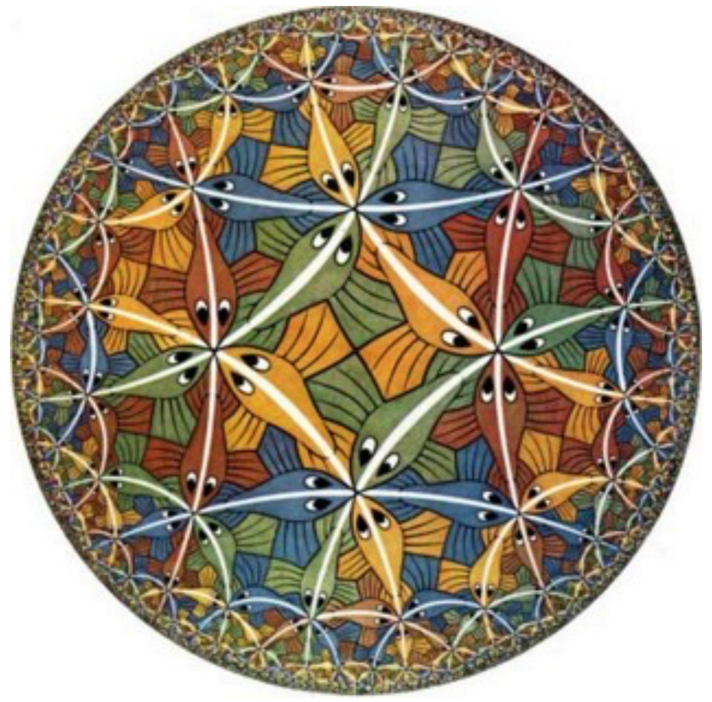


BTZ black hole

Reduce on φ circle:

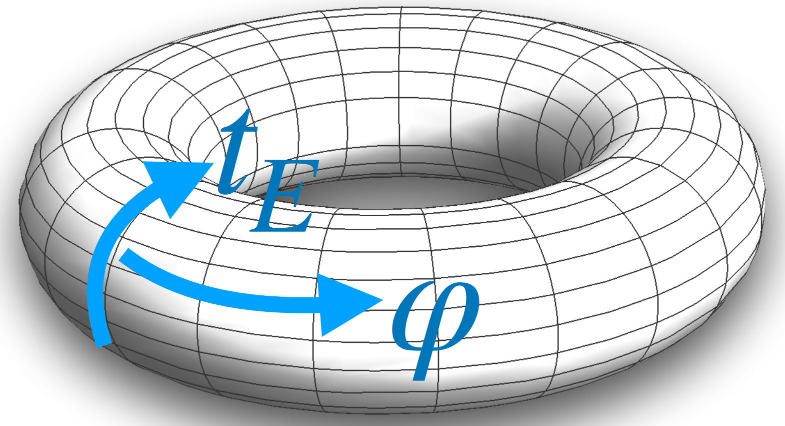


Near horizon: $AdS_2 \times S^1$, JT ✓

3D pure gravity

Negative density of states?

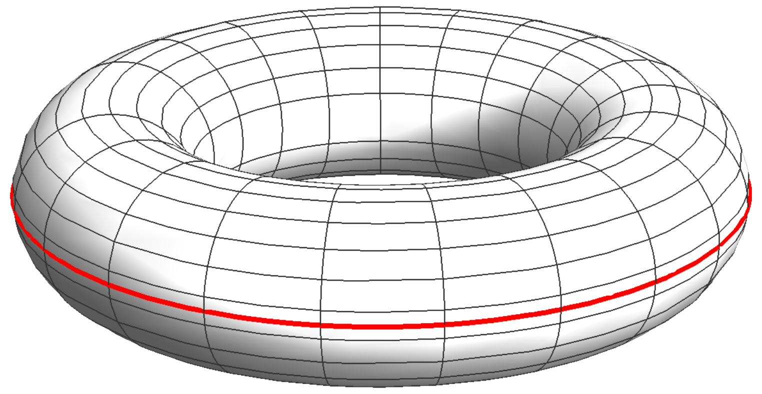
Boundary torus



3D geometry: "fill in" a cycle

Near extremal: $\rho_J(E) \sim e^{S_0(J)}\sqrt{E-|J|} + (-1)^J e^{\frac{S_0(J)}{2}} \frac{1}{\sqrt{E-|J|}}$

$S_0(J) = \frac{\text{Area}}{4G_N} \propto \sqrt{\frac{J}{G_N}}$



AdS_3

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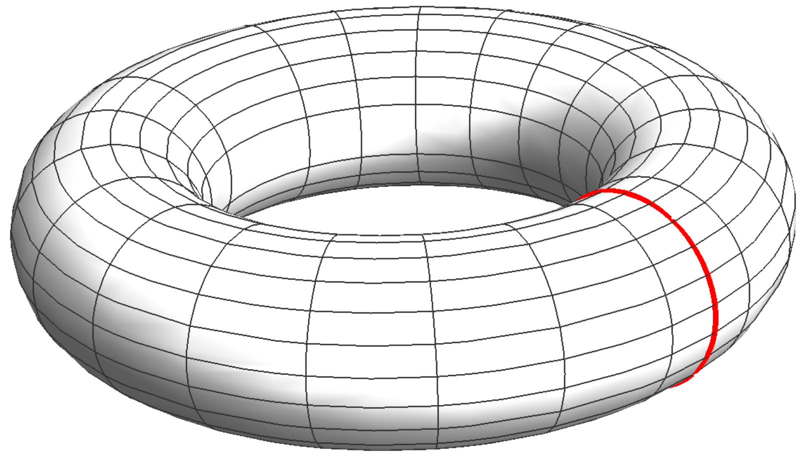

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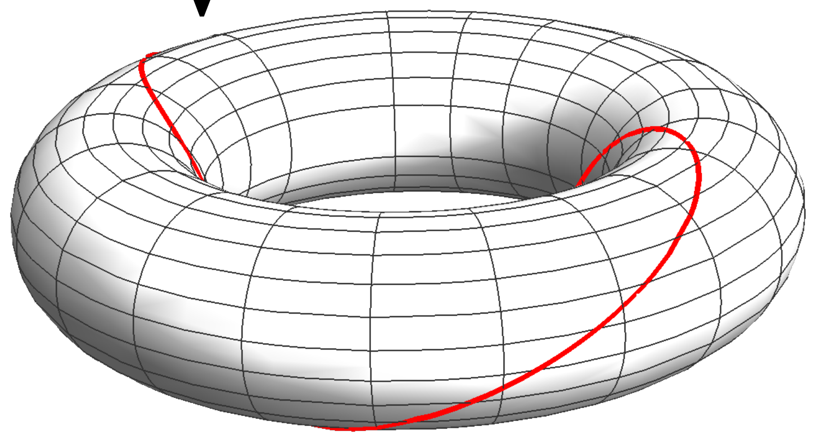


$SL(2,\mathbb{Z})$ black hole

Reduce on φ circle:

Near horizon: $AdS_2 \times S^1$, JT ✓

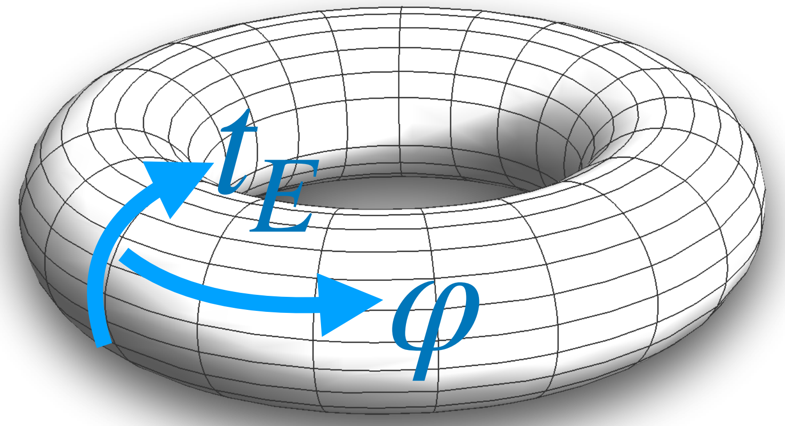
$\rho_J(E) < 0$ for $E - |J| \lesssim e^{-S_0/2}$!

[BenjaminOoguriShaoWang]

3D pure gravity

Negative density of states?

Boundary torus



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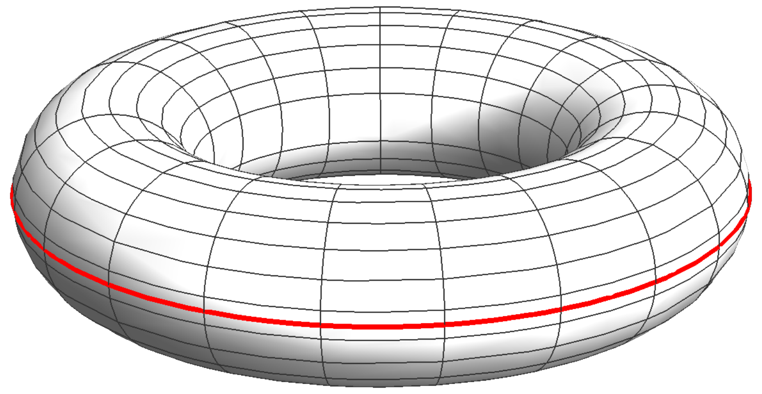

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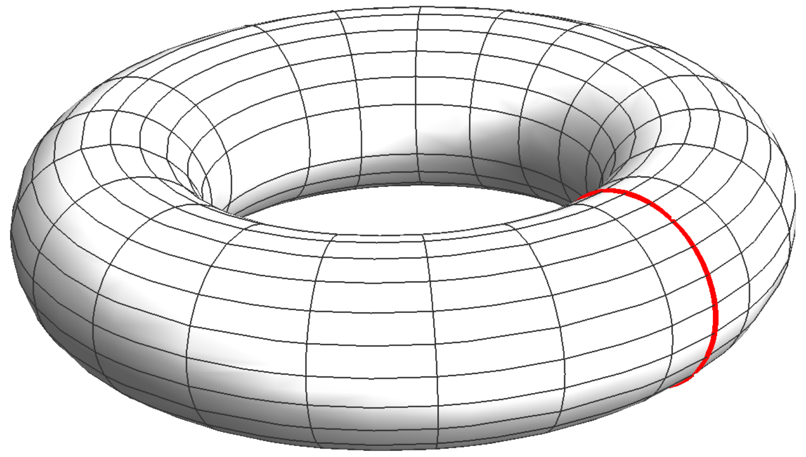

