A Mellin-Barnes Approach to Scattering in de Sitter Space

Charlotte Sleight

IAS & ULB

1906.12302 C.S.
1907.01143 C.S. and M. Taronna
+ to appear

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Based on work w. Massimo Taronna

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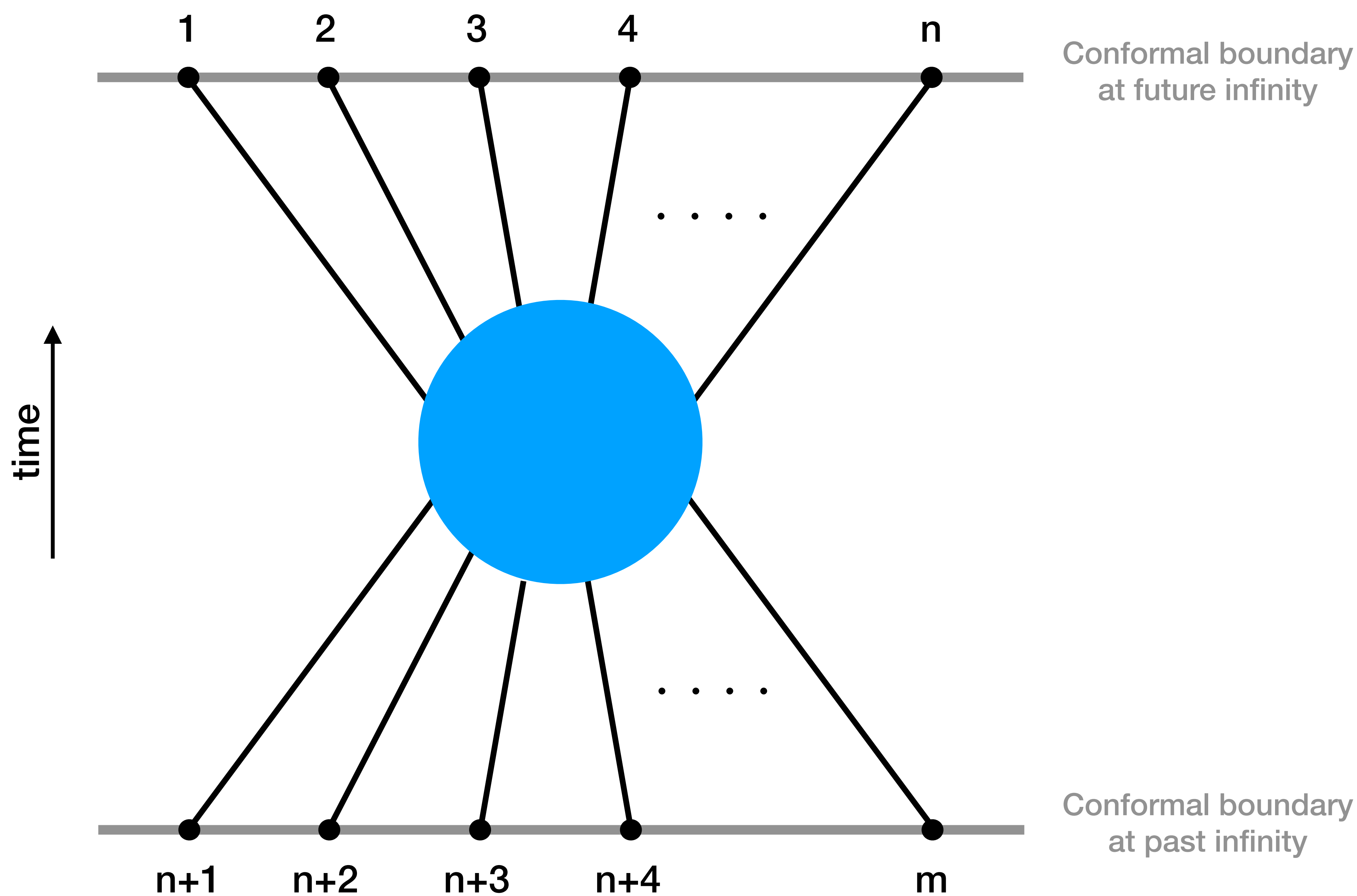
Scattering in de Sitter



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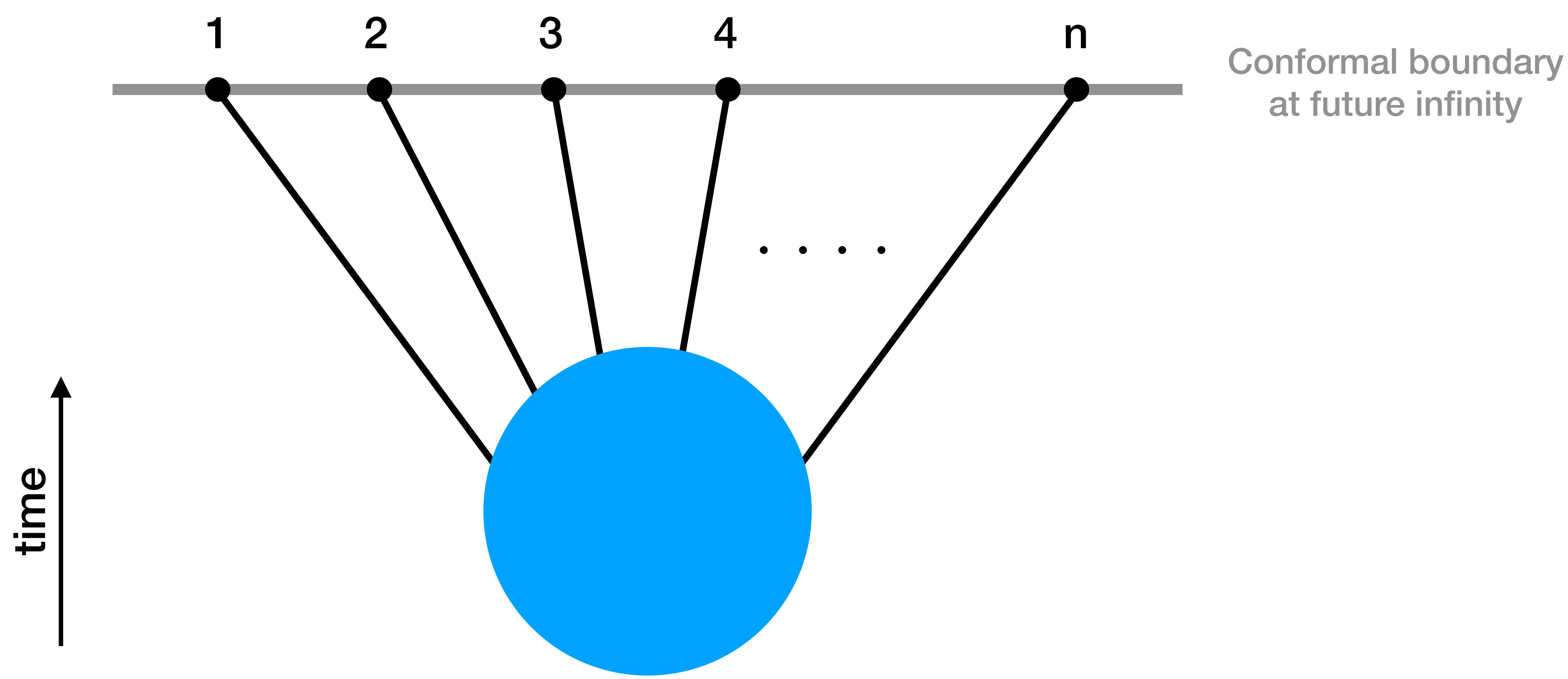
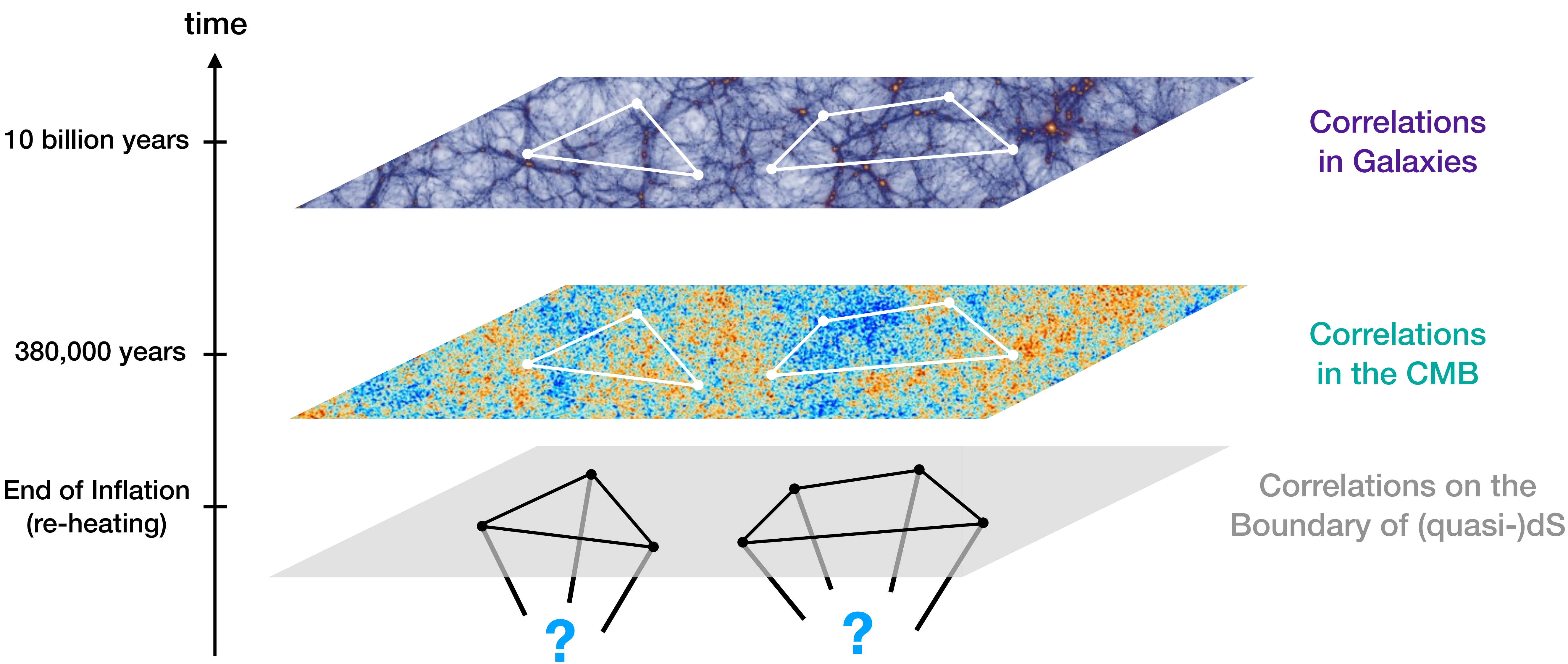


Cosmological Collider Physics

Many groups, e.g.: Chen and Wang 2009, Baumann and Green 2011, Noumi, Yamaguchi and Yokoyama 2013, Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018, …



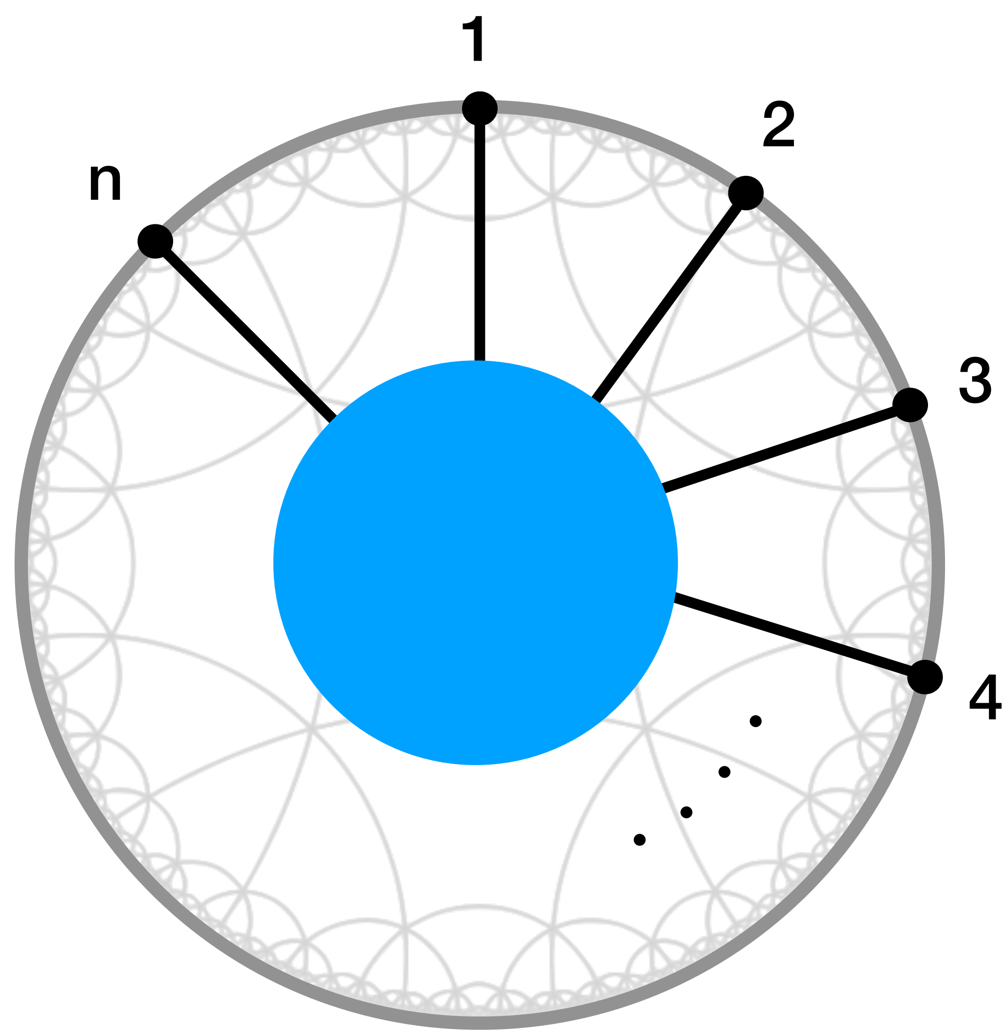
Task: Classify the effects of new degrees of freedom

Amplitudes meets Cosmology:

Arkani-Hamed, Benincasa, and Postnikov 2017; Benincasa 2018-2019; Arkani-Hamed, Baumann, Lee and Pimentel 2018; Baumann, Duaso-Pueyo, Joyce, Lee and Pimentel 2019-2020; Hillman 2019; Green and Pajer 2020…

Scattering in anti-de Sitter

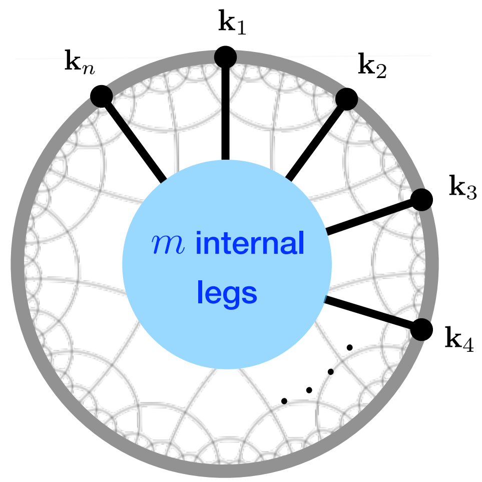
…in AdS we have a pretty good understanding.



Can we adapt existing successful AdS techniques to dS?

Bridging the Gap between EAdS and dS

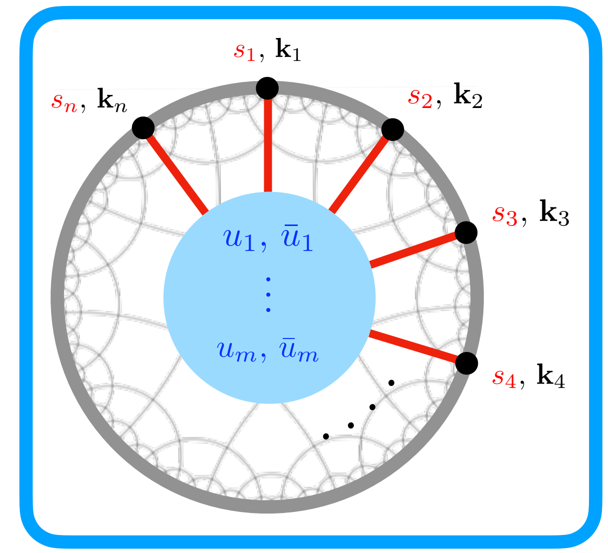
Mellin-Barnes representation in momentum space:



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs

internal legs



Mellin-Barnes representation

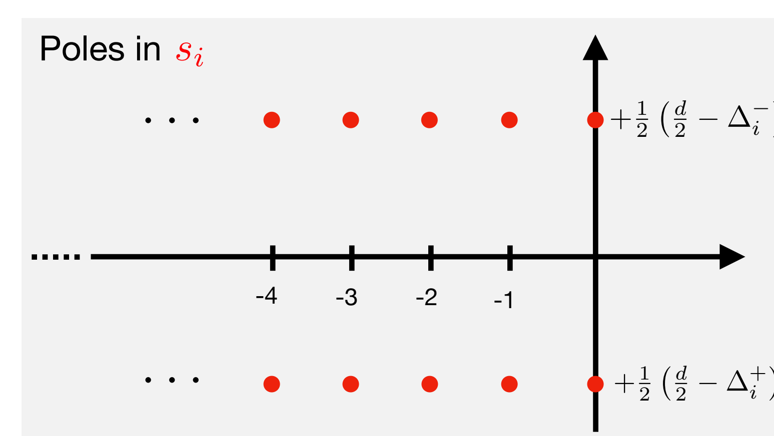
External leg, momentum $\mathbf{k}_i$ $\quad (|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ $\quad$ **external Mellin variable,** s_i

Translation invariance: $\mathbf{k}_1 + \ldots + \mathbf{k}_n = 0$

Dilatation Ward identities: $s_1 + \ldots + s_n = \text{const.}$

$$(2\pi)^d\, \delta^{(d)}\left(\mathbf{k}_1 + \ldots + \mathbf{k}_n\right) = \int d^d\mathbf{x}\, e^{i\mathbf{x}\cdot(\mathbf{k}_1+\ldots+\mathbf{k}_n)}$$

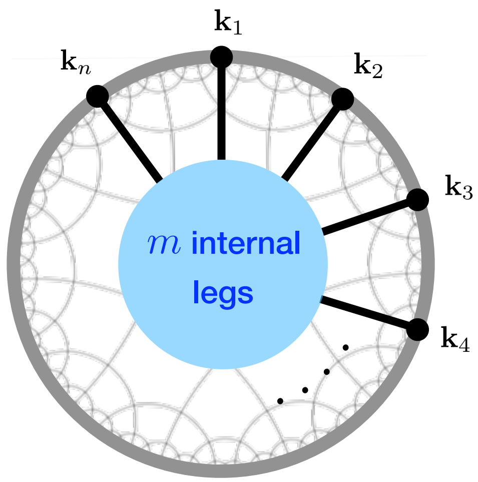
$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$



$$ds^2 = \left(\frac{R_{\text{AdS}}}{z}\right)^2 \left(dz^2 + d\mathbf{x}^2\right)$$

Bridging the Gap between EAdS and dS

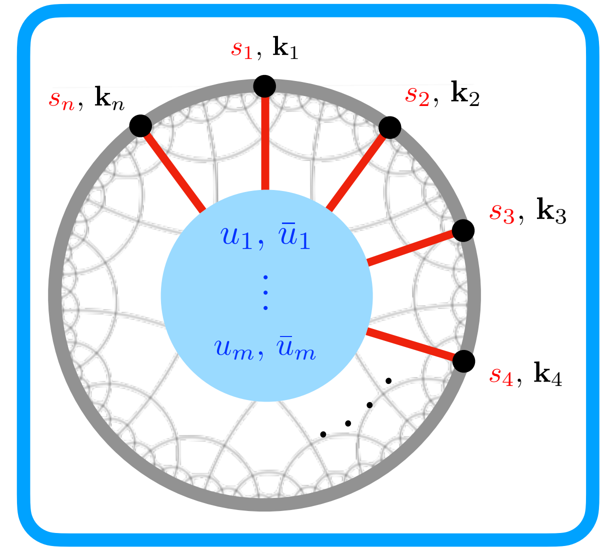
Mellin-Barnes representation in momentum space:



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs

internal legs



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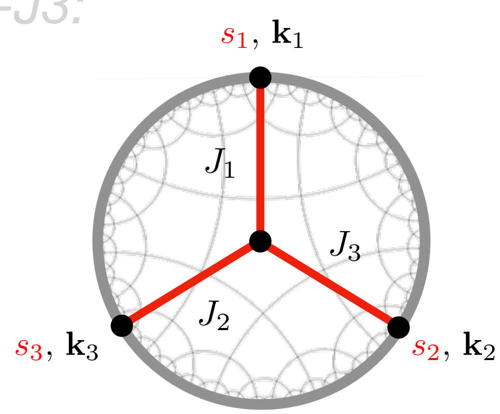
$$2\pi i\, \delta\left(\text{const.} - s_1 - \ldots - s_n\right) = \int_0^\infty \frac{dz}{z}\, z^{\text{const.}-2(s_1+\ldots+s_n)}$$

E.g. 3pt contact diagram, spins J1-J2-J3:

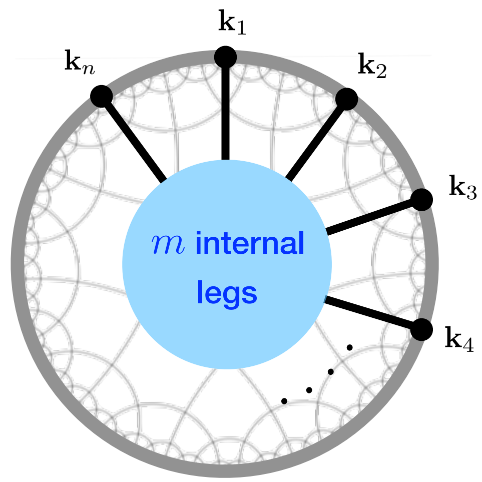
$$\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$$



$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

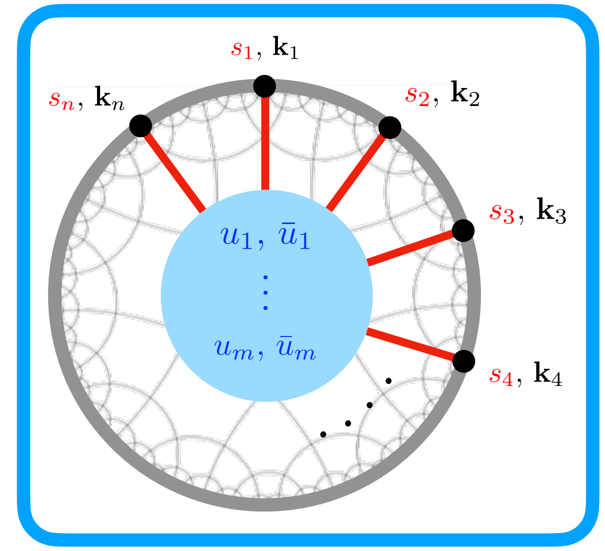
Bridging the Gap between EAdS and dS

Mellin-Barnes representation in momentum space:



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i} \cdots \frac{\mathrm{d}s_n}{2\pi i} \frac{\mathrm{d}u_1}{2\pi i} \frac{\mathrm{d}\bar{u}_1}{2\pi i} \cdots \frac{\mathrm{d}u_m}{2\pi i} \frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs



Mellin-Barnes representation

External leg, momentum $\mathbf{k}_i$ $\quad (|\mathbf{k}_i|)^{-2s_i+\Delta_i^+-\frac{d}{2}}$ $\quad$ external Mellin variable, s_i

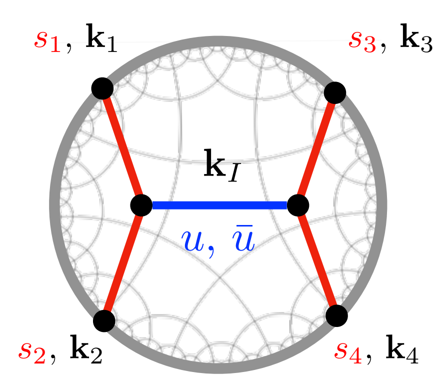
Translation invariance: $\mathbf{k}_1 + \ldots + \mathbf{k}_n = 0$ $\quad$ Dilatation Ward identities: $s_1 + \ldots + s_n = \text{const.}$

Internal leg, momentum $\mathbf{k}_I$ $\quad (|\mathbf{k}_I|)^{-2(u+\bar{u})}$ $\quad$ Two internal Mellin variables, $u, \bar{u}$

E.g. 4pt spin J exchange:

$$\mathbf{k}_1 + \mathbf{k}_2 = \mathbf{k}_I,$$

$$\mathbf{k}_3 + \mathbf{k}_4 = -\mathbf{k}_I$$

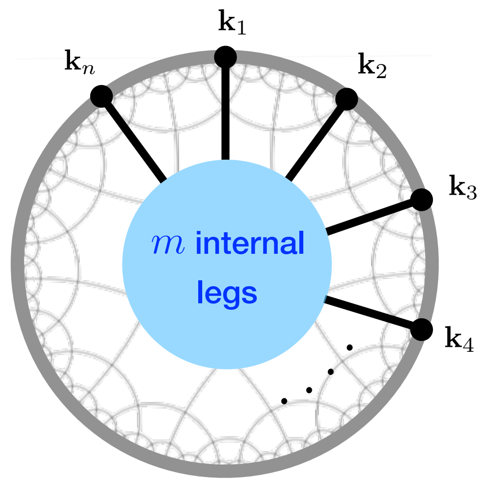


$$s_1 + s_2 + u = \frac{d + 2\,(J_1 + J_2 + J)}{4}$$

$$s_3 + s_4 + \bar{u} = \frac{d + 2\,(J + J_3 + J_4)}{4}$$

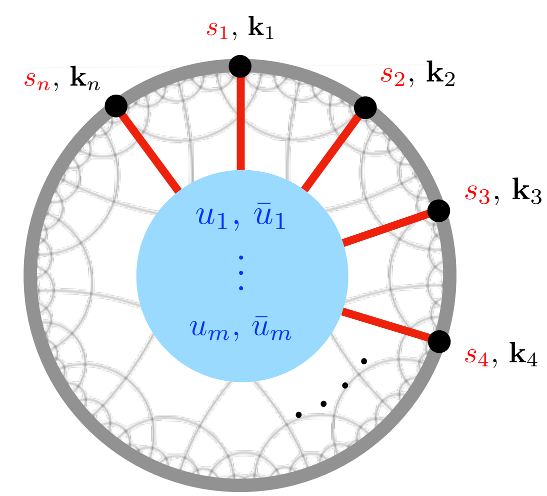
Bridging the Gap between EAdS and dS

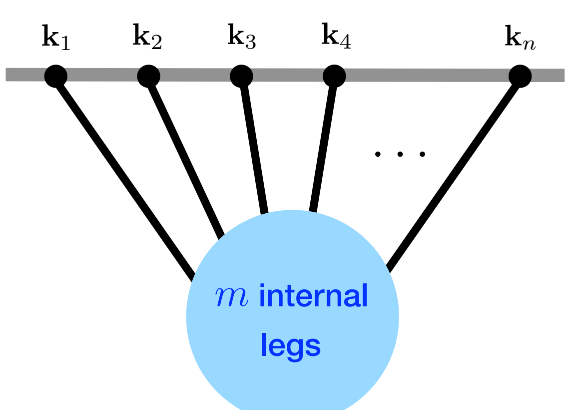
Mellin-Barnes representation in momentum space:



$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i}\cdots\frac{\mathrm{d}s_n}{2\pi i}\,\frac{\mathrm{d}u_1}{2\pi i}\,\frac{\mathrm{d}\bar{u}_1}{2\pi i}\cdots\frac{\mathrm{d}u_m}{2\pi i}\,\frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

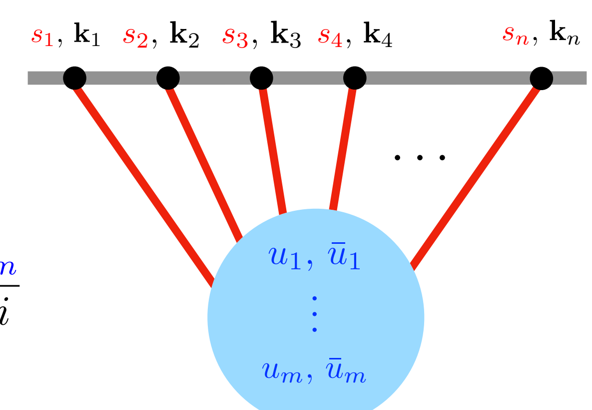
external legs internal legs





$$= \int_{-i\infty}^{i\infty} \frac{\mathrm{d}s_1}{2\pi i}\cdots\frac{\mathrm{d}s_n}{2\pi i}\,\frac{\mathrm{d}u_1}{2\pi i}\,\frac{\mathrm{d}\bar{u}_1}{2\pi i}\cdots\frac{\mathrm{d}u_m}{2\pi i}\,\frac{\mathrm{d}\bar{u}_m}{2\pi i}$$

external legs internal legs

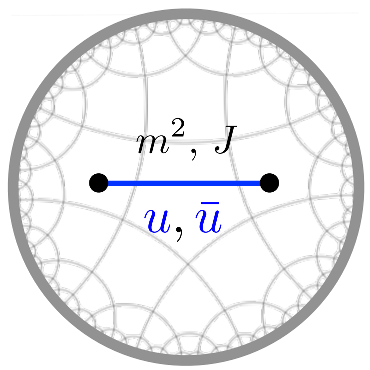


Bridging the Gap between EAdS and dS

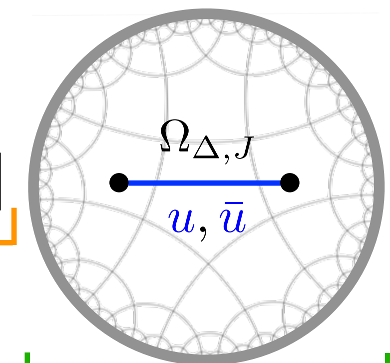
Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In EAdS we have:

Dirichlet Neumann



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha\, \omega_D\left(u,\bar{u}\right) + \beta\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

$\omega_{D/N}\left(u,\bar{u}\right)$ project onto Dirichlet/Neumann boundary conditions:

$m^2 R^2_{\text{AdS}} = -\left(\Delta_+\Delta_- + J\right)$

$$\omega_{D/N}\left(u,\bar{u}\right) = \frac{1}{2}\sin\left(\pi\left(u + \tfrac{1}{2}\left(\Delta_\mp - \tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u} + \tfrac{1}{2}\left(\Delta_\mp - \tfrac{d}{2}\right)\right)\right)$$

Recall the general solution to the wave equation near the boundary of EAdS, $z \to 0$:

$$\varphi\left(z,\mathbf{k}\right) = \alpha\, \underbrace{z^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right) + O\left(z^2\right)\right]}_{\text{Selected by } \omega_D\left(u,\bar{u}\right)} + \beta\, \underbrace{z^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right) + O\left(z^2\right)\right]}_{\text{Selected by } \omega_N\left(u,\bar{u}\right)}$$

$$ds^2 = \left(\frac{R_{\text{AdS}}}{z}\right)^2\left(dz^2 + d\mathbf{x}^2\right)$$

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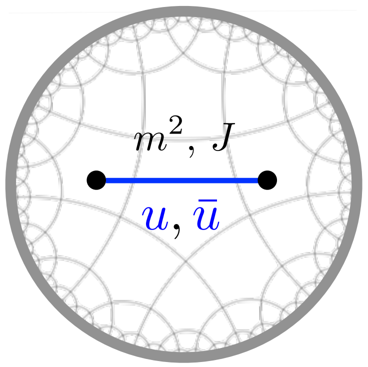
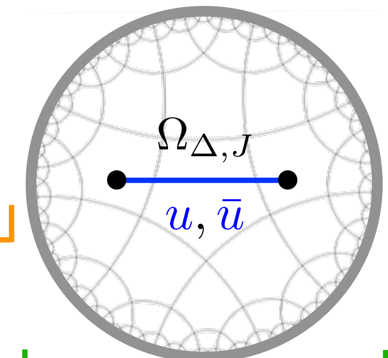


$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha\, \overset{\text{Dirichlet}}{\omega_D}\left(u,\bar{u}\right) + \beta\, \overset{\text{Neumann}}{\omega_N}\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



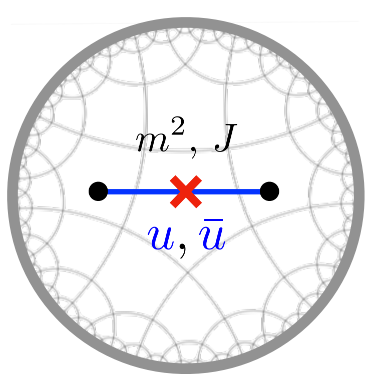
Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

$\omega_{D/N}\left(u,\bar{u}\right)$ project onto Dirichlet/Neumann boundary conditions:

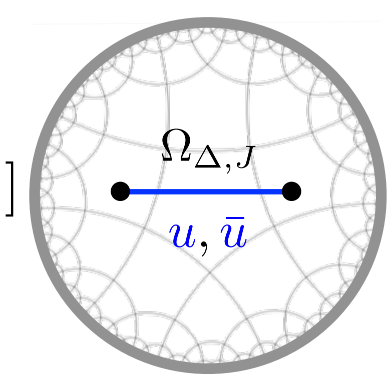
$m^2 R^2_{\text{AdS}} = -\left(\Delta_+\Delta_- + J\right)$

$$\omega_{D/N}\left(u,\bar{u}\right) = \frac{1}{2}\sin\left(\pi\left(u+\tfrac{1}{2}\left(\Delta_\mp - \tfrac{d}{2}\right)\right)\right)\sin\left(\pi\left(\bar{u}+\tfrac{1}{2}\left(\Delta_\mp - \tfrac{d}{2}\right)\right)\right)$$

On shell, the factor $\csc\left(\pi\left(u+\bar{u}\right)\right)$ gets cancelled:



$$= \left[\alpha\, \omega_D\left(u,\bar{u}\right) + \beta\, \omega_N\left(u,\bar{u}\right)\right]$$

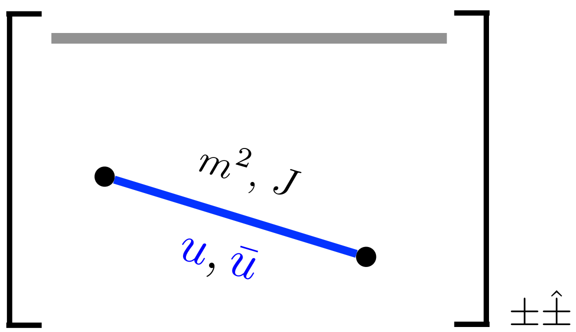


i.e. $\csc\left(\pi\left(u+\bar{u}\right)\right)$ is generated by the source term in the propagator equation.

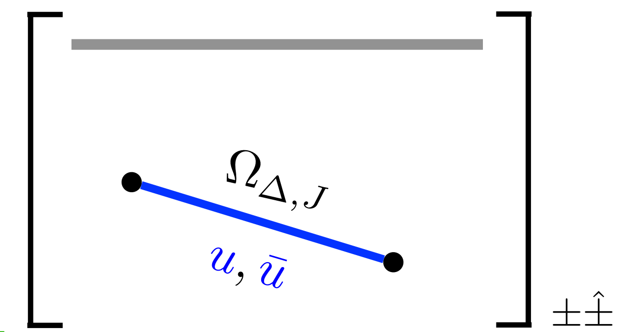
Bridging the Gap between EAdS and dS

Propagators in EAdS and dS take a universal form, constructed from 3 building blocks.

In dS, for the $\pm\hat{\pm}$ branch of the in-in contour, we have:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



Harmonic function, $\left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0$

$m^2 R_{\mathrm{dS}}^2=\left(\Delta_+\Delta_-+J\right)$

Recall the general solution to the wave equation near the boundary of dS, $\eta\to 0$

$$\varphi\left(\eta,\mathbf{k}\right)=\alpha_{\pm\hat{\pm}}\underbrace{\left(-\eta\right)^{\Delta_+}\left[\mathcal{O}_{\Delta_+}\left(\mathbf{k}\right)+O\left(\eta^2\right)\right]}_{\text{Selected by }\omega_D\left(u,\bar{u}\right)}+\beta_{\pm\hat{\pm}}\underbrace{\left(-\eta\right)^{\Delta_-}\left[\mathcal{O}_{\Delta_-}\left(\mathbf{k}\right)+O\left(\eta^2\right)\right]}_{\text{Selected by }\omega_N\left(u,\bar{u}\right)}$$

For the Bunch Davies (Euclidean) vacuum we have:

$$\alpha_{\pm\pm}=\beta_{\mp\mp}=\csc\left(\pi\left(\tfrac{d}{2}-\Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm-\tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp}=-\beta_{\mp\pm}=\pm\csc\left(\pi\left(\tfrac{d}{2}-\Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm-\tfrac{d}{2}\right)\pi i\right]$$

$$ds^2=\left(\frac{R_{\mathrm{dS}}}{\eta}\right)^2\left(-d\eta^2+d\mathbf{x}^2\right)$$

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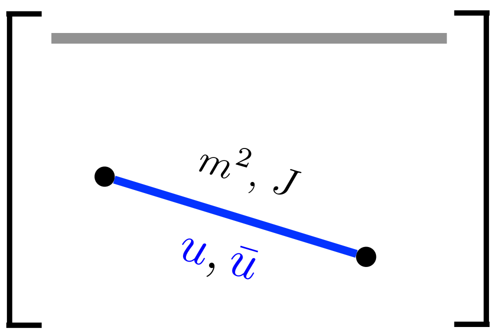
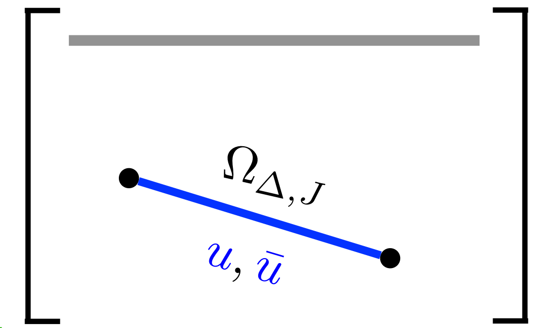
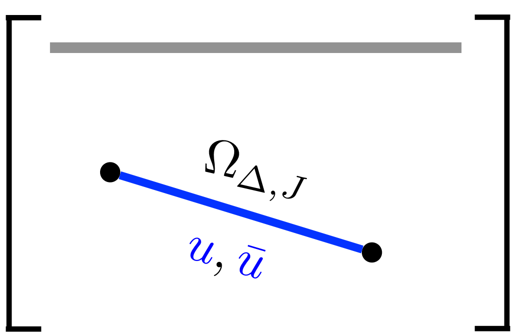


$$\left[\;\cdot\;\right]_{\pm\hat{\pm}} = \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}} \underbrace{\left[\;\cdot\;\right]_{\pm\hat{\pm}}}_{\text{Harmonic function, } \left(\nabla^2-m^2\right)\Omega_{\Delta,J}=0}$$

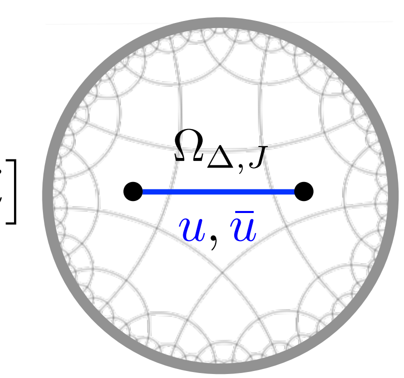


dS and EAdS Harmonic functions differ by a simple phase:

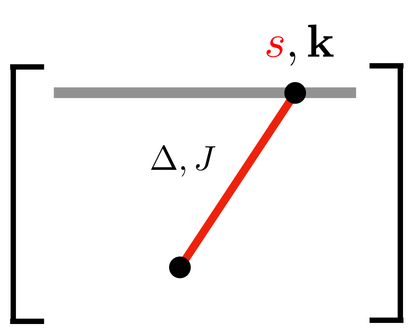
$m^2 R_{\text{dS}}^2 = \left(\Delta_+\Delta_- + J\right)$



