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Asymptotic Symmetries and Celestial CFT

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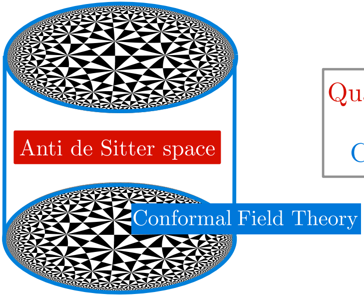
Holographic window into Quantum Gravity

Ground-breaking tool: **holographic principle** ['t Hooft'93] [Susskind'94]

AdS/**CFT** correspondence [Maldacena'97]

[Witten'98]

[Gubser,Klebanov,Polyakov'98]



Quantum gravity in Anti de Sitter space
=
Conformal Field Theory on boundary

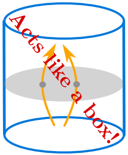
match of *symmetries* and *degrees of freedom*

e.g. black hole entropy from CFT

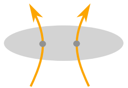
[Strominger,Vafa'96; Strominger'97]

Holography in the Universe ?

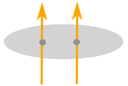
- **$\Lambda < 0$ Anti de Sitter:**
  spacetime near black holes with
  $M \gtrsim Q$ or $M^2 \gtrsim J$



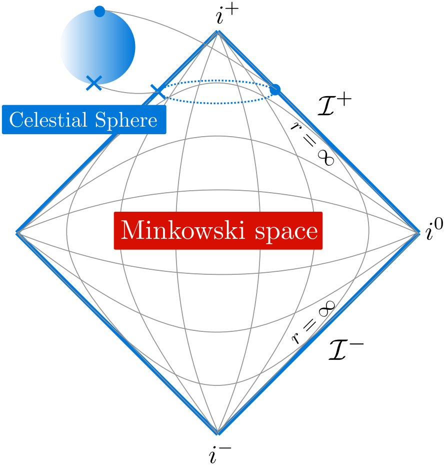
- **$\Lambda > 0$ de Sitter:**
  spacetime @ cosmological scale



- **$\Lambda = 0$ Minkowski:**
  spacetime @ scales > black hole throat
  and < cosmological



Flat Space Holography



What are the symmetries?

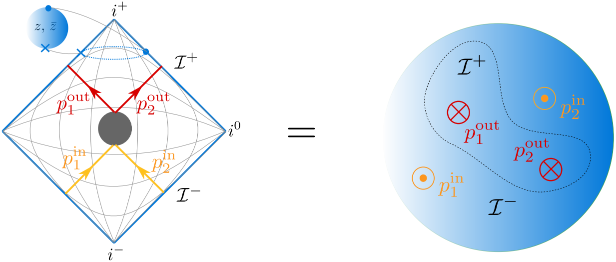
Is there a dual CFT?

What are its properties?

Is 4D spacetime physics encoded on conformal boundary?

Conformal Basis for QFT Amplitudes

4D Lorentz group $SO(1,3) \simeq SL(2,\mathbb{C})$ acts as global conformal group on celestial sphere: *4D scattering amplitudes recast as 2D correlators!*



- Constraints on Quantum Gravity $\mathcal{S}$-Matrix
- New insights into amplitude structures

Outline

Unified treatment of "soft" modes in new conformal basis for flat space holography:

- spectrum of primaries in 2D celestial CFT
- symplectic pairing of conformal primary wavefunctions
- conformal soft theorems for celestial amplitudes

Outline

UNIFIED TREATMENT OF "SOFT" MODES IN NEW CONFORMAL BASIS FOR FLAT SPACE HOLOGRAPHY:

- spectrum of primaries in 2D celestial CFT
- symplectic pairing of conformal primary wavefunctions
- conformal soft theorems for celestial amplitudes

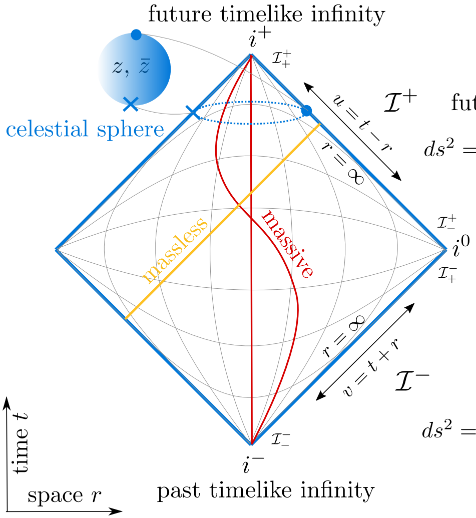
WHAT IS THE ASYMPTOTIC SYMMETRY GROUP OF EINSTEIN GRAVITY AT NULL INFINITY ?

- identification of conformally soft gravitons with generators of:
  - ⋆ BMS symmetry
  - ⋆ Celestial conformal symmetry
    - Virasoro symmetry
    - Diff(S^2) symmetry

Holographic screen of Minkowski spacetime

[Ashtekar et al'70s-80s]

Conformal boundary: $\mathcal{I} = \mathbb{R} \times \mathcal{C}S^2$



$\mathcal{I}^+$ future null infinity

$$ds^2 = -du^2 - 2du dr + \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$

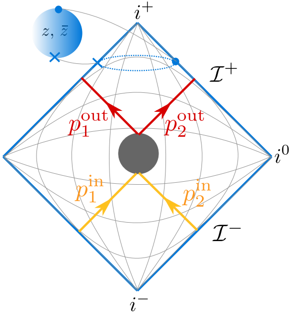
spacelike infinity

$\mathcal{I}^-$ past null infinity

$$ds^2 = -dv^2 + 2dv dr + \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$

A new basis for QFT scattering amplitudes

4D SPACETIME



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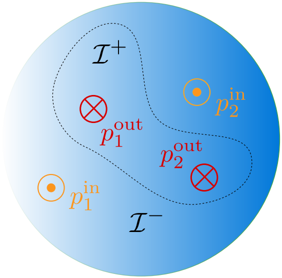
2D CELESTIAL SPHERE



