Taming Defects in N=4 Super-Yang-Mills

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Based on 2003.11016, 2004.09514 [with Shota Komatsu], 2005.07197 and works in progress

Motivations

$\mathcal{N} = 4$ SYM is perhaps the most well-studied interacting CFT

- Share many features of strongly coupled gauge theories in four dimensions
- A variety of methods: e.g. supersymmetric localization, integrability, bootstrap
- Close connection to quantum gravity via AdS/CFT
- In recent years, lots of progress in determining correlation functions of local operators

Motivations

Rich family of defects of different co-dimensions

- Sensitive to global structures
- New structure constants for the full operator algebra
- Refined consistency conditions on a full-fledged CFT
- Most works focus on (order-type) Wilson loops. More general disorder-type defects. See also Charlotte’s talk

Love defects!

A Program for Defects in $\mathcal{N} = 4$ SYM

- Classification of (conformal) defects

Many SUSY examples

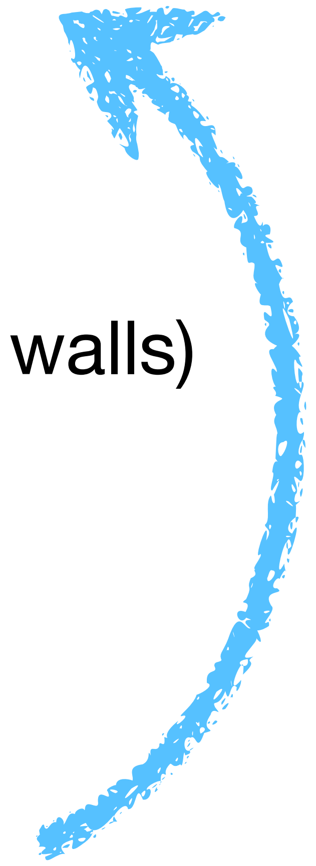
[Gaiotto-Witten, Gukov-Witten Kapustin, Kapustin-Witten,…]

(e.g. Wilson-’t Hooft loops, Gukov-Witten surface operators, Gaiotto-Witten walls)

Holographic descriptions

[Karch-Randall, Aharony-Karch, Dewolfe-Freedman-Ooguri, Drukker-Gomis-Matsuura, D’Hoker-Estes-Gutperle,…]

- Tools to extract defect observables



Bootstrap

Definition

Analytic methods + Bootstrap philosophy: formulate consistency conditions, constraints from unitarity and crossing

[Liendo-Rastelli-van Rees, Liendo-Meneghelli, Billo-Goncalves-Lauria-Meineri,…]

- Implications for the fully extended CFT

See also Clay’s talk

[Aharony-Seiberg-Tachikawa, Gaiotto-Kapustin-Seiberg-Willet…]

See Ibou’s talk

Higher form symmetries, anomalies through world-volume couplings, complete data to specify extended CFT?

A Program for Defects in $\mathcal{N} = 4$ SYM

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Bootstrap

Definition

Analytic
- Tools to extract defect observables

[Liendo-Rastelli-van Rees,
Liendo-Meneghelli,
[illegible]o-Goncalves-Lauria-Meineri,…]

This talk

Bootstrap: Formulate consistency conditions, constraints from unitarity and crossing

- Implications for the fully extended CFT

[Aharony-Seiberg-Tachikawa, Gaiotto-Kapustin-Seiberg-Willet…]

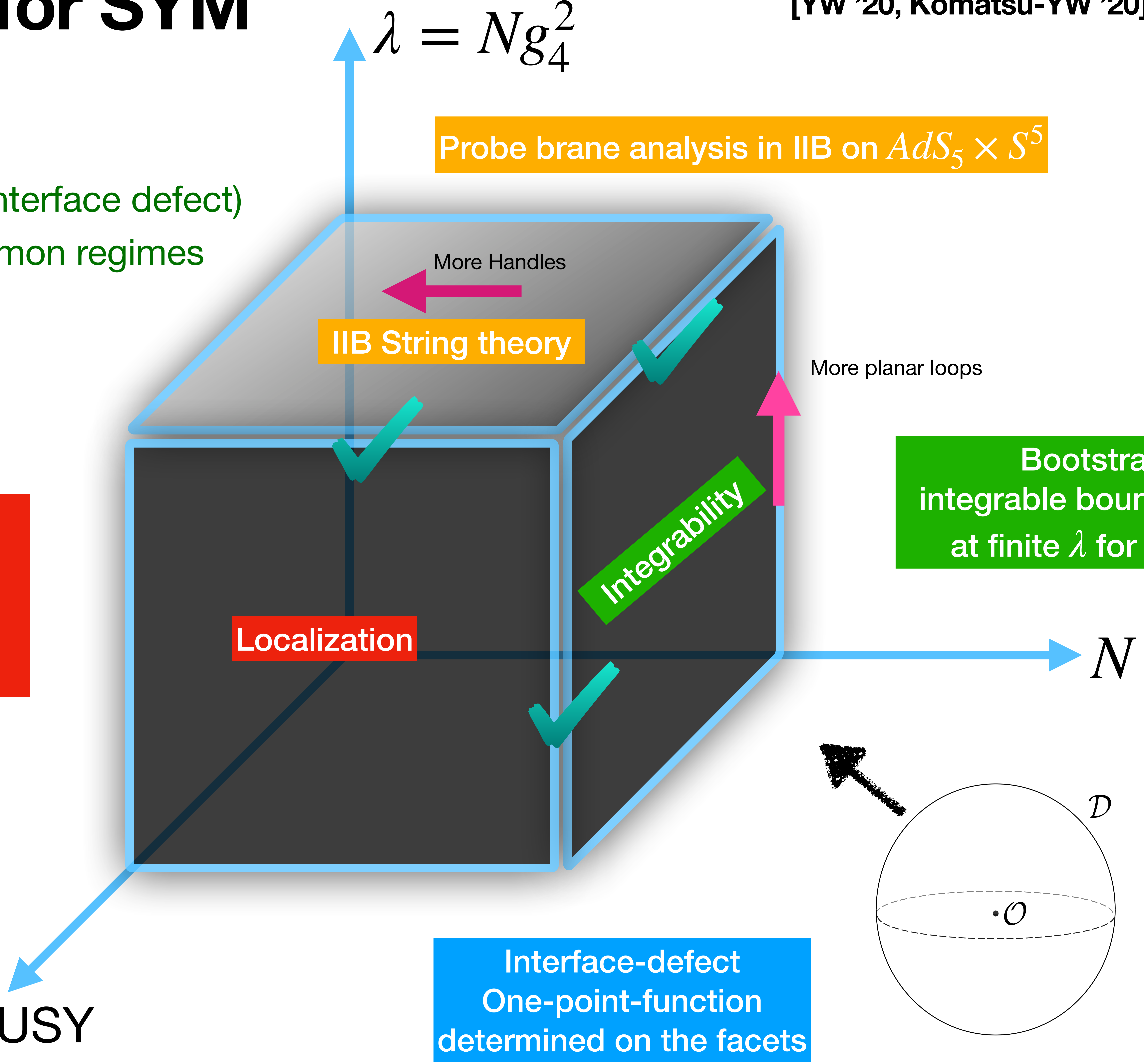
Higher form symmetries, anomalies through world-volume couplings,
complete data to specify extended CFT?

