Unitarity Methods in AdS/CFT

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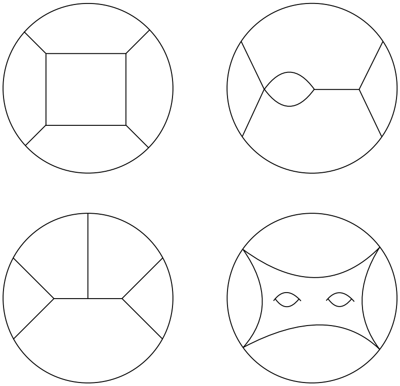
Caltech

Strings 2020

Based on:
1705.03453 w/ D. Poland & D. Li
1912.05580
1912.09521 w/ E. Perlmutter & A. Sivaramakrishnan
To appear w/ A. Sivaramakrishnan

Perturbation Theory in AdS

- **Goal**: Use causality and unitarity to simplify $\text{AdS}_{d+1}/\text{CFT}_d$ perturbation theory.



- Four-point correlators well-studied at tree-level in gravity limit. Correlators at higher points, higher loops, or finite α' are less understood.

Universality and AdS/CFT

- Which CFTs have a local, weakly-coupled, gravity dual?
- Conjecture (HPPS): Heemskerk, Penedones, Polchinski, Sully

$$N \gg 1 \;\; \& \text{ large gap } \Delta_{\text{gap}} \gg 1 \Longrightarrow \text{Local, weakly-coupled AdS dual.}$$

$\Delta_{\text{gap}} =$ Dimensions of higher-spin ($J >$2) single-traces.

- Imposing causality in the limit $\Delta_{\text{gap}} \gg 1$ picks out Einstein gravity:

Camanho, Edelstein, Maldacena, Zhiboedov

$$\langle \mathcal{O}_i T \mathcal{O}_j \rangle = \delta_{ij} \langle \mathcal{O}_i T \mathcal{O}_j \rangle_{\text{Einstein}} + O(\Delta_{\text{gap}}^{-2})$$

where $\mathcal{O}_i$ is any single-trace of spin $J \leq 2$, including T.

Afkhami-Jeddi, Hartman, Kundu, Tajdini; Kulaxizi, Parnachev, Zhiboedov; Costa, Hansen, Penedones; DM, Perlmutter; Kologlu, Kravchuk, Simmons-Duffin, Zhiboedov; Belin, Hofman, Mathys

Pushing Universality

Many natural questions to consider next:

1. What happens at higher points, e.g. can we fix all $\langle T ... T \rangle$ when $\Delta_{\text{gap}} \gg 1$? That is, can we fix all graviton scattering amplitudes in bulk?

   Chowdhury, Gadde, Gopalka, Halder, Janagal, Minwalla

2. What role do loops play in constraining space of theories? Sensitive to extra dimensions but loop-level constraints less explored.

   Alday, Caron-Huot; Alday, Perlmutter

3. Can we relax large Δ_{gap} condition? Space of weakly-coupled theories in AdS with higher-spin particles?

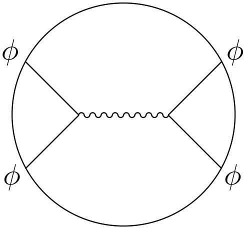
   Caron-Huot, Komargodski, Sever, Zhiboedov; Sever, Zhiboedov; Arkani-Hamed, Huang, Huang

By working in AdS can rigorously constrain theories with CFT bootstrap.

Polyakov; Rattazzi, Rychkov, Tonni, Vichi

AdS Amplitudes

- Price of working in AdS: computations are more difficult than in flat-space!



- At tree-level and with special masses, have diLogs in position space.

  D'Hoker, Freedman, Mathur, Matusis, Rastelli; D'Hoker, Freedman, Rastelli; Dolan, Osborn

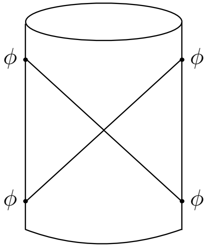
  By contrast, flat-space, tree-level S-matrix is very simple:

$$M(s,t) = \frac{g^2}{s - m^2}$$

- Can we understand AdS "amplitudes" as well as we understand S-matrix? Are there any hidden structures we are missing?

AdS and Flat-Space

- First hint: take the flat-space limit of AdS/CFT.
  Can set up AdS scattering to reproduce flat-space S-matrix.



$$d\text{- dimensional } \langle\phi\phi\phi\phi\rangle \implies (d+1)\text{-dimensional } \mathcal{T}(s,t)$$

Gary, Giddings, Penedones; Penedones; Maldacena, Simmons-Duffin, Zhiboedov

- Do flat-space structures generalize to AdS or only emerge in limit?