4D $\mathcal{S}$cattering matrix

$$\langle out|\mathcal{S}|in\rangle$$

$$p_k^\mu = \omega_k q^\mu(z_k, \bar{z}_k), \ell_k$$

4D Lorentz symmetry

2D correlation function

$$\langle \mathcal{O}_1(z_1, \bar{z}_1) \dots \mathcal{O}_n(z_n, \bar{z}_n) \rangle$$

$$(z_k, \bar{z}_k),\ \Delta_k,\ J_k$$

2D conformal symmetry

Mapping wavefunctions

[de Boer,Solodukhin'03]

MOMENTUM BASIS $\xrightarrow{\textit{Mellin transform}}$ CONFORMAL BASIS

translation symmetry — *conformal symmetry*

momentum eigenstates — boost eigenstates

$$e^{i\omega\, q\cdot X} \xrightarrow{\int_0^\infty d\omega\omega^{\Delta-1}} \frac{\mathcal{N}(\Delta)}{(-q\cdot X)^{\Delta}} \equiv \Phi^{\Delta}$$

$$p^\mu = \omega q^\mu(w,\bar{w})\,,\ \ell \qquad w = \frac{p^1+ip^2}{p^3+p^0}\,,\ \Delta\,,\ J=\ell$$

basis of $\omega \geqslant 0$ wavefunctions — basis of $\Delta \in 1+i\mathbb{R}$ wavefunctions

principal continuous series of $SL(2,\mathbb{C})$

[Pasterski,Shao'17]

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Φ^Δ transforms as conformal primary with $(h,\bar{h}) = \frac{1}{2}(\Delta,\Delta)$:

$$\Phi^\Delta\left(\Lambda^\mu{}_\nu X^\nu; \frac{aw+b}{cw+d}, \frac{\bar{a}\bar{w}+\bar{b}}{\bar{c}\bar{w}+\bar{d}}\right) = (cw+d)^{2h}(\bar{c}\bar{w}+\bar{d})^{2\bar{h}}\Phi^\Delta(X^\mu; w,\bar{w})$$

$ad - bc = 1$

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$ad-bc=1$

"State-operator correspondence":

massless particles ↔ operators on celestial sphere

Celestial amplitudes

[Pasterski,Shao,Strominger'17]
[Cheung,de la Fuente,Sundrum'17]

$$\text{4D AMPLITUDES} \xrightarrow{\textit{Mellin transform}} \text{2D CORRELATORS}$$

$$\mathcal{A}_{\ell_1 \ldots \ell_n}(\omega_1, z_1, \bar{z}_1, ..., \omega_n, z_n, \bar{z}_n) \xrightarrow{\prod_{k=1}^{n} \int_0^\infty d\omega_k \omega_k^{\Delta_k - 1}} \widetilde{\mathcal{A}}_{J_1 \ldots J_n}(\Delta_1, z_1, \bar{z}_1, ..., \Delta_n, z_n, \bar{z}_n)$$

$$= \langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{\Delta_n, J_n}(z_n, \bar{z}_n) \rangle$$

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$$= \langle \mathcal{O}_{\Delta_1, J_1}(z_1, \bar{z}_1) \ldots \mathcal{O}_{\Delta_n, J_n}(z_n, \bar{z}_n) \rangle$$

E.g. MHV 4 gluon amplitude

$$\mathcal{A}_{--++} \sim \frac{z_{12}^3}{z_{23} z_{34} z_{41}} \frac{\omega_1 \omega_2}{\omega_3 \omega_4} \qquad \xrightarrow{\prod_{k=1}^{4} \int_0^\infty d\omega_k \omega_k^{i\lambda_k}} \qquad \widetilde{\mathcal{A}}_{--++} \sim \prod_{i<j}^{4} z_{ij}^{\frac{h}{3} - h_i - h_j} \bar{z}_{ij}^{\frac{\bar{h}}{3} - \bar{h}_i - \bar{h}_j} z^{\frac{5}{3}} (1-z)^{-\frac{1}{3}}$$

$$\times\, \delta^{(4)}\Big(\sum_{k=1}^{4} \epsilon_k \omega_k q_k \Big) \qquad\qquad \times\, \delta(z - \bar{z}) \delta\Big(\sum_{k=1}^{4} \lambda_k \Big)$$

$$z_{ij} = z_i - z_j \,, \quad \epsilon_k = \pm 1 \qquad\qquad z = \frac{z_{12} z_{34}}{z_{13} z_{24}} \,, \quad h = \sum_{k=1}^{4} h_k \,, \quad \bar{h} = \sum_{k=1}^{4} \bar{h}_k$$

$$q^\mu(z_k, \bar{z}_k) = (1 + z_k \bar{z}_k, z_k + \bar{z}_k, -i(z_k - \bar{z}_k), 1 - z_k \bar{z}_k)$$

What is the appropriate spectrum ?