Toolkit for SYM

[YW ’20, Komatsu-YW ’20]

Results (e.g. interface defect) match at common regimes

Probe brane analysis in IIB on $AdS_5 \times S^5$



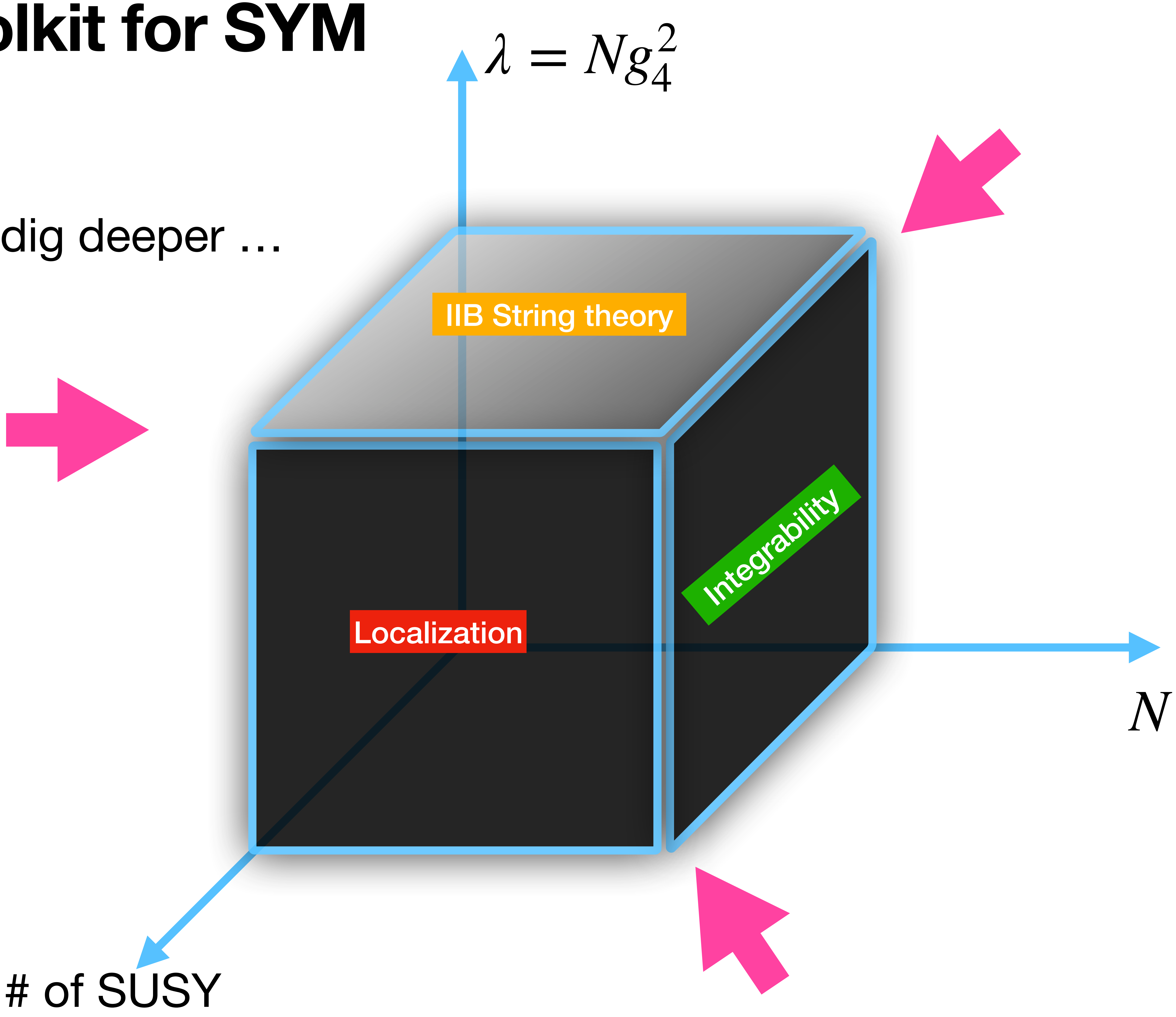
Localization of general 1/16 BPS defect networks
Emergent 2d/1d description

Bootstrap of integrable boundary states at finite λ for interface

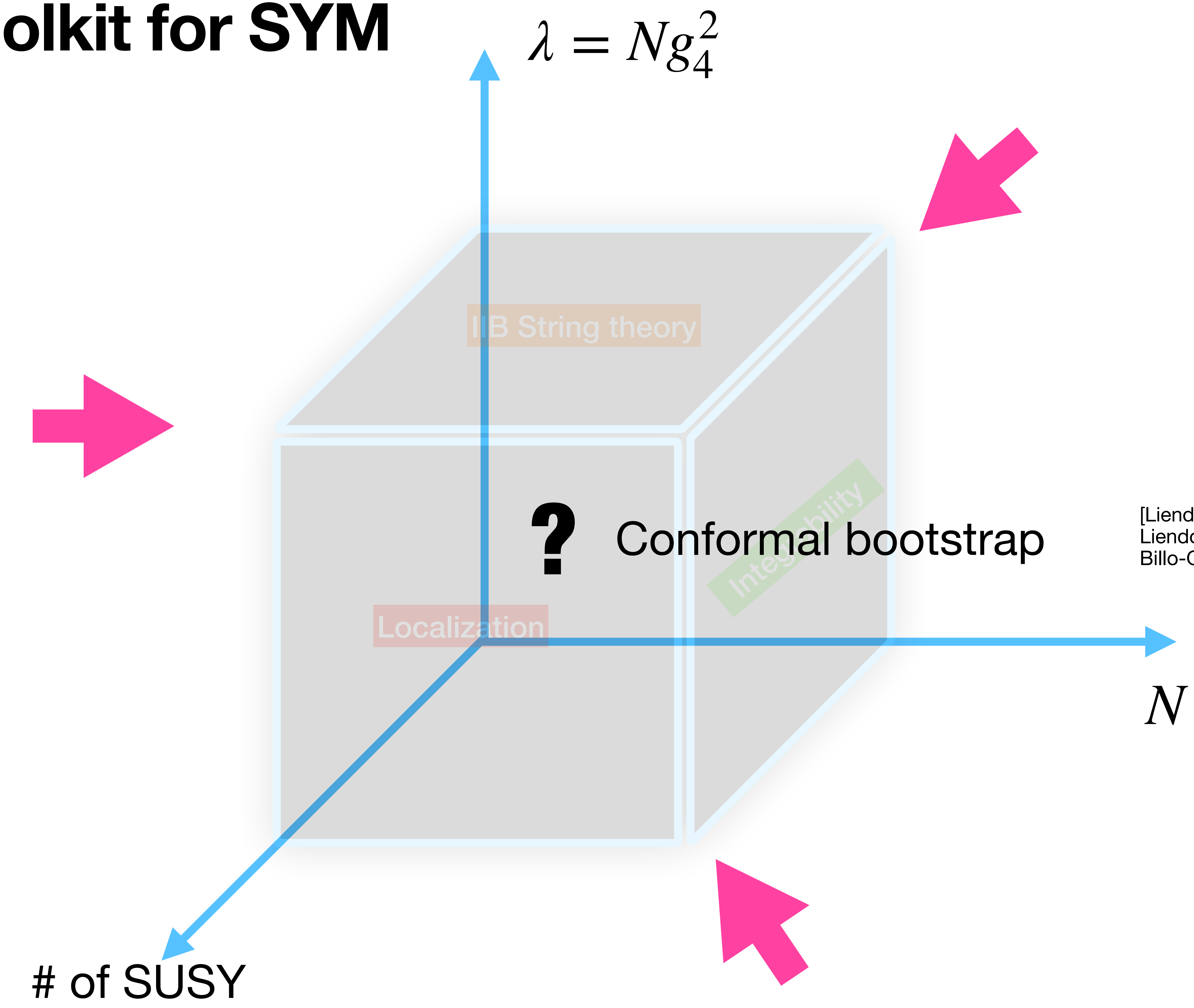
Interface-defect One-point-function determined on the facets