- Similar questions considered in dS and inflationary correlators.
  Arkani-Hamed, Maldacena; Arkani-Hamed, Baumann, Lee, Pimentel; Baumann, Pueyo, Joyce, Lee, Pimentel; Arkani-Hamed, Benincasa, Postnikov; Arkani-Hamed, Benincasa; Benincasa; Sleight; Sleight, Taronna

Holographic Correlators

- Want to answer simple question: What are the AdS Cutkosky rules?

- Present two new *bulk* unitarity methods for AdS loops.
  Matches earlier *boundary* method and generalizes S-matrix conditions to AdS.
  Restrict to scalar field theory in AdS for simplicity.

DM, Perlmutter, Sivaramakrishnan; DM, Sivaramakrishnan

- As an application, use unitarity to study Regge limit to all loop orders.
  Find eikonalization at large, but finite, Δ_{gap}.
  Consistent with previous work on bulk strings in AdS.

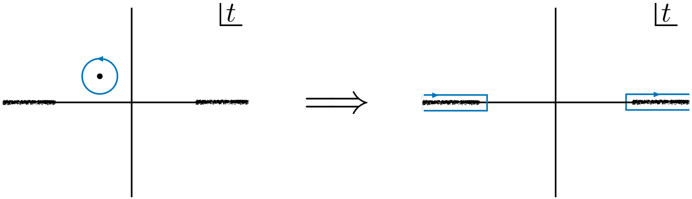
Li, DM, Poland; DM

Flat Space Unitarity

- Recall unitarity of S-matrix buys us a loop order:

$$\mathcal{S}^\dagger \mathcal{S} = 1 \quad \& \quad \mathcal{S} = \mathbf{1} + i\mathcal{T} \quad \Longrightarrow \quad 2\,\mathrm{Im}(\mathcal{T}) = \mathcal{T}^\dagger \mathcal{T}$$

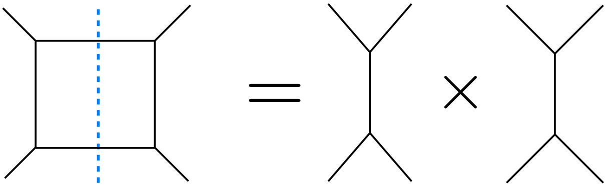
- Find amplitude $M(s,t)$ from discontinuities using dispersion relation.



$$M(s,t) \propto \int \frac{dt'}{t'-t} \mathsf{Disc}_{t'} M(s,t') + (\text{u-channel})$$

Assuming we can drop arc at infinity.

Cutkosky Rules



- In perturbation theory we use Cutkosky rules.
  Cut lines are put on-shell and integrals are trivialized.

$$\frac{1}{p^2 + m^2 - i\epsilon} \implies \delta(p^2 - m^2)\theta(p^0)$$

- Cut loop diagram = glue on-shell tree graphs via phase-space integral.

Cutkosky; Veltman

- What is analog of dispersion + cutting in AdS?

Dispersion Relation

- $\mathrm{Im}(\mathcal{T})$ for CFTs is a double-commutator: Caron-Huot

$$\mathsf{dDisc}_s[\langle\phi_1\phi_2\phi_3\phi_4\rangle] = \langle[\phi_1,\phi_2][\phi_3,\phi_4]\rangle$$

- Dispersion formula reconstructs $\langle\phi\phi\phi\phi\rangle$ from its dDisc_s.

$$\langle\phi(x_1)...\phi(x_4)\rangle = \int d^d y_i K(x_i,y_i)\mathsf{dDisc}_s[\langle\phi(y_1)...\phi(y_4)\rangle]$$

Caron-Huot; Carmi, Caron-Huot

- Holds for all CFTs, including weakly coupled theories or non-perturbative. In holographic theories corresponding cutting rules previously unknown.

A Unitarity Summary

1. **Boundary CFT Method:**

$$\langle[\phi,\phi][\phi,\phi]\rangle = \sum_{\Psi}\langle[\phi,\phi]|\Psi\rangle\langle\Psi|[\phi,\phi]\rangle$$

Find $\langle\Psi|[\phi,\phi]\rangle$ via CFT bootstrap equations.

Aharony, Alday, Bissi, Perlmutter; Caron-Huot

2. **Euclidean Bulk Method:**

Bulk, symmetry based method to compute $\langle\Psi|[\phi,\phi]\rangle$.
Wick rotate CFT to Lorentzian at end of calculation.

DM, Perlmutter, Sivaramakrishnan

3. **Lorentzian Bulk Method:**
Derive Cutkosky rules in directly in Lorentzian AdS.
Uses AdS Wightman functions + momentum space.

