STRINGS 2020

on Zoom

D-instanton Calculus in $c=1$ String Theory

based on work with

Bruno Balthazar & Victor Rodriguez

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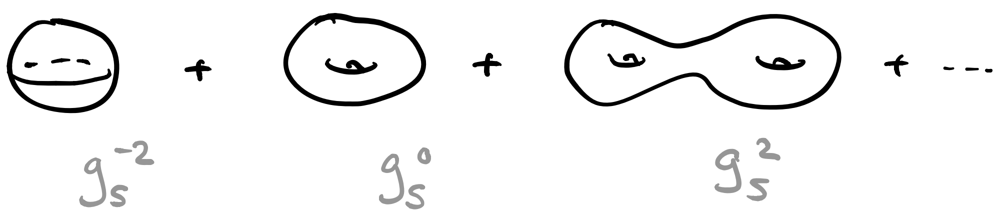
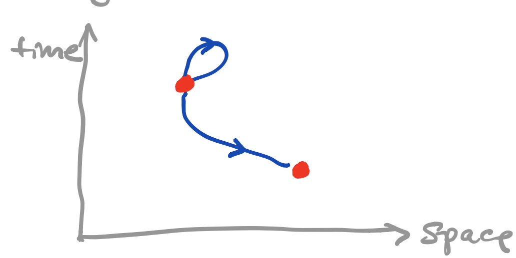
world-sheet formulation of string theory:

perturbative expansion



[asymptotic series
– resummation?]

"+" D-instantons $\quad e^{-\frac{1}{g_s}}$



Polchinski '94   Green, Gutperle '97   Sen '19

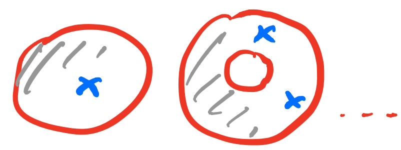
[+ gravitational instantons $e^{-\frac{1}{g_s^2}}$]

- No 1st principle derivation of D-instanton prescription (many indirect arguments for their contribution + consistency checks ...)

Anticipated structure:

$-S_{D\text{-inst}}$ correction in measure

$$\mathcal{N}\int_{\mathcal{M}_{D\text{-inst}}} dx \; e$$



closed string amplitude via worldsheet w/ bdry

overall normalization fixed by duality arguments

[S-duality in IIB, MQM in c=1 string]

- Divergence due to open string modes on D-instantons, cancel between diagrams of different topologies

Fischer, Susskind '86 Polchinski '94

Full treatment requires

Open + closed SFT Sen '19

Goal of this talk:

To learn the rule of D-instanton computation in $c = 1$ string theory

[a.k.a. 2D string theory]

from the world-sheet perspective,

guided by (a non-pert. completion of) the dual MQM.

$c = 1$ string theory

- The only known bosonic string theory that adimts
  - ✓ world-sheet description
  - ✓ space-time interpretation
  - ✓ consistent quantum pert. theory

[will extend beyond pert. theory]

world-sheet description:

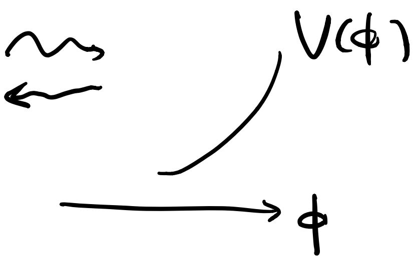
CFT

$$X^0 \oplus \underbrace{c=25 \text{ Liouville}} \oplus bc \text{ ghosts}$$

$$S_L[\phi] = \frac{1}{4\pi}\int (\partial\phi)^2 + QR\phi + \mu e^{2b\phi}$$

$$Q = b + \frac{1}{b}, \qquad c = 1 + 6Q^2$$

$$b \to 1 \qquad (c \to 25)$$



states / vertex operators are scattering waves off Liouville potential

$$V_P \overset{\phi\to-\infty}{\sim} S(P)^{-\frac{1}{2}} e^{(Q+2iP)\phi} + S(P)^{\frac{1}{2}} e^{(Q-2iP)\phi}$$

Liouville momentum $P \geq 0$

"leg-pole factor"

closed string asymptotic states:

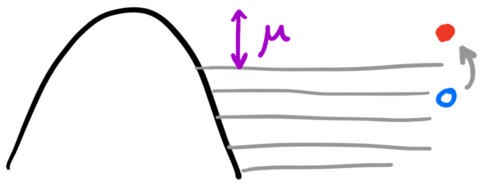
$$V_\omega^{\pm} = g_s \, e^{\pm i\omega X^0} \, V_{P=\frac{\omega}{2}}$$