Toolkit for SYM

To dig deeper …



Toolkit for SYM



? Conformal bootstrap

[Liendo-Rastelli-van Rees,
Liendo-Meneghelli,
Billo-Goncalves-Lauria-Meineri,...]

Emergent 2d/1d effective theory

Defect-Yang-Mills

for 1/16 BPS defect observables in $\mathcal{N} = 4$ SYM

Two important ingredients: 2d YM

Constrained to 0-instanton

$$\text{4d } \mathcal{N} = 4 \text{ SYM on } \mathbb{R}^4 \xrightarrow[g_{\text{YM}}^2 = -\frac{8\pi}{g_4^2 R^2}]{\text{Closed subsector}} \text{2d bosonic YM on } S_R^2$$

Compact 10d notation ($M = 1{,}2{,}3{,}4$ spacetime directions
$M = 5{,}6{,}7{,}8{,}9{,}0$ R-symmetry directions)

$$S_{\text{SYM}} = -\frac{1}{2g_4^2}\int_{\mathbb{R}^4} d^4x \, \text{tr}\left(\frac{1}{2}F_{MN}F^{MN} - \Psi\Gamma^M D_M \Psi\right) \longrightarrow S_{\text{YM}}(\mathscr{A}) \equiv -\frac{1}{g_{\text{YM}}^2}\int_{S^2} dV_{S^2}\, \text{tr}(\star\mathscr{F})^2$$

$$\mathscr{A} \equiv A + i\epsilon_{ijk}\phi_i x^k dx^j$$ (coupled to 3 of the 6 scalars: Φ_7, Φ_9, Φ_0)

$$\underbrace{\text{1/8-BPS Wilson loops}}_{\text{Interacting (complicated)}} \xrightarrow[\text{SUSY localization wrt } \mathscr{Q}]{\text{tr}_{\text{rep}}\, \text{P}e^{\oint \mathscr{A}}} \underbrace{\text{Ordinary Wilson loops}}_{\text{Solvable (quasi-topological)}}$$

Interacting
(complicated)

$$\mathscr{Q}^2 = -2(M_\perp - R_{56})$$

$$\mathfrak{su}(1|1) \subset \mathfrak{psu}(2{,}2|4)$$

Solvable
(quasi-topological)

[Drukker-Giombi-Ricci-Trancanelli, Pestun, Giombi-Pestun, Giombi-Komatsu…]

Two important ingredients: 1d TQM

$$\text{3d } \mathcal{N} = 4 \text{ SCFT on } S^3 \xrightarrow{\text{Closed subsector}} \text{1d TQM on a great } S^1_\varphi$$

$$\text{Free hypermultiplets } (q_{iI}, \tilde{q}^{jJ}) \xrightarrow[Q(\varphi) = u^a q_a(\varphi),\ \tilde{Q} = u^a \tilde{q}_a(\varphi)]{\text{Twist by } u^a = \left(\cos\frac{\varphi}{2} \ \ \sin\frac{\varphi}{2}\right)} S_{\text{TQM}} = \ell \int d\varphi \tilde{Q}_{iI} (D_{\mathscr{A}})^i{}_j Q^{jI}$$

$$\text{Coupling to vector multiplets} \longrightarrow \text{Gauged TQM}$$

Supersymmetric localization wrt $\mathcal{Q}_{3d}$

$$\mathcal{Q}_{3d}^2 = -2(M_\perp - R_C) \quad \mathfrak{su}(1|1) \subset \mathfrak{osp}(4|4)$$

Strongly-coupled
(nontrivial RG)

Solvable
(mini bootstrap)

[Chester-Lee-Pufu-Yacoby, Beem-Peelaers-Rastelli, Dedushenko-Pufu-Yacoby, Dedushenko-Fan-Pufu …]

A general picture

[YW ’20]

- The 3d supercharge $\mathcal{Q}_{3d}$ is the restriction of $\mathcal{Q}$ to a half-BPS interface?

$$\mathfrak{su}(1|1) \subset \mathfrak{osp}(4|4) \subset \mathfrak{psu}(2,2|4)$$ ✓

- Study $\mathcal{Q}$ cohomology among general supersymmetric defect observables in the SYM → A rich zoo!

- $\mathcal{Q}$ localization of the defect-enriched SYM gives rise to defect-enriched 2d YM.