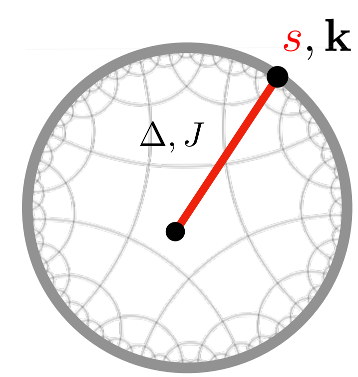
$$\left[\;\cdot\;\right]_{\pm\hat{\pm}} = \exp\left[\mp\left(u+\tfrac{1}{2}\left(\Delta_+-\tfrac{d}{2}\right)\right)\pi i\right]\exp\left[\hat{\mp}\left(\bar{u}+\tfrac{1}{2}\left(\Delta_--\tfrac{d}{2}\right)\right)\pi i\right]$$



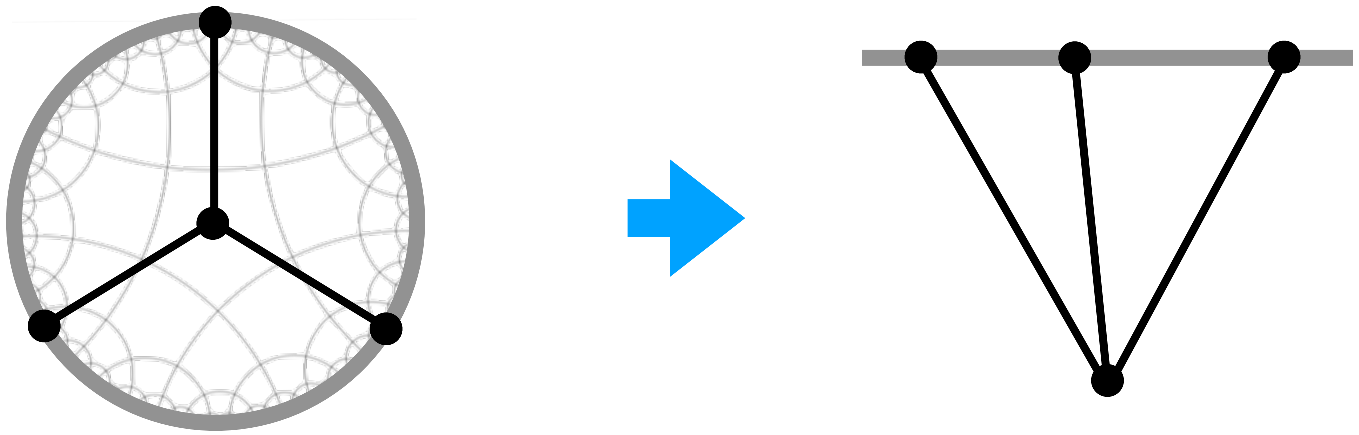
Also the bulk-boundary propagators:

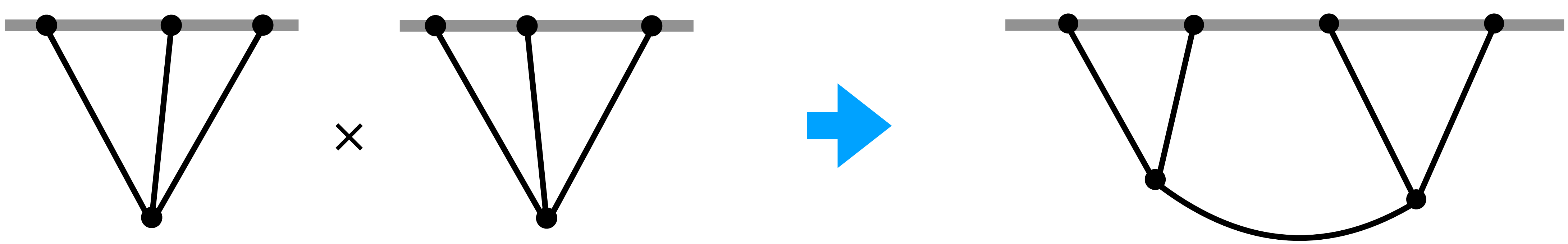


$$\left[\;\cdot\;\right]_{\pm} = \exp\left[\mp\left(s+\tfrac{1}{2}\left(\Delta-\tfrac{d}{2}\right)\right)\pi i\right]$$

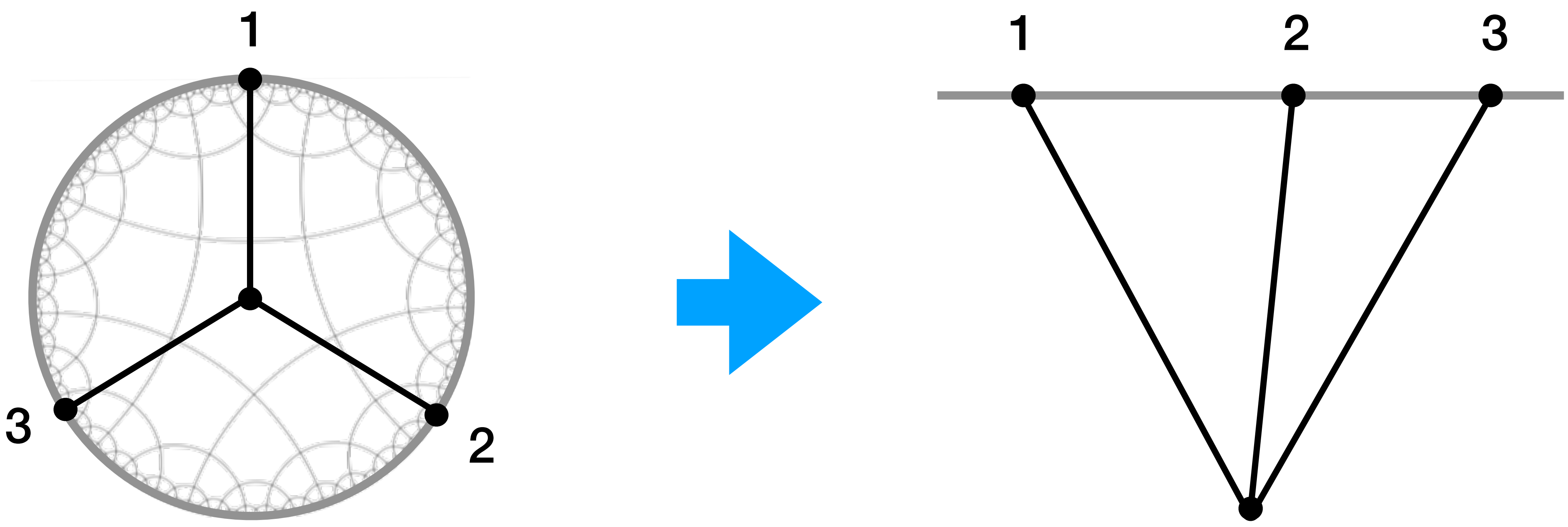


Outline

1. 

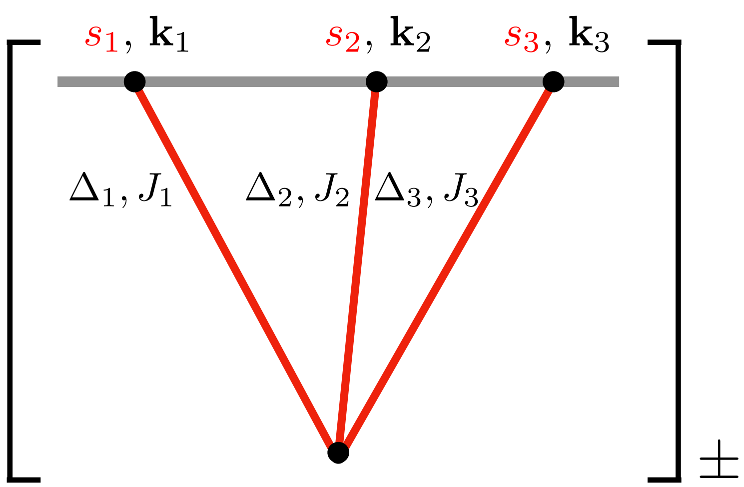
2. 

3pt Contact

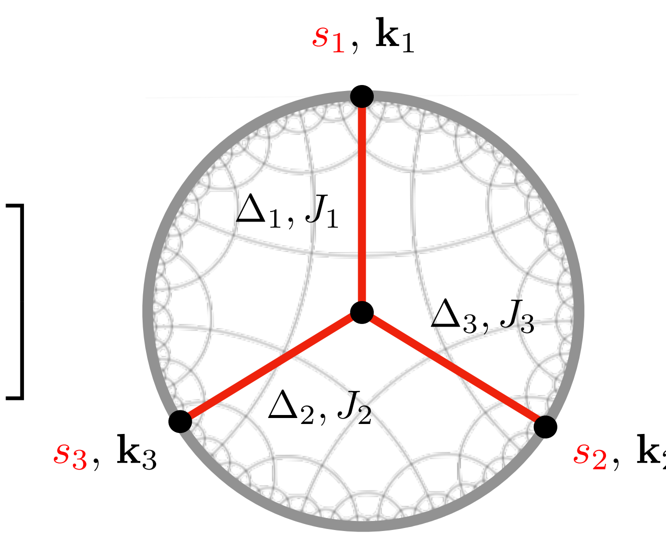


3pt Contact

Contact amplitudes in dS can be obtained directly from their EAdS counterparts:



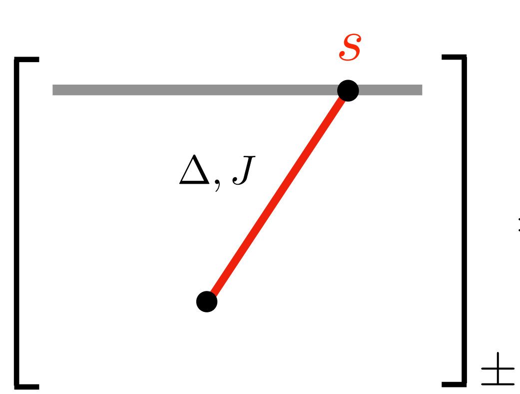
$$= \pm i \exp\left[\mp \pi i \sum_{j=1}^{3} \left(s_j + \tfrac{1}{2}\left(\Delta_j - \tfrac{d}{2}\right)\right)\right]$$



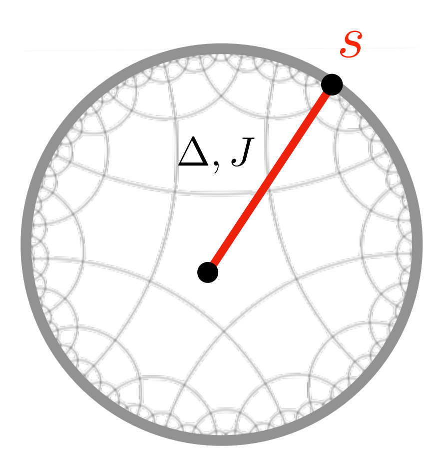
Overall phase is constant, as required by the Dilatation Ward identity, since:

$$s_1 + s_2 + s_3 = \frac{d + 2\left(J_1 + J_2 + J_3\right)}{4}$$

Above we simply used that:

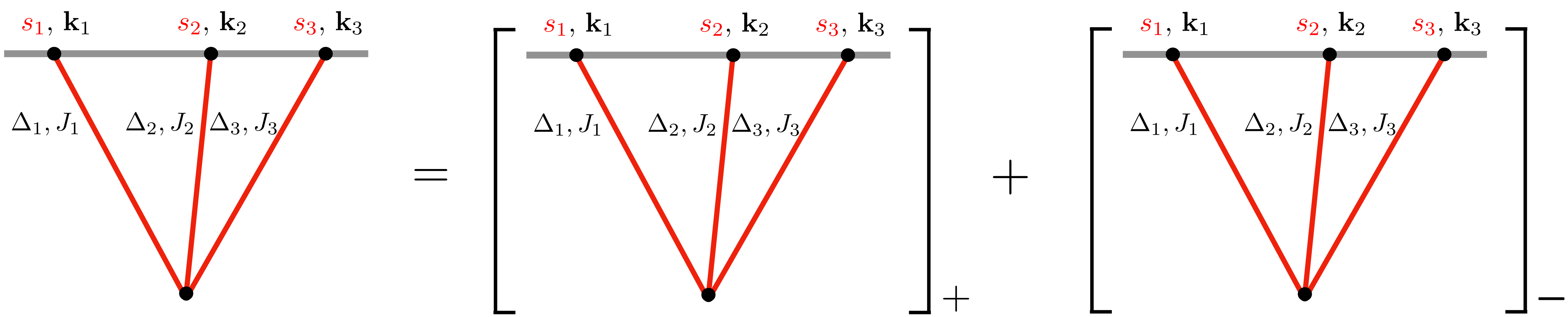


$$= \exp\left[\mp \left(s + \tfrac{1}{2}\left(\Delta - \tfrac{d}{2}\right)\right)\pi i\right]$$



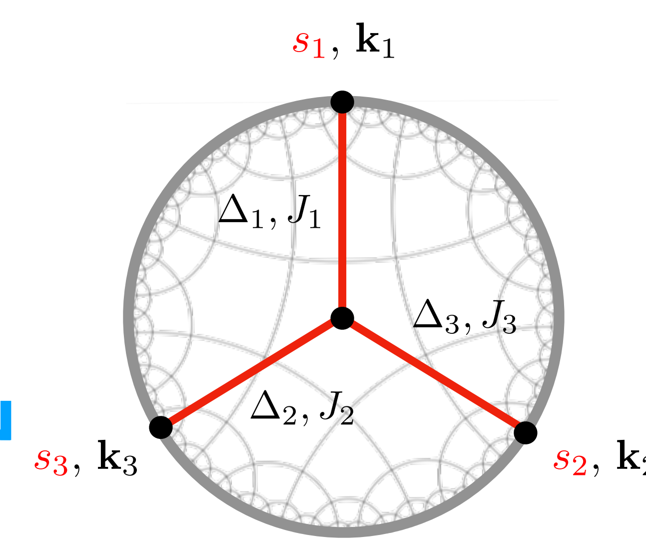
3pt Contact

The full de Sitter 3pt function is the sum from each branch of the in-in contour:

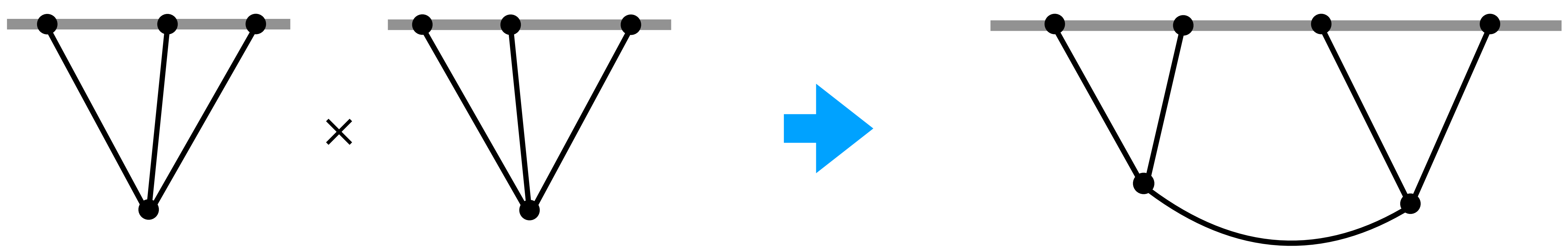


…which manifests an interference effect:

$$= \underbrace{\sin\left[\left(-d+\sum_{i=1}^{3}(\Delta_i+J_i)\right)\frac{\pi}{2}\right]}_{\text{interference factor}}$$

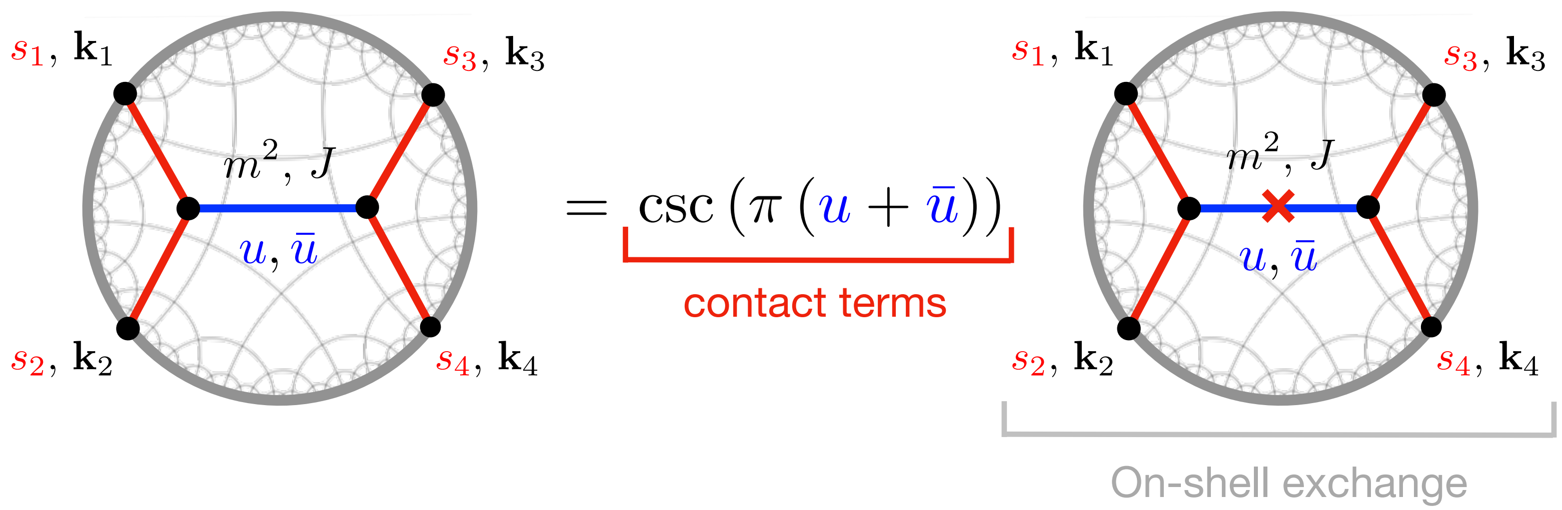


Exchanges



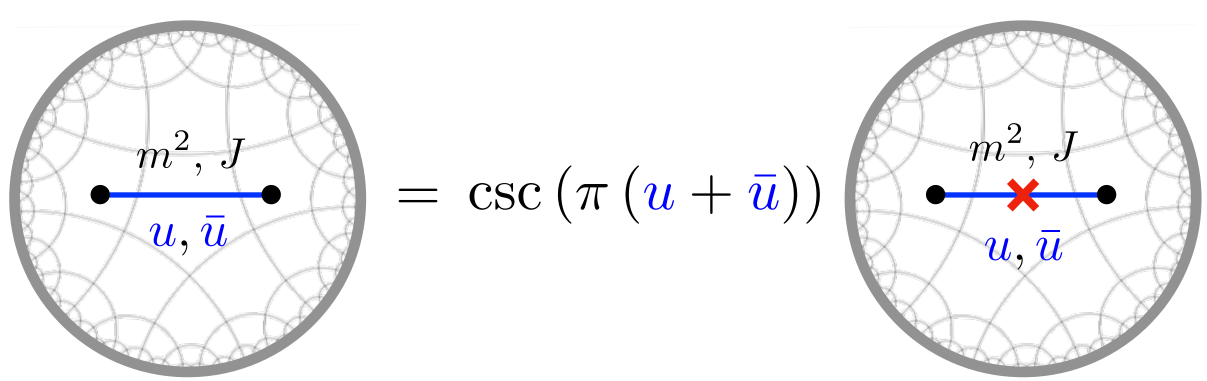
Exchanges in EAdS

Exchanges are straightforwardly reconstructed from their on-shell part:



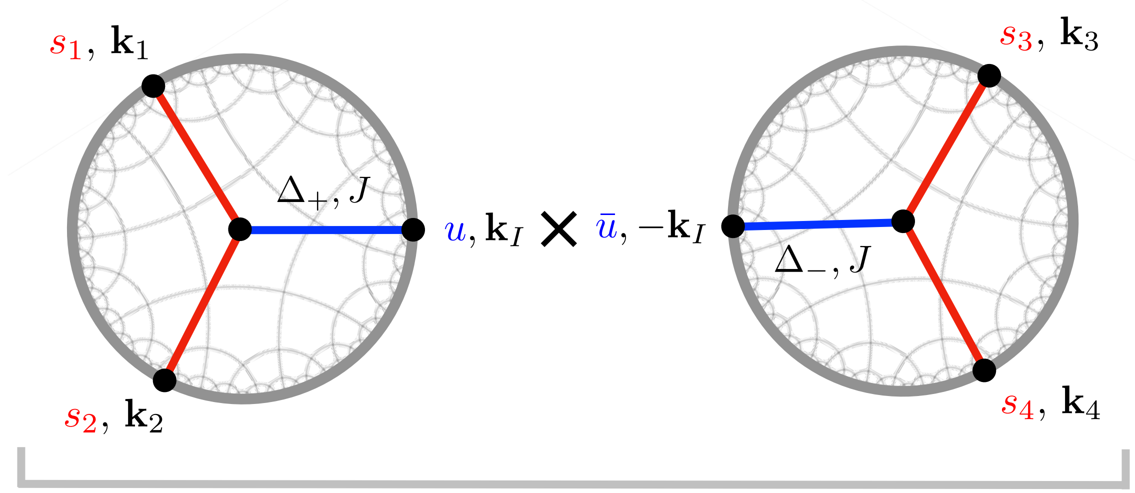
Constrained by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