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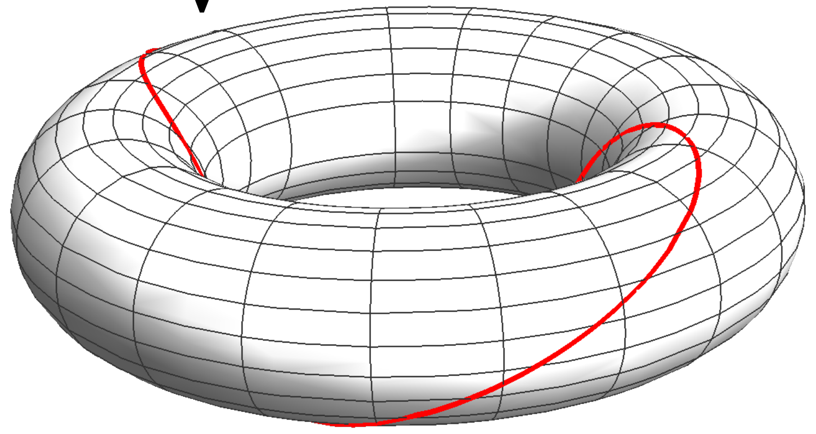
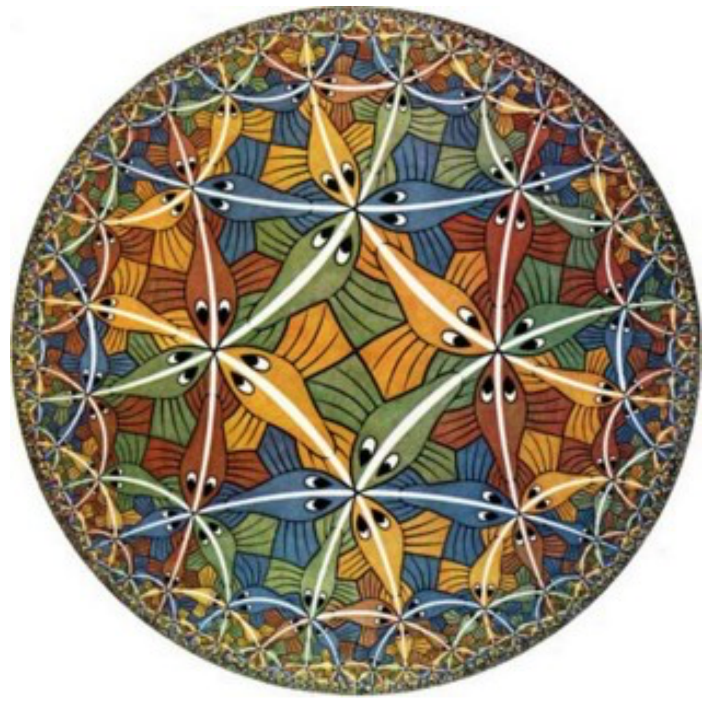


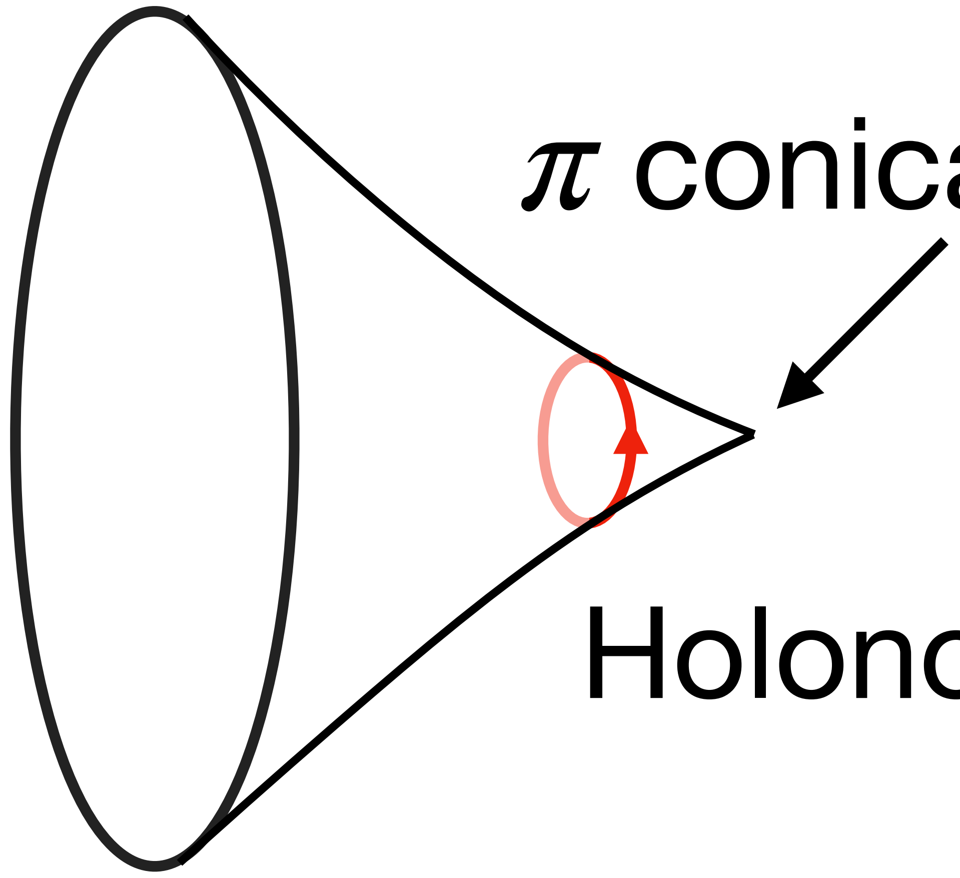
$SL(2,\mathbb{Z})$ black hole

Reduce on φ circle:



Near horizon: $AdS_2 \times S^1$, JT ✓

π conical defect!



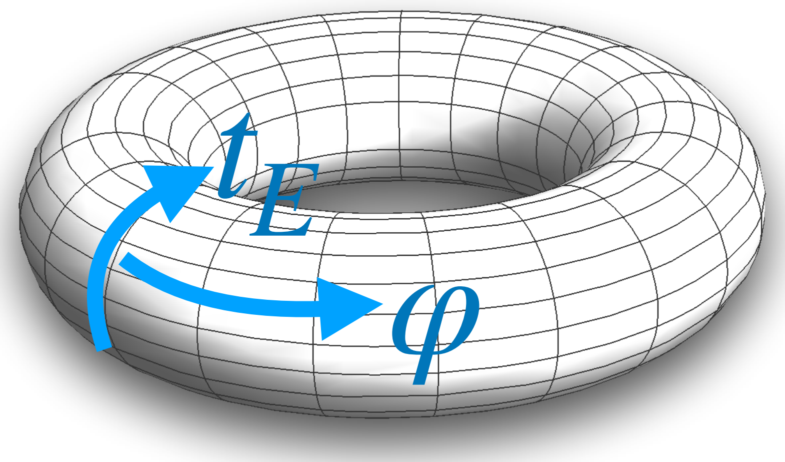
Holonomy $\int A = \pi$

Smooth in 3D, singular in 2D

3D pure gravity

Negative density of states?

Boundary torus



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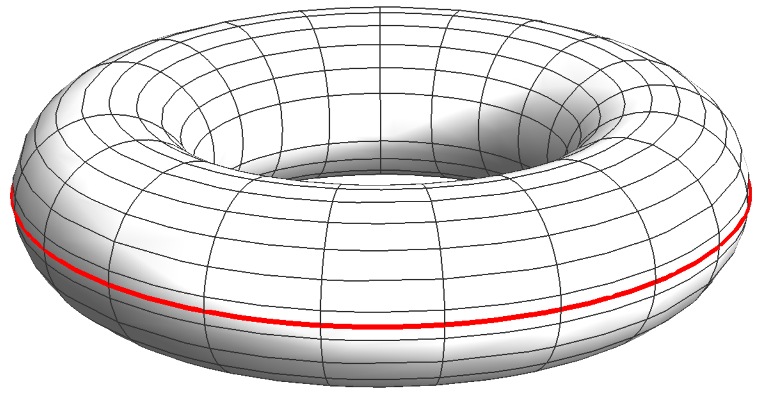

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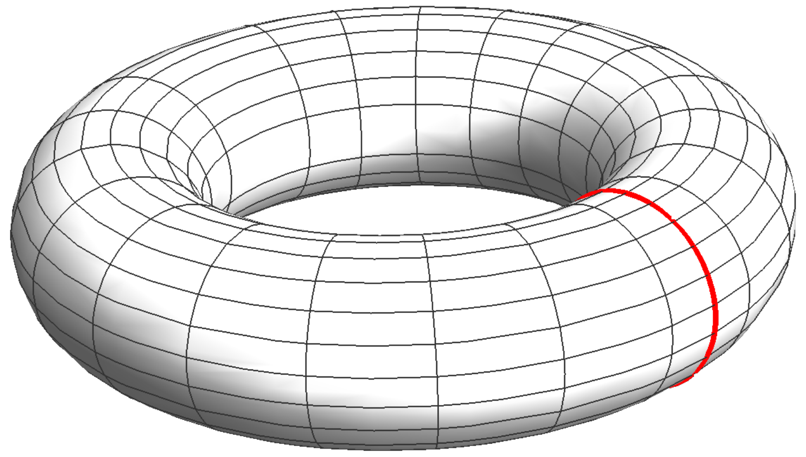

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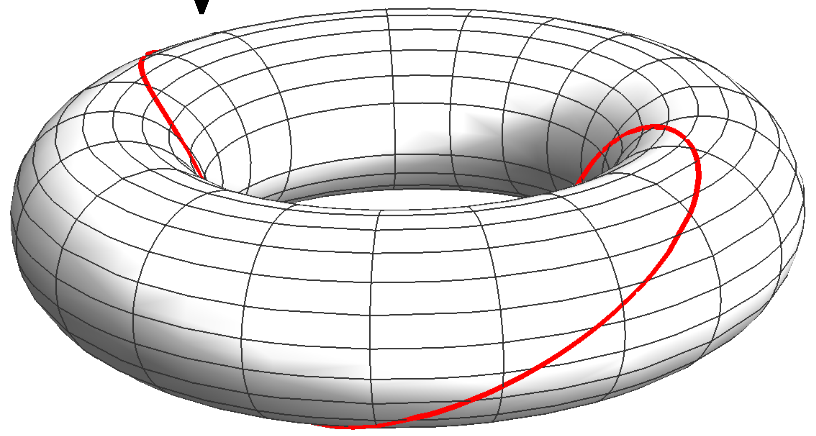