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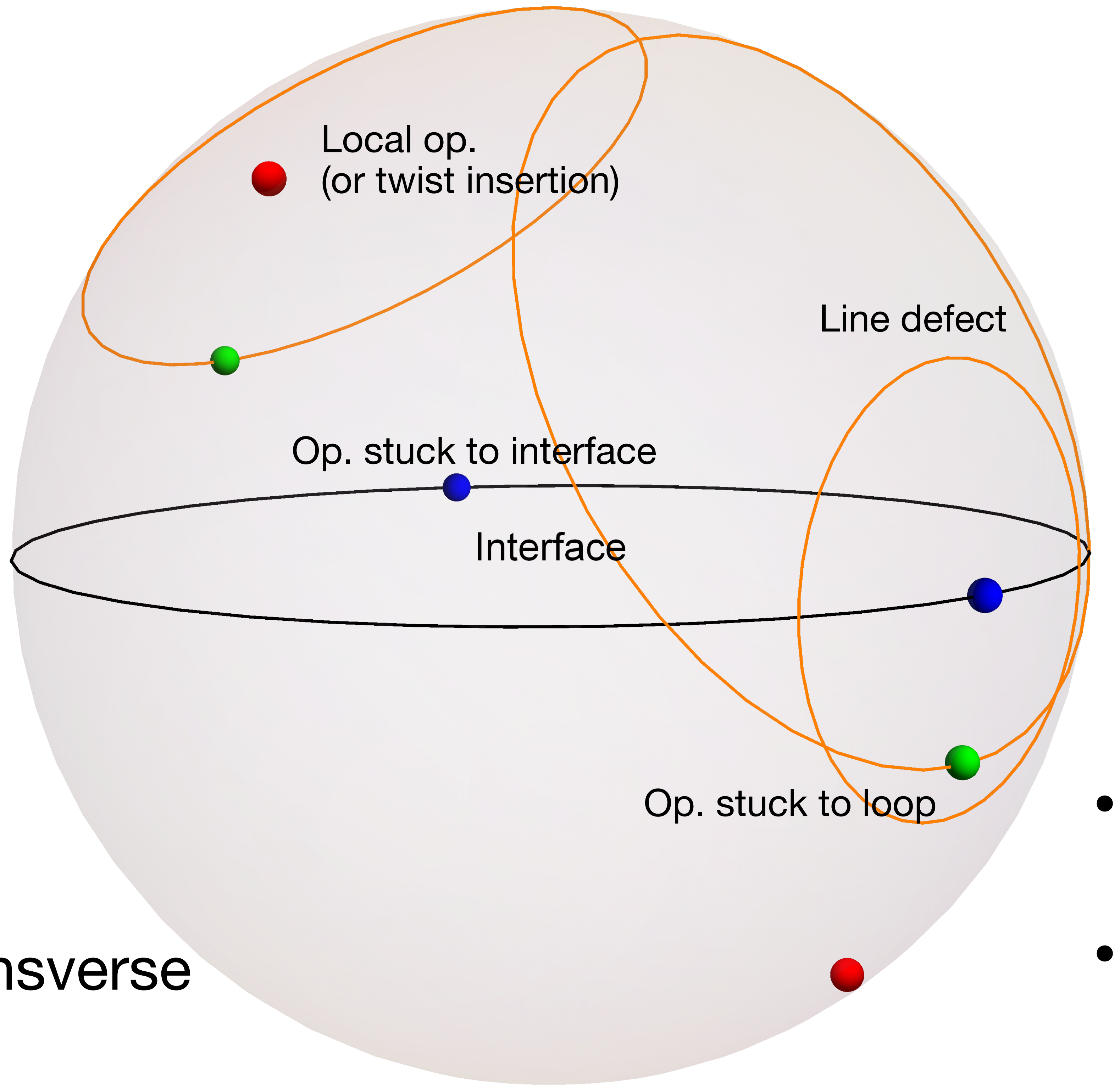
Defect-Yang-Mills

1/16 BPS defect networks

[YW ’20]

In general can have nontrivial topologies (sensitive to global structures): e.g. link of surface operators with line operators.



4d SYM on S^4

- Boundary/Interface
- Surface operators: transverse or longitudinal to S^2
- Line operators:
  Wilson, ’t Hooft lines
- Local operators
- Local operators on the defect world volume …

Dictionary →

2d dYM on S^2

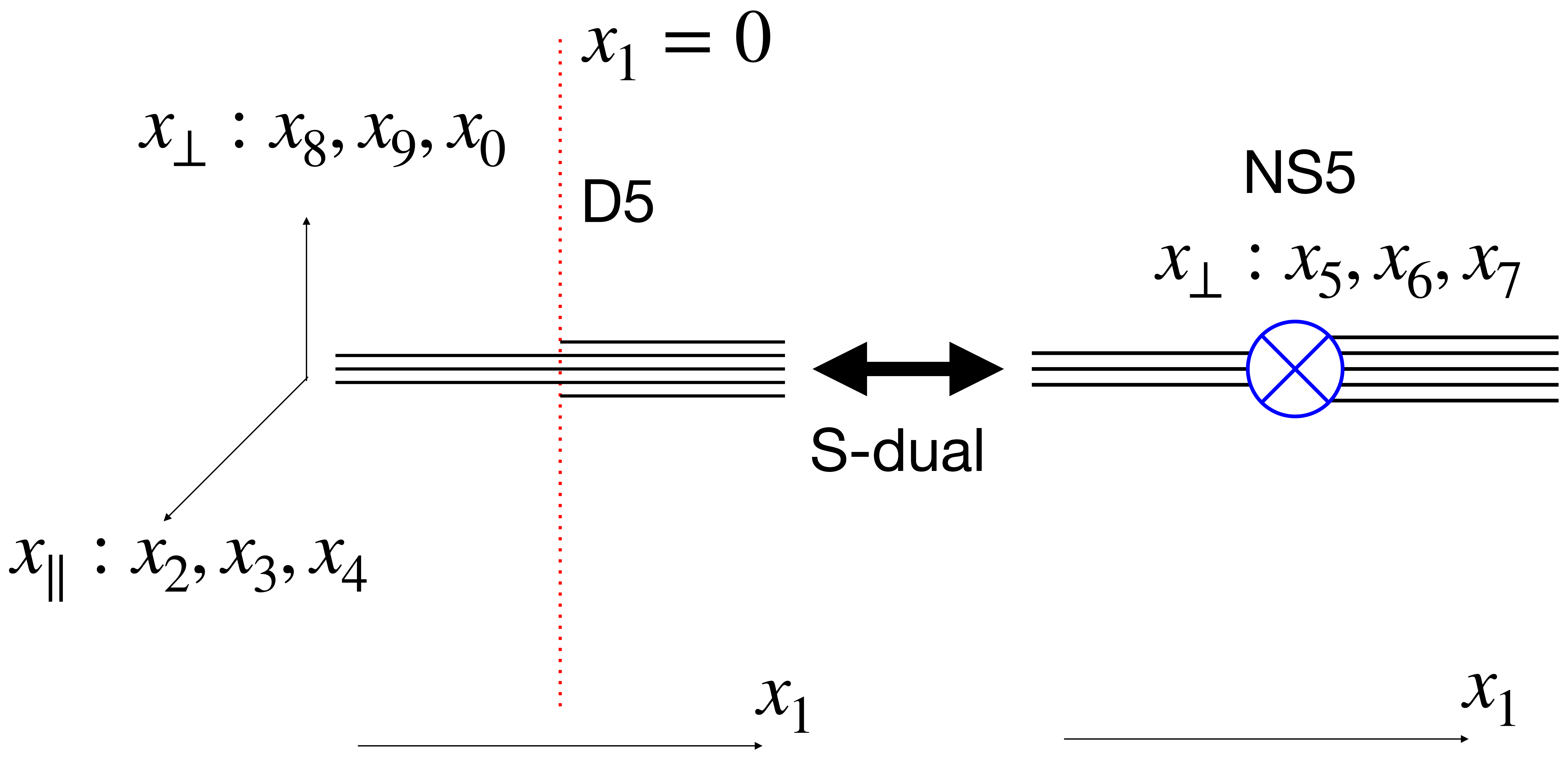
- Boundary/Interface **Focus here**
- Twist operators,
  or Higgsing the gauge group
- Wilson loops,
  or (unstable) 2d instantons
  [Giombi-Pestun]
- Field strength insertions
  [Giombi-Pestun]
- …
  [Giombi-Komatsu]