PLANE WAVEFUNCTIONS

$\omega \geqslant 0$

finite energy

$$\xrightarrow{\int_0^\infty d\omega \omega^{\Delta-1}}$$

CONFORMAL PRIMARY WAVEFUNCTIONS

$\boxed{\Delta \in 1 + i\mathbb{R}}$

<u>on</u> principal continuous series

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"*energetically* soft": $\omega \to 0$

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"*conformally* soft": $h \to 0$ or $\bar{h} \to 0$

[Donnay,AP,Strominger'18]

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PLANE WAVEFUNCTIONS

CONFORMAL PRIMARY WAVEFUNCTIONS

$$\omega \geqslant 0 \quad \xrightarrow{\int_0^\infty d\omega \omega^{\Delta-1}} \quad \boxed{\Delta \in 1 + i\mathbb{R}}$$

finite energy

on principal continuous series

"*energetically* soft": $\omega \to 0$

"*conformally* soft": $h \to 0$ or $\bar{h} \to 0$

[Donnay,AP,Strominger'18]

orders of ω

poles at $\boxed{\Delta \in \mathbb{Z}}$

off principal continuous series

$$\mathcal{A}_{n+1} \xrightarrow{\omega \to 0} \left(\underset{\Delta \to 1}{\omega^{-1} S^{(0)}} + \underset{\Delta \to 0}{\omega^{0} S^{(1)}} + \cdots \right) \mathcal{A}_n$$

Celestial gravitons with $\Delta \in 1 + i\mathbb{R}$

Solution to linearized Einstein: $\Box h_{\mu\nu} = 0$ [Pasterski,Shao'17]

in Lorenz gauge: $\nabla^\mu h_{\mu\nu} = 0$ $\epsilon_{\mu\nu;a} = \partial_a q_\mu \partial_a q_\nu$

outgoing/incoming

$$\boxed{h^{\Delta,\pm}_{\mu\nu;a}(X^\mu; w, \bar{w}) = \frac{\Delta - 1}{\Delta + 1} \frac{\epsilon_{\mu\nu;a}}{(-q \cdot X_\pm)^\Delta}}$$

helicity index: $a = ww$ gives $J = +2$, $a = \bar{w}\bar{w}$ gives $J = -2$

Celestial gravitons with $\Delta \in 1 + i\mathbb{R}$

Solution to linearized Einstein: $\Box h_{\mu\nu} = 0$ [Pasterski,Shao'17]

in Lorenz gauge: $\nabla^\mu h_{\mu\nu} = 0$ and radial gauge $X^\mu h_{\mu\nu} = 0$ $\epsilon_{\mu\nu;a} = \partial_a q_\mu \partial_a q_\nu$

outgoing/incoming $\zeta^{\Delta}_{\mu;a} = \frac{1}{\Delta+1}\left(\frac{\epsilon_{\mu\nu;a}X^{\nu}_{\pm}}{(-q\cdot X_{\pm})^{\Delta}} + \frac{q_\mu \epsilon_{\nu\rho;a}X^{\nu}_{\pm}X^{\rho}_{\pm}}{(-q\cdot X_{\pm})^{\Delta+1}}\right)$

↓ ↓

$$\boxed{h^{\Delta,\pm}_{\mu\nu;a}(X^\mu; w, \bar{w}) = \frac{\Delta-1}{\Delta+1}\frac{\epsilon_{\mu\nu;a}}{(-q\cdot X_{\pm})^{\Delta}} + \nabla_{(\mu}\zeta^{\Delta,\pm}_{\nu);a}}$$

↑

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Basis of conformal primary wavefunctions $h^{\Delta,\pm}_{\mu\nu;a}$ for $\Delta \in 1 + i\mathbb{R}$.

principal continuous series

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Basis of conformal primary wavefunctions $h^{\Delta,\pm}_{\mu\nu;a}$ for $\Delta \in 1 + i\mathbb{R}$.

principal continuous series

Shadow transform:

$$\widetilde{h^{\Delta,\pm}_{\mu\nu;\bar{a}}} = (-X^2_\pm)^{1-\Delta} h^{2-\Delta,\pm}_{\mu\nu;a} \equiv \widetilde{h}^{\widetilde{\Delta},\pm}_{\mu\nu;a} \quad \text{where} \quad \widetilde{\Delta} = 2 - \Delta$$

Celestial gravitons with $\Delta \in \mathbb{Z}$

[Donnay,Pasterski,AP'20]

arbitrary conformal dimension $\Delta \in \mathbb{C}$ principal continuous series $s \in 1 + i\mathbb{R}$

$$\overset{\downarrow}{h}{}^{\Delta,\pm}_{\mu\nu;a}(X^\mu; w, \bar{w}) = \lim_{\nu\to 0} \int_{1-i\infty}^{1+i\infty} (-ids)\, \boldsymbol{\delta}_{\nu,\gtrless}(i(\Delta - s))\, \overset{\downarrow}{h}{}^{s,\pm}_{\mu\nu;a}(X^\mu; w, \bar{w})$$

Celestial gravitons with $\Delta \in \mathbb{Z}$

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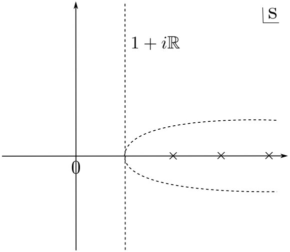
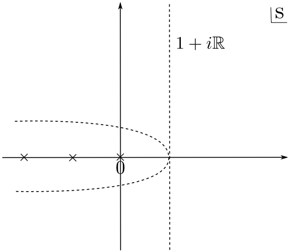
$$\boldsymbol{\delta}_{\nu,\gtrless}(i(\Delta - s)) = \frac{1}{2\pi}\int_0^\infty d\omega\omega^{\Delta-s-1} \left\{ \begin{array}{c} e^{-\nu\omega} \\ e^{-\nu/\omega} \end{array} \right\} = \frac{1}{2\pi}\nu^{\pm(s-\Delta)}\Gamma(\pm(\Delta - s))$$

regulator $\nu > 0$



Contour deformation for $\text{Re}(\Delta) > 1$.

Poles of $\Gamma(\Delta - s)$ at $s = \Delta + n$.



Contour deformation for $\text{Re}(\Delta) < 1$.

Poles of $\Gamma(s - \Delta)$ at $s = \Delta - n$.