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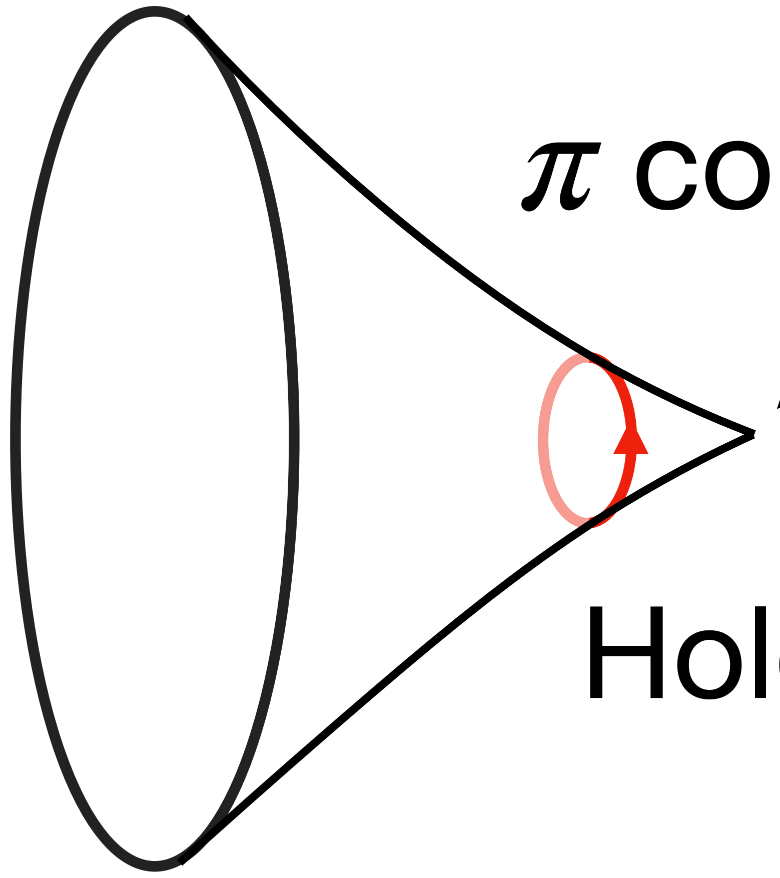


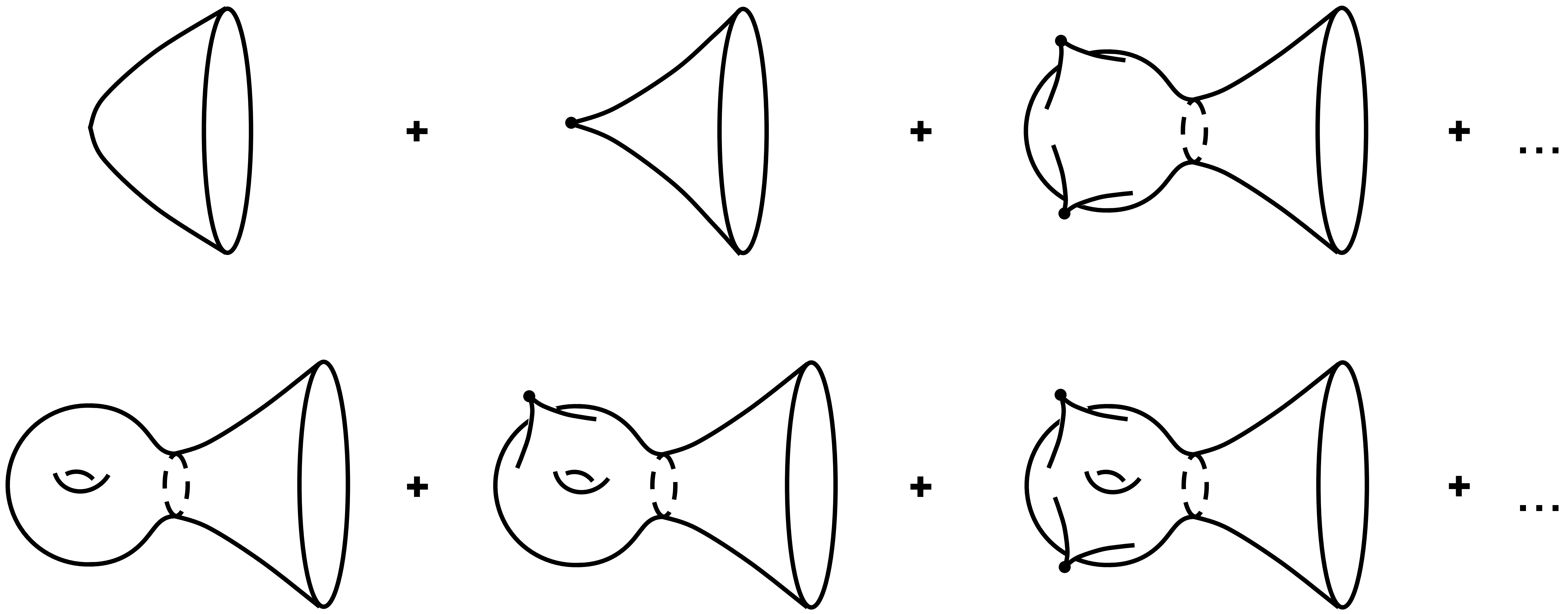
π conical defect!

Holonomy $\int A = \pi$

Smooth in 3D, singular in 2D

Include defects in near-horizon JT theory!

JT gravity with defects



Solving JT gravity

[Saad,Shenker,Stanford]

$$I_{\mathrm{JT}} = -S_0\chi - \frac{1}{2}\int d^2x\sqrt{g_2}\phi\left(R_2+2\right) - \int_\partial ds\,\phi(\kappa-1)$$

Asymptotic boundary circles: $L_\partial \to \infty$, $\phi_\partial \to \infty$, $\beta = \dfrac{L_\partial}{2\phi_\partial}$ held fixed.

Notation:

$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle = \int \mathscr{D}g_2\mathscr{D}\phi\, e^{-I_{\mathrm{JT}}[g_2,\phi]}$$

n asymptotic boundaries,
renormalised lengths $\beta_1, \ldots, \beta_n$

Solving JT gravity

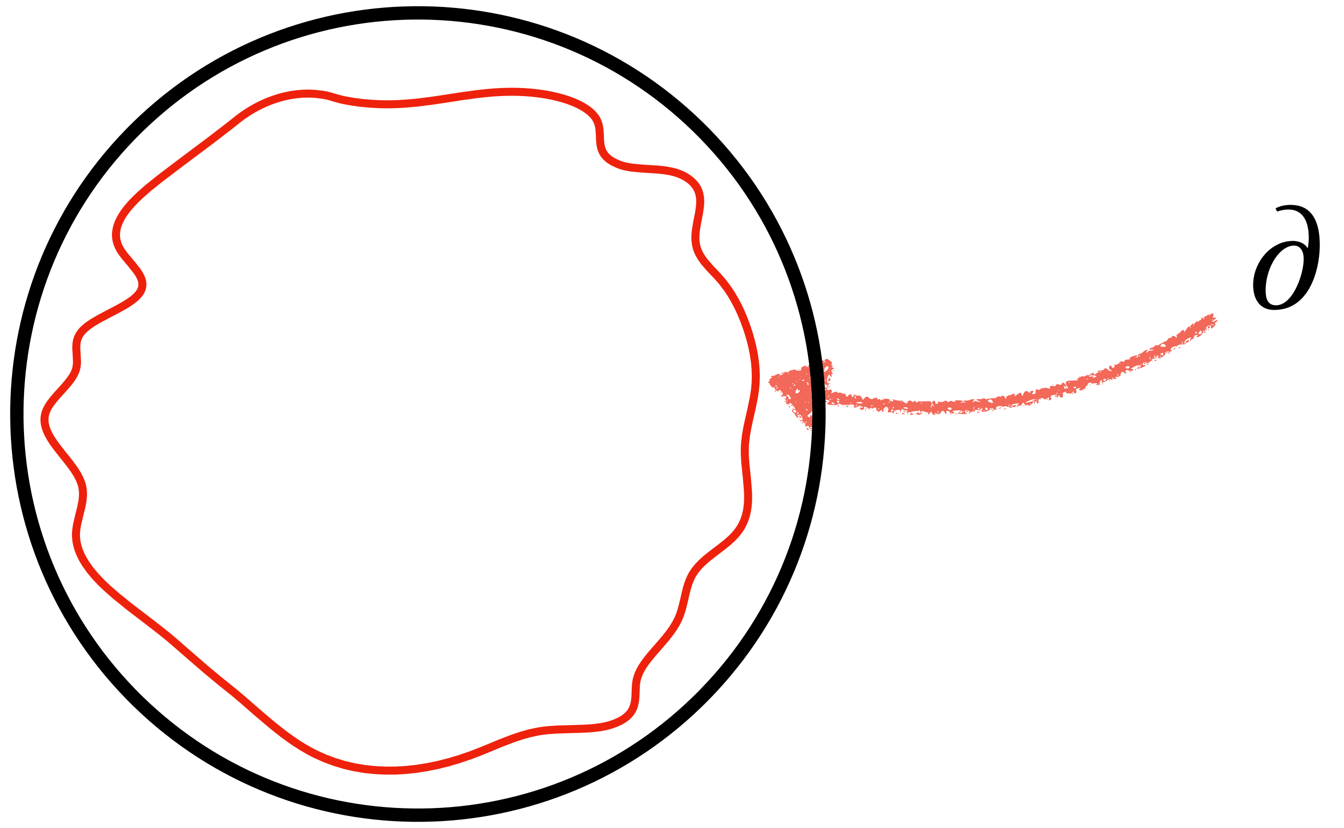
[Saad,Shenker,Stanford]

$$I_{\text{JT}} = -S_0\chi - \frac{1}{2}\int d^2x\sqrt{g_2}\phi\left(R_2+2\right) - \int_\partial ds\,\phi(\kappa-1)$$

ϕ is a Lagrange multiplier, enforcing $R_2 = -2$

Classical solution: the hyperbolic disc

Perturbative corrections: integrate over possible locations of boundary



Solving JT gravity

[Saad,Shenker,Stanford]

becomes Schwarzian action

$$I_{\text{JT}} = -S_0\chi - \frac{1}{2}\int d^2x\sqrt{g_2}\phi\left(R_2+2\right) - \int_\partial ds\,\phi(\kappa-1)$$

[Kitaev] [Jensen]
[MaldacenaStanfordYang]
[EngelsöyMertensVerlinde]

