Holographic null infinity and the information paradox in flat space

Suvrat Raju

International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

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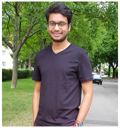
Collaborators

Bidisha Chakraborty



Pushkal Shrivastava

Alok Laddha

Olga Papadoulaki

Chandramouli Chowdhury

Kyriakos Papadodimas

Siddharth Prabhu

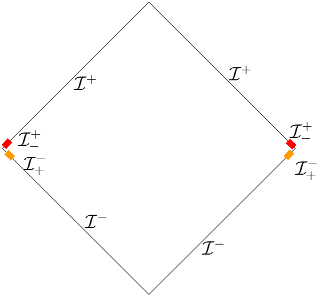
References

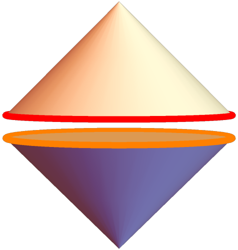
- 2002.02448, Alok Laddha, Siddharth Prabhu, S.R., Pushkal Shrivastava
- arXiv:2007.???? Chandramouli Chowdhury, Olga Papadoulaki, Kyriakos Papadodimas, S.R.
- arXiv:1903.11073, S.R.
- arXiv:1809.10154, S.R.
- arXiv:1603.02812 Souvik Banerjee, Jan-Willem Bryan, Kyriakos Papadodimas, S.R.

Holographic information in AdS

- In AdS, “Information is stored holographically” means that information is available from a single-slice of the boundary.
- In principle, we can immediately read off the bulk excitation, without waiting for its signals to emerge at late times.

Main result



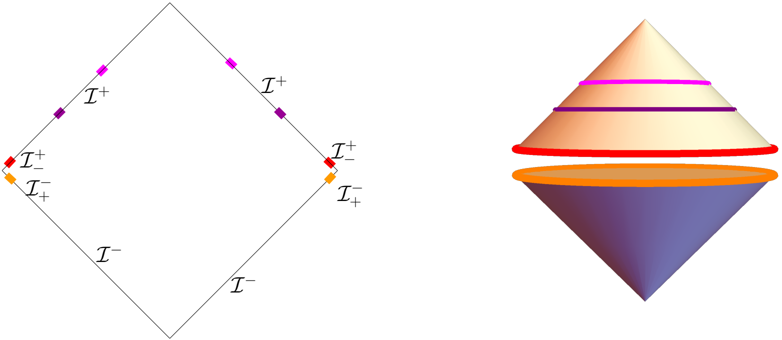


- We were able to establish a similar result for gravitational theories in four dimensional asymptotically flat spacetimes.

[Alok Laddha, Siddharth Prabhu, S.R., Pushkal Shrivastava, 2020]

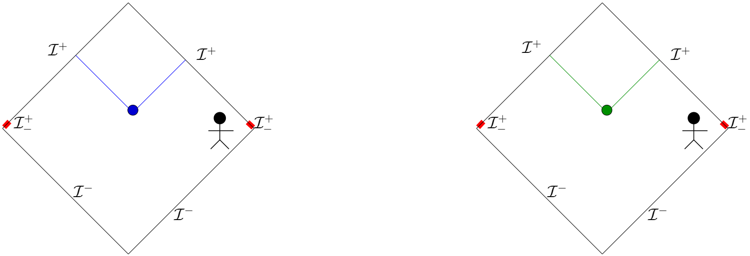
- All information about massless particles is present on the past boundary of future null infinity (or future boundary of past null infinity).

Differences with AdS



- Unlike AdS, we lose information as we move along the boundary.
- Information on later cuts is a subset of information on earlier cuts.

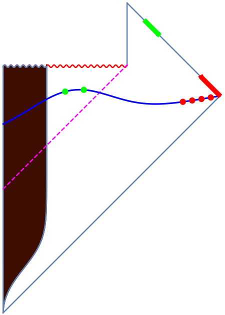
Perturbative verification



- This is not just an abstract result.
- It leads to a physical protocol for observers near the boundary to identify the bulk state.