Mode expansion and charge operators

[Donnay,Pasterski,AP'20]

General metric perturbations expanded on conformal basis:

$$h_{\mu\nu}(X) = \int d^2w \sum_{a=ww,\bar{w}\bar{w}} \int_{1-i\infty}^{1+i\infty} (-id\Delta) \Big[\mathcal{N}^{+}_{2-\Delta} h^{2-\Delta,+}_{\mu\nu;\bar{a}}(X^\mu; w, \bar{w}) \mathfrak{a}_a(\Delta; w, \bar{w}) + \mathcal{N}^{-}_{\Delta} h^{\Delta,-}_{\mu\nu;a}(X^\mu; w, \bar{w}) \mathfrak{a}_a(\Delta; w, \bar{w})^\dagger \Big]$$

Commutation relations:

$$[\mathfrak{a}_{j,a}(\Delta, w, \bar{w}), \mathfrak{a}_{j,a'}(\Delta', w', \bar{w}')^\dagger] = \delta_{aa'} \delta^{(2)}(w - w') \boldsymbol{\delta}(i(\Delta + \bar{\Delta}' - 2))$$

Mode expansion and charge operators

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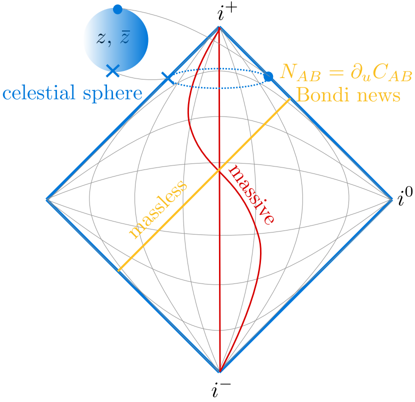
Analytically continued mode operator:

$$\boxed{Q^{\Delta}_a \equiv i(h, h^{\bar{\Delta},+}_{\bar{a}}(w,\bar{w}))_\Sigma = i(\mathcal{N}^{-}_{\Delta})^{-1}\mathfrak{a}_a(\Delta; w, \bar{w})}$$

- ⋆ shifts the metric as $[Q^{\Delta}_a(w,\bar{w}), h_{\mu\nu}(X)] = ih^{\Delta,-}_{\mu\nu;a}(X^\mu; w, \bar{w})$
- ⋆ produces single particle states with $h^{\Delta,-}_{\mu\nu;a}$ when acting on the vacuum

$Q^{\Delta=0,1}_a$: asymptotic **soft charges** at null infinity

Asymptotic Symmetry Group



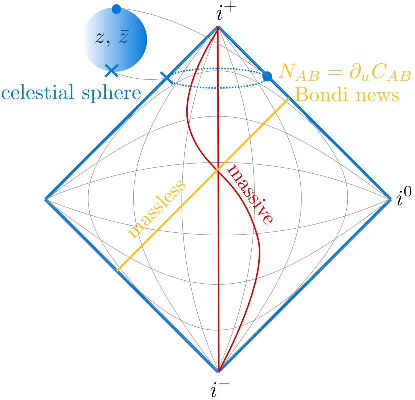
Boundary conditions:

$$ds^2 = -du^2 - 2dudr + \tfrac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2} + \ldots$$

radiative spacetime:
$\{m_B, N_A, C_{AB}\}$
free data

$$ds^2 = -dv^2 + 2dvdr + \tfrac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2} + \ldots$$

Asymptotic Symmetry Group



Boundary conditions:

$$ds^2 = -du^2 - 2dudr + \frac{4r^2 dz d\bar{z}}{(1+z\bar{z})^2}$$
$$+ \ldots$$

radiative spacetime:
$\{m_B, N_A, C_{AB}\}$
free data

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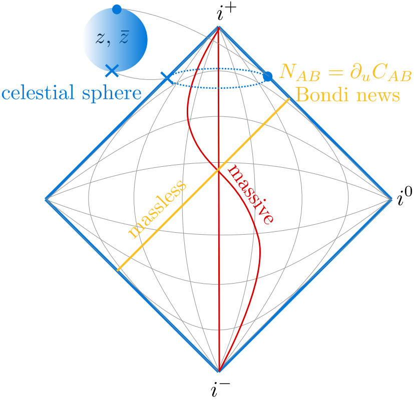
$$\text{Asymptotic Symmetry Group} = \frac{\text{allowed gauge symmetry}}{\text{trivial gauge symmetry}}$$

Asymptotic Symmetry Group



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Guess: Poincaré group (Lorentz transformations + translations)

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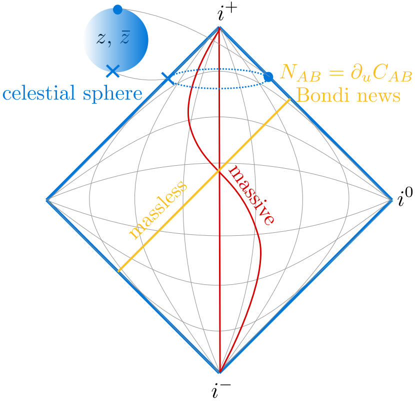

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BMS group

Boundary conditions: [Bondi,van der Burg,Metzner,Sachs'62]

$$g_{uu} = -1 + \mathcal{O}(1/r)\,, \quad g_{uA} = \mathcal{O}(1)\,, \quad g_{AB} = r^2\gamma_{AB} + rC_{AB} + \mathcal{O}(1)$$

Radial metric components determined by radial gauge condition.