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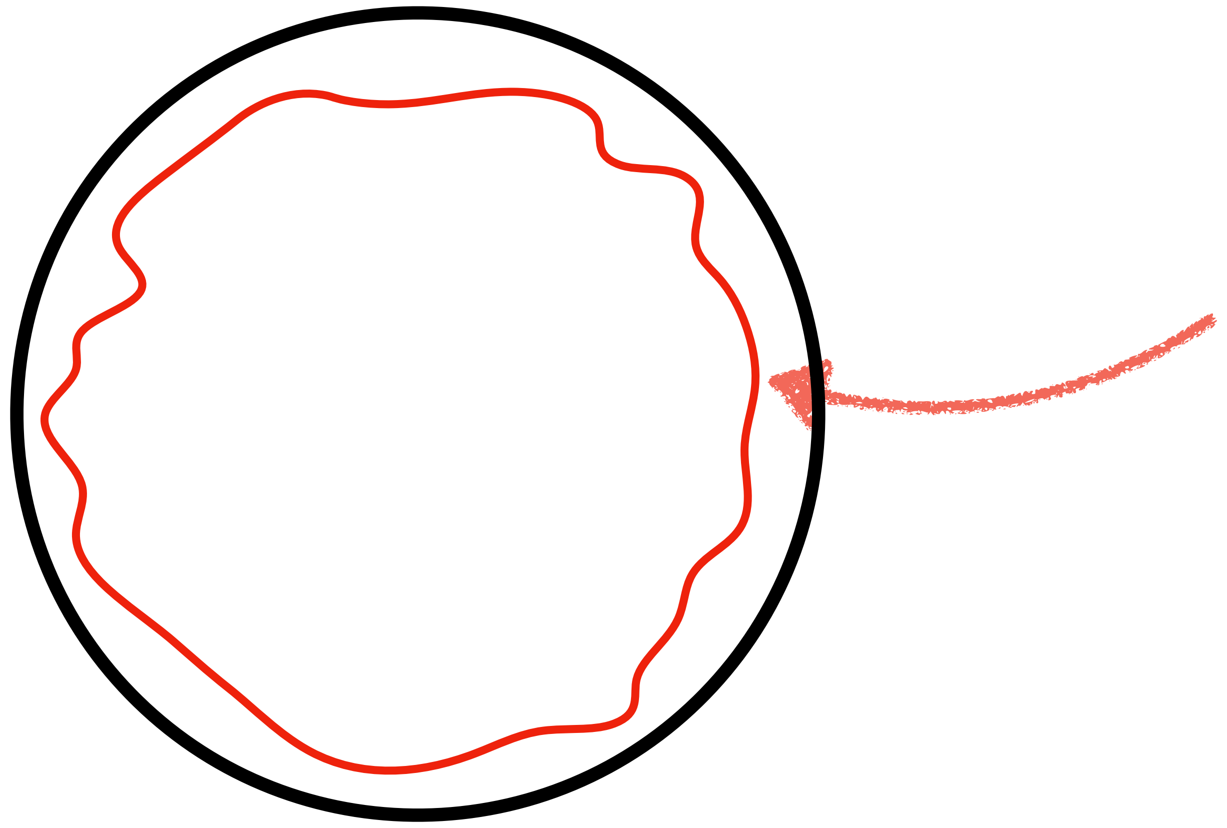


∂

$$Z_{\text{disc}}(\beta) = e^{S_0}\frac{e^{\frac{\pi^2}{\beta}}}{\sqrt{16\pi\beta^3}}$$

$$\rho_{\text{disc}}(E) = \frac{e^{S_0}}{(2\pi)^2}\sinh\left(2\pi\sqrt{E}\right)$$

Solving JT gravity

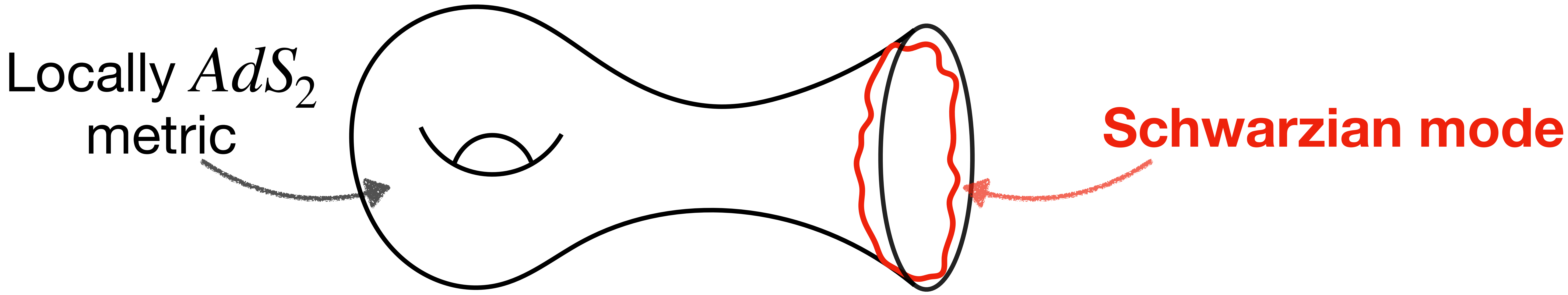
[Saad,Shenker,Stanford]

ϕ is a Lagrange multiplier, enforcing $R_2 = -2$

Higher topologies: path integral $\longrightarrow$ finite-dimensional integral

$$\int \mathcal{D}g_2 \mathcal{D}\phi \, e^{-I_{\mathrm{JT}}[g_2,\phi]} \longrightarrow \int_{\mathcal{M}} \mu_{\mathrm{WP}}$$

$\mathcal{M}$ is a space of hyperbolic surfaces,
μ_{WP} is the Weil-Petersson measure

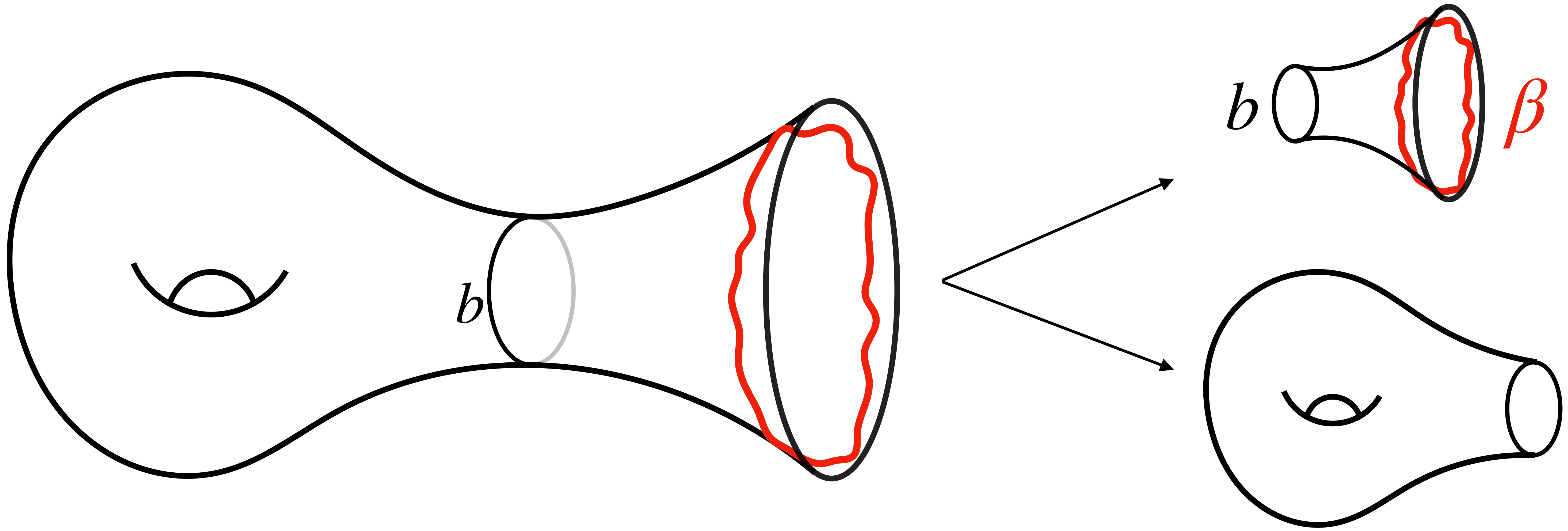


Solving JT gravity

[Saad, Shenker, Stanford]

$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle_{\text{conn.}} = \sum_{g=0}^{\infty} e^{-(2g+n-2)S_0} Z_{g,n}(\beta_1,\cdots,\beta_n)$$

$Z_{g=1,n=1}(\beta)$:



$= Z_{\text{trumpet}}(\beta, b)$

$= V_{g=1,n=1}(b)$

Weil-Petersson volume

$$Z_{g,n}(\beta_1,\cdots,\beta_n) = \int b_1 db_1 Z_{\text{trumpet}}(\beta_1,b_1)\cdots\int b_n db_n Z_{\text{trumpet}}(\beta_n,b_n)\, V_{g,n}(b_1,\ldots,b_n)$$

Solving JT gravity

[Saad,Shenker,Stanford]

$V_{g,n}(b_1, \ldots, b_n)$ obey a recursion relation *[Mirzakhani]*

Close relation to "topological recursion": **matrix integrals** *[EynardOrantin]*

$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle_{\text{Matrix}} = \int dH\, e^{-L\,\mathrm{Tr}\,V(H)} \left(\mathrm{Tr}\, e^{-\beta_1 H}\right)\cdots\left(\mathrm{Tr}\, e^{-\beta_n H}\right)$$

Integral has a genus expansion: *['t Hooft]*

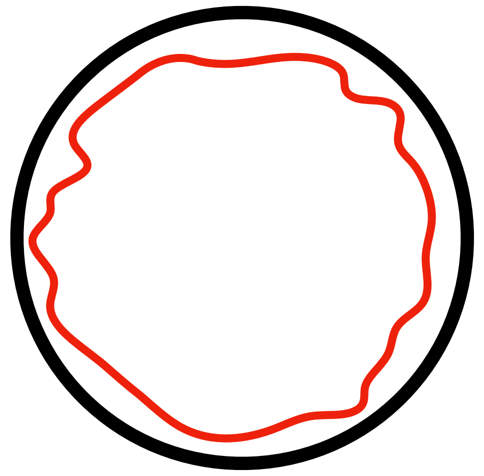
$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle_{\text{conn.,Matrix}} = \sum_{g=0}^{\infty} e^{-(2g+n-2)S_0}\, Z_{g,n}^{\text{Matrix}}(\beta_1, \cdots, \beta_n)$$

Topological recursion: all $Z_{g,n}^{\text{Matrix}}(\beta_1, \cdots, \beta_n)$ uniquely determined by $Z_{0,1}^{\text{Matrix}}(\beta)$

The matrix integral dual of JT gravity

$$\int dH\, e^{-L\,\mathrm{Tr}\,V(H)} \left(\mathrm{Tr}\, e^{-\beta_1 H}\right) \cdots \left(\mathrm{Tr}\, e^{-\beta_n H}\right) \longleftrightarrow \int_{n-\text{boundary}} \mathcal{D}g_2 \mathcal{D}\phi\, e^{-I_{\mathrm{JT}}[g_2,\phi]}$$