Application

D5-brane interface defects

D5-brane interface for SYM: brane picture

- Engineered by intersection of one D5 brane with D3 branes where k D3 branes can end



D5-brane interface for SYM: field theory

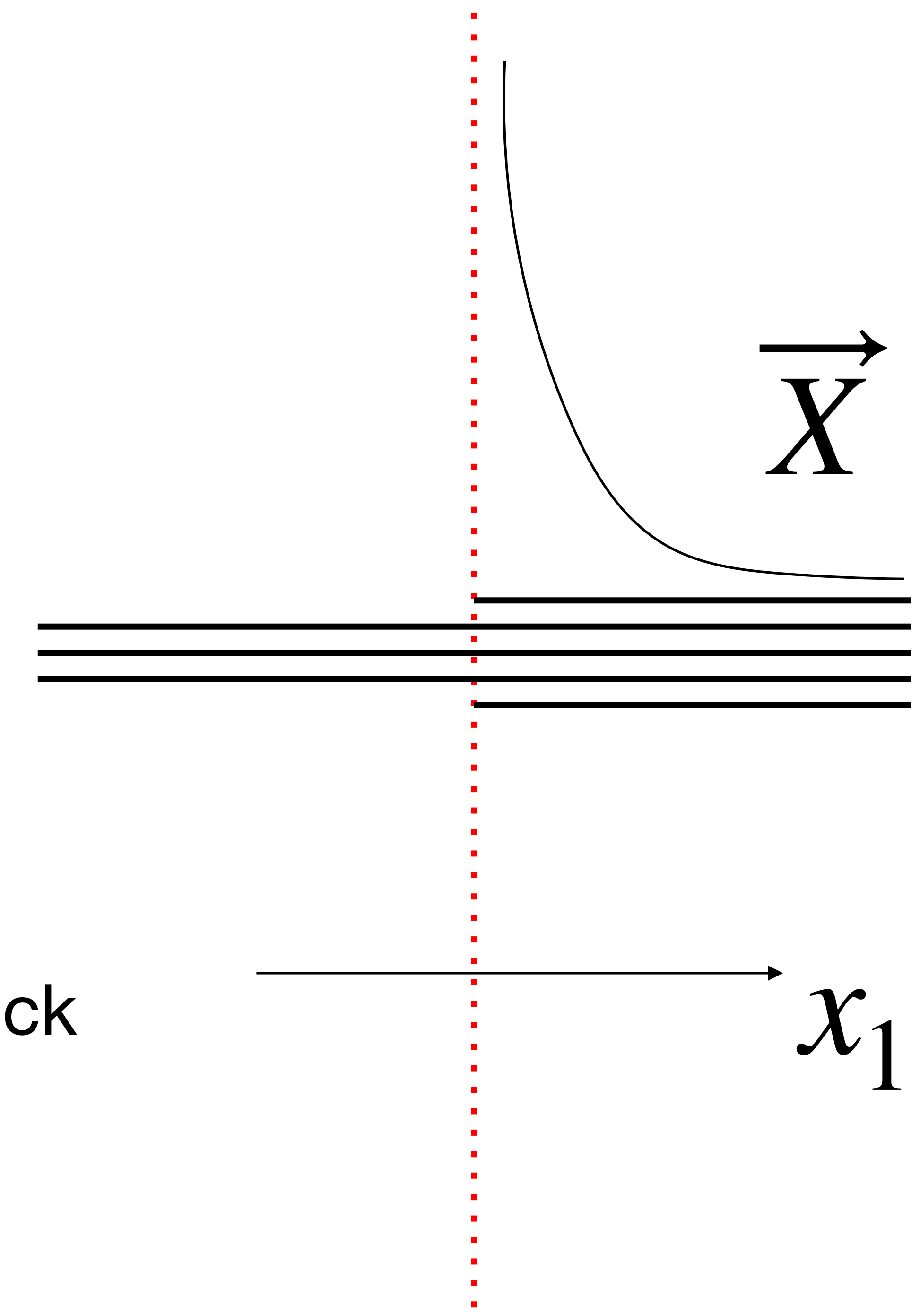
- Interpolate between $U(N)$ and $U(N+k)$ SYM at the same coupling

- Singular Nahm pole configuration
(related to usual N.P. bc by folding trick)

$$\vec{X} = (\Phi_8, \Phi_9, \Phi_0), \quad \vec{Y} = (\Phi_5, \Phi_6, \Phi_7)$$

$$A_\mu^+ = \begin{pmatrix} A_\mu^- & \cdot \\ \cdot & \cdot \end{pmatrix}, \quad \vec{Y}^+ = \begin{pmatrix} \vec{Y}^- & \cdot \\ \cdot & \cdot \end{pmatrix}, \quad \vec{X}^+ = \begin{pmatrix} \vec{X}^- & \cdot \\ \cdot & \frac{-\vec{t}}{x_1} \end{pmatrix}$$

$k \times k$ block



Nahm
Pole
Config

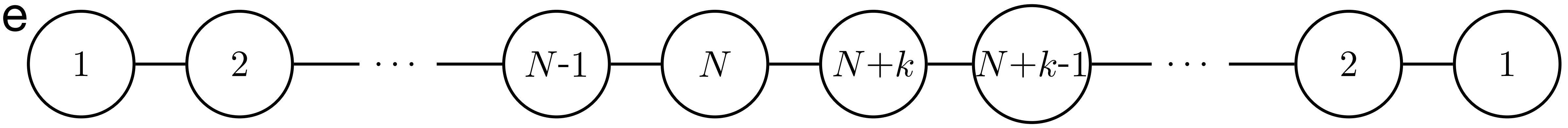
$$\begin{cases} [t_i, t_j] = \epsilon_{ijk} t_k \text{ given by } SU(2) \text{ reps} \\ t_3^{k\times k} = -\frac{i}{2}\text{Diag}[k-1, k-3, \ldots, 1-k] \end{cases}$$

D5-brane interface for SYM: field theory

- Hard: disorder-type defect, “non-Lagrangian”
- Use Mirror description [Gaiotto,Witten]

Superconformal NP BC
as IR fixed point

Interface
theory:



“Bad quiver”

- Global symmetry $U(N+k) \times U(N)$ (enhancement)