DM, Sivaramakrishnan

Conformal Primer

- Euclidean method works by factorizing diagrams via exchanged *states*. Need conformal block expansion for $\mathcal{A} = \langle \phi\phi\phi\phi \rangle$.

$$\begin{aligned} \mathcal{A}(x_i) &= \sum_{\mathcal{O},\partial\mathcal{O},\ldots} \langle \phi\phi|\mathcal{O}\rangle\langle\mathcal{O}|\phi\phi\rangle \\ &= \sum_{\mathcal{O}} \lambda^2_{\phi\phi\mathcal{O}} g_{\mathcal{O}}(x_i) \end{aligned}$$

- Blocks $g_{\mathcal{O}}$ sum primary and its descendants.

$$\phi \times \phi \supset \mathcal{O}, \partial_\mu \mathcal{O}, \partial^2 \mathcal{O}, \ldots$$

$\lambda_{\phi\phi\mathcal{O}} =$ OPE coefficient.

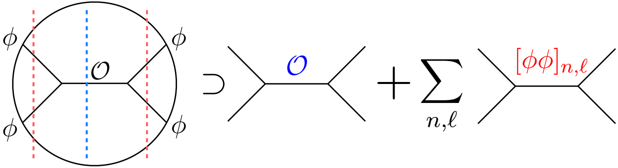
Ferrara, Grillo, Parisi, Gatto; Dolan, Osborn

- In large N CFT will have single trace ϕ, $\mathcal{O}$, etc. and double-traces:

$$\begin{aligned} [\phi\phi]_{n,\ell} &= \phi \partial^{2n} \partial^{\mu_1} \ldots \partial^{\mu_\ell} \phi \\ \Delta_{n,\ell} &= 2\Delta_\phi + 2n + \ell + \gamma_{n,\ell}/N^2 \end{aligned}$$

Example: Tree-Level Exchange

- Simplest example is tree level-exchange, $\mathcal{W}^{(s)}_{\mathcal{O}}$:



- See factorization via block expansion:

$$\mathcal{W}^{(s)}_{\mathcal{O}}\big|_{\mathcal{O}} = \lambda^2_{\phi\phi\mathcal{O}} g_{\mathcal{O}}$$

$$\mathcal{W}^{(s)}_{\mathcal{O}}\big|_{[\phi\phi]_{n,\ell}} = \sum_{n,\ell} (\lambda^{(0)}_{n,\ell})^2 \left(\delta\lambda^{(1)}_{n,\ell} + \frac{1}{2}\gamma^{(1)}_{n,\ell}\partial_n\right) g_{n,\ell}$$

- Internal cut fixed by $1/N$, *3-point* coupling $\lambda_{\phi\phi\mathcal{O}}$.

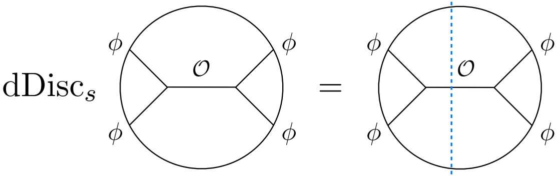
  External cut contains new $1/N^2$ data, $\gamma^{(1)}_{n,\ell}$ and $\delta\lambda^{(1)}_{n,\ell}$.

Factorization via dDisc

- dDisc removes double-traces $[\phi\phi]_{n,\ell}$:

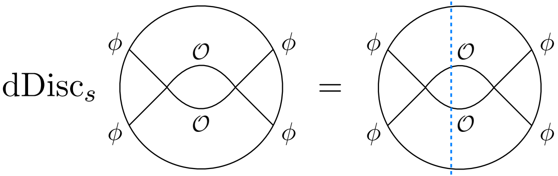
$$\text{dDisc}_s[g_{\mathcal{O}}(x_i)] = 2\sin^2\left(\frac{\pi}{2}(\Delta_{\mathcal{O}} - \ell - 2\Delta_\phi)\right) g_{\mathcal{O}}(x_i)$$

- At tree-level we have:



Caron-Huot

- Euclidean bulk method generalizes to arbitrary loops. For example:



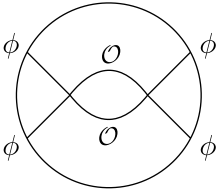
DM, Perlmutter, Sivaramakrishnan

Bubble Diagram

More precisely, make following two claims:

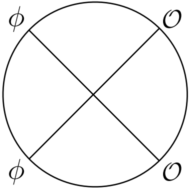
1. Acting with dDisc_s projects onto $[\mathcal{O}\mathcal{O}]$ exchange.

dDisc_s



$$\propto \sum_{n,\ell} \lambda^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} g_{[\mathcal{O}\mathcal{O}]_{n,\ell}}(x_i)$$