Characterised by leading order partition function $Z^{\mathrm{Matrix}}_{g=0,n=1}(\beta)$ $\longleftrightarrow$ Disc:



$$Z_{0,1}(\beta) = \frac{e^{\frac{\pi^2}{\beta}}}{\sqrt{16\pi\beta^3}}$$

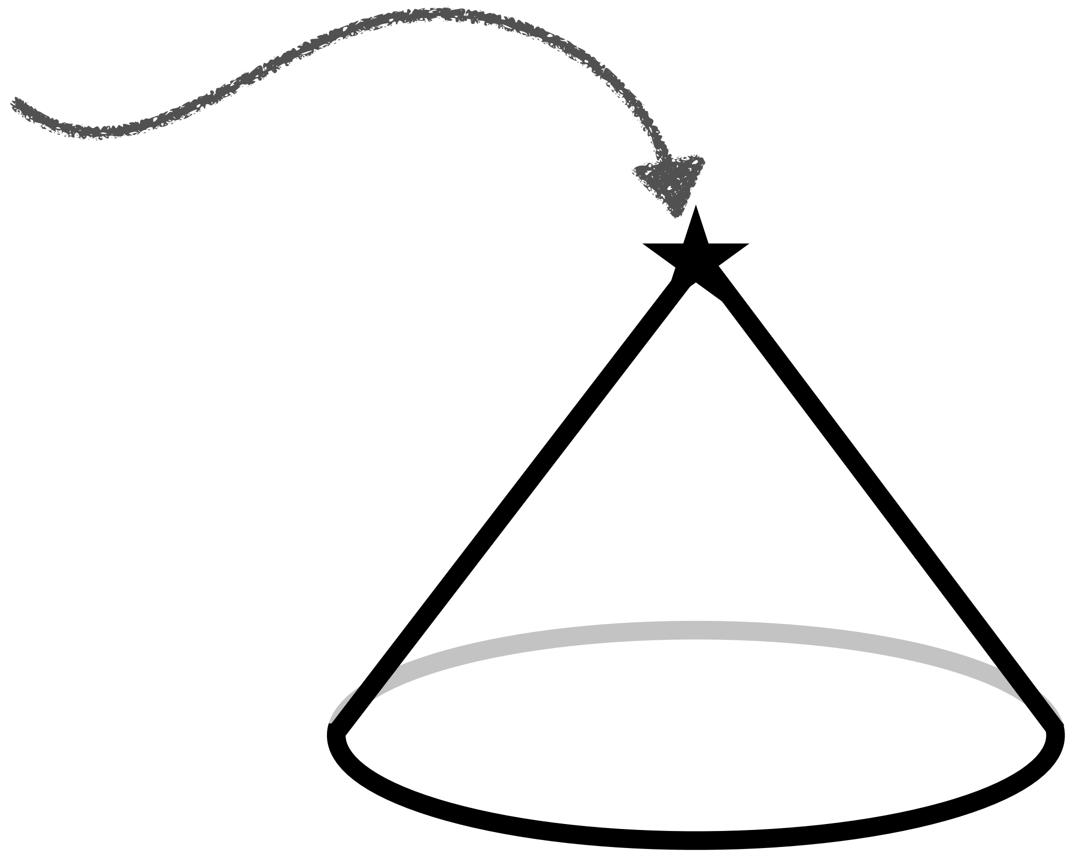
All other amplitudes: Topological recursion $Z^{\mathrm{Matrix}}_{g,n}(\beta)$ $\longleftrightarrow$ Mirzakhani's recursion

A correspondence to all orders in genus expansion

Matrix integral = random Hamiltonian of JT boundary theory

JT gravity with defects

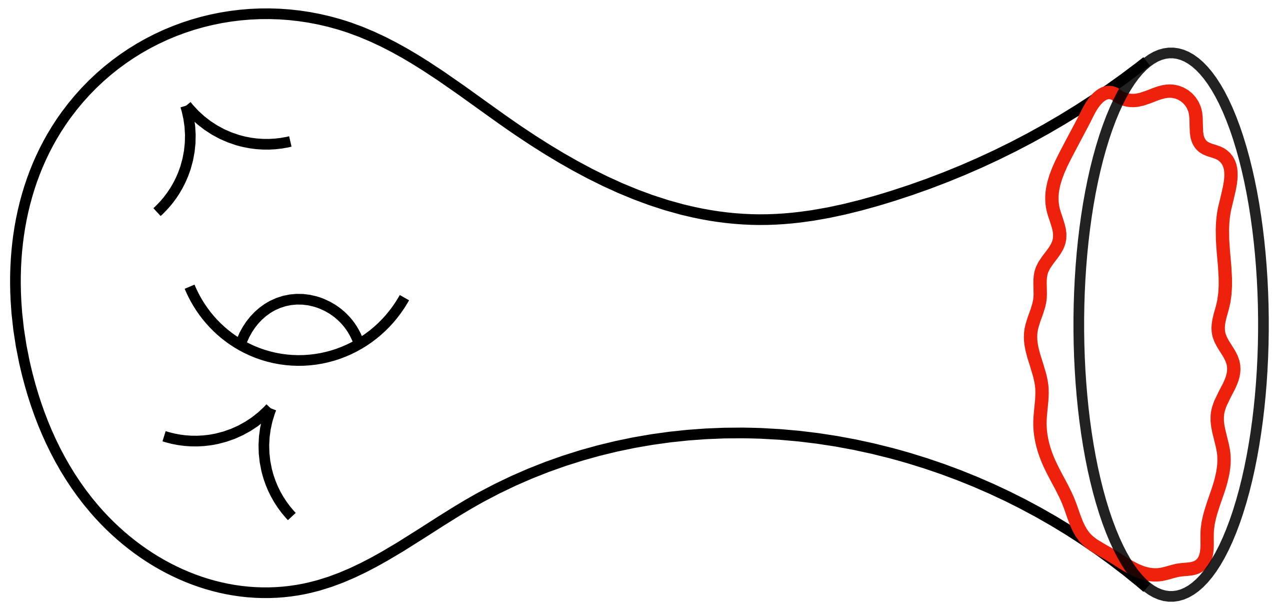
Generalise JT: include dynamical defects



Defects appear with amplitude λ

Source deficit angle $2\pi(1-\alpha)$

Integrate over hyperbolic surfaces with k cone points

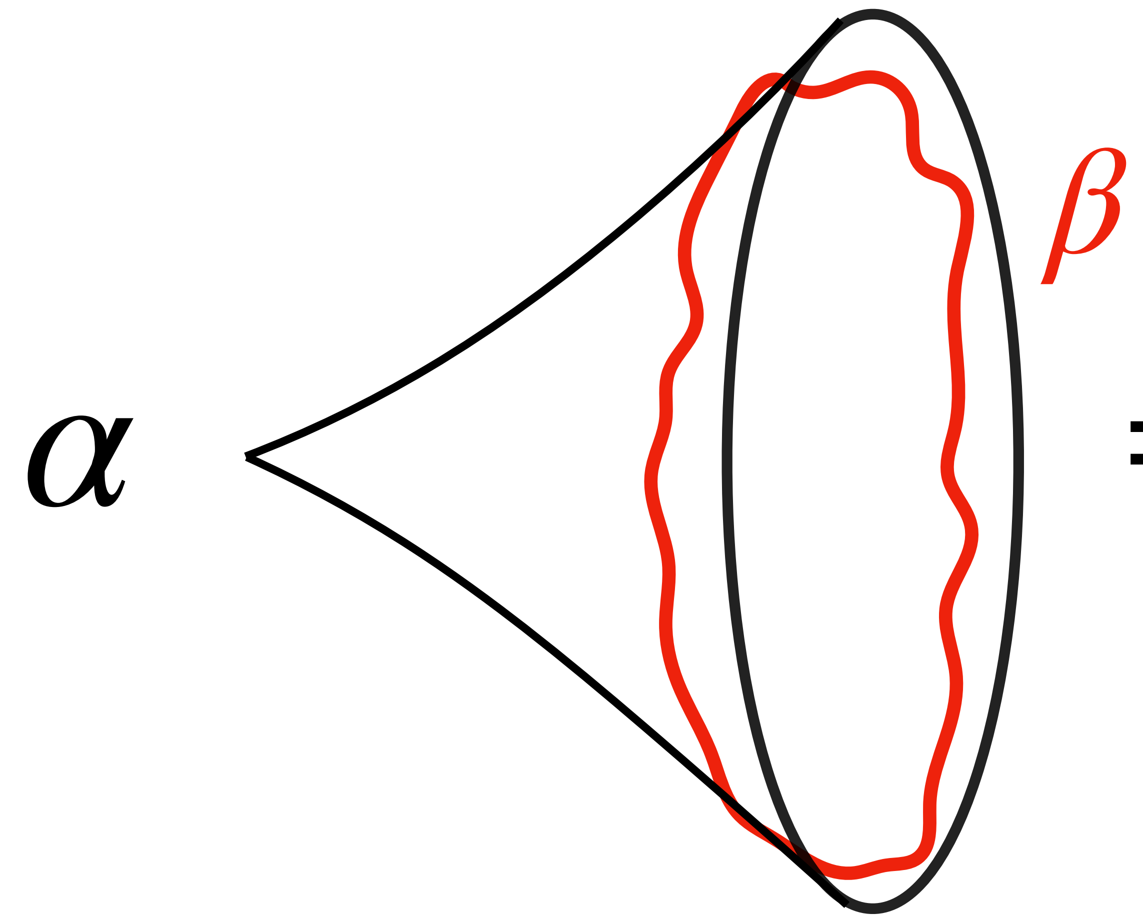


Two-parameter expansion:

$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle_{\text{conn.}} = \sum_{g=0}^{\infty}\sum_{k=0}^{\infty} e^{-(2g+n-2)S_0}\frac{\lambda^k}{k!} Z_{g,n,k}(\beta_1,\cdots,\beta_n;\alpha,\ldots,\alpha)$$

JT gravity with defects

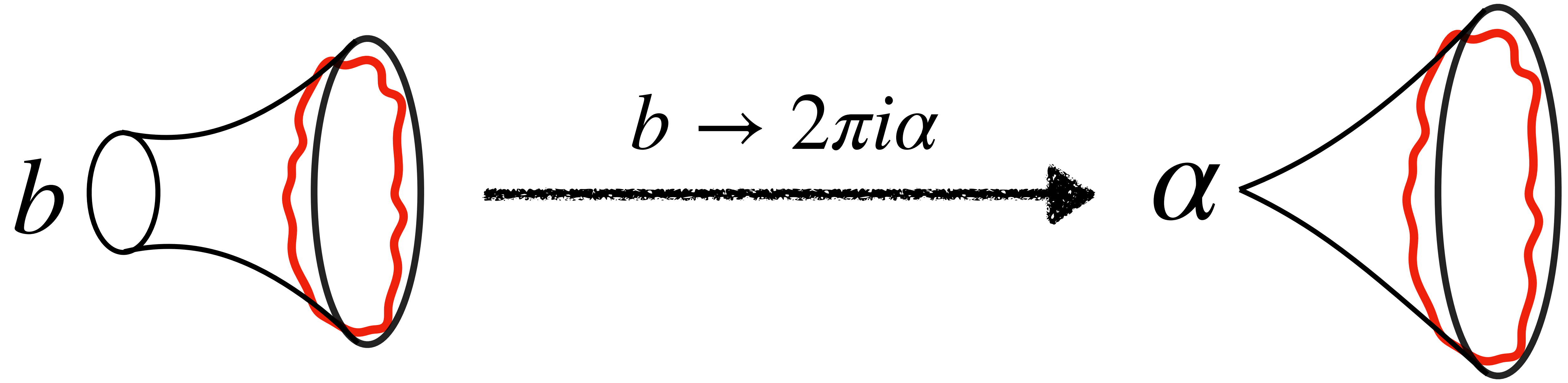
A new special case: $g = 0,\ n = 1,\ k = 1$