Gluing by 3d vectors

$U(N+k)$ SYM
with Dirichlet BC

$U(N)$ SYM
with Dirichlet BC

- A UV Lagrangian for SYM on S^4 coupled to the D5 interface at equator that preserves $\mathcal{Q} \in \mathfrak{osp}(4|4,\mathbb{R}) \subset \mathfrak{psu}(2,2|4)$

For SUSY observables this suffices by non-renormalization

[YW ’20, Komatsu-YW ’20]

Defect-Yang-Mills for D5-brane interface

- Run localization by $\mathscr{Q}$ to find 2d/1d description

$$Z_{\mathrm{dYM}} = \int_{S^2_{\mathrm{YM}}} D\mathscr{A}\,|_{HS^2_-} D\mathscr{A}'\,|_{HS^2_+} DQD\tilde{Q}\,|_{S^1_{\mathrm{TQM}}}\, e^{-S_{\mathrm{YM}}(\mathscr{A})-S_{\mathrm{YM}}(\mathscr{A}')-S_{\mathrm{TQM}}(\mathscr{A},\mathscr{A}',Q,\tilde{Q})}$$

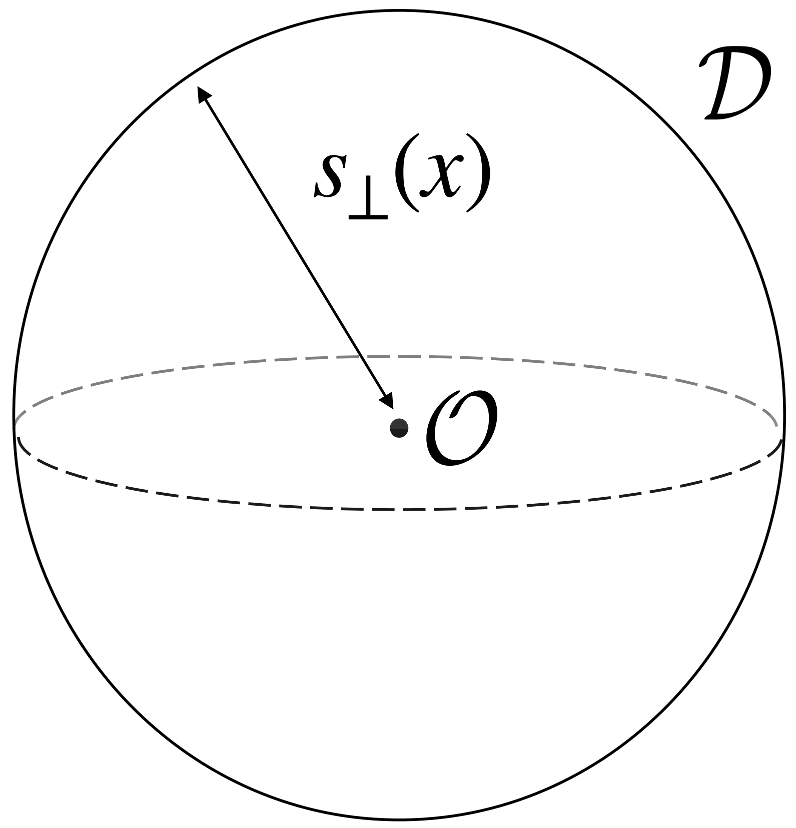
$$S_{\mathrm{YM}}(\mathscr{A}) \equiv -\frac{1}{g^2_{\mathrm{YM}}}\int_{HS^2_-} dV_{S^2}\,\mathrm{tr}(\star\mathscr{F})^2$$

- Ready to use to compute general 1/16 BPS SYM observables in the presence of the D5-brane interface

[YW ’20, Komatsu-YW ’20]

Defect One-point function

[YW ’20, Komatsu-YW ’20]



Basic defect observables:

higher point correlators determined by bulk OPE

$$\langle \mathcal{O}(x) \rangle_{HS^4} = \frac{\boxed{h_{\mathcal{O}}}}{s_\perp(x)^{\Delta_{\mathcal{O}}}}$$

Fixed by conformal Ward identity

Take $\mathcal{O}_J \equiv \operatorname{tr}(\Phi_7 + i\Phi_8)^J$ half-BPS

$$\langle \mathcal{O}_J \rangle_{\text{dSYM}} = \frac{(-i)^J}{Z_{\text{dYM}}} \int D\mathscr{A} DQD\tilde{Q} \operatorname{tr}(\star\mathscr{F})^J e^{-S_{\text{YM}}(\mathscr{A}) - S_{\text{TQM}}(Q,\tilde{Q},\mathscr{A})}$$

Denote gauge fields on the two hemispheres collectively by $\mathscr{A}$

- Compute by first integrate out 1d fields $(Q, \tilde{Q})$
- Obtain matrix model for $\langle \mathcal{O}_J \rangle_{\text{dSYM}}$ by 2d gauge theory techniques

Full D5 brane matrix model at any (N, λ)

One-point-function of BPS operator

deg-J polynomial from G.S. procedure

$$\langle \mathscr{D}\mathscr{O}_J \rangle_k = \int [da] \frac{\Delta(a)^2}{\prod_{j=1}^{N} 2\cosh \pi \left(a_j + \frac{ki}{2}\right)} \left[\sum_{j=1}^{N} f_J(a_j) + \sum_{s=-\frac{k-1}{2}}^{\frac{k-1}{2}} f_J(is) \right]$$

Between $U(N)$ and $U(N+k)$

$$\times \prod_{s=-\frac{k-1}{2}}^{\frac{k-1}{2}} \prod_{n} \left(a_j - is\right) e^{-\frac{8\pi^2 N}{\lambda} \sum_{i=1}^{N} a_i^2}$$

Normalize by taking ratio

One-point-function of the interface

$$\langle \mathscr{D} \rangle_k = \int [da] \frac{\Delta(a)^2}{\prod_{j=1}^{N} 2\cosh \pi \left(a_j + \frac{ki}{2}\right)} \prod_{s=-\frac{k-1}{2}}^{\frac{k-1}{2}} \prod_{j=1}^{N} \left(a_j - is\right) e^{-\frac{8\pi^2 N}{\lambda} \sum_{i=1}^{N} a_i^2}$$

[Komatsu-YW ’20]

Large N One-point functions and checks

[YW ’20, Komatsu-YW ’20]

$$\langle \mathcal{O}_J \rangle_k = \frac{i^J}{2^{\frac{J}{2}}\sqrt{J}} \left(\texttt{integral} + \texttt{sum} \right)$$

$$g = \frac{\sqrt{\lambda}}{4\pi}$$

$$\texttt{integral} = \begin{cases} \oint \frac{dx}{2\pi i} g \left(1 - \frac{1}{x^2}\right) x^J \, \pi \coth \left[\pi g \left(x + \frac{1}{x} \right) \right] & k \in 2\mathbb{Z} + 1 \\ \oint \frac{dx}{2\pi i} g \left(1 - \frac{1}{x^2}\right) x^J \, \pi \tanh \left[\pi g \left(x + \frac{1}{x} \right) \right] & k \in 2\mathbb{Z} \end{cases}$$

$$\texttt{sum} = \sum_{s=-\frac{k-1}{2}}^{\frac{k-1}{2}} \left(x_s^J + \delta_{J,2} \right)$$

$$x_u = \frac{u + \sqrt{u^2 - 4g^2}}{2g}$$

Zhukovsky
Hint for integrability!