BMS supertranslations:

$$\zeta_f = f\partial_u + \tfrac{1}{2}D^2 f\partial_r - \tfrac{1}{r}D^A f\partial_A + \ldots$$

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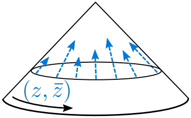
Action of large diffeormorphism ζ_f on the free gravitational data C_{AB}:

$$\delta_f C_{AB} = f\partial_u C_{AB} - 2D_A D_B f + \gamma_{AB} D^2 f$$



$$\delta_f^{\text{shift}} C_{zz} = -2D_z^2 f(z,\bar{z})$$

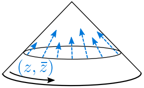
⇒ energy conserved *at every angle*

4 global translations $\xrightarrow{\text{enhanced}}$ angle-dependent supertranslations $f(z,\bar{z})$

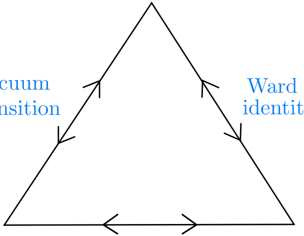
⇒ Poincaré group enhanced to BMS group

IR connections

[Bondi,van der Burg,Metzner,Sachs'62]



BMS supertranslations



vacuum transition

Ward identity

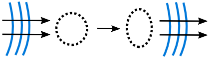
[Zel'dovich,Polnarev'74]
[Braginski,Thorne'87]
[Christodoulou'91]
[Blanchet,Damour'92]

[Weinberg'65]

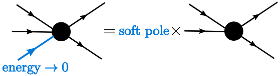
displacement memory

Fourier transform

leading soft graviton



position-space



momentum-space

Celestial $\Delta = 1$ and $\widetilde{\Delta} = 1$ gravitons

Conformal spin-two basis contains a pure diffeo: [Pasterski,Shao'17]

$$h^{\Delta=1}_{\mu\nu;a} = \nabla_{(\mu}\zeta^{1}_{\nu);a} \quad \rightarrow \quad C^{1}_{zz;ww} = -2D^{2}_{z}f_{ww} \text{ with } f_{ww} = -\frac{1}{4}\frac{(\bar{z}-\bar{w})}{(z-w)(1+z\bar{z})}$$

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[Donnay,AP,Strominger'18]

⇒ **Goldstone mode** of spontaneously broken **BMS symmetry**!

[He,Lysov,Mitra,Strominger'14]

$$\boxed{Q_f^{soft} = -\frac{1}{16\pi G}\int dud^2z\sqrt{\gamma}\,(D_z^2\,f\,N^{zz} + D_{\bar{z}}^2\,f\,N^{\bar{z}\bar{z}})}$$

Bondi news $N_{zz} = \partial_u C_{zz}$

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Puzzle: $\left(h^{\Delta=1+i\lambda}_a(w,\bar{w}), h^{\Delta'=1+i\lambda'}_{a'}(w',\bar{w}')\right)_\Sigma \sim \delta(\lambda-\lambda')\delta_{aa'}\delta^{(2)}(w-w')$

Conformal primary pairing between $1+i\lambda$ and $1-i\lambda$.

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Conformal primary pairing between $1+i\lambda$ and $1-i\lambda$.

What is the symplectic partner for $h^{\Delta=1}_{\mu\nu;a} = \widetilde{h}^{\widetilde{\Delta}=1}_{\mu\nu;a}$*?*

In the momentum basis the partner of the BMS Goldstone mode is the soft graviton. Very subtle to see from the Mellin transform!

Symplectically paired set of zero-modes

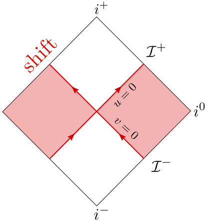
Hidden logarithmic branch: [Donnay,AP,Strominger'18]

$$h^{\log,\pm}_{\mu\nu;a} = \lim_{\Delta\to 1} \partial_\Delta \left(h^{\Delta,\pm}_{\mu\nu;a} + \widetilde{h}^{\widetilde{\Delta},\pm}_{\mu\nu;a}\right)$$

yields a new $\Delta = 1$ conformal primary.

$\Delta = 1$ Conformally soft graviton

$$h^{\mathrm{CS}}_{\mu\nu;a} = [\Theta\left(X^2\right) + \log[X^2](q\cdot X)\delta(q\cdot X)]h^1_{\mu\nu;a}$$



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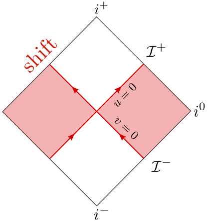

$$\left(h^{CS}_{ww}(w,\bar{w}), {h'}^{1}_{w'w'}(w',\bar{w}')\right)_\Sigma \sim \gamma_{w\bar{w}}\delta^{(2)}(w-w') \checkmark$$

Symplectically paired set of zero-modes (h^{CS}, h^1).

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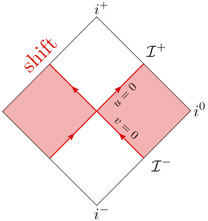


Symplectically paired set of zero-modes (h^{CS}, h^1).

$P_w = 4D^w Q^{\Delta=1}_{a=ww}$ is the $(h,\bar{h}) = (\frac{3}{2},\frac{1}{2})$ BMS supertranslation current.
It generates BMS **supertranslation** symmetry.

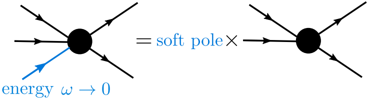
[Barnich,Troessaert'09-'11] [Strominger'13]

Conformal soft theorem in gravity

BMS supertranslation current P with $(\Delta, J) = (2, \pm 1)$ in amplitudes:

MOMENTUM BASIS: $\omega \to 0$

$$P_w \mathcal{O}_\omega(z) \sim \frac{\omega}{z-w} \mathcal{O}_\omega(z)$$



Weinberg's soft graviton theorem

[Weinberg'65]

$$\lim_{\omega_n \to 0} \omega_n \, \mathcal{A}_n(\omega_1, ..., \omega_k, ..., \omega_n)$$

$$= S^{(0)} \mathcal{A}_{n-1}(\omega_1, ..., \omega_k, ..., \omega_{n-1})$$

$$S^{(0)} = \sum_{k=1}^{n-1} \omega_k s(z_k, z_n)$$

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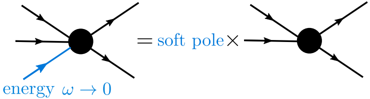