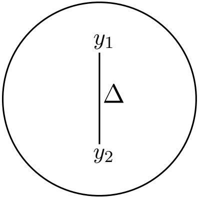
2. $\lambda_{\phi\phi[\mathcal{O}\mathcal{O}]}$ is fixed by the *tree-level* contact sub-diagram:



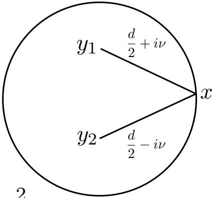
Fitzpatrick, Kaplan; Penedones; DM, Perlmutter, Sivaramakrishnan

Split Representation

- Euclidean unitarity method uses split representation for bulk propagators:



$$= \int_{\partial AdS} d^d x \int_{-\infty}^{\infty} d\nu P(\nu, \Delta)$$



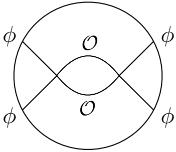
$$P(\nu, \Delta) = \frac{1}{\nu^2 + (\Delta - \frac{d}{2})^2} \frac{\nu^2}{\pi}$$

- Bulk to boundary legs have dimension $\frac{d}{2} \pm i\nu \Longrightarrow$ Off-shell.
- P has poles at $\frac{d}{2} \pm i\nu = \Delta \Longrightarrow$ On-shell, single-trace pole.
- $d\nu$ integral is analogous to momentum integral in flat-space.
  $P(\nu, \Delta)$ is analogous to flat-space propagator.

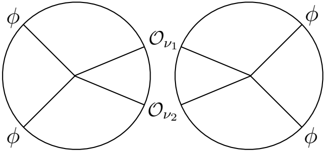
Leonhard, Manvelyan, Ruehl; Liu, Tseytlin; Penedones; Costa, Goncalves, Penedones

Splitting a Bubble

- Using split representation appears to cut bubble:



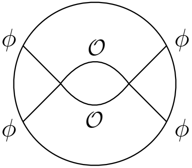
$$= \int \prod_{i=1,2} d\nu_i dx_i P(\nu_i, \Delta_{\mathcal{O}})$$



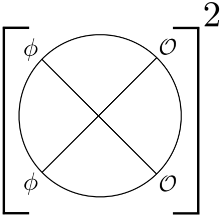
Here $\mathcal{O}_\nu$ has the off-shell dimension $\Delta_\nu = d/2 + i\nu$.

- Taking dDisc$_s$ (roughly) puts ν on-shell:

$$\mathrm{dDisc}_s$$



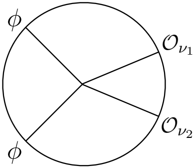
$$\sim$$



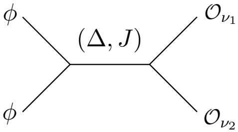
- To make concrete, we need conformal partial wave expansion.

Partial Wave Expansion

- Expand sub-diagrams using partial waves, $\Psi_{\Delta,J}$ for $\Delta \in d/2 + i\mathbb{R}$.



$$= \sum_J \int d\Delta \; c_{\nu_i}(\Delta, J)$$



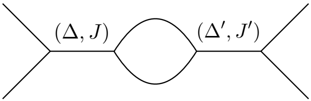
- Partial waves are an orthogonal, single-valued basis in Euclidean signature.

$$\Psi_{\Delta,J} = g_{\Delta,J} + \# g_{d-\Delta,J}$$

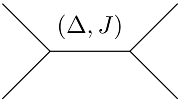
Can close Δ contour to recover block expansion.

Dobrev, Mack, Petkova, Petrova, Todorov

- Partial waves trivialize boundary integrals:



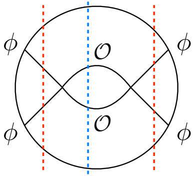
$$\propto \delta(\Delta - \Delta')\delta_{JJ'}$$



Gadde; Kravchuk, Simmons-Duffin; Karateev, Kravchuk, Simmons-Duffin

dDisc of Bubble

- Two sets of poles appear in CPW expansion:



$\supset [\phi\phi]_{n,\ell}, \ [\mathcal{O}\mathcal{O}]_{n,\ell}$

- But dDisc_s projects out $[\phi\phi]_{n,\ell}$ and we are left with $[\mathcal{O}\mathcal{O}]_{n,\ell}$.

$$\text{dDisc}_s[\mathcal{A}_{\text{bubble}}] = \sum_{n,\ell} \lambda^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} g_{[\mathcal{O}\mathcal{O}]_{n,\ell}}$$

$$\lambda^2_{\phi\phi[\mathcal{O}\mathcal{O}]_{n,\ell}} \propto \underset{\Delta=\Delta_{[\mathcal{O}\mathcal{O}]_{n,\ell}}}{\text{Res}} c_{\nu_i}(\Delta, J) c_{-\nu_i}(\Delta, J) \Big|_{d/2+\nu_i=\Delta_{\mathcal{O}}}$$

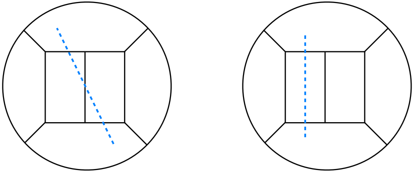
- One-loop dDisc fixed by on-shell, tree-level OPE data from internal cuts.
- Only on-shell poles in ν from $P(\nu, \Delta)$ contribute.
  Can ignore ∞ number of poles in c_{ν_i}.