[C Chowdhury, K. Papadodimas, O. Papadoulaki, S.R., in-progress]

Implications for black hole information



This suggests a physical origin of the idea that degrees of freedom on a nice slice must be identified with each other. Complementary to Euclidean path-integrals or the RT formula.

[Papadodimas, S.R(2012), Verlinde^2(2012), Nomura(2012) Maldacena, Susskind(2013),. . . islands(2019–20). . .]

- Suggests information is always outside a black hole, even before the Page time!
- Suggests the naive Page curve does not describe the von Neumann entropy of the state on $\mathcal{I}^+$ for black holes in flat space.

Philosophy

- We observe some interesting properties of the semi-classical theory.
- We assume that the full UV-theory shares some low-energy properties of the semi-classical theory.
- This allows us to extrapolate our results to the full UV-theory.

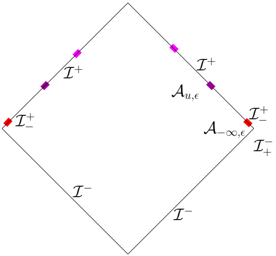
No claim that the semi-classical gravity is a complete theory by itself.

Claim is merely that semi-classical gravity has robust lessons about how quantum information is stored holographically in quantum gravity.

Asymptotic boundary conditions

We consider four dimensional asymptotically flat spacetimes

$$ds^2 = -du^2 - 2dudr + r^2\gamma_{AB}d\Omega^A d\Omega^B$$
$$+ rC_{AB}d\Omega^A d\Omega^B + \frac{2m_{\rm B}}{r}du^2 + \gamma^{DA}D_D C_{AB}dud\Omega^B + \ldots$$



The algebra of observables between the cuts $[u, u+\epsilon]$ is $\mathcal{A}_{u,\epsilon}$. Includes the Bondi news, Bondi mass

$$N_{AB} = \partial_u C_{AB}; \qquad M(u) = \int \sqrt{\gamma}\, m_{\rm B}(u,\Omega) d^2\Omega,$$

and all possible products and linear combinations.

$\mathcal{A}_{-\infty,\epsilon}$ is the algebra near $u \to -\infty$.

Hilbert space

The vacuum is infinitely degenerate

$$\mathcal{Q}_{\ell,m}|\{s\}\rangle = s_{\ell,m}|\{s\}\rangle.$$

and on top of each vacuum we can build a Fock space

$$\mathcal{H}_{\{s\}} = \text{span of}\{N(f_1)N(f_2)\ldots N(f_n)\}|\{s\}\rangle,$$

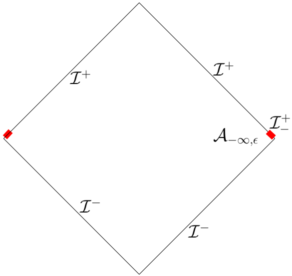
[Ashtekar, Faddeev, Kulish, Strominger, He, Lysov, Mitra, Pasterski, Compere, Laddha, Campiglia . . . , 1981–2020]

The full Hilbert space of massless particles is

$$\mathcal{H} = \bigoplus_{\{s\}} \mathcal{H}_{\{s\}},$$

Our statements will be confined to this Hilbert space (which excludes massive excitations)

Result : Information at $\mathcal{I}^+_-$



We will now describe.

Any two distinct states in $\mathcal{H}$ can be distinguished just by observables in $\mathcal{A}_{-\infty,\epsilon}$

[Laddha, Prabhu, S.R., Shrivastava, 2020]

[Marolf, 2006–13]

[de Boer, Solodukhin, 2003]

[Bagchi, Grumiller, Pasterski, Shu-Heng Shao, Strominger, 2016–19]

Step 1: Squeezing the Hilbert space

The positivity of the full Hamiltonian guarantees that any state $|n\rangle \in \mathcal{H}_{\{s\}}$ can be approximated arbitrarily well as

$$|n\rangle \doteq X_n |\{s\}\rangle$$

where $X_n \in \mathcal{A}_{-\infty,\epsilon}$.