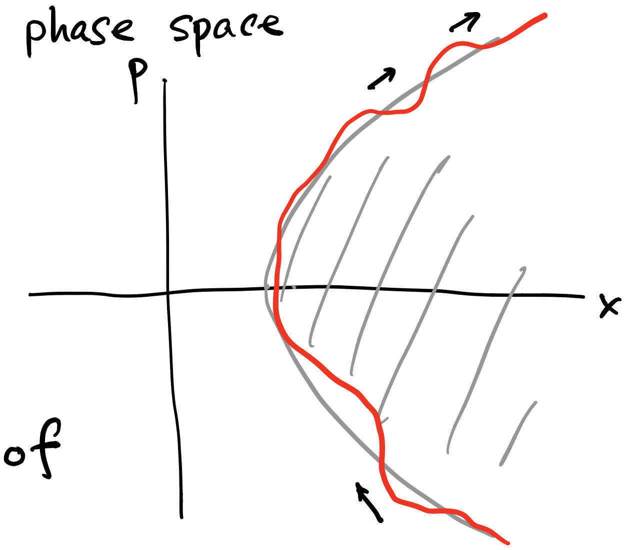
energy

Dual matrix quantum mechanics

= scaling limit of free fermions

$$H = \frac{1}{2}p^2 - \frac{1}{2}x^2$$





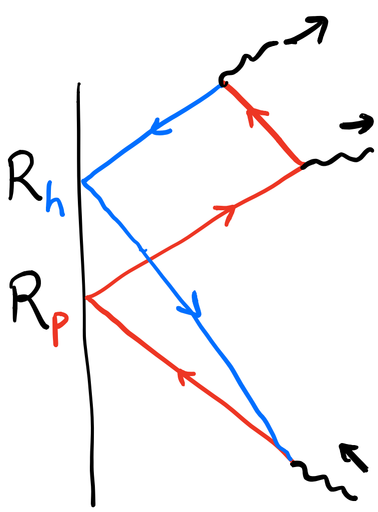
closed string

= collective excitation of fermi surface

= particle / hole pair

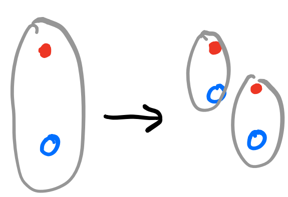
Effective interaction strength

$$g_s = \frac{1}{2\pi\mu} \qquad (\hbar = 1)$$



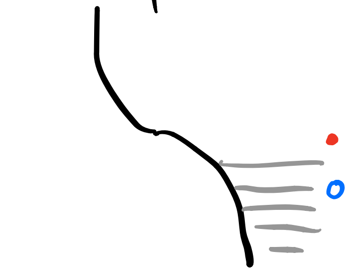
(1 → 2 scattering)

Moore, Plesser, Ramgoolam '91



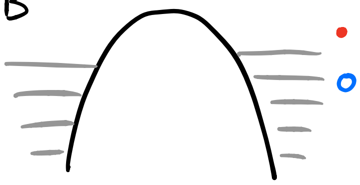
Non-perturbative Completion of MQM

"?"



$$R_h(E) = \left(R_p(E)\right)^* = \left(R_p(E)\right)^{-1}$$

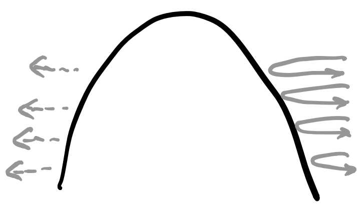
"type 0B"



$$R_h(E) = \left(R_p(E)\right)^*$$

$$|R_h(E)| = |R_p(E)| < 1$$

"c=1"

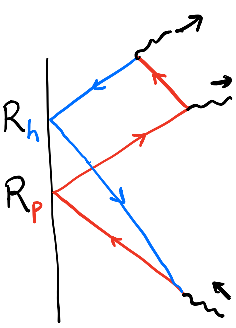


$$R_h(E) = \left(R_p(E)\right)^{-1} \qquad \checkmark$$

$$|R_p(E)| < 1, \quad |R_h(E)| > 1$$

$$R_p(E) = i\,\mu^{iE}\sqrt{\underbrace{\frac{1}{1+e^{2\pi E}}}_{\text{"non-pert"}}\;\underbrace{\frac{\Gamma(\frac{1}{2}-iE)}{\Gamma(\frac{1}{2}+iE)}}_{\text{"pert"}}}$$

$E \sim -\mu + \mathcal{O}(1)$



$$\underset{\omega\to\{\omega_1,\dots,\omega_k\}}{\mathcal{A}_{1\to k}} = \sum_{\substack{\{\omega_1,\dots,\omega_k\}\\ = S_1 \sqcup S_2}} (-)^{|S_2|-1}\int_0^{\omega(S_2)} dx \; R_p(-\mu+\omega-x)\, R_h(-\mu-x)$$

structure of particle/hole amplitude

"closed string expansion"

$$\mathcal{A}_{1\to k} = \sum_{g=0}^{\infty} \left(\frac{1}{\mu}\right)^{k-1+2g} \mathcal{A}_{1\to k}^{\text{pert},(g)}$$