$= Z_{\text{trumpet}}(\beta, b = 2\pi i \alpha)$ *[MertensTuriaci]*

Schwarzian integral over elliptic coadjoint orbit

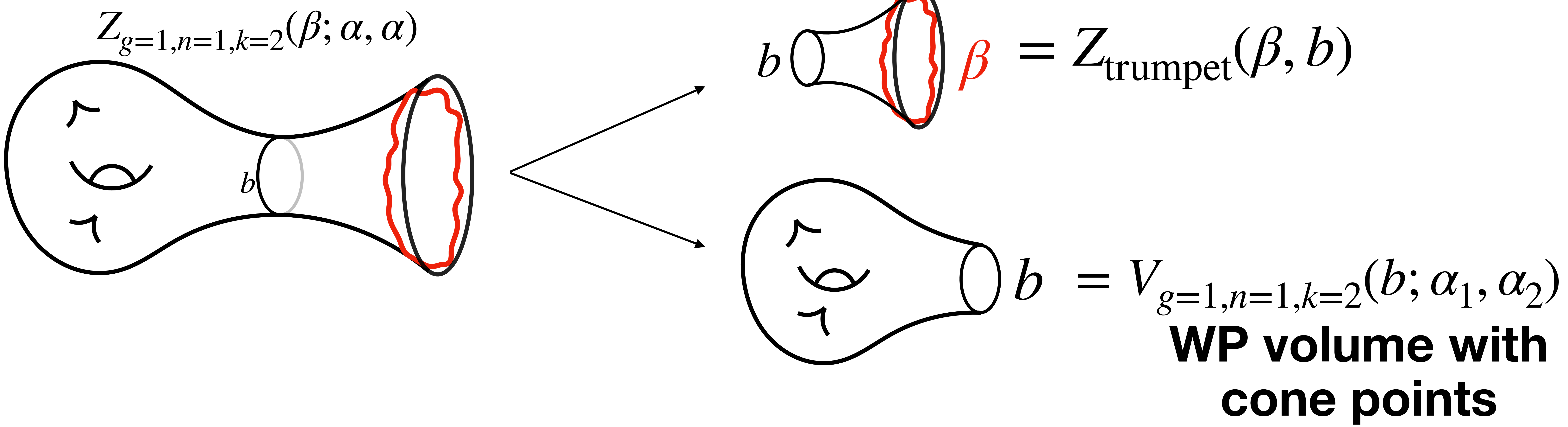
Note:



Defect ~ geodesic of imaginary length!

JT gravity with defects

$$\left\langle Z(\beta_1)\cdots Z(\beta_n)\right\rangle_{\text{conn.}} = \sum_{g=0}^{\infty}\sum_{k=0}^{\infty} e^{-(2g+n-2)S_0}\frac{\lambda^k}{k!} Z_{g,n,k}(\beta_1,\cdots,\beta_n;\alpha,\ldots,\alpha)$$

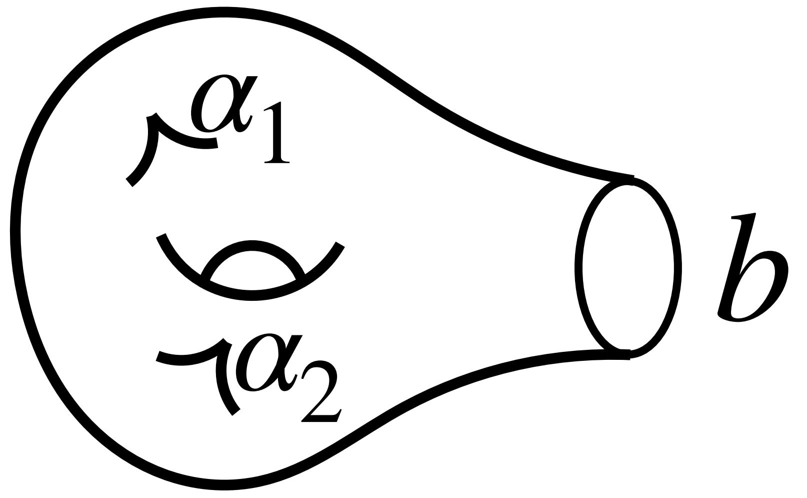


$$Z_{g,n,k}(\beta_1,\cdots,\beta_n;\alpha_1,\ldots,\alpha_k) = \int b_1 db_1 Z_{\text{trumpet}}(\beta_1,b_1)\cdots\int b_n db_n Z_{\text{trumpet}}(\beta_n,b_n)\, V_{g,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k)$$

Requires $\alpha \leq \frac{1}{2}$ so a splitting geodesic always exists for $k \geq 2$

JT gravity with defects

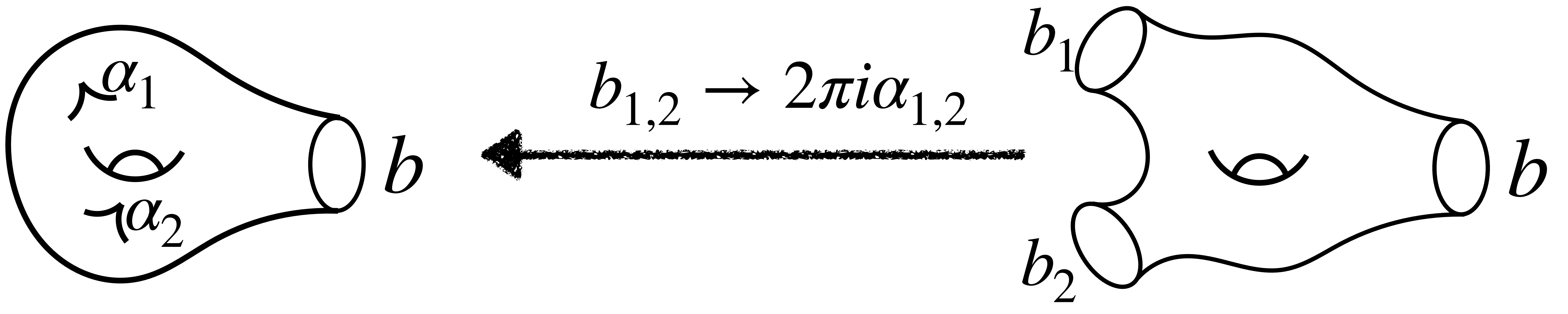
Weil-Petersson volumes with conical defects

$V_{g=1,n=1,k=2}(b;\alpha_1,\alpha_2) =$ 

JT gravity with defects

Weil-Petersson volumes with conical defects

$$V_{g=1,n=1,k=2}(b;\alpha_1,\alpha_2) =$$



Defect $\sim$ geodesic of imaginary length

$$V_{g,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k) = V_{g,n+k}(b_1,\ldots,b_n,2\pi i\alpha_1,\ldots,2\pi i\alpha_k)$$

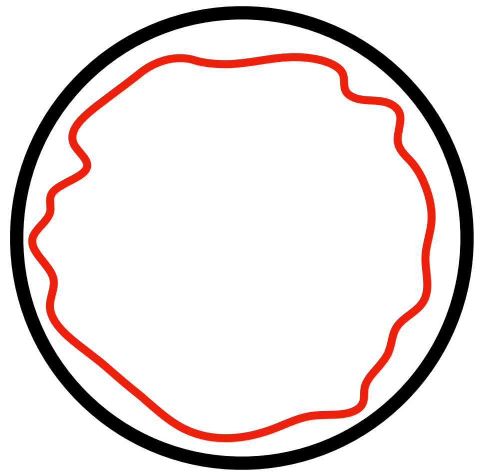
Usual WP volumes with boundaries of imaginary length! *[TanWongZhang][DoNorbury]*

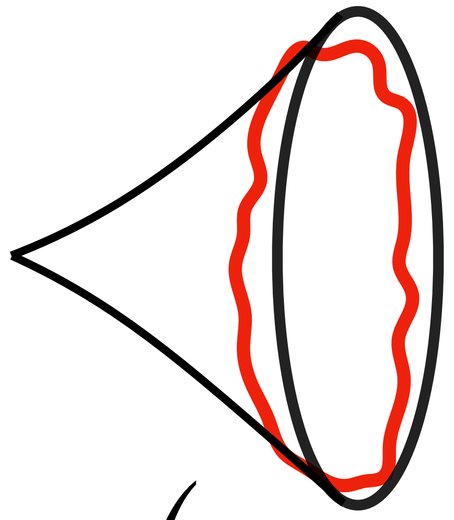
(Proven for $\alpha \leq \frac{1}{2}$)

Used recently for dS_2: *[CotlerJensenMaloney]*

The disc with defects

Start by only allowing topologies with classical solutions: just two!



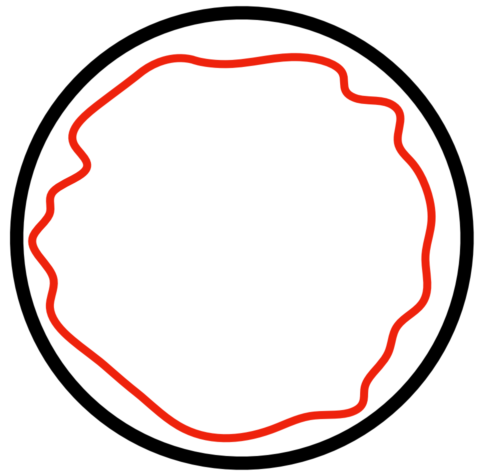


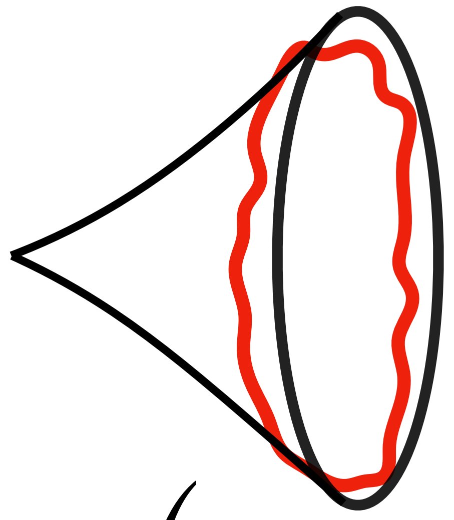
$$\rho_{\text{naive}}(E) = e^{S_0}\left[\frac{1}{(2\pi)^2}\sinh\left(2\pi\sqrt{E}\right) + \frac{\lambda}{2\pi\sqrt{E}}\cosh\left(2\pi\alpha\sqrt{E}\right)\right] \sim \frac{e^{S_0}}{2\pi}\left[\sqrt{E} + \frac{\lambda}{\sqrt{E}}\right]$$

Is $\lambda < 0$ inconsistent? $\rho < 0$ for $0 < E < |\lambda|$

The disc with defects

Start by only allowing topologies with classical solutions: just two!