This is a map between states $\Rightarrow$ operators.

⚠ Caution: This intermediate technical step does not even require dynamical gravity, and is not the main point of the argument.

Step 2: Projector onto all vacua

- Both the Hamiltonian and supertranslation charges are in $\mathcal{A}_{-\infty,\epsilon}$.

- If we measure a supertranslation charge, $\mathcal{Q}_{\ell,m}$ in state $|\Psi\rangle$ by standard rules of QM, we get an answer between $s + \Delta s$ with probability proportional to

$$\langle\Psi|\mathcal{P}_{\ell,m}[s]|\Psi\rangle$$

- So these projectors, and also the projector on the manifold of vacua

$$\mathcal{P}_{\Omega} = \int \left(\prod_{\ell,m} ds_{\ell,m}\right) |\{s\}\rangle\langle\{s\}| \in \mathcal{A}_{-\infty,\epsilon}.$$

  are observables and part of $\mathcal{A}_{-\infty,\epsilon}$.

Projector onto a specific vacuum

So, using a measure and discard procedure we can select a specific soft vacuum from near $\mathcal{I}^+_-$.

$$\mathcal{P}_\Omega \prod_{\ell,m} \mathcal{P}_{\ell,m}[s_{\ell,m}] = |\{s\}\rangle\langle\{s\}| \in \mathcal{A}_{-\infty,\epsilon}.$$

The fact that one can select a specific vacuum using observables from the boundary of the spacetime is a unique feature of gravity. Not true in any other theory including non-gravitational gauge theories.

Using projectors and simple operators, we can construct

$$T_{\{s\},\{s'\}} = |\{s\}\rangle\langle\{s'\}| \in \mathcal{A}_{-\infty,\epsilon}.$$

Argument for result

- Any operator, $\mathcal{H} \to \mathcal{H}$, can be written as

$$\begin{aligned} A &= \sum_{s,s',n,m} c(n,m,s,s')|n_{\{s\}}\rangle\langle m_{\{s'\}}| \\ &\doteq \sum c(n,m,s,s') X_n |\{s\}\rangle\langle\{s'\}| X_m^\dagger \\ &= \sum c(n,m,s,s') X_n T_{\{s\},\{s'\}} X_m^\dagger. \end{aligned}$$

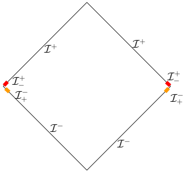
  The RHS is manifestly in $\mathcal{A}_{-\infty,\epsilon}$

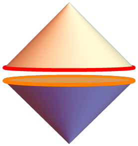
- So any operator, $\mathcal{H} \to \mathcal{H}$, can be approximated arbitrarily well by an operator near $\mathcal{I}^+_-$.

This result extends to the full theory of quantum gravity, if we assume

1. Vacua in the full theory are still labelled by operators near $\mathcal{I}^+_-$.
2. Operators that map the space of vacua back to itself are contained in $\mathcal{A}_{-\infty,\epsilon}$.

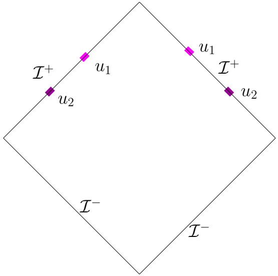
So, if the full theory shares low-energy properties of the semi-classical theory, all information about massless particles is available near $\mathcal{I}^+_-$ (or near $\mathcal{I}^-_+$).

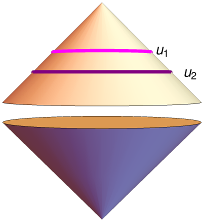




A stronger result

All information accessible through $\mathcal{A}_{u_1,\epsilon}$ is also available through $\mathcal{A}_{u_2,\epsilon}$ for any $u_2 < u_1$.