Euclidean Summary

- To recap, procedure is:
  1. Use split representation to produce lower-loop, off-shell bulk diagrams.
  2. Use conformal integrals + partial waves to perform boundary integrals.
  3. "Cutting" lines corresponds to putting ν parameters put on-shell. Picks out exchanged states in the OPE.

- Output are well-studied CFT objects, e.g. $6j$ symbol and conformal blocks. Dispersion formula for blocks and their sums are well-known.

Liu, Perlmutter, Rosenhaus, Simmons-Duffin; Carmi, Caron-Huot



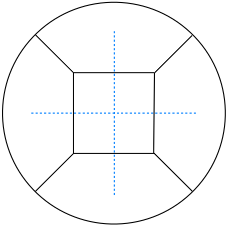
- Generalizable to higher-loops via higher-point partial waves.

DM, Perlmutter, Sivaramakrishnan

Euclidean Summary

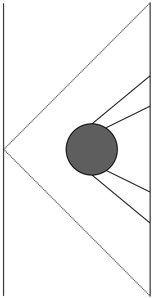
But there are still unanswered questions:

- Found factorization at the level of exchanged states.
  Do AdS correlators factorize directly in terms of AdS sub-correlators?
- How do we classify the allowed cuts?
- Defining cut required choice of OPE channel.
  How do we define simultaneous cuts?



Euclidean to Lorentzian

- Will derive diagrammatic rules to find dDisc in Lorentzian AdS.



- New ingredients: Use Poincaré patch of AdS and use CFT momentum space.

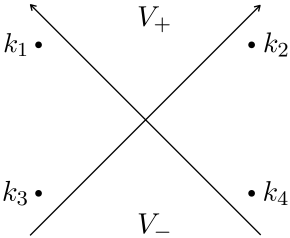
A CFT Optical Theorem

- The unitarity condition for time-ordered correlator is:

$$-2\,\mathrm{Re}\,\langle T[\phi(k_1)...\phi(k_4)]\rangle = \langle [\phi(k_3),\phi(k_4)]_A[\phi(k_1),\phi(k_2)]_R\rangle$$

Here $[\phi(x_1),\phi(x_2)]_R = [\phi(x_1),\phi(x_2)]\theta(t_1-t_2)$.

- Holds for special kinematics:



$V_\pm$ = Forwards/backwards lightcone.

Polyakov; Gillioz, Lu, Luty; Gillioz; Polyakov

A CFT Optical Theorem

- How does this relation help?

$$-2\,\mathrm{Re}\,\langle T[\phi_1 ... \phi_4]\rangle = \langle [\phi_3, \phi_4]_A [\phi_1, \phi_2]_R \rangle$$

- LHS is natural generalization of $\mathrm{Im}(\mathcal{T})$ to full correlation function. Can use Veltman's derivation of cutting rules to compute $\mathrm{Re}\langle T[...]\rangle$.

Veltman

- The causal, double-commutator appears in the CFT inversion formula.

Caron-Huot; Simmons-Duffin, Stanford, Witten; Kravchuk, Simmons-Duffin

- Equality for special kinematics gives cutting rules for double-commutator.

DM, Sivaramakrishnan

AdS Propagators

- In AdS we work in Poincaré patch:

$$ds^2 = 1/z^2 \left(dz^2 + \eta_{\mu\nu} dx^\mu dx^\nu\right)$$

  Will later Fourier transform in x^μ to go to k-space.

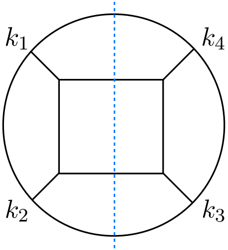
- For bulk scalar Φ we use two propagators, time-ordered and Wightman. For bulk-to-bulk propagator we have:

$$G_{BB}(x_1, z_1; x_2, z_2) = \langle T[\Phi(x_1, z_1)\Phi(x_2, z_2)]\rangle$$
$$G^+_{BB}(x_1, z_1; x_2, z_2) = \langle \Phi(x_1, z_1)\Phi(x_2, z_2)\rangle$$

- Also have the corresponding boundary to bulk propagators:

$$G_{\partial B}(x_1; x_2, z_2) = \lim_{z_1 \to 0} z_1^{-\Delta} G_{BB}(x_1, z_1; x_2, z_2)$$