Large λ: check with IIB string theory answers [Nagasaki-Yamaguchi] ✓

- Probe D5 brane on $AdS_4 \times S^2$ with k units of w.v. flux
- One-point function from w.v. couplings to closed string KK modes

Small λ: check perturbation theory ✓

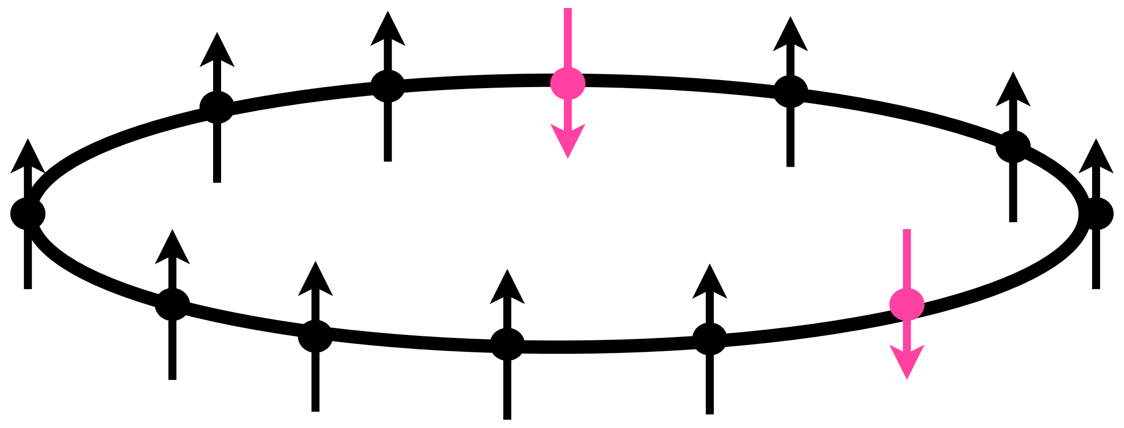
[Buhl-Mortensen-de Leeuw-Ipsen-Kristjansen-Wilhelm,Kristjansen-Müller-Zarembo]]

Bootstrap approach to

Integrable boundary states

for interface defects in $\mathcal{N} = 4$ SYM

D5 brane and Integrable spin chain



Traditional Spin chain approach [Minahan-Zarembo]:

$\mathcal{O} = \mathrm{tr}(\ldots ZYZ\ldots ZYZ\ldots)$ "Y magnons"

- $|\mathcal{D}\rangle$ represented by a matrix-product state
- $|\mathcal{O}\rangle$ represented by Bethe states, labelled by rapidities (momentum) u_m of the magnons
- One-point function from overlap
- Limitation to weak coupling (nearest-neighbor interactions, fixed length)

[Buhl-Mortensen, Gimenez Grau, Ipsen, de Leeuw, Linardopoulos, Kristjansen, Pozsgay, Volk, Widen, Wilhelm, Zarembo …]

D5 brane and Integrable field theory

D5-brane interface defect ⟷ Boundary state $\mathscr{D}$ on IIB worldsheet in $AdS_5 \times S^5$ (lightcone G.S.)

Large N

- Single trace operators (non-BPS) - closed string state $|\Psi\rangle$
- One-point function from overlap $\langle \mathscr{O} \rangle_{\mathscr{D}} = \langle \mathscr{D} | \Psi \rangle$

Weak coupling evidences for defect observable:

1) Rapidities pair up $(u_1, -u_1, u_2, -u_2, \ldots, u_{M/2}, -u_{M/2})$, i.e. no particle production

2) Overlaps involve ratio of Gaudin determinants

[Jiang-Komatsu-Vescovi]

↓

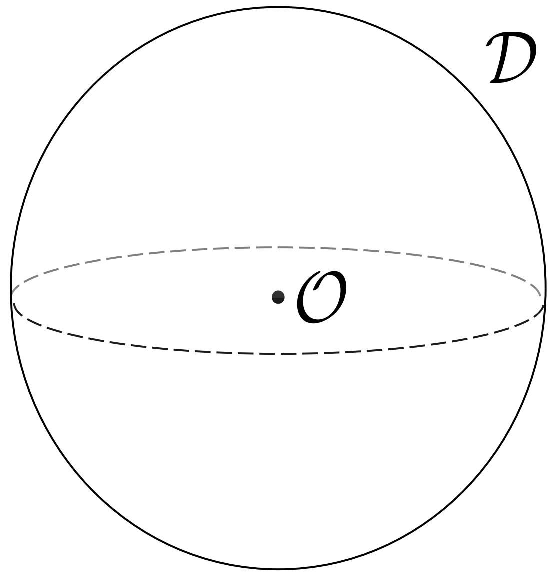
$\mathscr{D}$ is Integrable

[Komatsu-YW ’20]

General Strategy for non-perturbative boundary state

Apply bootstrap philosophy to integrable boundary state
[Ghoshal-Zamolodchikov, Jiang-Komatsu-Vescovi]

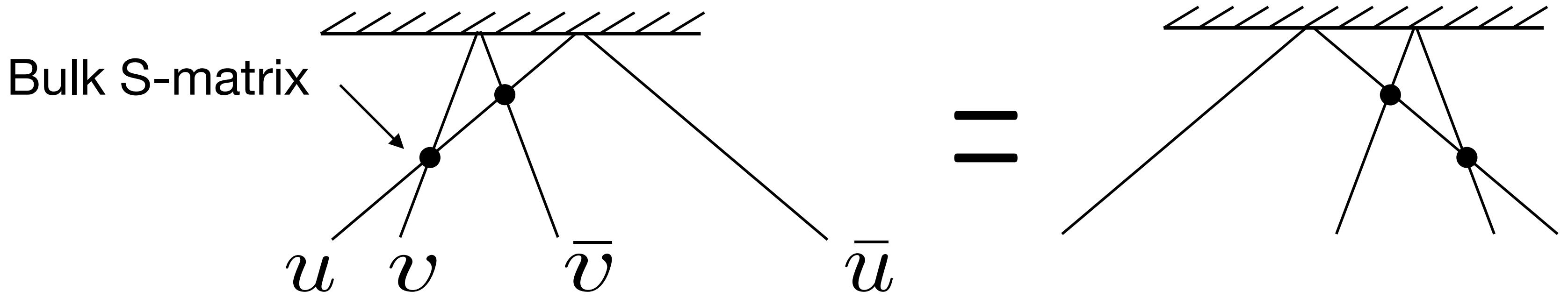
- Integrability implies general overlaps determined by two particle overlaps e.g. $\langle \mathscr{D} | Y(u) Y(\bar{u}) \rangle$