Simply follows from:



Exchanges in EAdS

Factorisation and Conformal Symmetry:



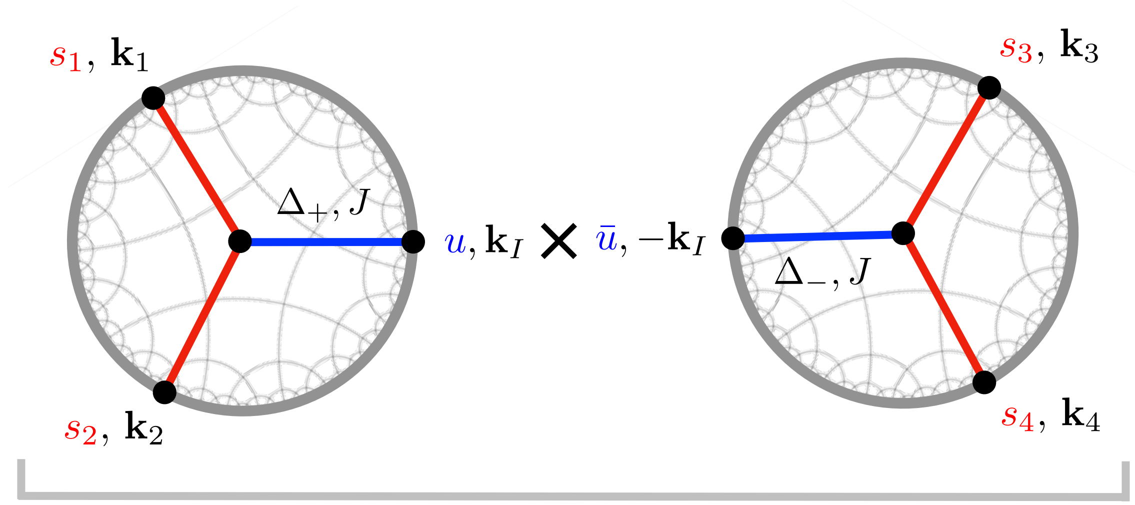
"Conformal Partial Wave",
single valued Eigenfunction of Conformal Casimirs

Mack, Dobrev, Petkova, Petrova, Todorov, 1974-7

$$m^2 R^2_{\mathrm{AdS}} = -\left(\Delta_+ \Delta_- + J\right)$$

Exchanges in EAdS

Factorisation and Conformal Symmetry:

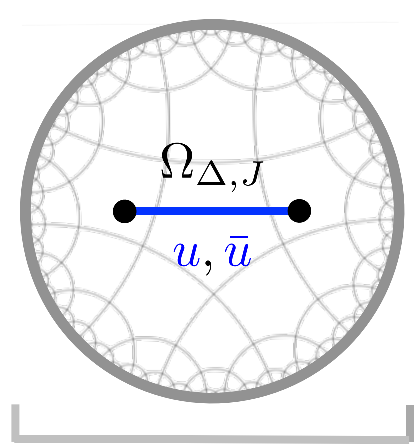


"Conformal Partial Wave",
single valued Eigenfunction of Conformal Casimirs

Mack, Dobrev, Petkova, Petrova, Todorov, 1974-7

$m^2 R^2_{\text{AdS}} = -\left(\Delta_+\Delta_- + J\right)$

attach/remove external legs

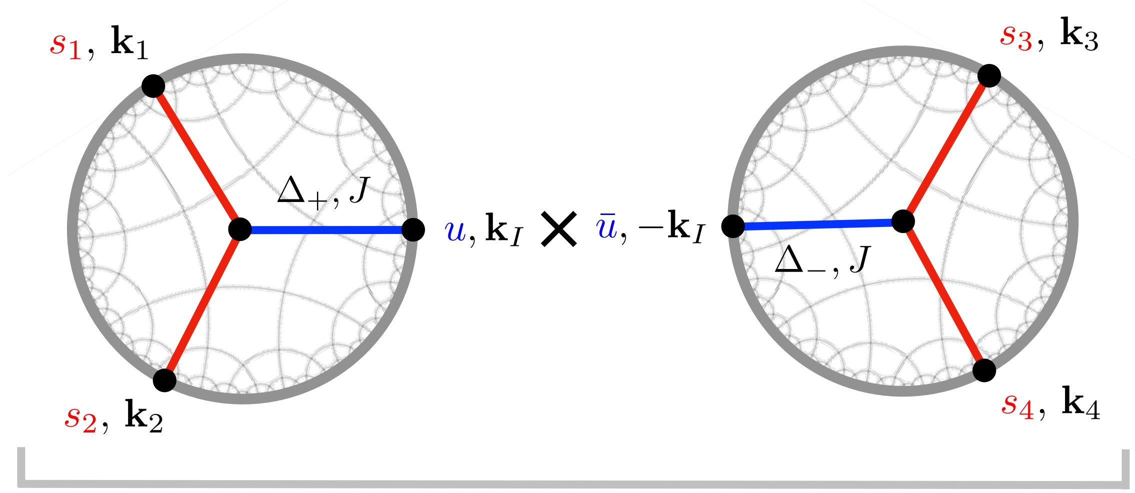


Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

Exchanges in EAdS

Factorisation and Conformal Symmetry:



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$m^2 R^2_{\text{AdS}} = -\left(\Delta_+ \Delta_- + J\right)$

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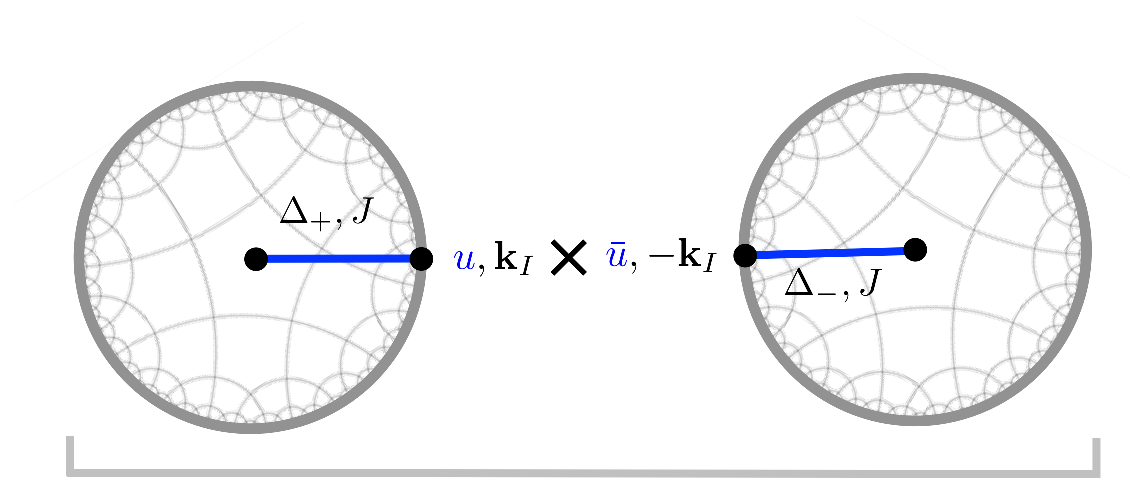


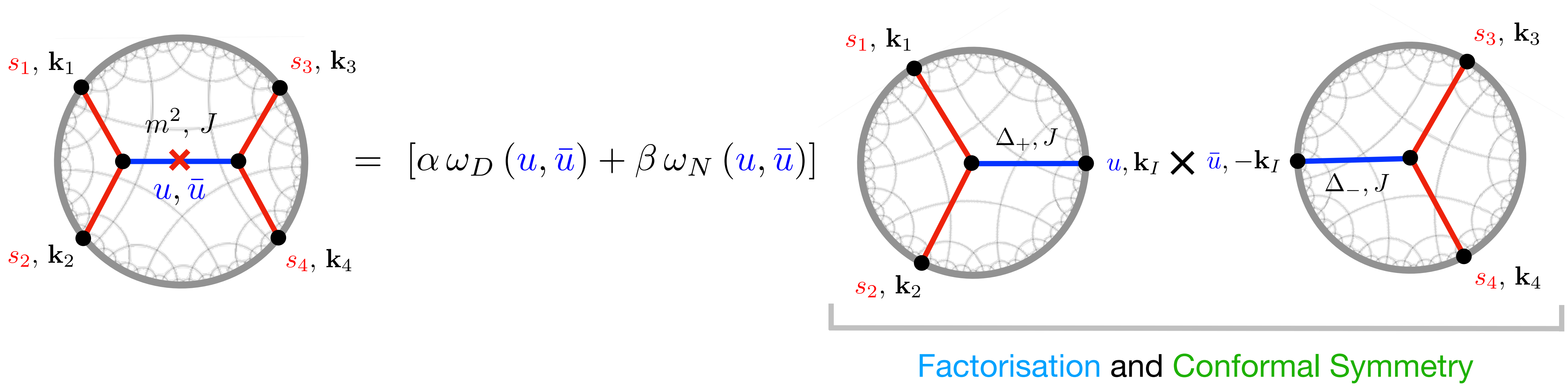
Harmonic function, $\left(\nabla^2 - m^2\right)\Omega_{\Delta,J} = 0$

This duality is made manifest by the "split representation" of $\Omega_{\Delta,J}$

e.g. Leonhardt, Manvelyan, Rühl 2003;
Costa, Gonçalves, Penedones 2014

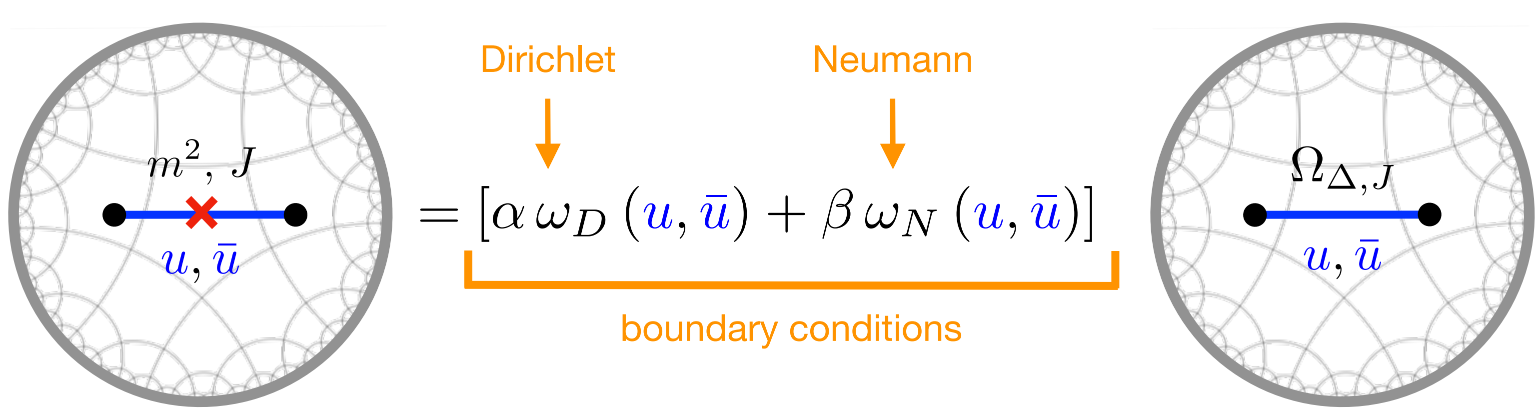
Exchanges in EAdS

Factorisation, Conformal Symmetry and boundary conditions:



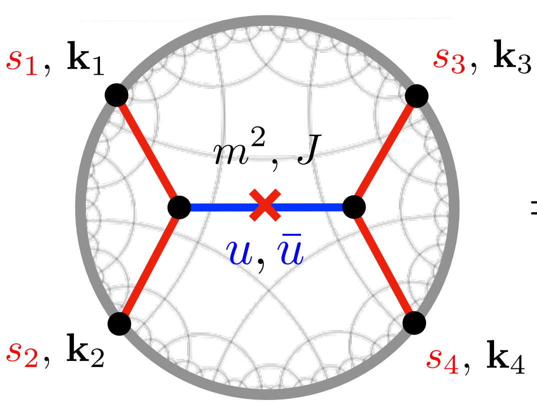
$m^2 R_{\text{AdS}}^2 = -\left(\Delta_+ \Delta_- + J\right)$

attach/remove external legs

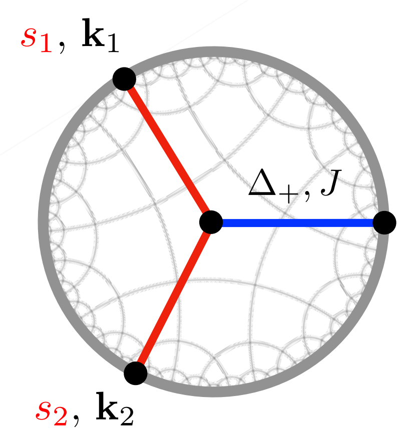


Exchanges in EAdS

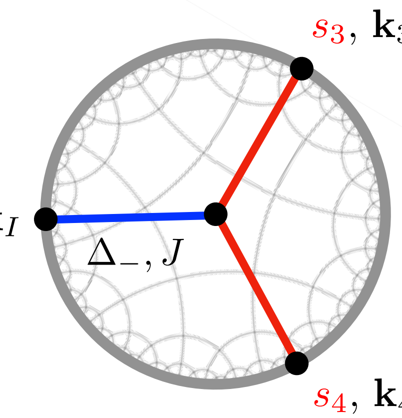
Factorisation, Conformal Symmetry and boundary conditions:



$$= \underbrace{\left[\alpha\, \omega_D\left(u, \bar{u}\right) + \beta\, \omega_N\left(u, \bar{u}\right)\right]}_{\text{boundary conditions}}$$



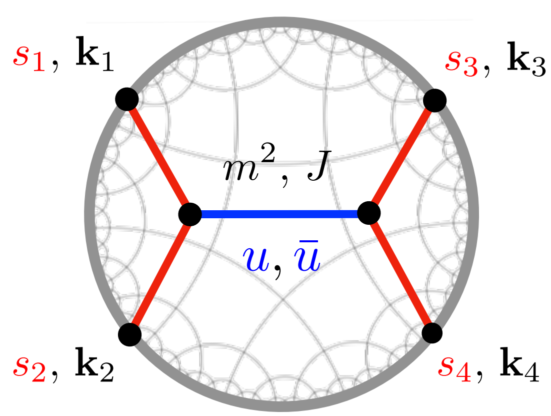
$u, \mathbf{k}_I \times \bar{u}, -\mathbf{k}_I$



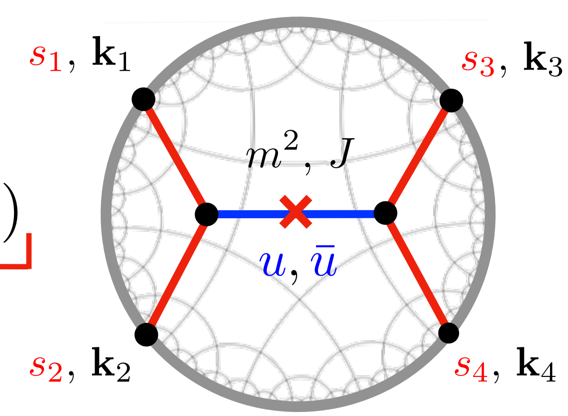
Factorisation and Conformal Symmetry

The full exchange is reconstructed via:

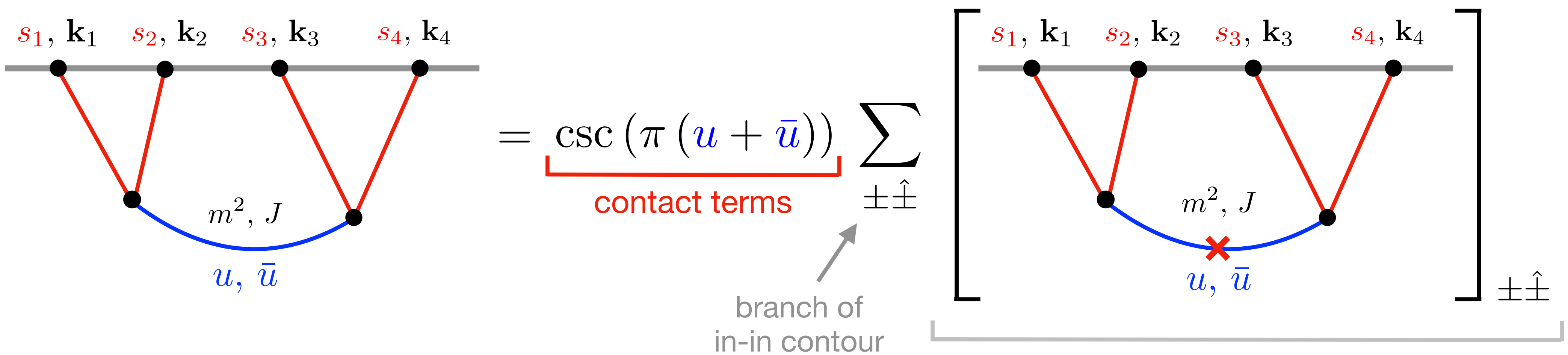
$$m^2 R^2_{\text{AdS}} = -\left(\Delta_+ \Delta_- + J\right)$$



$$= \underbrace{\csc\left(\pi\left(u + \bar{u}\right)\right)}_{\text{contact terms}}$$



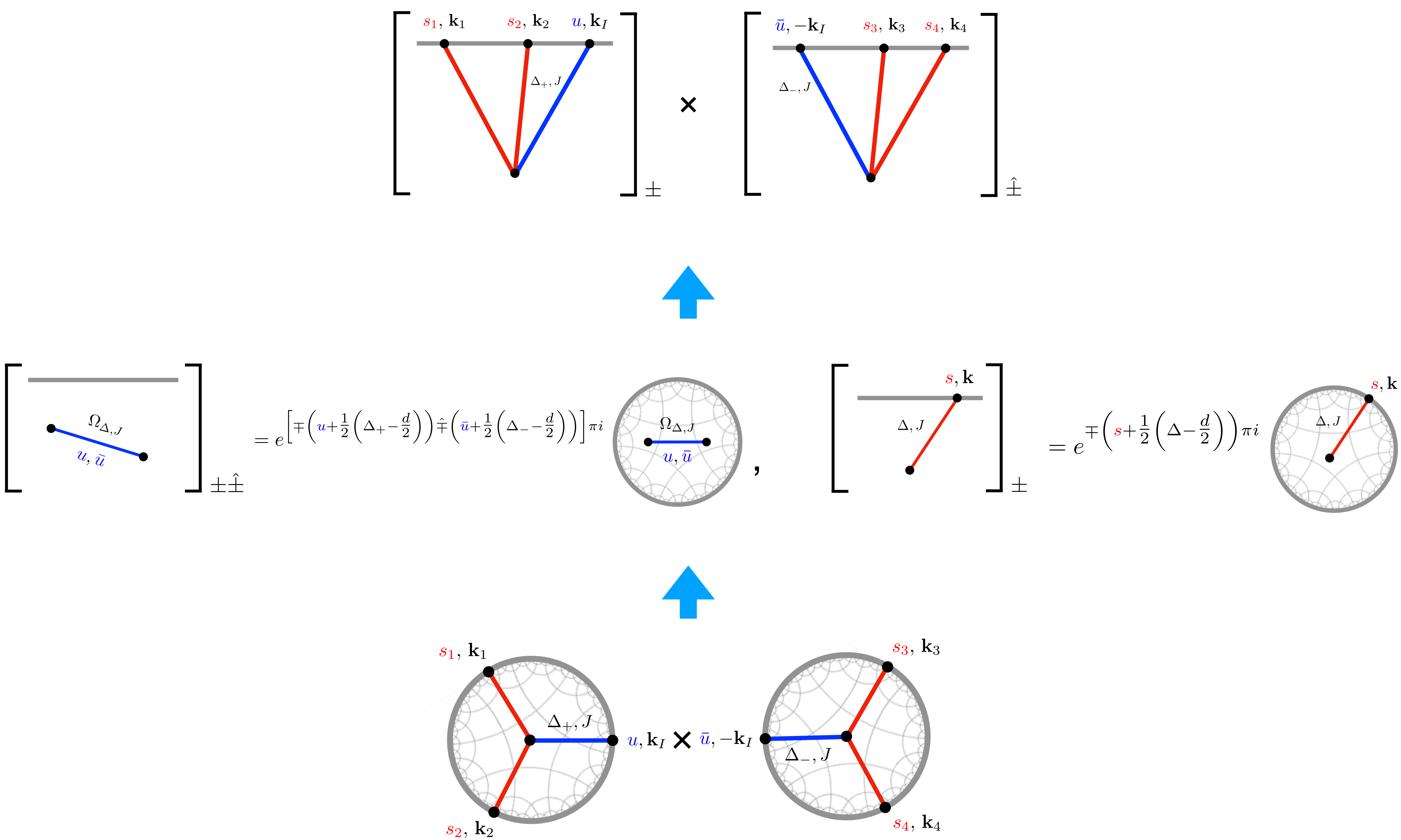
Exchanges in dS



Fixed by:
- Factorisation
- Conformal Symmetry
- Boundary Conditions

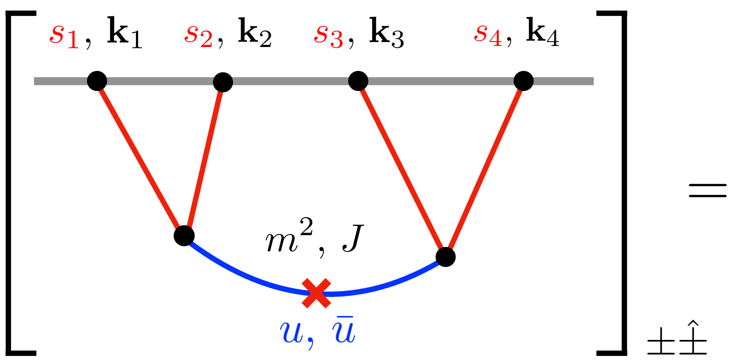
Exchanges in dS

Factorisation and Conformal Symmetry:

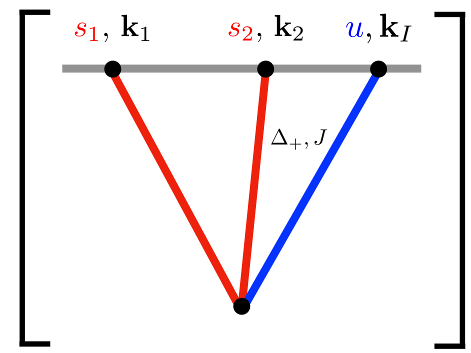


Exchanges in dS

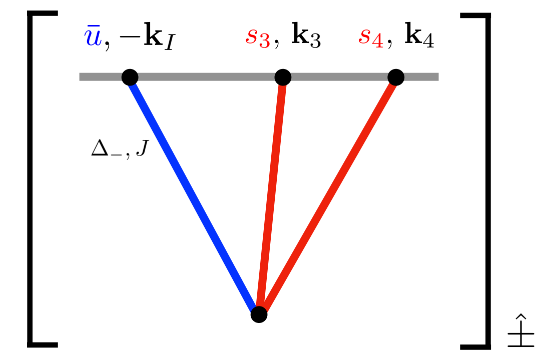
Factorisation, Conformal Symmetry and boundary conditions:



$$= \left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$

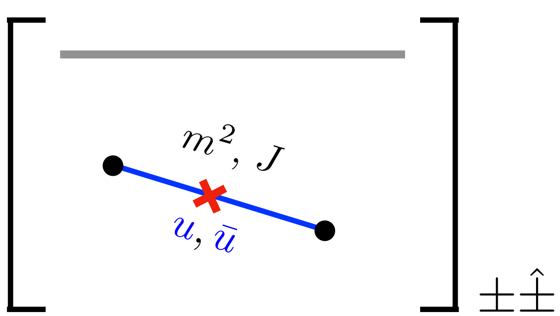


$\times$

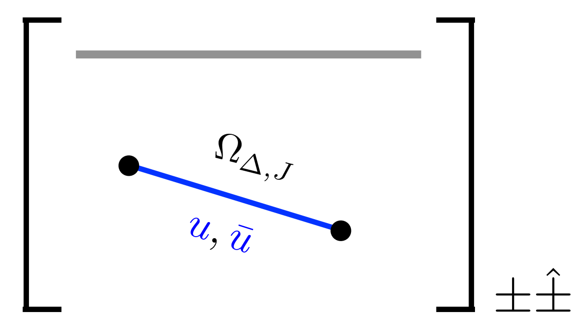


Factorisation + Conformal Symmetry

attach/remove
external legs



$$= \underbrace{\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$

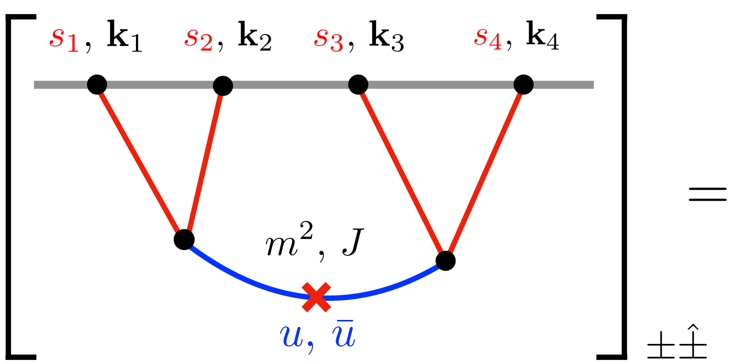


For the Bunch Davies (Euclidean) vacuum:

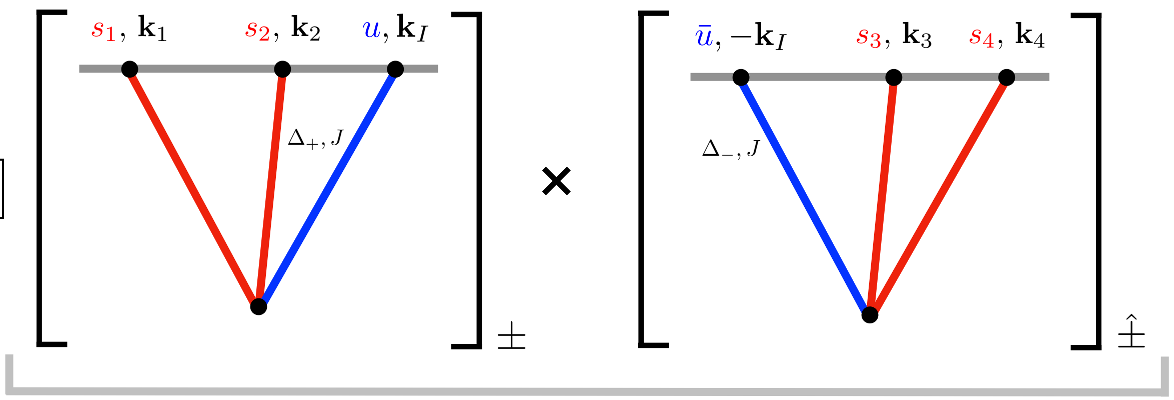
$$\alpha_{\pm\pm} = \beta_{\mp\mp} = \csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[-\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right], \qquad \alpha_{\pm\mp} = -\beta_{\mp\pm} = \pm\csc\left(\pi\left(\tfrac{d}{2} - \Delta_\pm\right)\right)\exp\left[\left(\Delta_\pm - \tfrac{d}{2}\right)\pi i\right]$$

Exchanges in dS

Factorisation, Conformal Symmetry and boundary conditions:

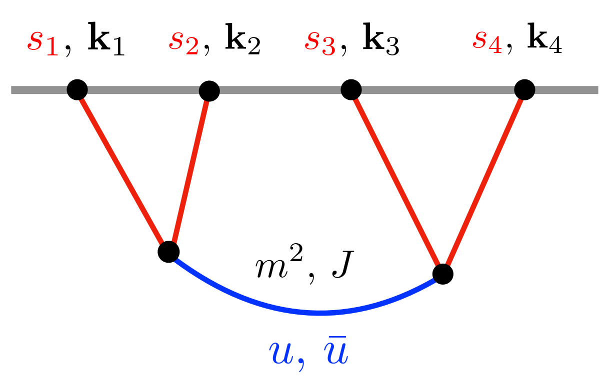


$$= \left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



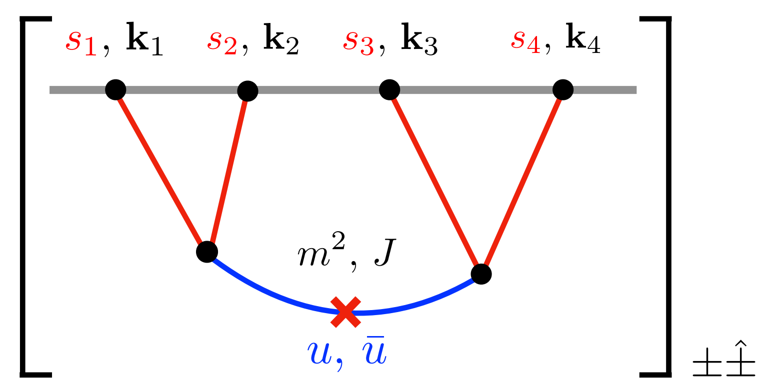
Factorisation + Conformal Symmetry

The full exchange is reconstructed via:



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \sum_{\pm\hat{\pm}}$$

branch of in-in contour



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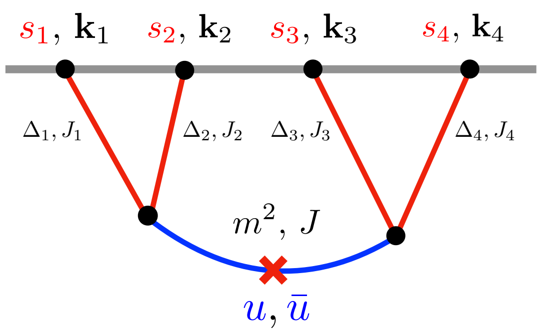
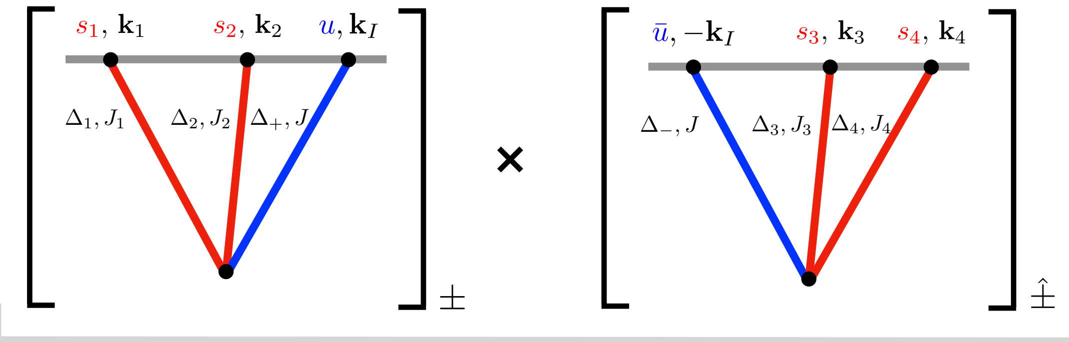
Exchanges in dS

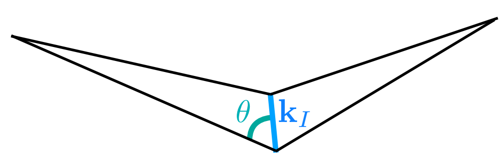


$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\,(u,\bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N\,(u,\bar{u})\right]$$

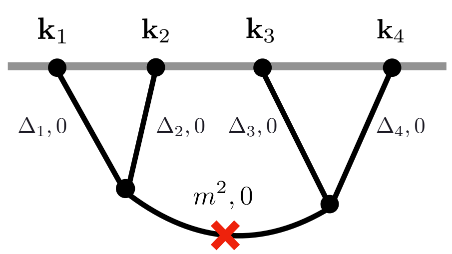


$$\sim (|\mathbf{k}_I|)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)



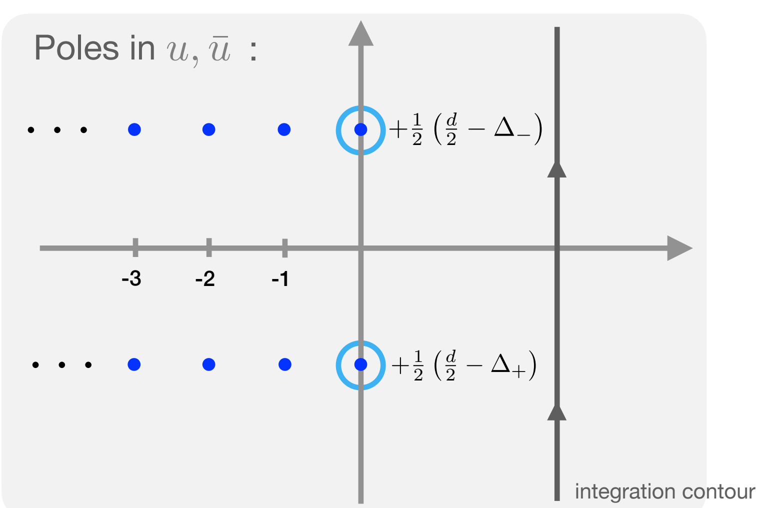
The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If all the fields are **scalars**:



non-analytic

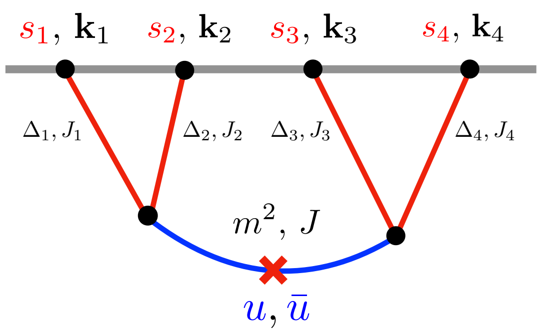
$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_+-\frac{d}{2}} \left[1+\mathcal{O}\left(k_I^2\right)\right]$$

$$+\ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_--\frac{d}{2}} \left[1+\mathcal{O}\left(k_I^2\right)\right] + \ldots$$

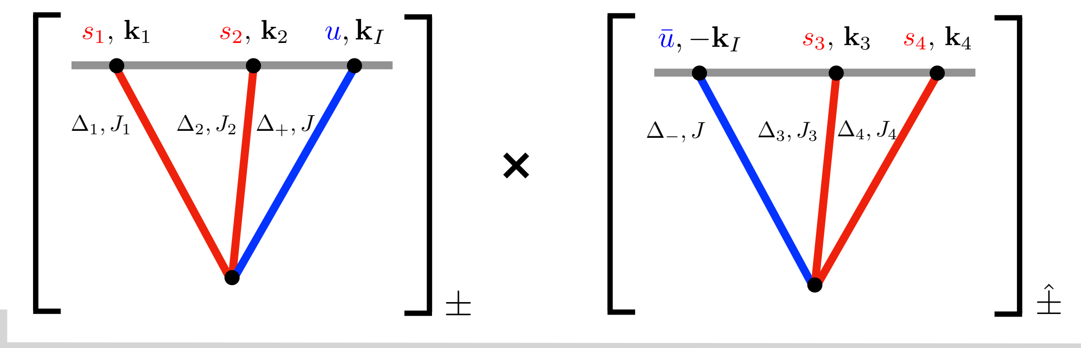


External conformally coupled/massless scalars:
Arkani-Hamed and Maldacena 2015;
Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS

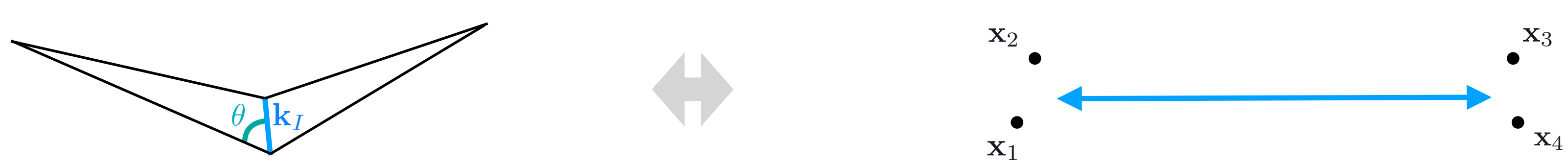


$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \cancel{\beta_{\pm\hat{\pm}}}\, \omega_N\left(u,\bar{u}\right)\right]$$

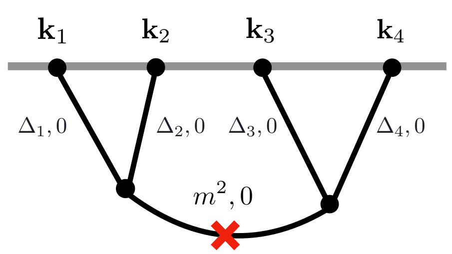


$$\sim \left(|\mathbf{k}_I|\right)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)



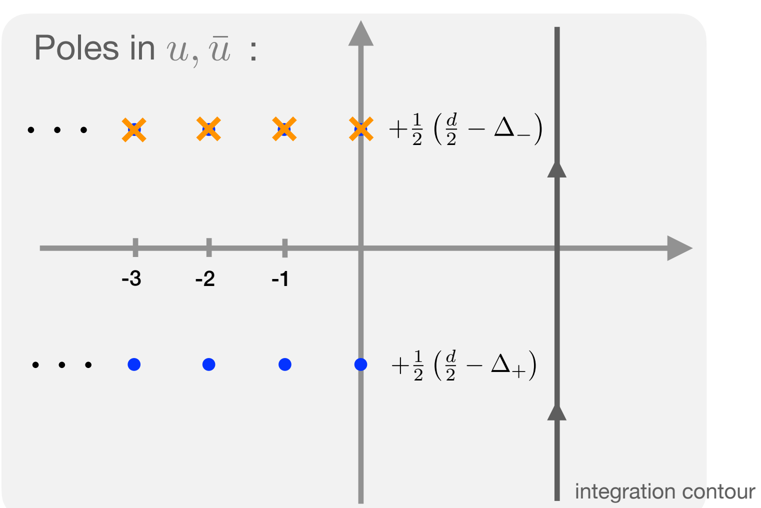
The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If all the fields are **scalars**:



non-analytic

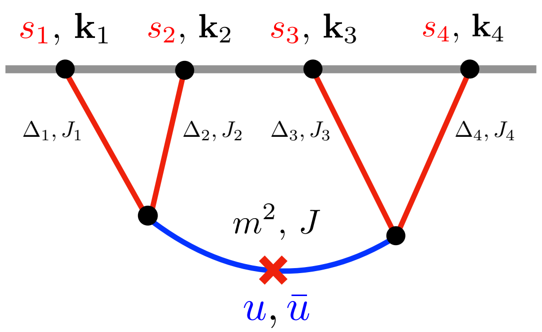
$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_+ - \frac{d}{2}} \left[1 + \mathcal{O}\left(k_I^2\right)\right]$$

$$\cancel{+ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_- - \frac{d}{2}} \left[1 + \mathcal{O}\left(k_I^2\right)\right]} + \ldots$$

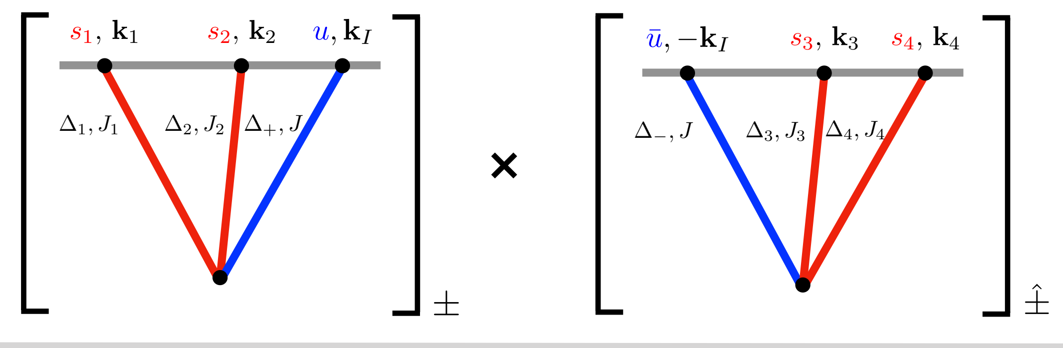


External conformally coupled/massless scalars:
Arkani-Hamed and Maldacena 2015;
Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS

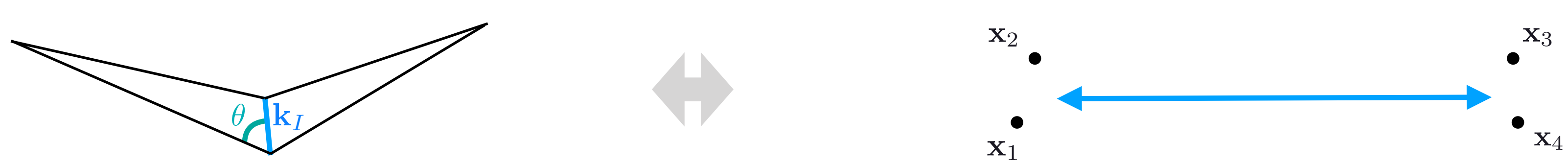


$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



$$\sim \left(|\mathbf{k}_I|\right)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)



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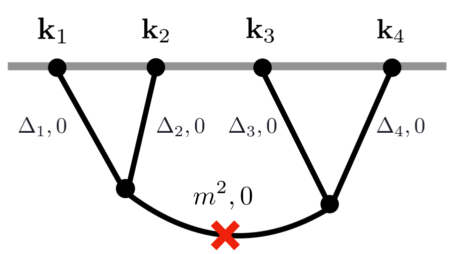
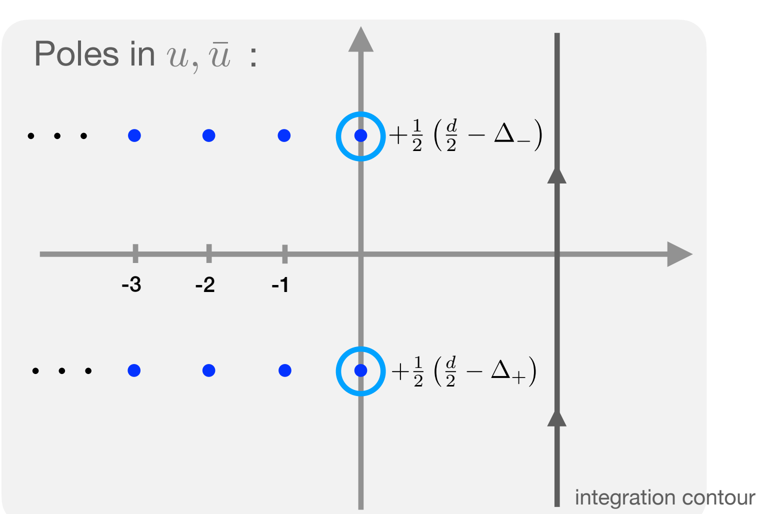


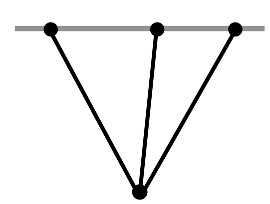
non-analytic

$$\overset{|\mathbf{k}_I| \ll |\mathbf{k}_i|}{\sim} \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_+-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_+-d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_+-\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right]$$

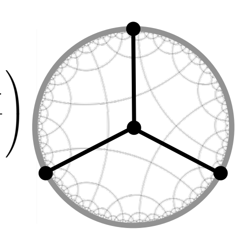
$$+\ \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_--d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_--d}{2}\right)\pi\right)\left(k_I^2\right)^{\Delta_--\frac{d}{2}}\left[1+\mathcal{O}\left(k_I^2\right)\right]\ +\ldots$$

Factorisation





$$= \sin\left(\left(-d+\sum_{j=1}^{3}\Delta_j\right)\frac{\pi}{2}\right)$$



External conformally coupled/massless scalars:
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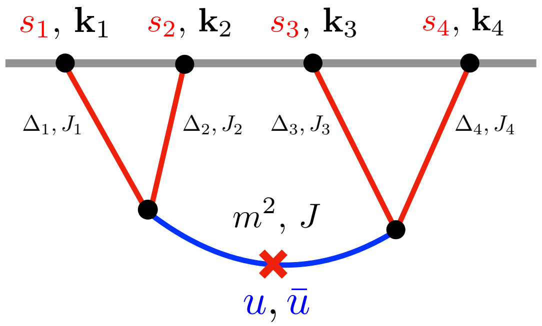
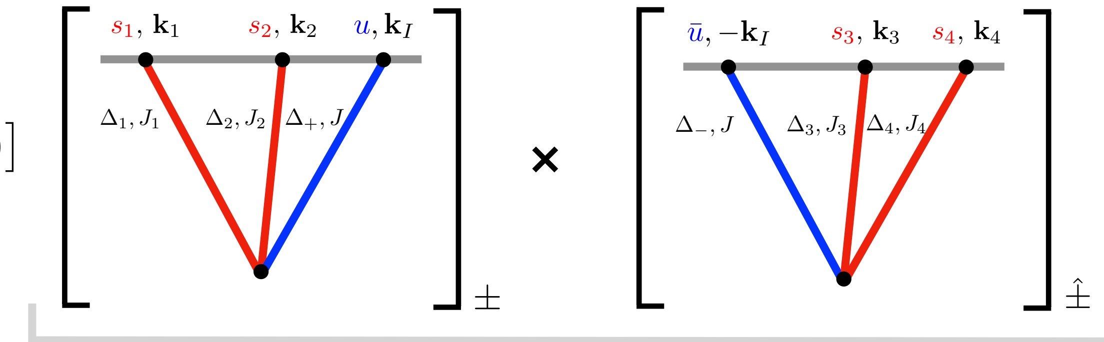
Exchanges in dS



$$= \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\,(u,\bar{u}) + \beta_{\pm\hat{\pm}}\, \omega_N\,(u,\bar{u})\right]$$



$$\sim (|\mathbf{k}_I|)^{-2(u+\bar{u})}$$

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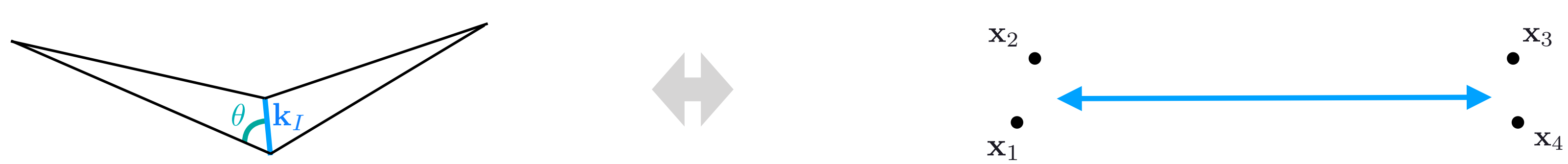
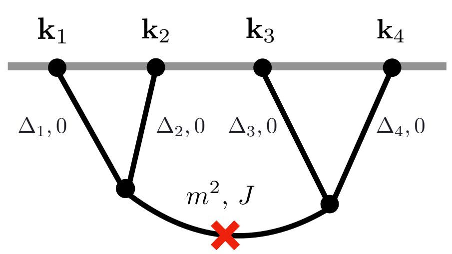


The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If the exchanged field has **spin J**:



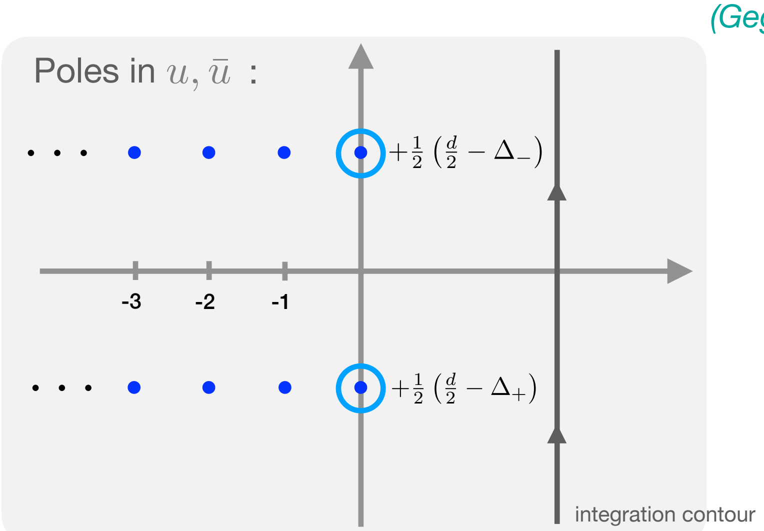
non-analytic

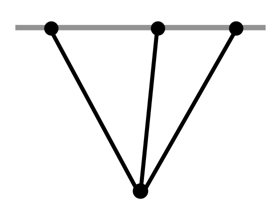
$$\overset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta)\left[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_+-\frac{d}{2}} + \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right)\sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right)(k_I^2)^{\Delta_--\frac{d}{2}}\right]$$

angular dependence, (Gegenbauer polynomial)

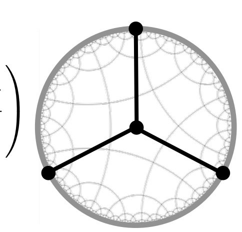
Factorisation

$+\dots$



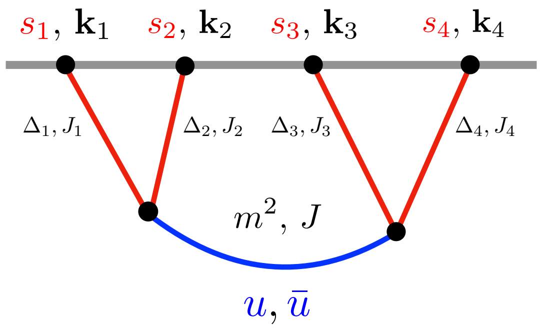


$$= \sin\left(\left(-d+J+\sum_{j=1}^{3}\Delta_j\right)\frac{\pi}{2}\right)$$

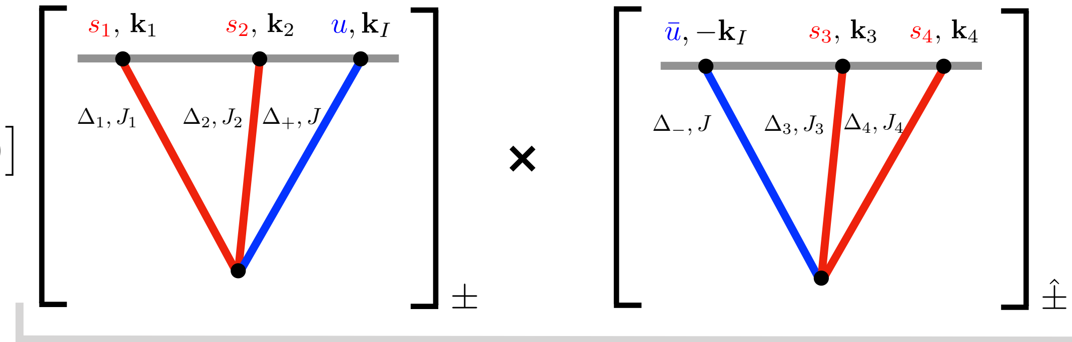


External conformally coupled/massless scalars:
Arkani-Hamed and Maldacena 2015;
Arkani-Hamed, Baumann, Lee and Pimentel 2018

Exchanges in dS



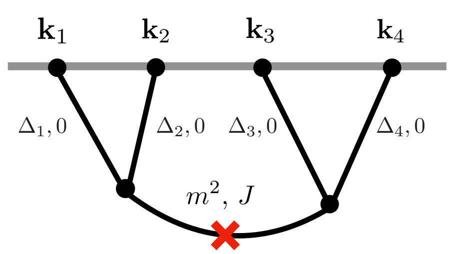
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{contact terms}} \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



$$\sim \left(|\mathbf{k}_I|\right)^{-2(u+\bar{u})}$$

The imprints of a particle exchange are particularly sharp in the limit $|\mathbf{k}_I| \ll |\mathbf{k}_j|$ (OPE limit)

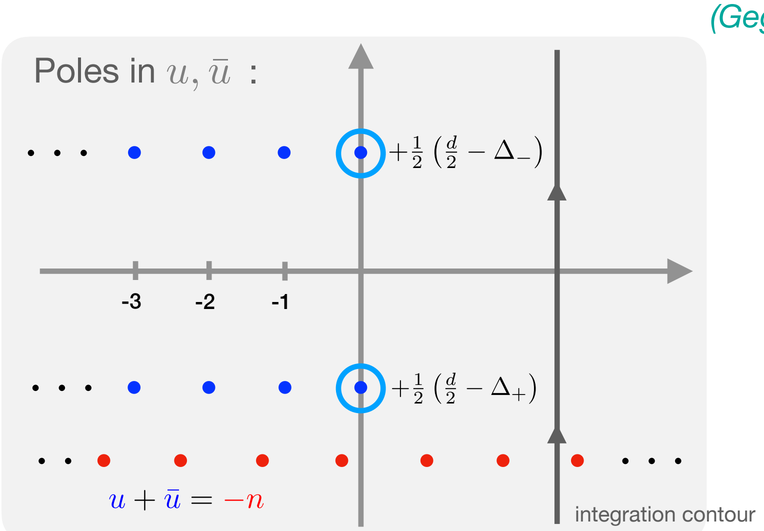
The expansion in this limit is generated by residues of poles in $u, \bar{u}$. If the exchanged field has **spin J**:



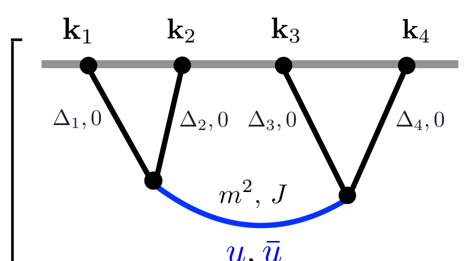
non-analytic

$$\overset{|\mathbf{k}_I|\ll|\mathbf{k}_i|}{\sim} \underbrace{C_J^{\left(\frac{d-2}{2}\right)}(\cos\theta)}_{\text{angular dependence, (Gegenbauer polynomial)}} \left[\sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_++J-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_++J-d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_+-\frac{d}{2}} \right.$$

$$\left. + \sin\left(\left(\frac{\Delta_1+\Delta_2+\Delta_-+J-d}{2}\right)\pi\right) \sin\left(\left(\frac{\Delta_3+\Delta_4+\Delta_-+J-d}{2}\right)\pi\right) \left(k_I^2\right)^{\Delta_--\frac{d}{2}} \right]$$



$$+\sum_{n=0}^{\infty} \mathrm{Res}_{u+\bar{u}=-n}\left[\ \cdots\ \right] \qquad + \ldots$$



analytic $\propto \left(k_I^2\right)^n$

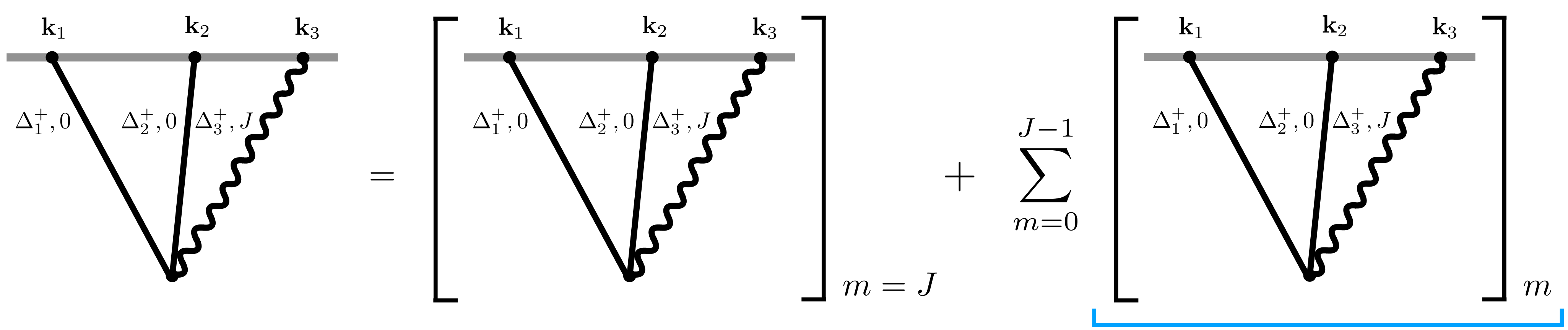
$\sim\ H^2/m^2$, EFT expansion

External conformally coupled/massless scalars: Arkani-Hamed and Maldacena 2015; Arkani-Hamed, Baumann, Lee and Pimentel 2018

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Decomposition into helicities m = 0, 1, …, J :



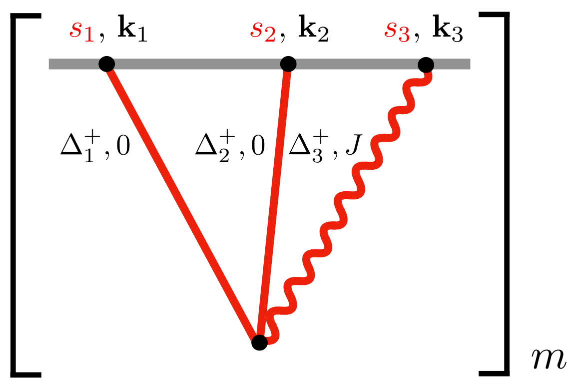
gauge invariance → lower helicity components cannot contain bulk contact terms

i.e. no singularity in $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| \to 0$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

For each helicity component we have:

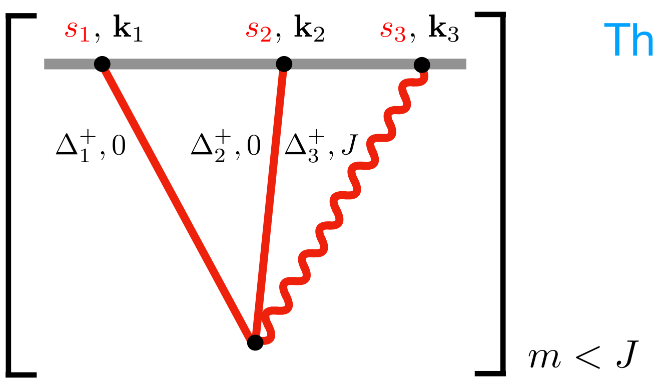


$$\propto \underbrace{\delta\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)}_{\text{Dilatation Ward identities}}$$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Gauge invariance requires that for the lower helicity components m < J we have:



This combination cancels the bulk contact singularity and replaces it with a boundary term:

$$\propto \underbrace{\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)\delta\left(\tfrac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3\right)}$$

$$\lim_{\eta_0 \to 0} \int_{-\infty}^{\eta_0} d\eta\, \partial_\eta \left(\eta^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right) = \lim_{\eta_0 \to 0} \left(\eta_0^{\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3}\right)$$

cf. $\mathbf{k}\,\delta(\mathbf{k}) = \int d\mathbf{x}\,(-i\partial_{\mathbf{x}})\left(e^{i\mathbf{k}\cdot\mathbf{x}}\right)$

Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✔

Scalars of unequal mass ✘

(Consistent with Berends, Burgers and van Dam 1986)

Constraints on Massless Particles

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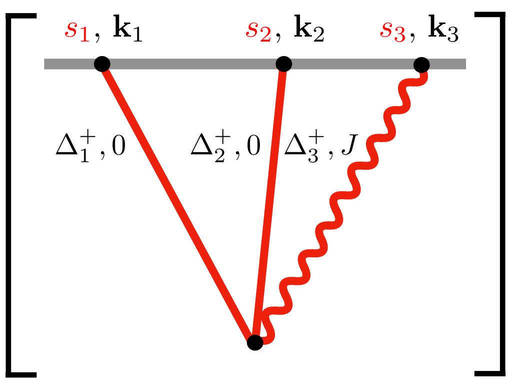

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Scalars of equal mass $\Delta_1 = \Delta_2 = \Delta$ ✔

cf. $\mathbf{k}\,\delta(\mathbf{k}) = \int d\mathbf{x}\,(-i\partial_{\mathbf{x}})\left(e^{i\mathbf{k}\cdot\mathbf{x}}\right)$

A non-trivial Ward-Takahashi identity is generated by the finite number of poles that satisfy:

$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

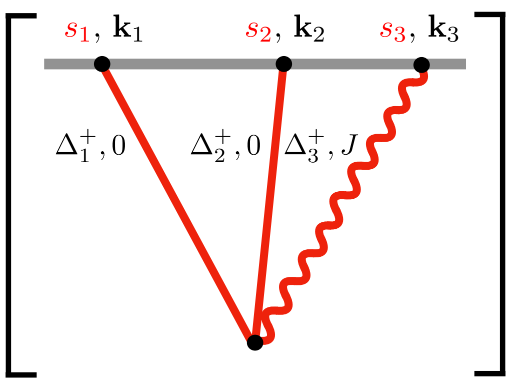
which are: $s_1 = \pm\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_1, \quad s_2 = \mp\frac{1}{2}\left(\Delta - \frac{d}{2}\right) - n_2, \quad s_3 = \frac{1}{2}\left(\Delta_3^+ - \frac{d}{2}\right) - n_3, \quad n_i \in \mathbb{N}$

with $(n_1 + n_2 + n_3) = (J - 1 - m)$

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.