Cutting Rules

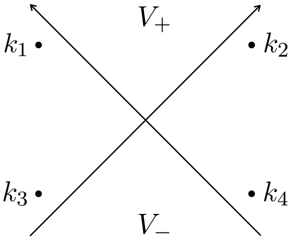


Assuming $k_1 + k_2 \in V_+$ cutting rules for $\mathrm{Re}\langle T[\phi(k_1)...\phi(k_4)]\rangle$ are:

1. For each cut line let $G_{BB,\,\partial B} \to G^{+}_{BB,\,\partial B}$.
2. For left half use regular Feynman rules.
   For right half use complex conjugated Feynman rules.
3. Multiply by (-1) for each external point on right half.
4. Sum over all cuts consistent with momentum conservation.

Cutting Rules cont.d

- With rules for $\mathrm{Re}\langle T[\phi(k_1)...\phi(k_4)]\rangle$ go to special kinematics:



to derive cutting rules for $\langle[\phi(k_3), \phi(k_4)]_A[\phi(k_1), \phi(k_2)]_R\rangle$.

- Causal restrictions in position space $\rightarrow$ Analyticity in k-space.

  Can analytically continue to general k_i for double-commutator.

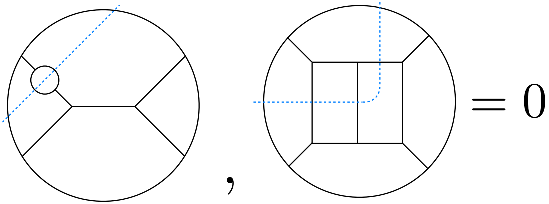
Polyakov; DM, Sivaramakrishnan

Comments on Cuts

Few comments on cutting rules:

- Cuts found here for $\langle [\phi, \phi]_A [\phi, \phi]_R \rangle$ are the same as previous method.

- $G^{+}_{\partial B, BB}(k; z)$ have support only in forward lightcone, $k \in V_{+}$.
  Setting $k_i^2 > 0 \implies$ external lines cannot be put on-shell.



- Double-commutator has less cuts than $\mathrm{Re}\langle T[\phi ... \phi] \rangle$.

DM, Sivaramakrishnan

Cutting a Propagator

- How do these cuts help? Full propagator is:

$$G_{BB}(k; z_1, z_2) \propto \int_0^\infty dp\, p \frac{J_{\Delta-d/2}(pz_1) J_{\Delta-d/2}(pz_2)}{k^2 + p^2 - i\epsilon}$$

Putting line on-shell removes p integral:

$$G^+_{BB}(k; z_1, z_2) \propto J_{\Delta-d/2}(|k|z_1) J_{\Delta-d/2}(|k|z_2) \theta(-k^2) \theta(k^0)$$

- Gives split representation for on-shell propagators:

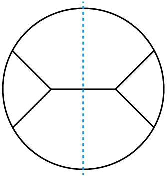
$$G^+_{BB}(k; z_1, z_2) \propto G^+_{B\partial}(k; z_1) \frac{1}{|k|^{2\Delta-d}} G^+_{\partial B}(k; z_2)$$

Cutting a diagram factorizes it into on-shell sub-diagrams.

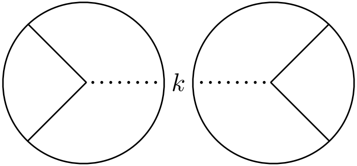
DM, Sivaramakrishnan

Transition Amplitudes

- Cut diagrams have interpretation as sum over states.



$$= \int_{V_+} \frac{d^d k}{\langle \mathcal{O}(-k)\mathcal{O}(k)\rangle}$$



- Undotted lines $= G_{B\partial}(k;z)$ or non-normalizable bulk mode.
  Dotted lines $= G^{+}_{B\partial}(k;z)$ or normalizable bulk mode.

$$G_{B\partial}(k;z) \propto z^{\frac{d}{2}} K_{\Delta-d/2}(\sqrt{k^2}z)$$
$$G^{+}_{B\partial}(k;z) \propto z^{\frac{d}{2}} J_{\Delta-d/2}(\sqrt{-k^2}z)$$

- Witten diagram with normalizable bulk modes = transition amplitude.

Raju; Balasubramanian, Giddings, Lawrence

- Integral over forward lightcone, V_+, = Sum over states.
  Derivation was diagrammatic, but answer matches physical expectations.

DM, Sivaramakrishnan

Lorentzian Summary

- To recap, Lorentzian procedure is:
  1. Use spacelike momenta to relate $\mathrm{Re}\langle T[\phi\phi\phi\phi]\rangle$ to $\langle[\phi,\phi]_A[\phi,\phi]_R]\rangle$.
  2. Compute cuts for $\mathrm{Re}\langle T[\phi\phi\phi\phi]\rangle$ using standard techniques.
  3. Analytically continue in momenta for causal double-commutator.