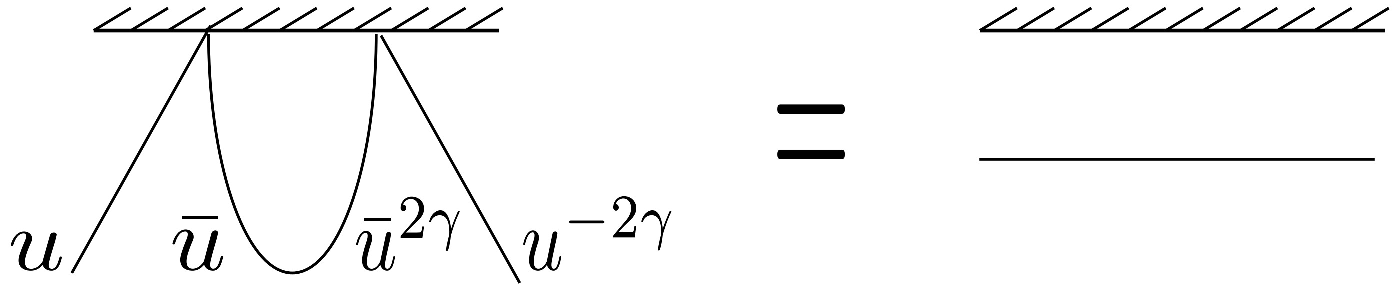
- Explore symmetry constraints under

$$SU(2|1) \times SU(2|1) \subset SU(2|2) \times SU(2|2)_{\text{c.e.}}$$

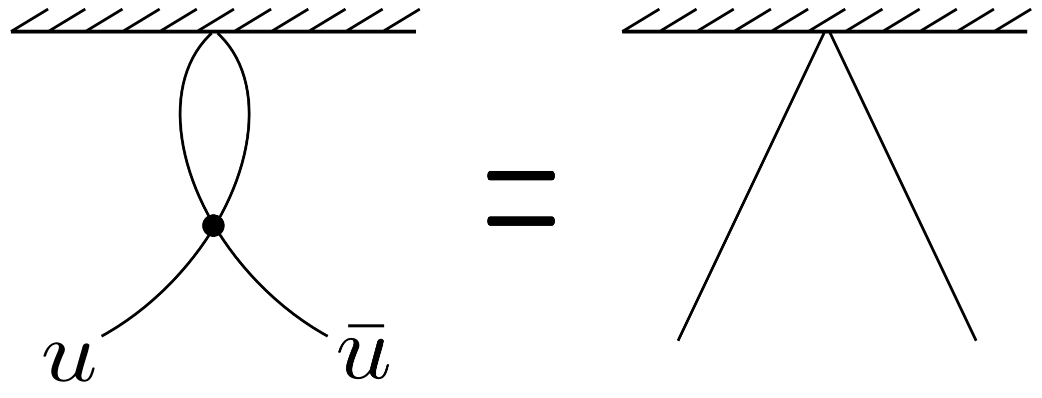
- Impose consistency conditions



Boundary Young-Baxter



Boundary unitarity



Watson (crossing)

[Komatsu-YW ’20]

See also [Gombor-Bajnok]

Large J R-charge non-BPS one-point function

$SU(2)$ sector:
single trace operators of length L
built out of $Z = \Phi_7 + i\Phi_8$ and
excitations $Y = \Phi_5 + i\Phi_9$

$$\langle \mathcal{O}_{\mathbf{u}}(x)\rangle_{\mathcal{D}_k} = \frac{c_{\mathbf{u}}^{(k)}}{2^J\sqrt{L}x_\perp^\Delta}$$

$$c_{\mathbf{u}}^{(k)} \equiv \sum_{a=-\frac{k-1}{2}}^{\frac{k-1}{2}} \frac{\langle \mathcal{D}_k^{(a)}|\mathbf{u}\rangle}{\sqrt{\langle\mathbf{u}|\mathbf{u}\rangle}} = \mathbb{T}_k \frac{\sqrt{Q(\frac{i}{2})Q(0)}}{Q(\frac{ik}{2})} \sqrt{\left(\prod_{m=1}^{M} \sigma_B(u_m)\right)\frac{\det G_+}{\det G_-}}$$

Extend to SO(6) in [Gombor-Bajnok]

$$\mathbb{T}_k \equiv \sum_{a=-\frac{k-1}{2}}^{\frac{k-1}{2}} (x_a)^{J+M} \frac{\left(Q(\frac{ik}{2})\right)^2}{Q^-(ia)Q^+(ia)} \prod_{m=1}^{M} \tilde{\sigma}_k^{(a)}(u_m)$$

$$Q(u) = \prod_{m=1}^{M}(u-u_m)$$

Important: excited boundary stares
(bound states with bdy)

- Zero magnon $M = 0$ case gives large J BPS one point function ✓
- Weak coupling expansion agrees with spin chain method

More checks in [Kristjansen-Müller-Zarembo]

- Finite J (non-pert.) from wrapping corrections (using TBA and analytic continuation)

Summary

A toolkit for defects in $\mathcal{N} = 4$ SYM

- Localization: 1/16 BPS, arbitrary N, arbitrary λ
- Integrability: non-BPS, large N, arbitrary λ
- Application: Interface-defect one-point-function

More applications and future directions

- Correlation functions of both bulk and defect operators [YW '20]
- Unorientable spacetimes, e.g. SYM $\mathbb{RP}^4$ governed by YM on $\mathbb{RP}^2$ [YW '20]
- Surface defects (linked with line defects)
- Discrete theta angles and global structures of gauge theories:
  from 4d SYM to 2d dYM
- Classification of integrable boundary states in IIB string on $AdS_5 \times S^5$
  e.g. D7, D-instanton, more exotic ones?
- Topological defects (bi-branes) on w.s. sigma model are known to generate boundary states [Fuchs-Schweigert-Waldorf, Kojita-Maccaferri-Masuda-Schnabl]
  Relation to SYM interfaces?
- $T\bar{T}$ deformation:
  2d YM has solvable $T\bar{T}$ [Sentilli-Tierz]
  learn about $T\bar{T}$ deformations 4d $\mathcal{N} = 4$ SYM [Caetano-Rastelli-Peelaers]
- $SL(2,\mathbb{Z})$ properties:
  Recent progress in $SL(2,\mathbb{Z})$ invariant four point functions [Chester-Green-Pufu-YW-Wen]
  including defects which transform nontrivially under $SL(2,\mathbb{Z})$?
- Extend the defect analysis to other AdS/CFT pairs?
  e.g. deriving AdS_3/CFT_2 beyond local correlators by incorporating defects [See Rajesh's talk]

Thank you!