Gauge invariance requires that for the lower helicity components m < J we have:



$m < J$

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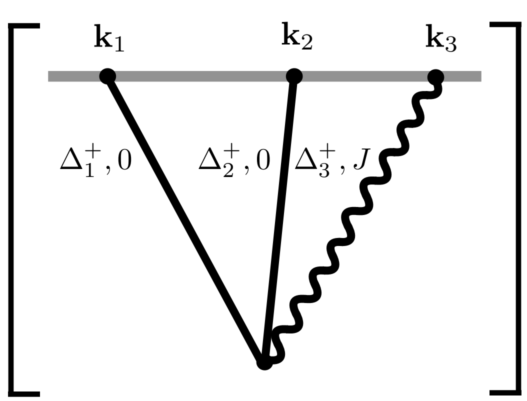
A non-trivial Ward-Takahashi identity is generated by the finite number of poles that satisfy:

$$\frac{d+2J}{4} - (J-m) - s_1 - s_2 - s_3 = 0$$

For example:

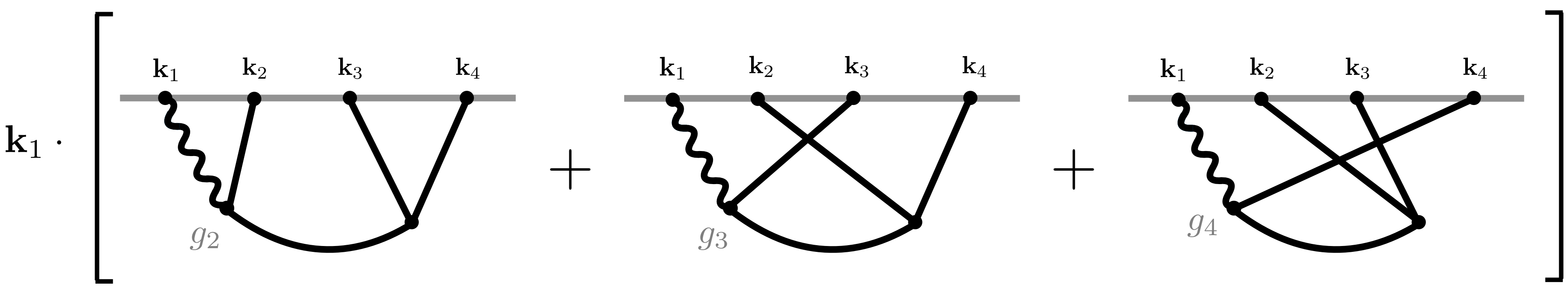


$m = J-1$

$$\sim \quad (\xi\cdot\mathbf{k}_{12})^{J-1}\langle\mathcal{O}_\Delta(\mathbf{k}_1)\,\mathcal{O}_\Delta(-\mathbf{k}_1)\rangle - (\xi\cdot\mathbf{k}_{12})^{J-1}\langle\mathcal{O}_\Delta(\mathbf{k}_2)\,\mathcal{O}_\Delta(-\mathbf{k}_2)\rangle$$

Constraints on Massless Particles

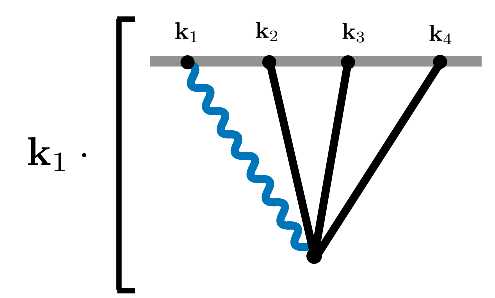
Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.



~ [Ward-Takahashi identity] + [Bulk contact singularities $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4| \to 0$]

Comes from the on-shell exchange, inherited from the gauge invariant 3pt functions

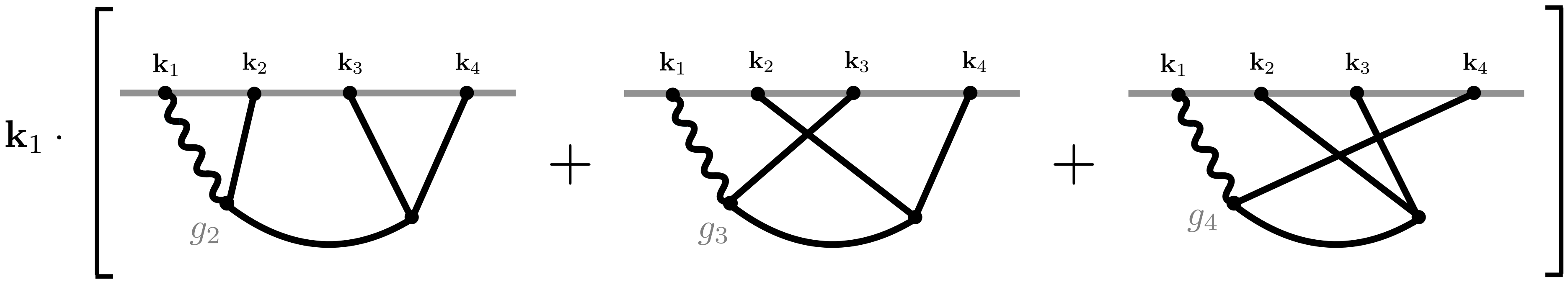
Violates gauge invariance. *Can they be compensated by:*



?

Constraints on Massless Particles

Toy model: Cubic coupling of a $\underbrace{\text{massless spin-J field}}_{\Delta_3^+ = d-2+J}$ to scalars.



$\sim$ [Ward-Takahashi identity] $+$ [Bulk contact singularities $E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4| \to 0$]

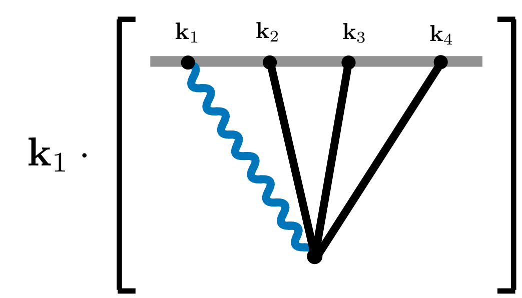
Comes from the on-shell exchange, inherited from the gauge invariant 3pt functions

Violates gauge invariance. *Can they be compensated by:*

Some singularities in E_T

cannot be cancelled!

(by local quartic vertices)



These singularities must therefore cancel by themselves $\rightarrow$ constrains g_i

The helicity-(J-1) component this gives the constraint:

$$\left[\sum_{i=2}^{4} g_i \, (\xi \cdot \mathbf{k}_i)^{J-1}\right] (E_T)^{\#} = 0$$

cf. Weinberg soft theorem in flat space!

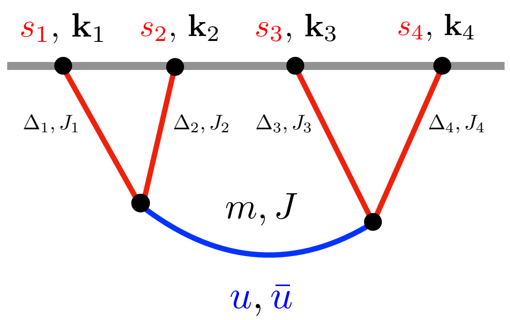
$J = 1: \; g_2 + g_3 + g_4 = 0,$ *charge conservation*

$J = 2: \; g_2 = g_3 = g_4,$ *equivalence principle*

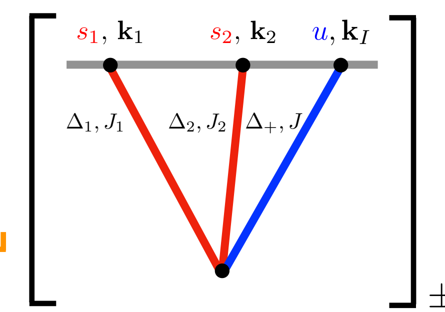
See also Baumann et al. May 2020

$J > 2: \; g_2 = g_3 = g_4 = 0,$ *no consistent coupling (in local theories)*

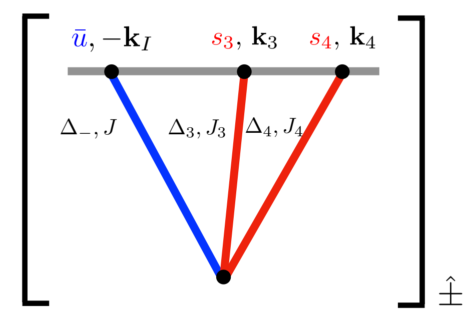
Summary



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{EFT}} \underbrace{\sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



$\times$



Factorisation + Conformal Symmetry

Comments:

- The Mellin-Barnes representation captures the full analytic structure of boundary correlators

c.f. Gauss Hypergeometric function:

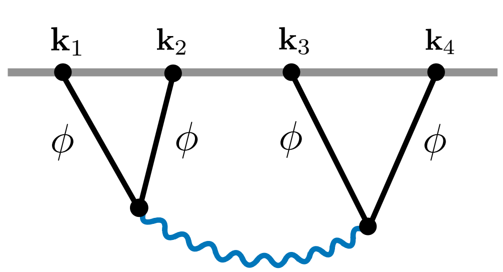
$${}_2F_1\left(a,b;c;z\right) = \frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)} \int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)} \left(-z\right)^s$$

$$= \sum_{n=0}^{\infty} \frac{\left(a\right)_n \left(b\right)_n}{\left(c\right)_n} \frac{z^n}{n!}, \quad |z| < 1$$

- For some special representations the Mellin-Barnes integrals can be lifted

  Includes: (partially-)massless fields, conformally coupled scalar

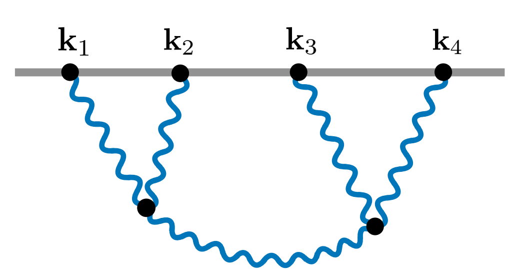
E.g.



massless spin-J

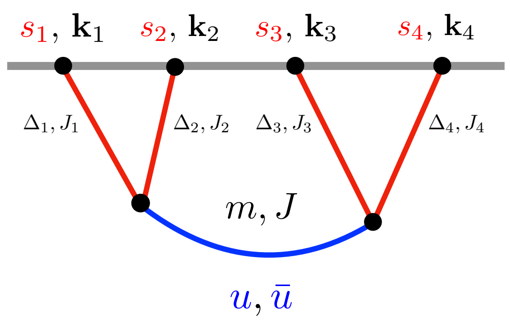
$\phi =$ conformally coupled or massless scalar

and

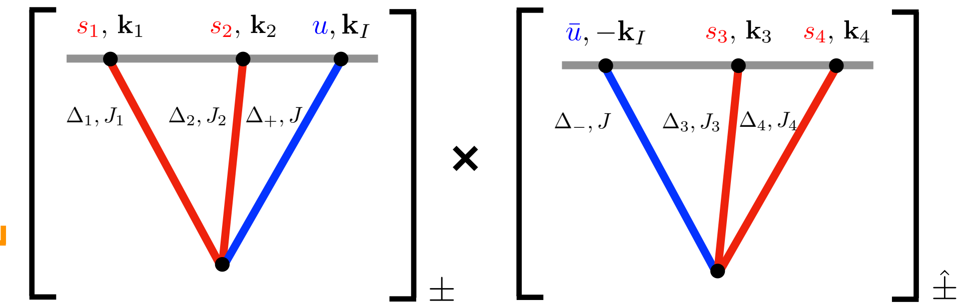


in Yang Mills and Gravity

Summary



$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{EFT}} \underbrace{\sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



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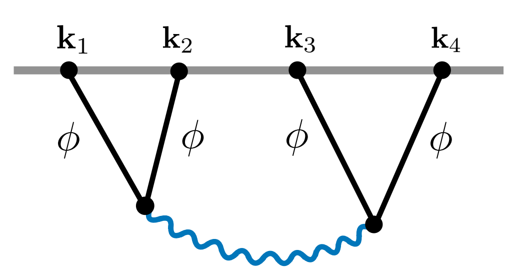
$$= \sum_{n=0}^{\infty} \frac{\left(a\right)_n \left(b\right)_n}{\left(c\right)_n} \frac{z^n}{n!}, \quad |z|<1$$

e.g.

- For some special representations the Mellin-Barnes integrals can be lifted

  Includes: (partially-)massless fields, conformally coupled scalar

E.g.



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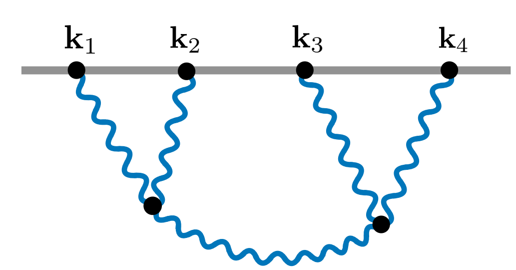
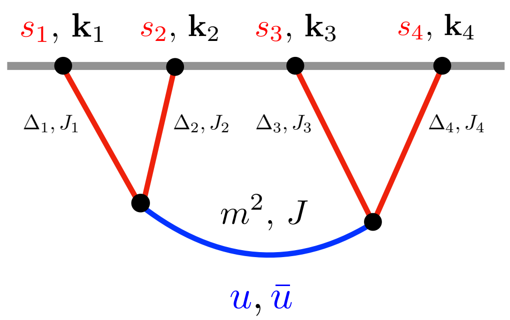
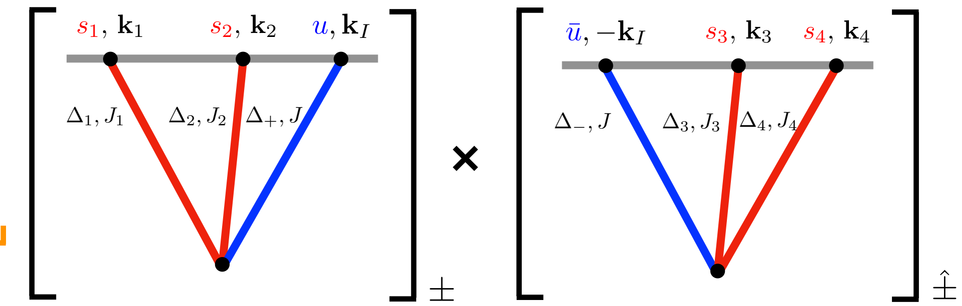


in Yang Mills and Gravity

Summary



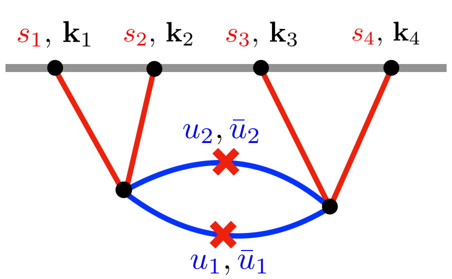
$$= \underbrace{\csc\left(\pi\left(u+\bar{u}\right)\right)}_{\text{EFT}} \underbrace{\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]}_{\text{boundary conditions}}$$



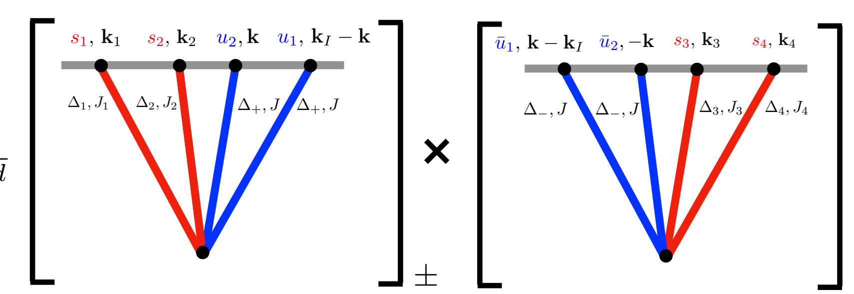
Factorisation + Conformal Symmetry

Plenty of diverse directions for the future!

- Higher points and Loops. Nice parallel with generalised unitarity methods/Cutkosky rules:



$$= \sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u_1,\bar{u}_1\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u_1,\bar{u}_1\right)\right]\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u_2,\bar{u}_2\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u_2,\bar{u}_2\right)\right]\int\frac{d^d\mathbf{k}}{(2\pi)^d}$$



similar in spirit to existing generalised unitarity methods in AdS/CFT — next talk by David Meltzer.

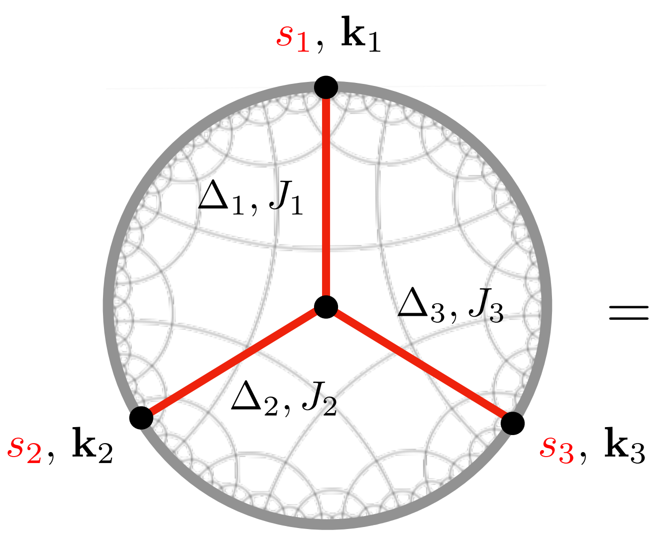
- Celestial Amplitudes?

- Conformal Bootstrap?