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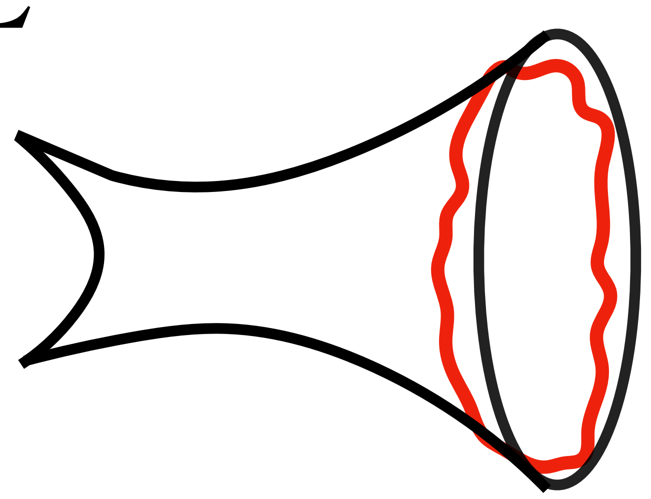
Is $\lambda < 0$ inconsistent? $\rho < 0$ for $0 < E < |\lambda|$

Include multiple defects:
$$\rho_{\text{disc}}(E) \sim \frac{e^{S_0}}{2\pi}\left[\sqrt{E} + \frac{\lambda}{\sqrt{E}} - \frac{\lambda^2}{2E^{3/2}} + \cdots\right]$$

Expansion in defects divergent for $E \lesssim |\lambda|$



The disc with defects

Leading order for k defects at low energy: $V_{g=0,n=1,k} \sim \frac{1}{(k-2)!}\left(\frac{b^2}{2}\right)^{k-2}$ $\quad (b \gg 1)$

Sum: $$\rho_{\text{disc}}(E) \sim \frac{e^{S_0}}{2\pi}\left[\sqrt{E} + \frac{\lambda}{\sqrt{E}} - \frac{\lambda^2}{2E^{3/2}} + \cdots\right] \sim \frac{e^{S_0}}{2\pi}\sqrt{E+2\lambda}, \quad E \sim \lambda \ll 1$$

Shift of threshold: $E > E_0(\lambda) \sim -2\lambda$

The disc with defects

Leading order for k defects at low energy: $V_{g=0,n=1,k} \sim \frac{1}{(k-2)!}\left(\frac{b^2}{2}\right)^{k-2}$ $(b \gg 1)$

Sum: $\rho_{\text{disc}}(E) \sim \frac{e^{S_0}}{2\pi}\left[\sqrt{E} + \frac{\lambda}{\sqrt{E}} - \frac{\lambda^2}{2E^{3/2}} + \cdots\right] \sim \frac{e^{S_0}}{2\pi}\sqrt{E - E_0(\lambda)}, \quad E \sim \lambda \ll 1$

Shift of threshold: $E > E_0(\lambda) = -2\lambda - 2\pi^2(1-2\alpha^2)\lambda^2 - \frac{2}{3}\pi^4(5 - 18\alpha^2 + 15\alpha^4)\lambda^3 + \mathcal{O}(\lambda^4)$

Strong constraint on volume polynomials!

$$V_{g=0,n=1,k} = \underbrace{\frac{1}{(k-2)!}\left(\frac{b^2}{2}\right)^{k-2} + \#b^{2(k-3)} + \cdots + \#b^4}_{\text{First } k-3 \text{ terms fixed by lower orders}} + \#b^2 + \#$$

Changes "shape" of $\rho_{\text{disc}}(E)$

New contribution to E_0 at order λ^k

The disc: all orders in λ

[BanksDouglasSeibergShenker],…
[Okuyama,Sakai][Johnson]

An alternative way to write the disc density of states:

$$\left\langle Z(\beta)\right\rangle_{g=0} = \frac{e^{S_0}}{\sqrt{4\pi\beta}}\int_0^\infty dx\, e^{-\beta u(x)}$$

$u(x) \Longleftrightarrow$ genus zero $\rho(E)$: defines a matrix integral

For JT: $u(x)$ implicitly from $\frac{1}{2\pi}\sqrt{u_{\text{JT}}(x)}\; I_1\left(2\pi\sqrt{u_{\text{JT}}(x)}\right) = x$ "string equation"

Inverse Laplace, change variables: $\rho_{\text{disc}}(E) = \frac{e^{S_0}}{2\pi}\int_{E_0}^{E}\frac{du}{\sqrt{E-u}}\frac{dx}{du}$

Threshold $E_0 = u(0)$, $\rho_{\text{disc}} \sim \sqrt{E-E_0}$ (generically)

The disc: all orders in λ

An explicit expression for genus zero WP volumes *[MertensTuriaci]*:

$$V_{0,n}(b_1, \ldots, b_n) = \frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{n-3}\left[J_0\left(b_1\sqrt{u_{\rm JT}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\rm JT}(x)}\right)u'_{\rm JT}(x)\right]\Bigg|_{x=0}$$

The disc: all orders in λ

An explicit expression for genus zero WP volumes **with defects**:

$$V_{0,n,k}(b_1, \ldots, b_n; \alpha_1, \ldots, \alpha_k) = \frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{k+n-3} \left[J_0\left(b_1\sqrt{u_{\rm JT}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\rm JT}(x)}\right) I_0\left(2\pi\alpha_1\sqrt{u_{\rm JT}(x)}\right)\cdots I_0\left(2\pi\alpha_k\sqrt{u_{\rm JT}(x)}\right) u'_{\rm JT}(x)\right]\Bigg|_{x=0}$$

→ Explicit expression for $Z_{0,1,k}(\beta;\alpha)$ for all k

Perform sum over k using “Lagrange reversion theorem”

The disc: all orders in λ

An explicit expression for genus zero WP volumes **with defects**:

$$V_{0,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k) = \frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{k+n-3}\left[J_0\left(b_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\mathrm{JT}}(x)}\right) I_0\left(2\pi\alpha_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots I_0\left(2\pi\alpha_k\sqrt{u_{\mathrm{JT}}(x)}\right) u'_{\mathrm{JT}}(x)\right]\Bigg|_{x=0}$$

→ Explicit expression for $Z_{0,1,k}(\beta;\alpha)$ for all k

Perform sum over k using "Lagrange reversion theorem"

Lagrange reversion theorem

From Wikipedia, the free encyclopedia

In mathematics, the **Lagrange reversion theorem** gives series or formal po

Let *v* be a function of *x* and *y* in terms of another function *f* such that

$$v = x + yf(v)$$

Then for any function *g*, for small enough *y*:

$$g(v) = g(x) + \sum_{k=1}^{\infty} \frac{y^k}{k!}\left(\frac{\partial}{\partial x}\right)^{k-1}\left(f(x)^k g'(x)\right).$$

The disc: all orders in λ

An explicit expression for genus zero WP volumes **with defects**:

$$V_{0,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k)=\frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{k+n-3}\left[J_0\left(b_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\mathrm{JT}}(x)}\right)I_0\left(2\pi\alpha_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots I_0\left(2\pi\alpha_k\sqrt{u_{\mathrm{JT}}(x)}\right)u'_{\mathrm{JT}}(x)\right]\Bigg|_{x=0}$$

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Let v be a function of x and y in terms of another function f such that

$$v=x+yf(v)$$ String equation

Then for any function g, for small enough y:

$$g(v)=g(x)+\sum_{k=1}^{\infty}\frac{y^k}{k!}\left(\frac{\partial}{\partial x}\right)^{k-1}\left(f(x)^k g'(x)\right).$$ Sum over defects

The disc: all orders in λ

An explicit expression for genus zero WP volumes **with defects**:

$$V_{0,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k)=\frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{k+n-3}\left[J_0\left(b_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\mathrm{JT}}(x)}\right)I_0\left(2\pi\alpha_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots I_0\left(2\pi\alpha_k\sqrt{u_{\mathrm{JT}}(x)}\right)u'_{\mathrm{JT}}(x)\right]\Bigg|_{x=0}$$

⟶ Explicit expression for $Z_{0,1,k}(\beta;\alpha)$ for all k

Perform sum over k using "Lagrange reversion theorem"

Result: a linear deformation of the string equation!

$$\frac{\sqrt{u(x)}}{2\pi}\,I_1\left(2\pi\sqrt{u(x)}\right)+\lambda\,I_0\left(2\pi\alpha\sqrt{u(x)}\right)=x$$

The disc: all orders in λ

An explicit expression for genus zero WP volumes **with defects**:

$$V_{0,n,k}(b_1,\ldots,b_n;\alpha_1,\ldots,\alpha_k)=\frac{1}{2}\left(-\frac{\partial}{\partial x}\right)^{k+n-3}\left[J_0\left(b_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots J_0\left(b_n\sqrt{u_{\mathrm{JT}}(x)}\right)I_0\left(2\pi\alpha_1\sqrt{u_{\mathrm{JT}}(x)}\right)\cdots I_0\left(2\pi\alpha_k\sqrt{u_{\mathrm{JT}}(x)}\right)u'_{\mathrm{JT}}(x)\right]\Bigg|_{x=0}$$

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Perform sum over k using "Lagrange reversion theorem"

Result: a linear deformation of the string equation!

$$\frac{\sqrt{u(x)}}{2\pi}\,I_1\left(2\pi\sqrt{u(x)}\right)+\sum_i\lambda_i\,I_0\left(2\pi\alpha_i\sqrt{u(x)}\right)=x$$

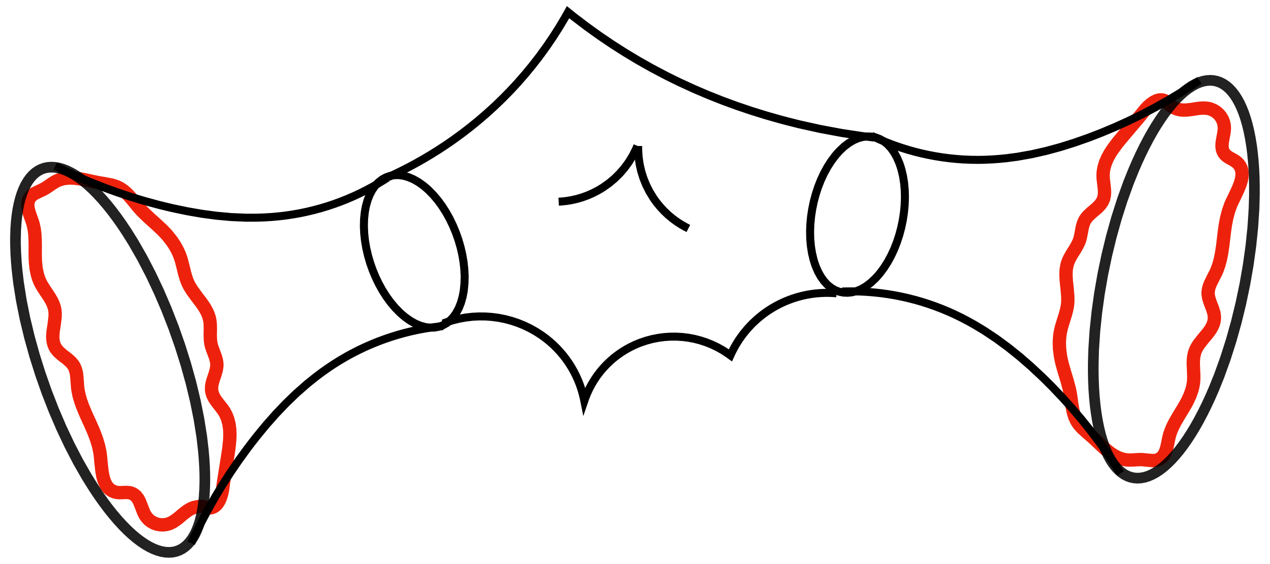
Exact genus zero density of states

The matrix integral dual

The double trumpet: a smoking gun

Two boundaries: double-scaled matrix integrals have a ***universal*** genus 0 answer:

$$\left\langle Z(\beta_1)Z(\beta_2)\right\rangle_{\text{conn.,g=0,Matrix}} = \frac{1}{2\pi}\frac{\sqrt{\beta_1\beta_2}}{\beta_1+\beta_2}e^{-E_0(\beta_1+\beta_2)}$$

$Z_{g=0,n=2,k}(\beta_1,\beta_2) =$  gives precisely this answer, order by order in λ!