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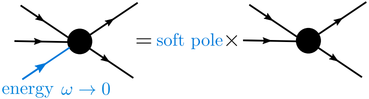
$$P_w \mathcal{O}_\omega(z) \sim \frac{\omega}{z-w} \mathcal{O}_\omega(z) \qquad \longrightarrow \qquad P_w \mathcal{O}_{(\Delta,J)}(z) \sim \frac{1}{z-w} \mathcal{O}_{(\Delta+1,J)}(z)$$

[Donnay,AP,Strominger'18]



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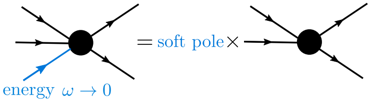

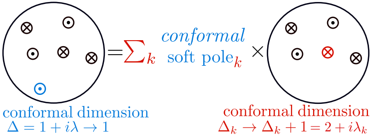

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Conformal soft graviton theorem

[Adamo,Mason,Sharma'19][AP'19][Guevara'19]

$$\lim_{\Delta_n \to 1} (\Delta_n - 1) \widetilde{\mathcal{A}}_n(\Delta_1, ..., \Delta_k, ..., \Delta_n)$$

$$= \sum_{k=1}^{n-1} \widetilde{S}_k^{(0)} \widetilde{\mathcal{A}}_{n-1}(\Delta_1, ..., \Delta_k + 1, ..., \Delta_{n-1})$$

$$\widetilde{S}_k^{(0)} = s(z_k, z_n)$$

BMS group

local enhancement

= Lorentz and ~~translations~~

Boundary conditions:

$$g_{uu} = -1 + \mathcal{O}(1/r)\,, \quad g_{uA} = \mathcal{O}(1)\,, \quad g_{AB} = r^2\gamma_{AB} + rC_{AB} + \mathcal{O}(1)$$

Extended BMS group

local enhancement ? local enhancement

= ~~Lorentz~~ and ~~translations~~

Boundary conditions: [Banks'03] [Barnich,Troessaert'09-'11]

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Virasoro and Diff(S^2) superrotations:

$Y(z)$ and $Y(z,\bar{z})$ allowed to be singular on S^2

$$\xi_Y = u\alpha\partial_u - \left(\alpha r + u\left(\frac{D^2}{2}+2\right)\alpha\right)\partial_r + \left(Y^A + \frac{u}{2r}((D^2+1)\,Y^A - 2D^A\alpha)\right)\partial_A + \ldots$$

where $\alpha = \frac{1}{2}D_C Y^C$

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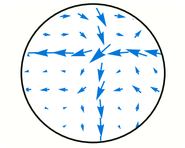
Action of large diffeormorphism ξ_Y on γ_{AB} and C_{AB}:



$$\delta_Y^{\text{shift}}\gamma_{\bar{z}\bar{z}} = 2\gamma_{z\bar{z}}D_{\bar{z}}\,Y^z(z,\bar{z})$$
$$\delta_Y^{\text{shift}}\,C_{zz} = -uD_z^3\,Y^z(z,\bar{z})$$

⇒ angular momentum conserved *at every angle*

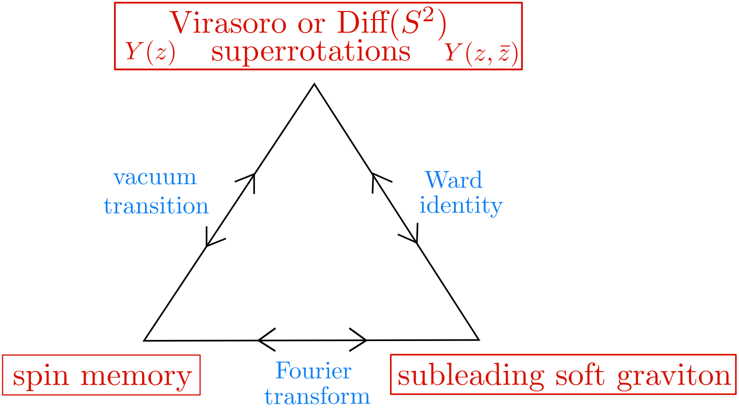
6 global rotations and boosts $\xrightarrow{\text{enhanced}}$ angle-dependent superrotations $Y^A(z,\bar{z})$

⇒ BMS group enhanced to extended BMS group

IR connections

[Banks'03]
[Barnich,Troessaert'09-'11]

[Campiglia,Laddha'15]
[Compère,Fiorucci,Ruzziconi'18]
[Campiglia,Peraza'20]



Virasoro or Diff(S^2)
$Y(z)$ superrotations $Y(z,\bar{z})$

vacuum transition

Ward identity

spin memory

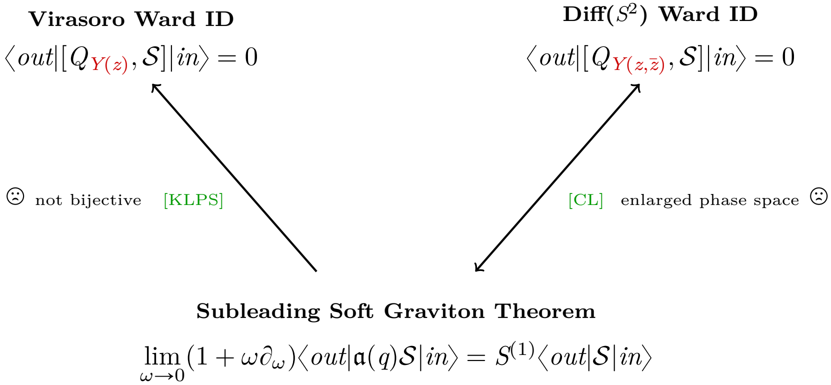
Fourier transform

subleading soft graviton

[Pasterski,Strominger,Zhiboedov'15]

[Cachazo,Strominger'14]

Virasoro *or* Diff(S^2) superrotations ?