But this requires stronger assumptions about the UV-theory.

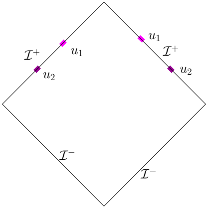
Assumptions for result 2

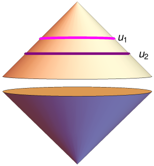
- In the semiclassical theory, the Bondi mass and dynamical fields obey

$$[M(u), C_{AB}(u', \Omega)] = -4\pi G i \partial_{u'} C_{AB}(u', \Omega)\theta(u' - u),$$
$$[M(u), O(u', \Omega)] = -4\pi G i \partial_{u'} O(u', \Omega)\theta(u' - u).$$

- These asymptotic commutators can be derived in nonlinear general relativity because they depend only on the weak field limit. So we could assume that they carry over to the full UV theory.

Argument for result 2



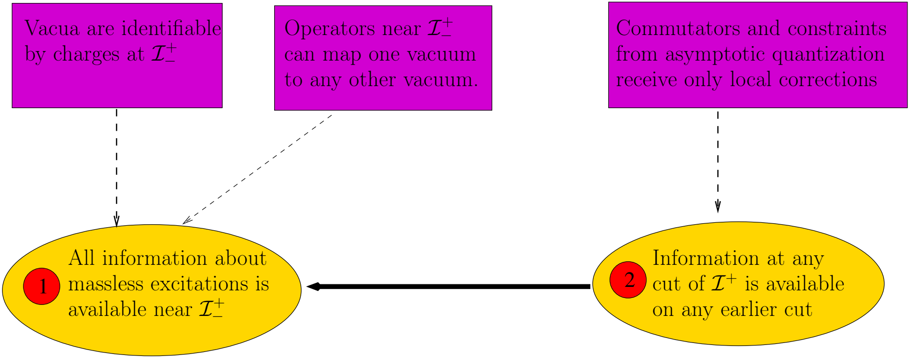


- Since $M(u)$ acts as a "Hamiltonian" for operators at larger values of u

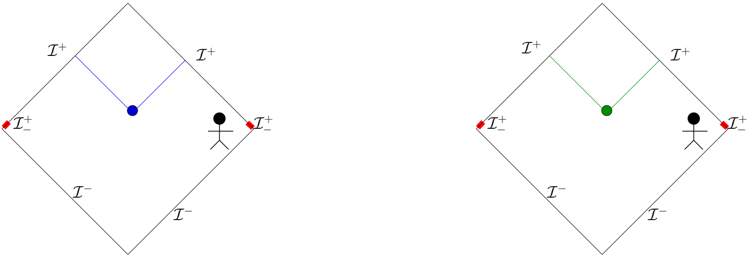
$$C_{AB}(u_1, \Omega) = e^{\frac{iM(u_2)}{4\pi G}(u_1 - u_2)} C_{AB}(u_2, \Omega) e^{\frac{-iM(u_2)}{4\pi G}(u_1 - u_2)};$$

- Unlike AdS, we seem to lose information as we move up on $\mathcal{I}^+$.

Recap of assumption and result



Perturbative verification

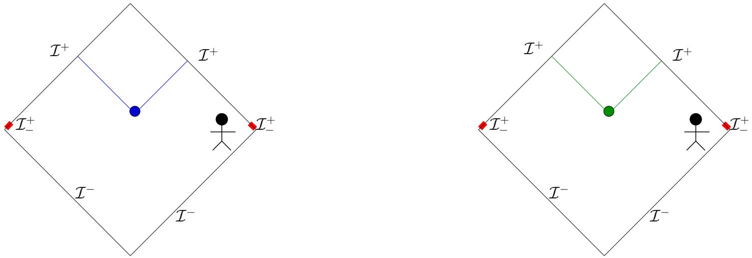


- This may seem like an abstract argument but can be verified in perturbation theory.
- With $f(u, \Omega)$ localized near u = 0,

$$|f\rangle = e^{i\lambda \int du d^2\Omega \sqrt{\gamma} N_{AB}(u,\Omega) f^{AB}(u,\Omega)} |\Omega\rangle.$$

- **Challenge:** At small λ, working only near $\mathcal{I}^+_-$, figure out the form of $f^{AB}(u, \Omega)$.