- Explains how to factorize directly in terms of bulk diagrams.
  Gives classifications of bulk cuts and relation to states.

- Cutting rules generalize to higher-points.
  Missing ingredient is corresponding CFT dispersion formula.

High Energy Scattering

- Can use unitarity methods to study AdS high-energy scattering.

Conformal Regge Theory $\Longleftrightarrow$ High-energy, fixed impact parameter scattering in AdS.

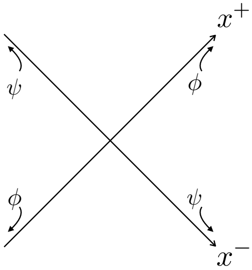
Sensitive to high-spin spectrum, e.g graviton + higher-spin particles.

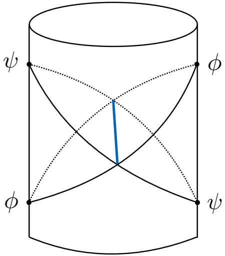
Costa, Goncalves, Penedones; Caron-Huot, Komargodski, Sever, Zhiboedov

- Recall higher-spin scale denoted as $\Delta_{\text{gap}} \approx M_{HS}$.
  Take bulk energy S and Δ_{gap}^2 large and study to all orders in $1/N$.

Li, DM, Poland; DM

High Energy Kinematics





- Spacetime configuration for $\langle \phi\psi\psi\phi \rangle$ is:

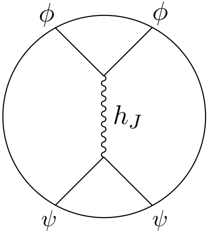
$$x_1 = -x_4 = (\sqrt{\rho}, \sqrt{\bar{\rho}})$$
$$x_2 = -x_3 = (1/\sqrt{\rho}, 1/\sqrt{\bar{\rho}})$$

where $\rho,\ 1/\bar{\rho} \to \infty$ with $\rho\bar{\rho}$ fixed.

Costa, Goncalves, Penedones

Exchange Diagram

- Exchange diagrams for spin J give growing contribution in the Regge limit:



$$\langle \phi\psi\psi\phi \rangle \sim \rho^{J-1}, \qquad \text{for} \;\; \rho \gg 1$$

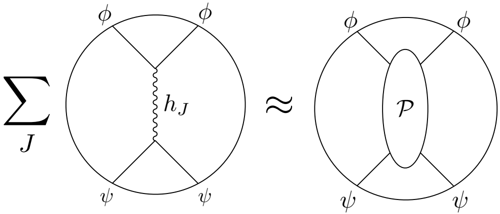
- Tree-level anomalous dimensions for $[\phi\psi]_{n,\ell}$ also grow:

$$\gamma^{(1)}_{n,\ell} \sim \big(n(n+\ell)\big)^{J-1} \qquad \text{where} \;\; n,\ell \gg 1 \;\; \text{with} \;\; n/\ell \text{ fixed.}$$

Cornalba, Costa, Penedones; Li, DM, Poland

Reggeization

- Infinite sum over leading trajectory particles reggeizes.



- Find Pomeron, $\mathcal{P}$, with spin $j_* = 2 - \#/\Delta^2_{\text{gap}}$.

$$\langle \phi\psi\psi\phi \rangle \ \sim \ \frac{1}{N^2}\rho^{j_*-1},$$

$$\gamma^{(1)}_{n,\ell} \ \sim \ (n(n+\ell))^{j_*-1}, \qquad \text{where} \ \ n,\ell \gg 1 \ \ \text{with} \ \ n/\ell \ \text{fixed.}$$

- Will assume for leading trajectory $J(\Delta)$ is polynomial in Δ and:

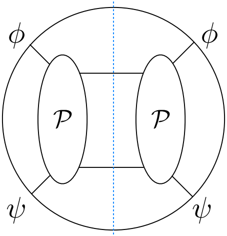
$$J(\Delta) \sim \Delta^2 \quad \text{for} \quad \Delta \sim \Delta_{\text{gap}} \gg 1$$

Costa, Goncalves, Penedones; Brower, Polchinski, Strassler, Tan; Maldacena, Shenker, Stanford; Caron-Huot, Komargodski, Sever, Zhiboedov

Li, DM, Poland

Higher-Spins and Loops

- Glue together tree answer and take Regge limit to get Pomeron box:



- One-loop dDisc_s given by sum over $[\phi\psi]_{n,\ell}$:

$$\text{dDisc}_s[\langle\phi\psi\psi\phi\rangle] \propto \frac{1}{N^4}\sum_{n,\ell}\left(\lambda^{(0)}_{n,\ell}\gamma^{(1)}_{n,\ell}\right)^2 g_{n,\ell}(\rho,\bar{\rho})$$

- CFT dispersion formula gives:

$$\langle\phi\psi\psi\phi\rangle \sim \frac{1}{N^4}\rho^{2j_*-2}$$

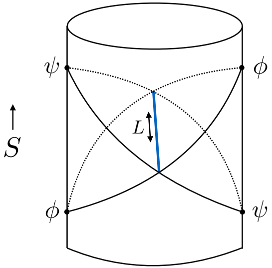
Regge growth at one-loop is twice as strong as tree-level result.