[Kapec,Lysov,Pasterski,Strominger'14]

Virasoro Ward ID

$$\langle out|[Q_{Y(z)}, \mathcal{S}]|in\rangle = 0$$

☹ not bijective [KLPS]

[Campiglia,Laddha'14]

Diff(S^2) Ward ID

$$\langle out|[Q_{Y(z,\bar{z})}, \mathcal{S}]|in\rangle = 0$$

[CL] enlarged phase space ☹

Subleading Soft Graviton Theorem

$$\lim_{\omega \to 0}(1 + \omega\partial_\omega)\langle out|\mathfrak{a}(q)\mathcal{S}|in\rangle = S^{(1)}\langle out|\mathcal{S}|in\rangle$$

[Cachazo,Strominger'14]

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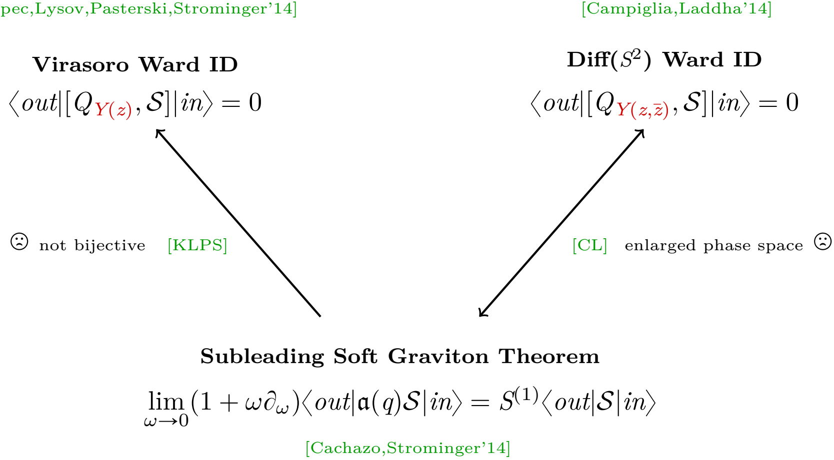

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[Cachazo,Strominger'14]

Virasoro and Diff(S^2) asymptotic symmetry group proposals are related from celestial CFT perspective!

Celestial $\widetilde{\Delta} = 2$ and $\Delta = 0$ gravitons

$\widetilde{\Delta} = 2 - \Delta = 2$ **Goldstone mode** [Pasterski,Shao'17]

$$\widetilde{h}^{\widetilde{\Delta}=2}_{\mu\nu;a} = \nabla_{(\mu}\xi^{2}_{\nu);a} \quad \rightarrow \quad \tilde{C}^{\,2}_{zz;ww} = -uD_z^3\, Y^z_{ww} \text{ with } Y^z_{ww}(z) = \tfrac{1}{6(z-w)}$$

[Donnay,AP,Strominger'18]

$\Delta = 0$ **Goldstone mode** [Pasterski,Shao'17]

$$h^{\Delta=0}_{\mu\nu;a} = \nabla_{(\mu}\xi^{0}_{\nu);a} \quad \rightarrow \quad C^{\,0}_{zz;\bar{w}\bar{w}} = -uD_z^3\, Y^z_{\bar{w}\bar{w}} \text{ with } Y^z_{\bar{w}\bar{w}}(z,\bar{z}) = -\tfrac{(z-w)^2}{2(\bar{z}-\bar{w})}$$

[Donnay,Pasterski,AP'20]

Celestial $\widetilde{\Delta} = 2$ and $\Delta = 0$ gravitons

$\widetilde{\Delta} = 2 - \Delta = 2$ **Goldstone mode** [Pasterski,Shao'17]

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[Donnay,AP,Strominger'18]

T_{ww} is the $(h, \bar{h}) = (2, 0)$ 2D stress tensor for 4D gravity.
It generates Virasoro **superrotation** symmetry.

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[Donnay,Pasterski,AP'20]

$\widetilde{T}_{\bar{w}\bar{w}}$ is the $(h,\bar{h}) = (-1,1)$ shadow of 2D stress tensor.
It generates Diff(S^2) **"shadow superrotation"** symmetry.

[Donnay,Pasterski,AP'20]

Soft Virasoro and Diff(S^2) charges

Iyer-Wald symplectic structure

$$\Omega[\delta g, \delta_Y^{\text{shift}} g; g] = \int_{\mathcal{I}^+} \omega[\delta g, \delta_Y^{\text{shift}} g; g]$$

for variations δg and $\delta_Y^{\text{shift}} g = h^0$ or $\widetilde{h}^2$ around fixed background g.

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- ⋆ Divergence due to $\delta_Y^{\text{shift}} \gamma_{\bar{z}\bar{z}} = \begin{cases} \gamma_{z\bar{z}} \frac{(z-w)^2}{(\bar{z}-\bar{w})^2} & h^0_{\bar{w}\bar{w}} \\ \frac{2\pi}{3} \gamma_{z\bar{z}} \delta^{(2)}(z-w) & \widetilde{h}^2_{ww} \end{cases}$ (previously dropped)

  requires *renormalization procedure*: [Compère,Fiorucci,Ruzziconi'18]

$$\Omega^{ren}[\delta g, \delta g_Y^{\text{shift}}; g] \equiv -\delta \mathcal{Q}_Y$$

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Soft charge associated with Virasoro and Diff(S^2) superrotations:

$$\boxed{Q_Y^{soft} = \frac{1}{16\pi G} \int du d^2 z \sqrt{\gamma}\, u D_z^3 Y^z N^{zz}}$$

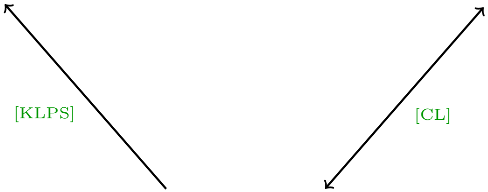
$$D_z^3 Y^z = \begin{cases} -2\pi \delta^{(2)}(z-w) & h^0_{\bar{w}\bar{w}} \\ -\frac{1}{(z-w)^4} & \widetilde{h}^2_{ww} \end{cases}$$

[Donnay,Pasterski,AP'20]

Bondi news $N_{zz} = \partial_u C_{zz}$

Agrees with results in Bondi gauge for Virasoro [Kapec,Lysov,Pasterski,Strominger'14] and for Diff(S^2) [Campiglia,Laddha'15] [Compère,Fiorucci,Ruzziconi'18] .