Perturbative verification



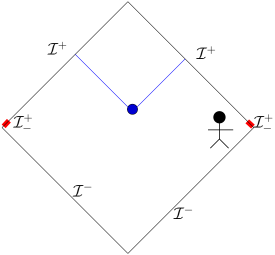
- Compute

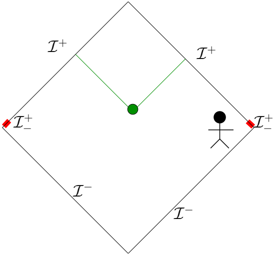
$$\langle f | M(u \to -\infty)\, N_{CD}(u, \Omega') | f \rangle$$

$M(u \to -\infty)$ is the Hamiltonian.

- Measure quantum correlations of the Hamiltonian with other fields.

[Chowdhury, Papadoulaki, Papadodimas, S.R., in-progress]

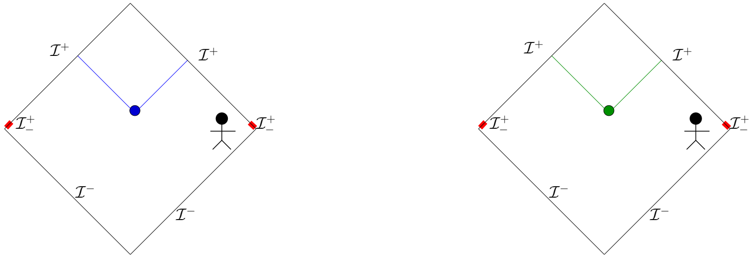




$$\langle f|M(-\infty)N_{CD}(u,\Omega')|f\rangle$$
$$= \lambda \int dx 16G \frac{f^{AB}(x,\Omega')}{(x-u-i\epsilon)^3}[\,\gamma_{A(M}\gamma_{N)B} - \frac{1}{2}\gamma_{AB}\gamma_{MN}\,] + \mathrm{O}\left(\lambda^2\right).$$

If we know RHS for $u \in (-\infty, -\frac{1}{\epsilon})$ we can reconstruct f^{AB}

The importance of gravity

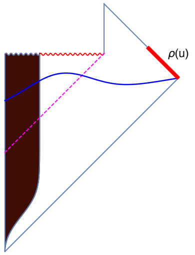


- This trick cannot work without gravity.
- Non-gravitational gauge theories contain exactly local gauge-invariant bulk operators which commute with all elements of $\mathcal{A}_{-\infty,\epsilon}$.

$$|\Omega\rangle \quad \text{and} \quad e^{i\mathrm{Tr}(F^2)(0)}|\Omega\rangle$$

cannot be distinguished by any measurement near $\mathcal{I}^+_-$ without gravity.

Revisiting the Page curve



Common to define a state on $\mathcal{I}^+$ satisfying,

$$\mathrm{Tr}(\rho(u)b) = \langle b \rangle.$$

where b and $\rho(u)$ are both in the algebra on $[-\infty, u]$.

But, we can always choose

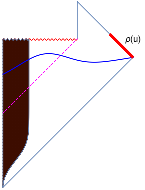
$$\rho(u) = \sigma \in \mathcal{A}_{-\infty,\epsilon},$$

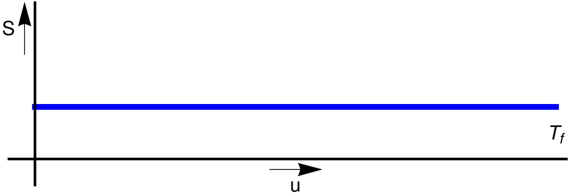
So

$$S = -\mathrm{Tr}(\rho(u)\log(\rho(u)))$$

is independent of u!