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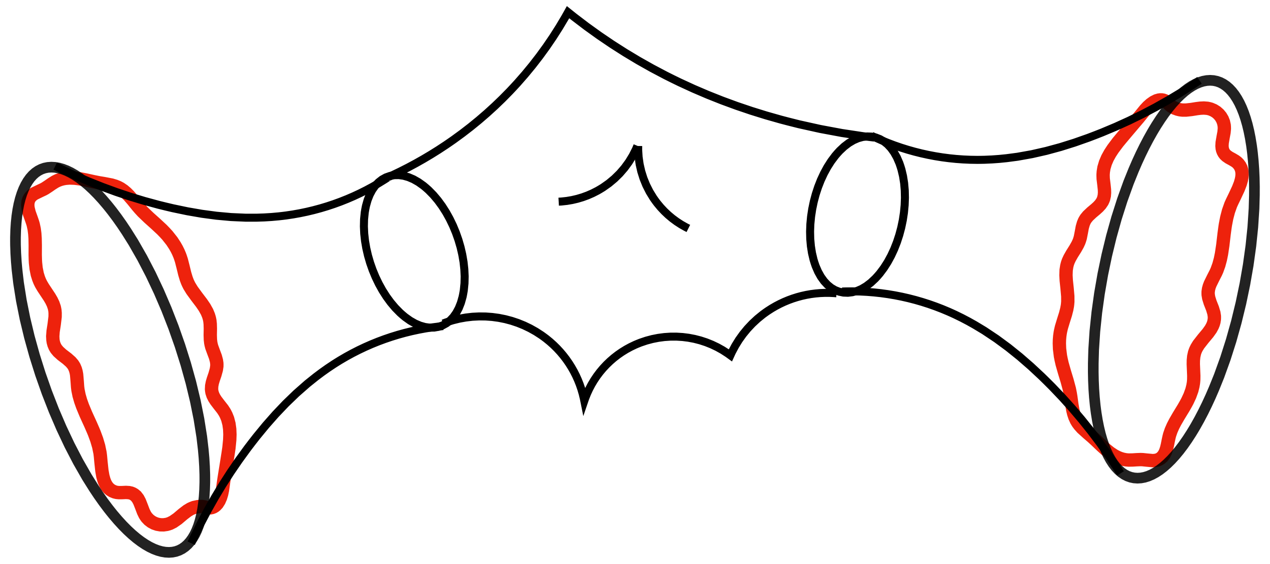
$Z_{g=0,n=2,k}(\beta_1,\beta_2) =$  gives precisely this answer, order by order in λ!

Explicit checks for $Z_{g=0,n,k}$, and some at higher genus: all match matrix integral!

Proof that topological recursion is obeyed using deformation theorem of *[Eynard,Orantin]*

As a deformation of dilaton potential

The “defect gas”

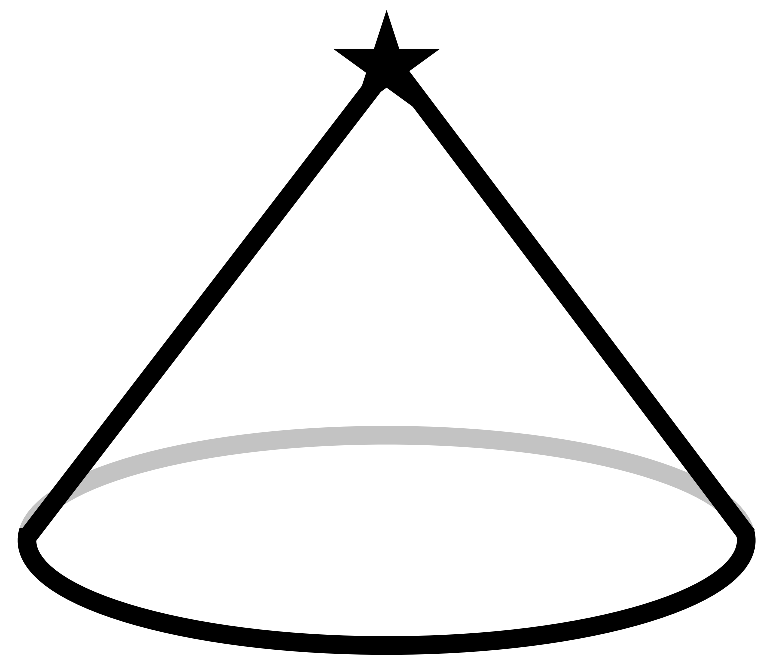
Defect: insertion in path integral

$$\int \mathscr{D}g_2 \mathscr{D}\phi \, e^{-I_{\mathrm{JT}}[g_2,\phi]} \underbrace{\int d^2x \sqrt{g_2} e^{-2\pi(1-\alpha)\phi(x)}}_{\text{Defect insertion}}$$

Integrate out dilaton: imposes delta-function curvature

$$I_{\mathrm{JT}} \longrightarrow -\frac{1}{2}\int d^2x \sqrt{g_2}\phi\left(R_2 + 2 - 4\pi(1-\alpha)\delta_{\mathrm{defect}}\right) \implies R_2 = -2 + 4\pi(1-\alpha)\delta_{\mathrm{defect}}$$

Conical singularity at location of defect ✓



As a deformation of dilaton potential

The “defect gas”

Sum over defect insertions:

$$\sum_k \frac{\lambda^k}{k!} \int \mathscr{D} g_2 \mathscr{D}\phi \, e^{-I_{\mathrm{JT}}[g_2,\phi]} \left(\int d^2x \sqrt{g_2} e^{-2\pi(1-\alpha)\phi(x)} \right)^k$$

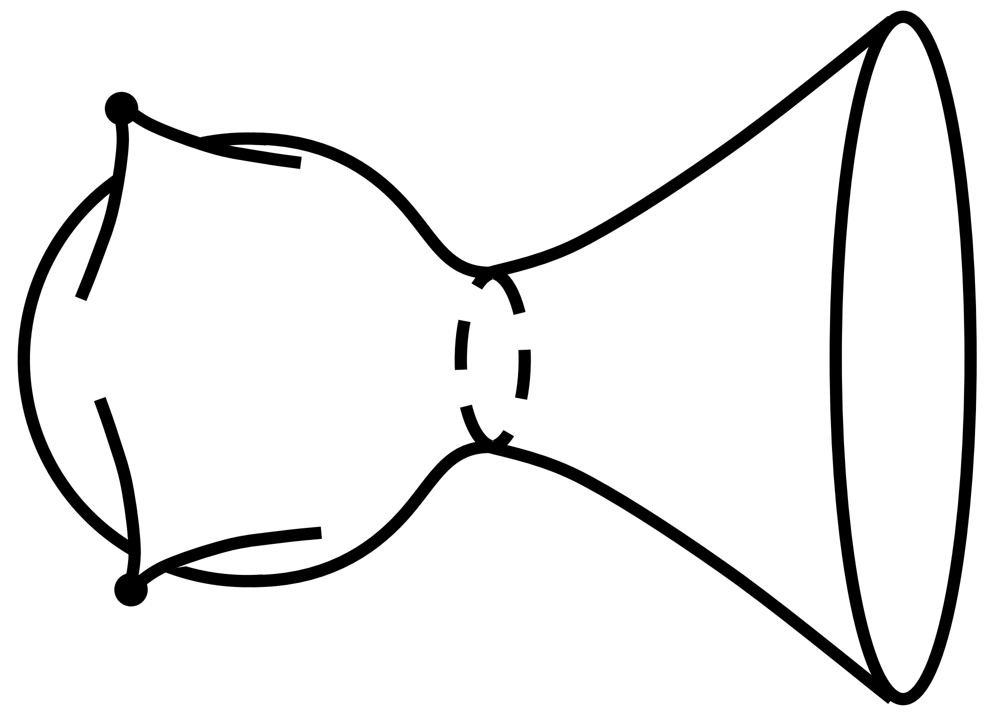
Exponentiates: extra local term in action

$$I = -\frac{1}{2} \int d^2x \sqrt{g_2} \left(\phi R_2 + U(\phi) \right) \qquad U(\phi) = 2\phi + \sum_i \Lambda_i \, e^{-2\pi(1-\alpha_i)\phi}$$

A family of dilaton gravity theories with matrix integral duals

JT with defects

Summary



- Exact path integral for JT gravity generalises to models with defects
- For N_F “flavours” of defect, $2N_F$ parameters λ_i, α_i
- Exact formula for disc density of states
- Matrix integral to all orders in the genus expansion
- Dilaton gravity with deformed potential $U(\phi) = 2\phi + \sum_i \Lambda_i \, e^{-2\pi(1-\alpha_i)\phi}$

Back to three dimensions

Back to three dimensions

A proposal to cure negative density of states

Near-extremal density of primary states well-described by JT + KK instantons!

Negativity: $\rho_J(E) \sim e^{S_0(J)}\sqrt{E-|J|} + (-1)^J e^{\frac{S_0(J)}{2}} \frac{1}{\sqrt{E-|J|}} \longrightarrow e^{S_0(J)}\sqrt{E-E_0(J)}$

replaced by nonperturbative shift of BTZ extremality bound:

$$E_0(J) - |J| \sim -(-1)^J e^{-\frac{S_0(J)}{2}}$$

Multiple KK instantons in 2D $\longrightarrow$ Seifert manifolds in 3D

New topologies to include in path integral, with no classical solutions

A similar perturbative shift of BTZ extremality for generic CFTs (bootstrap) *[HM]*

Back to three dimensions

An ensemble dual for 3D pure gravity?

- “Spacetime wormholes”: gravity joins disconnected Euclidean boundaries
- An ensemble of dual CFTs?! *[Cotler,Jensen] [Belin,de Boer]*

  [MaloneyWitten][Afkhami-Jeddi,Cohn,Hartman,Tajdini]
- A reason to expect continuous $\rho_J(E)$
- A matrix integral near extremality, but more structure from locality, correlators
- Closely related to recent discussions of Page curve…
  [Penington][AlmheiriEngelhardtMarolfHM][PeningtonShenkerStanfordYang][AlmheiriHartmanMaldacenaShaghoulianTajdini]
- … and the Hilbert space of closed “baby” universes *[Coleman][GiddingsStrominger][MarolfHM]*
- A new paradigm: semiclassical gravity as an averaged theory?