Virasoro *or* Diff(S^2) superrotations ?



[Kapec,Lysov,Pasterski,Strominger'14]

Superrotation Ward ID

$$\langle out|[Q_{Y^z=\frac{1}{6(z-w)}},\mathcal{S}]|in\rangle = 0$$

[Campiglia,Laddha'14]

Shadow Superrotation Ward ID

$$\langle out|[Q_{Y^z=-\frac{(z-w)^2}{2(\bar{z}-\bar{w})}},\mathcal{S}]|in\rangle = 0$$

[KLPS]

[CL]

Subleading Soft Graviton Theorem

$$\lim_{\omega\to 0}(1+\omega\partial_\omega)\langle out|\mathfrak{a}_-(q)\mathcal{S}|in\rangle = S^{(1)-}\langle out|\mathcal{S}|in\rangle$$

[Cachazo,Strominger'14]

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Superrotation Ward ID

Shadow Superrotation Ward ID

$$\begin{tikzcd}
\langle out|[Q_{Y^z=\frac{1}{6(z-w)}},\mathcal{S}]|in\rangle = 0 \arrow[rr, leftrightarrow, "\textbf{shadow}"] & & \langle out|[Q_{Y^z=-\frac{(z-w)^2}{2(\bar{z}-\bar{w})}},\mathcal{S}]|in\rangle = 0 \arrow[dl, leftrightarrow, "{[CL]}"] \\
& \lim\limits_{\omega\to0}(1+\omega\partial_\omega)\langle out|\mathfrak{a}_-(q)\mathcal{S}|in\rangle = S^{(1)-}\langle out|\mathcal{S}|in\rangle \arrow[ul, "{[KLPS]}"] &
\end{tikzcd}$$

Subleading Soft Graviton Theorem

[Cachazo,Strominger'14]

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\end{tikzcd}$$

[Cachazo,Strominger'14]

Asymptotic symmetry group of Einstein gravity at null infinity should include closure of Virasoro under shadows within Diff(S^2).

Celestial CFT perspective

$$\begin{tikzcd}
\tilde{Q}^{\tilde{\Delta}=2}_{ww} \arrow[rrr, leftrightarrow, "\text{shadow}"] & & & Q^{\Delta=0}_{\bar{w}\bar{w}}
\end{tikzcd}$$

Celestial CFT perspective

$$\begin{tikzcd}
\widetilde{Q}^{\widetilde{\Delta}=2}_{ww} \arrow[r, leftrightarrow, "\text{shadow}"] \arrow[d, "\text{renormalize}"'] & Q^{\Delta=0}_{\bar{w}\bar{w}} \arrow[d, "\text{renormalize}"'] \\
\widetilde{Q}^{2\;ren}_{ww} = Q^{soft}_{Y}[\,Y^{z}_{ww} = \frac{1}{6(z-w)}] & Q^{0\;ren}_{\bar{w}\bar{w}} = Q^{soft}_{Y}[\,Y^{z}_{\bar{w}\bar{w}} = -\frac{1}{2}\frac{(z-w)^2}{(\bar{z}-\bar{w})}]
\end{tikzcd}$$

Virasoro superrotations

Diff(S^2) shadow superrotations

Celestial CFT perspective

$$\begin{tikzcd}
\widetilde{Q}^{\widetilde{\Delta}=2}_{ww} \arrow[rr, leftrightarrow, "\text{shadow}"] \arrow[d, "\text{renormalize}"'] & & Q^{\Delta=0}_{\bar{w}\bar{w}} \arrow[d, "\text{renormalize}"'] \\
T_{ww} \arrow[rr, leftrightarrow, "\text{shadow}"] & & \widetilde{T}_{\bar{w}\bar{w}}
\end{tikzcd}$$

$$\widetilde{Q}^{2\,ren}_{ww} = Q^{soft}_{Y}[\,Y^{z}_{ww} = \tfrac{1}{6(z-w)}] \qquad Q^{0\,ren}_{\bar{w}\bar{w}} = Q^{soft}_{Y}[\,Y^{z}_{\bar{w}\bar{w}} = -\tfrac{1}{2}\tfrac{(z-w)^2}{(\bar{z}-\bar{w})}]$$

Virasoro superrotations and $\mathrm{Diff}(S^2)$ shadow superrotations
are on equal footing via the shadow transform

Conclusion

Unified treatment of conformally soft Goldstone modes of spontaneously broken asymptotic symmetries for flat space holography.

Massless particles described as conformal primary wavefunctions with (Δ, J) under $SL(2,\mathbb{C})$ Lorentz group in celestial CFT:

- finite energy → principal continuous series $\Delta \in 1 + i\mathbb{R}$
- energetically soft → conformally soft $\Delta \in \mathbb{Z}$

contour integral

- symplectically paired set of zero-modes
- conformal soft theorems for celestial amplitudes

	$\widetilde{A}_\mu^{\widetilde{\Delta}}$	$\widetilde{h}_{\mu\nu}^{\widetilde{\Delta}}$	
$\widetilde{\Delta}$	1	1	2
symmetry	large $U(1)$	supertranslation	Virasoro superrotation
soft charge	$\mathcal{J}$	P	T

	A_μ^{Δ}	$h_{\mu\nu}^{\Delta}$	
Δ	1	1	0
symmetry	large $U(1)$	supertranslation	Diff(S^2) shadow superrotation
soft charge	$\mathcal{J}$	P	$\widetilde{T}$

Flat space holography

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a developing story

[see also talks by Raju and Strominger]

Flat space holography

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Thank you!