The Page curve in flat space



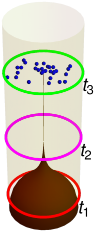


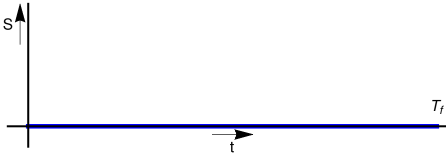
- The Page curve fails since it is based on known incorrect assumptions about factorization of the Hilbert space.
- Even the algebra on $\mathcal{I}^+$ does not factorize.
- Perhaps, appropriate restriction of algebra will yield Page curve.

Physical point is that information is always available outside a black hole in flat space even before the Page time.

Page curve in AdS/CFT

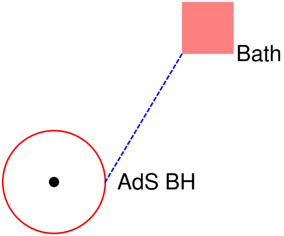
This is also the prediction for small black holes forming and evaporating in AdS/CFT.





If we consider the fine-grained entropy of the bulk region next to the boundary, this is always zero.

- This is not inconsistent with recent work, which considers gravity coupled to a non-gravitational bath.



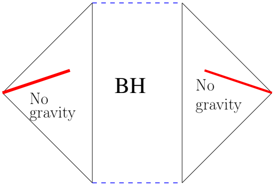
[Pennington, Almheiri, Mahajan, Maldacena, Hartman,]
[Shagoulian, Tajdini, Stanford, Shenker, Yang..., 2019]

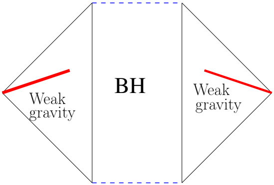
- The Hilbert space factorizes

$$\mathcal{H} = \mathcal{H}_{\text{bh}} \otimes \mathcal{H}_{\text{bath}}$$

- And both S_{bh} and S_{bath} follow a Page curve.

Real black holes?





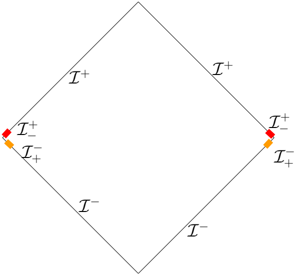
For fine-grained quantum-information questions

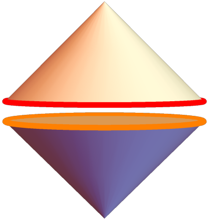
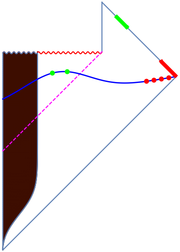
weak effects in gravity can conspire to give radically different answers from local theories

So

- So in systems where gravity is non-dynamical beyond some region, information may emerge only after Page time.
- For more realistic black holes, information is always outside.

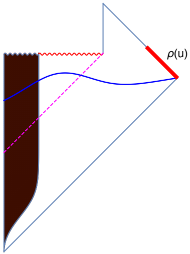
Conclusion: Result

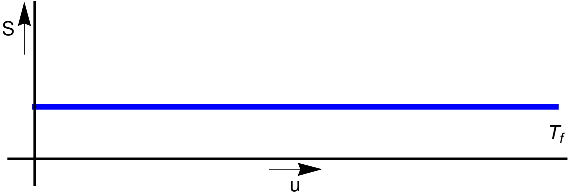


- We argued that all information on future (past) null infinity is present near its past (future) boundary.
- This is a precise version of the claim that degrees-of-freedom on parts of a nice slice must be identified.

Conclusion: Implications





- Main physical lesson: an outside observer can always determine the state of even a "young" black hole with suitably complicated measurements.

- Not inconsistent with the recent Page curve derivations. But suggests that these derivations are inapplicable when gravity is everywhere dynamical.