⋮

Back up

3pt Contact



$$= M\left(s_i, \boldsymbol{\epsilon}_i \cdot \mathbf{k}_j, \boldsymbol{\epsilon}_i \cdot \boldsymbol{\epsilon}_j\right) \underbrace{\prod_{i=1}^{3} \Gamma\left(s_i + \tfrac{1}{2}\left(\Delta_i^{+} - \tfrac{d}{2}\right)\right) \Gamma\left(s_i + \tfrac{1}{2}\left(\Delta_i^{-} - \tfrac{d}{2}\right)\right) \left(\frac{|\mathbf{k}_i|}{2}\right)^{-2s_i + \Delta_i^{+} - \frac{d}{2}}}_{\text{External propagators}}$$

$$M\left(s_i, \boldsymbol{\epsilon}_i \cdot \mathbf{k}_j, \boldsymbol{\epsilon}_i \cdot \boldsymbol{\epsilon}_j\right) = \delta^{(d)}\left(\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3\right) \delta\left(\frac{d+2(J_1+J_2+J_3)}{4} - s_1 - s_2 - s_3\right) \underbrace{\mathcal{M}\left(s_i, \boldsymbol{\epsilon}_i \cdot \mathbf{k}_j, \boldsymbol{\epsilon}_i \cdot \boldsymbol{\epsilon}_j\right)}_{\text{polynomial}}$$

$J_1 = J_2 = J_3 = 0:$ $\quad \mathcal{M}\left(s_i, \epsilon_i \cdot \mathbf{k}_j, \epsilon_i \cdot \epsilon_j\right) = 1$

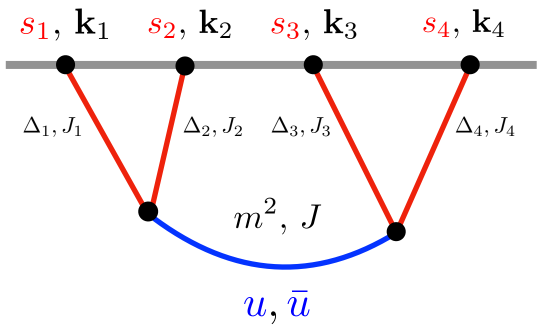
Yang Mills, highest helicity component:

$$\mathcal{M}\left(s_i, \epsilon_i \cdot \mathbf{k}_j, \epsilon_i \cdot \epsilon_j\right) = \underbrace{\left(\epsilon_1 \cdot \mathbf{k}_2\, \epsilon_2 \cdot \epsilon_3 + \epsilon_2 \cdot \mathbf{k}_3\, \epsilon_3 \cdot \epsilon_1 + \epsilon_3 \cdot \mathbf{k}_1\, \epsilon_1 \cdot \epsilon_2\right)}_{\text{3pt Yang-Mills amplitude in flat space}}$$

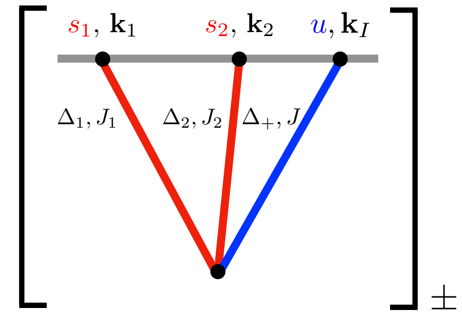
Gravity, highest helicity component:

$$\mathcal{M}\left(s_i, \epsilon_i \cdot \mathbf{k}_j, \epsilon_i \cdot \epsilon_j\right) = \left(\epsilon_1 \cdot \mathbf{k}_2\, \epsilon_2 \cdot \epsilon_3 + \epsilon_2 \cdot \mathbf{k}_3\, \epsilon_3 \cdot \epsilon_1 + \epsilon_3 \cdot \mathbf{k}_1\, \epsilon_1 \cdot \epsilon_2\right)^2$$

Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



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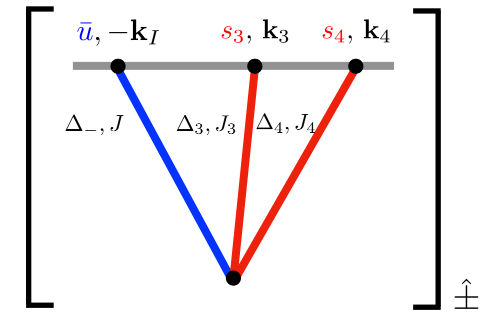


The Mellin-Barnes representation above captures the full analytic structure of the boundary correlators.

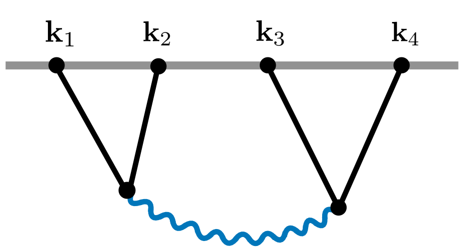
cf. Mellin-Barnes representation of the Gauss Hypergeometric function

$${}_2F_1\left(a,b;c;z\right) = \frac{\Gamma\left(c\right)}{\Gamma\left(a\right)\Gamma\left(b\right)} \int_{-i\infty}^{i\infty} \frac{ds}{2\pi i} \frac{\Gamma\left(a+s\right)\Gamma\left(b+s\right)\Gamma\left(-s\right)}{\Gamma\left(c+s\right)} \left(-z\right)^s \overset{c=b}{=} \left(1-z\right)^{-a}$$

cf.

For some special representations [e.g. (partially)-massless fields] the Mellin-Barnes integrals can be lifted.

e.g. massless spin-J exchange between conformally coupled scalars:



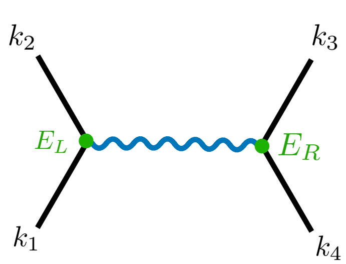
$$\propto \left(2\pi\right)^3 \delta^{(3)}\left(\sum_{i=1}^{4} \mathbf{k}_i\right) \frac{1}{\left(E_L\right)^J \left(E_R\right)^J \left(E_T\right)^{2J-1}} \left[\sum_{n=0}^{J-1} c_n \left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n} \left(|\mathbf{k}_I| E_T\right)^n\right] \Xi_J + \ldots$$

helicity-J

Simple integer coefficients.

Lower helicity, contact terms

cf. corresponding amplitude in (d+1)-dimensional flat space:



$$= \left(2\pi\right)\delta\left(E_T\right)\left(2\pi\right)^3 \delta^{(3)}\left(\sum_{i=1}^{4} \mathbf{k}_i\right) \frac{1}{\mathrm{s}}\, \Xi_J$$

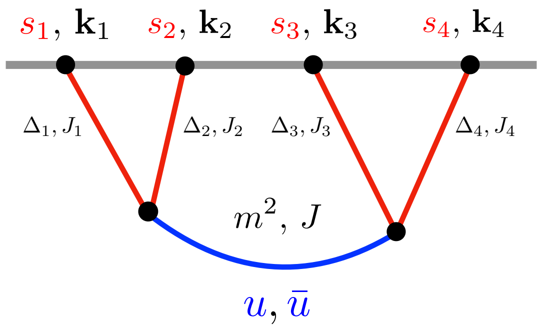
(d+1)-dim. Mandelstam variable

(d+1)-dim. null momentum: $k_i = \left(|\mathbf{k}_i|, \mathbf{k}_i\right)$

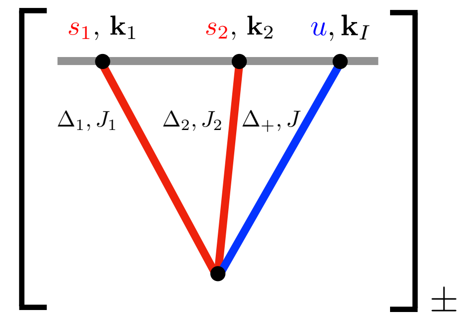
$E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4|$

$E_L = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_I|, \quad E_R = |\mathbf{k}_I| + |\mathbf{k}_3| + |\mathbf{k}_4|$

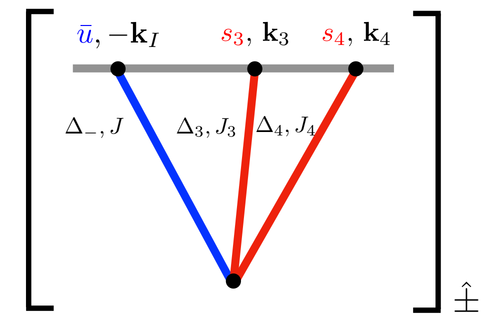
Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right)\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



$\times$



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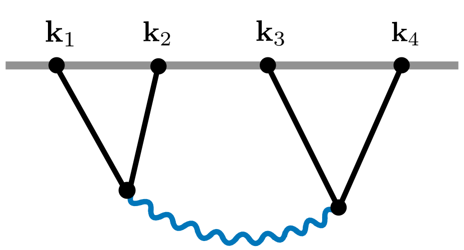


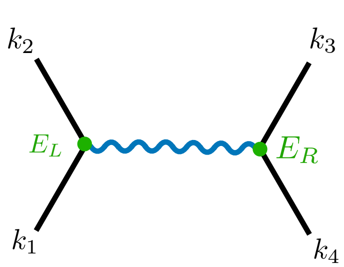
$$\propto (2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\frac{1}{\left(E_L\right)^J\left(E_R\right)^J\left(E_T\right)^{2J-1}}\left[\sum_{n=0}^{J-1}c_n\left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n}\left(|\mathbf{k}_I|E_T\right)^n\right]\Xi_J+\dots$$

helicity-J

Lower helicity, contact terms

Maldacena, Pimentel & Raju 2011-2012

$E_T \to 0$



$$\times\ \frac{1}{\left(E_T\right)^{\text{power}}}$$

(d+1)-dim. null momentum: $k_i = (|\mathbf{k}_i|, \mathbf{k}_i)$

$$E_T = |\mathbf{k}_1|+|\mathbf{k}_2|+|\mathbf{k}_3|+|\mathbf{k}_4|$$

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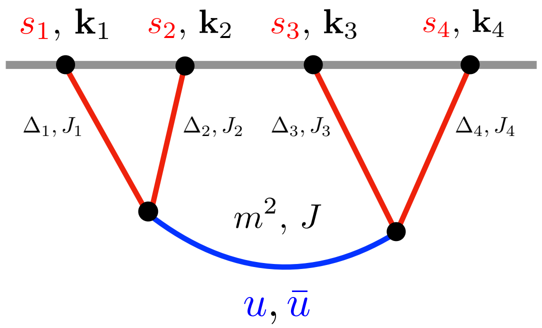
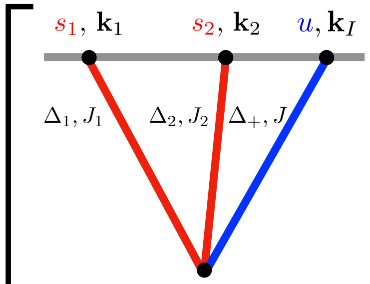
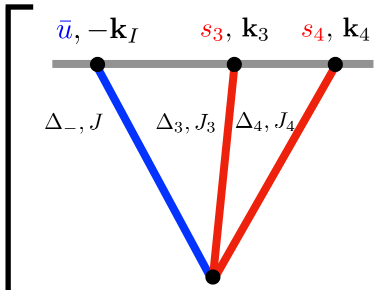
Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right] \Big[\ \cdots\ \Big]_{\pm} \times \Big[\ \cdots\ \Big]_{\hat{\pm}}$$





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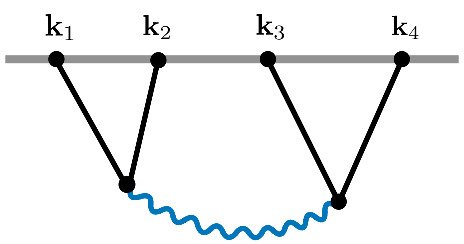


$$\propto \left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \frac{1}{\left(E_L\right)^J \left(E_R\right)^J \left(E_T\right)^{2J-1}} \left[\sum_{n=0}^{J-1} c_n \left(\left(|\mathbf{k}_1|+|\mathbf{k}_2|\right)\left(|\mathbf{k}_3|+|\mathbf{k}_4|\right)+|\mathbf{k}_I|^2\right)^{J-1-n} \left(|\mathbf{k}_I| E_T\right)^n\right] \Xi_J + \ldots$$

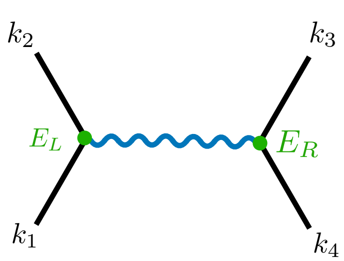
helicity-J

Lower helicity, contact terms

$E_T \to 0$

$$\left(E_L E_R\right)^{J-1} = \mathsf{s}^{J-1}$$

$E_T \to 0$



$$\times \frac{1}{\left(E_T\right)^{\text{power}}} = \left(2\pi\right)^3 \delta^{(3)}\Big(\sum_{i=1}^{4} \mathbf{k}_i\Big) \frac{\Xi_J}{\mathsf{s}} \times \frac{1}{\left(E_T\right)^{2J-1}}$$

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$$E_T = |\mathbf{k}_1| + |\mathbf{k}_2| + |\mathbf{k}_3| + |\mathbf{k}_4|$$

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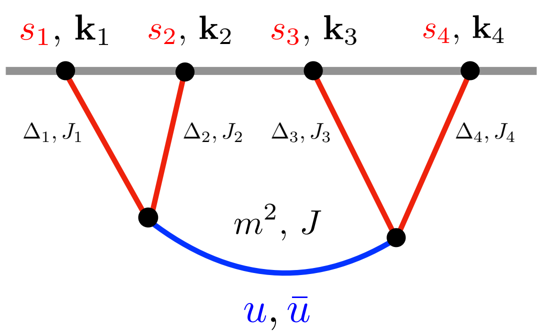
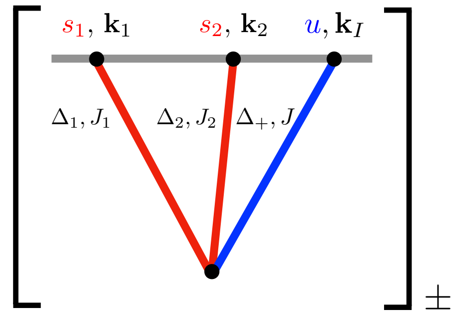
Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right)\sum_{\pm\hat{\pm}}\left[\alpha_{\pm\hat{\pm}}\,\omega_D\left(u,\bar{u}\right)+\beta_{\pm\hat{\pm}}\,\omega_N\left(u,\bar{u}\right)\right]$$



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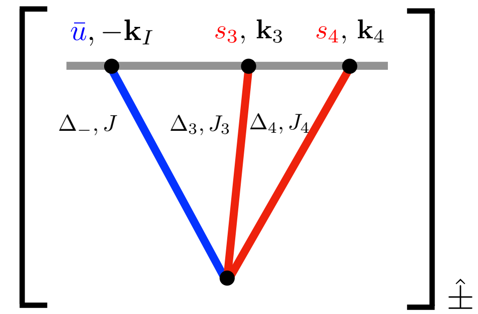
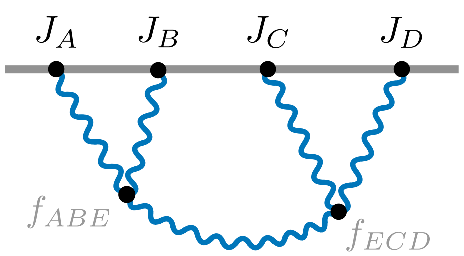


For some special representations [e.g. (partially)-massless fields] the Mellin-Barnes integrals can be lifted.

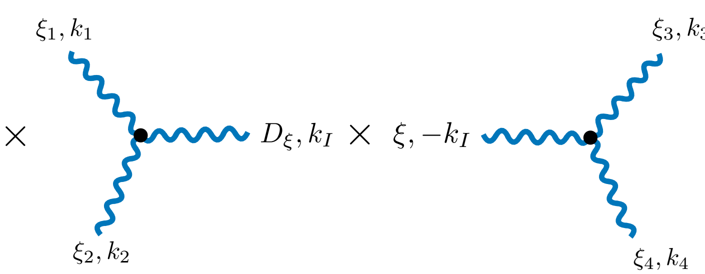
e.g. 4pt exchange in Yang-Mills theory



$$\propto\ f_{ABE}f_{ECD}\left(2\pi\right)^3\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\times\frac{1}{|\mathbf{k}_I|E_LE_RE_T}\times\left(|\mathbf{k}_I|+E_T\right)\times\underbrace{\mathsf{p}_{+}^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,D_\xi\right)\mathsf{p}_{-}^{\mathrm{YM}}\left(-\mathbf{k}_I,\mathbf{k}_3,\mathbf{k}_4;\xi,\xi_3,\xi_4\right)}_{\text{contraction of the 3pt tensorial structures}}$$

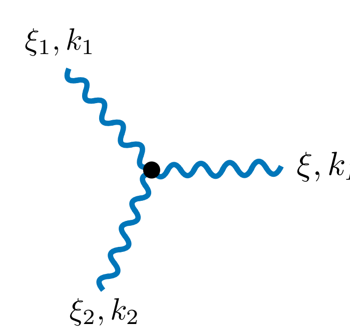
$$\Big\downarrow\ E_T\to 0$$

$$f_{ABE}f_{ECD}\left(2\pi\right)^3\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big)\frac{1}{\mathsf{s}}\ \times$$

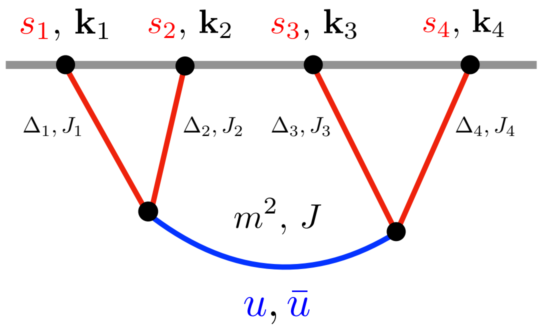


The highest helicity component is simple and given by the 3pt Yang-Mills amplitude in flat space:

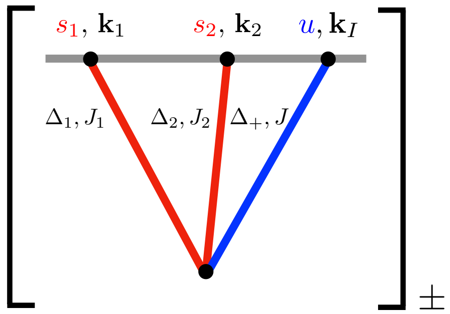
$$\mathsf{p}_{\pm}^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,\xi\right)\Big|_{\lambda=\pm1}=-i\left[\left(\xi_1\cdot\mathbf{k}_2\right)\left(\xi_2\cdot\xi\right)+\left(\xi_2\cdot\mathbf{k}_I\right)\left(\xi\cdot\xi_1\right)+\left(\xi\cdot\mathbf{k}_1\right)\left(\xi_1\cdot\xi_2\right)\right]\quad=$$



Exchanges in dS



$$= \csc\left(\pi\left(u+\bar{u}\right)\right) \sum_{\pm\hat{\pm}} \left[\alpha_{\pm\hat{\pm}}\, \omega_D\left(u,\bar{u}\right) + \beta_{\pm\hat{\pm}}\, \omega_N\left(u,\bar{u}\right)\right]$$



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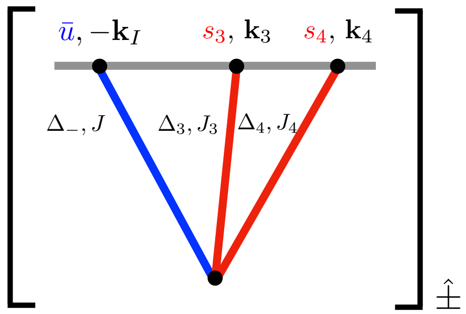

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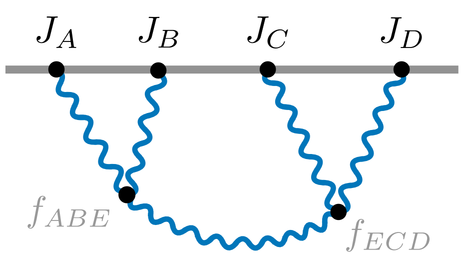
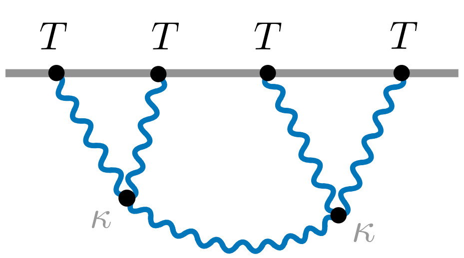
e.g. 4pt exchange in Yang-Mills theory



$$\propto f_{ABE}f_{ECD}\,(2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big) \times \frac{1}{|\mathbf{k}_I|E_L E_R E_T} \times \left(|\mathbf{k}_I| + E_T\right) \times \underbrace{\mathsf{p}_{+}^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,D_\xi\right) \mathsf{p}_{-}^{\mathrm{YM}}\left(-\mathbf{k}_I,\mathbf{k}_3,\mathbf{k}_4;\xi,\xi_3,\xi_4\right)}_{\text{contraction of the 3pt tensorial structures}}$$

e.g. 4pt exchange in Gravity



$$\propto \kappa^2\,(2\pi)^3\,\delta^{(3)}\Big(\sum_{i=1}^{4}\mathbf{k}_i\Big) \times \frac{\mathrm{Poly}\left(|\mathbf{k}_i|, |\mathbf{k}_I|\right)}{|\mathbf{k}_I|^3\left(E_L\right)^2\left(E_R\right)^2\left(E_T\right)^3} \times \mathsf{p}_{+}^{\mathrm{GR}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,D_\xi\right) \mathsf{p}_{-}^{\mathrm{GR}}\left(-\mathbf{k}_I,\mathbf{k}_3,\mathbf{k}_4;\xi,\xi_3,\xi_4\right)$$

The double-copy structure of flat scattering amplitudes is encoded in dS correlators:

$$\mathsf{p}_{\pm}^{\mathrm{GR}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,\xi\right)\Big|_{\lambda=\pm 2} = \left(\mathsf{p}_{\pm}^{\mathrm{YM}}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_I;\xi_1,\xi_2,\xi\right)\Big|_{\lambda=\pm 1}\right)^2$$

See also: Farrow, Lipstein & McFadden 2018 for a similar observation at the level of 3pt correlators