Impact Parameter Space

- Stronger Regge growth at higher-loops may appear problematic. Non-perturbative, finite N, bound on Regge limit is $j_* \leq 1$.

Hartman, Jain, Kundu

- Resolution is correlator should eikonalize when we sum over all loops. Clearest in AdS impact-parameter space.



- Study correlator in impact-parameter space, $\mathcal{B}(S, L)$, with:
  $S =$ Energy^2
  $L =$ Impact parameter on H_{d-1}.

Cornalba, Costa, Penedones; Brower, Strassler, Tan

Eikonalization

- Eikonal ansatz: impact parameter correlator, $\mathcal{B}(S,L)$, exponentiates.
  If $\delta(S,L)$ is tree-level result then ansatz is:

$$\mathcal{B}_{\text{eik}}(S,L) \propto e^{i\delta(S,L)}$$

- Does this match prediction from unitarity?
  At one-loop we find:

$$\mathrm{Re}\left[\mathcal{B}_{\text{eik}} - \mathcal{B}\right]\Big|_{\text{1-loop}} \sim \Delta_{\text{gap}}^{-4} \quad \text{where} \quad \mathrm{Re}[\mathcal{B}]\Big|_{\text{1-loop}} \sim \Delta_{\text{gap}}^{0}$$
$$\mathrm{Im}\left[\mathcal{B}_{\text{eik}} - \mathcal{B}\right]\Big|_{\text{1-loop}} \sim \Delta_{\text{gap}}^{-6} \quad \text{where} \quad \mathrm{Im}[\mathcal{B}]\Big|_{\text{1-loop}} \sim \Delta_{\text{gap}}^{-2}$$

- Ansatz works for large Δ_{gap} at one-loop and higher.
  Matches previous bulk Pomeron analysis in strong coupling limit.

Brower, Polchinski, Strassler, Tan; Brower, Strassler, Tan

- Eikonalization at higher orders in $1/\Delta_{\text{gap}}$ imposes constraints.
  New contributions to dDisc have to be finely tuned.

DM

Conclusion

Summary:

- Euclidean method uses harmonic analysis to factorize AdS diagrams.
- Lorentzian method directly generalizes Cutkosky rules to AdS/CFT.
- New window into AdS high-energy scattering.

More Applications

- Map between boundary CFT method and bulk loops.
- Lorentzian method generalizes to weakly coupled CFTs.
- Bootstrap in Regge limit maps bulk phase shift to double-trace OPE data.

What Next?

- AdS/CFT correlators are not that different from flat-space amplitudes!
  Can we import generalized unitarity, double-copy, polytopes etc. to AdS?

  Bern, Dixon, Dunbar, Kosower; Bern, Carrasco, Johansson; Arkani-Hamed, Trnka; Raju

- Applications to other spacetimes, e.g., De Sitter or inflationary spaces?

  Maldacena; Maldacena, Pimentel; Sleight, Taronna; Mata, Raju, Trivedi; Ghosh, Kundu, Raju, Trivedi; Kundu, Shukla, Trivedi; Arkani-Hamed, Maldacena; Arkani-Hamed, Baumann, Lee, Pimentel; Baumann, Pueyo, Joyce, Lee, Pimentel

- Connection to correlators in Mellin space? Mack
  Holographic correlators in Mellin space have remarkably simple form.

  Penedones; Fitzpatrick, Penedones, Raju, van Rees; Paulos; Zhou; Rastelli, Zhou; Rastelli, Roumpedakis, Zhou; Alday, Zhou; Caron-Huot, Trinh; Alday, Bissi, Perlmutter; Alday; Chester, Pufu, Yin; Binder, Chester, Pufu; Binder, Chester, Pufu, Wang; Binder, Chester, Pufu; Chester, Pufu

- Other high-energy limits, e.g., high-energy, fixed-angle scattering.
  Conditions on CFT for Gross-Mende behavior?

  Gross, Mende; Maldacena, Simmons-Duffin, Zhiboedov; Cardona; Ooguri, Dodelson; Bagheer, Coronado, Vieira

Thank you!