Borel-resummed

$$+ \sum_{n=1}^{\infty} e^{-2\pi n\mu} \sum_{L=0}^{\infty} \left(\frac{1}{\mu}\right)^{L} \mathcal{A}_{1\to k}^{n\text{-inst},(L)}$$

D-instanton expansion

"open string expansion"

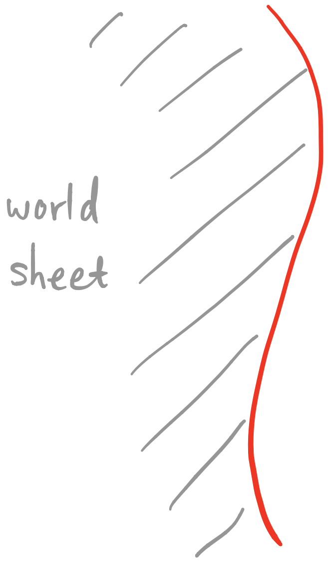
[Absence of $\mathcal{O}(e^{-\frac{1}{g_s^2}})$ effects

— an accident of $c=1$ string theory

... at least in closed string sector

without FZZT-branes...]

We now turn to the world-sheet formulation of D-instanton-mediated (exclusive) closed string amplitudes



D-instanton boundary condition

In $c=1$ string theory,

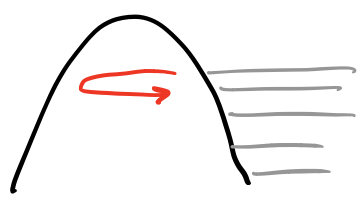
the simplest class of D-instantons are "ZZ-instantons"

$(\text{Dirichlet in } X^0) \otimes (\text{ZZ-brane in Liouville})$

[Note: FZZT-brane $\rightarrow$ infinite action]

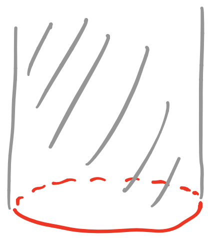
$S_{ZZ\text{-inst}} = -$ 

$= \frac{1}{g_s} = 2\pi\mu$



"fermion bounce"

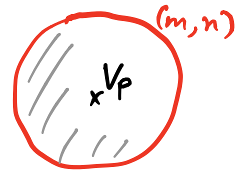
Zamolodchikov^2 '99



ZZ (m,n)

There is a more general family of ZZ boundary states

$$|ZZ(m,n)\rangle\rangle = \int_0^\infty \frac{dp}{\pi}\, \Psi^{(m,n)}(p)\ |V_p\rangle\rangle$$



$$\Psi^{(m,n)}(p) = 2^{\frac{5}{4}}\sqrt{\pi}\ \frac{\sinh(2\pi m p)\sinh(2\pi n p)}{\sinh(2\pi p)}$$

$m, n \in \mathbb{Z}_{\geq 1}$

" (m,n) ZZ - instanton "

$$S_{(m,n)\ ZZ\text{-inst}} = \lim_{p\to i} \frac{\Psi^{(m,n)}(p)}{\Psi^{(1,1)}(p)}\, S_{(1,1)}$$

$$= \frac{mn}{g_s}$$

[By comparison with MQM, we will conclude that (m,1) ZZ-instanton contributes, along with multiple instantons, for all $m \geq 1$]

1 - instanton contribution

to e.g. closed string $1 \to 1$ amplitude

(1) leading order $\sim e^{-\frac{1}{g_s}}$

same ZZ-instanton

$$\mathcal{N} \int dx^0 \; e^{-S_{ZZ}} \; \langle V_\omega^+ \rangle^{D^2}_{ZZ, x^0} \; \langle V_{\omega'}^- \rangle^{D^2}_{ZZ, x^0}$$

D-instanton moduli space

" e° "

$e^{i\omega x^0} \sinh(\pi\omega)$

$e^{-i\omega' x^0} \sinh(\pi\omega)$

$$= \mathcal{N} \, e^{-\frac{1}{g_s}} \, 2\pi \delta(\omega - \omega') \cdot 4 \underbrace{\sinh^2(\pi\omega)}$$

fix once for all by matching w/ MQM

$\mathcal{N} = -\frac{1}{8\pi^2}$ for (1,1) ZZ-instanton

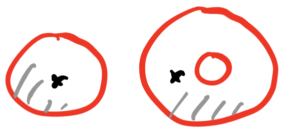
ω-dependence agrees w/ "c=1" MQM

[disagrees w/ "type 0B" MQM etc.]

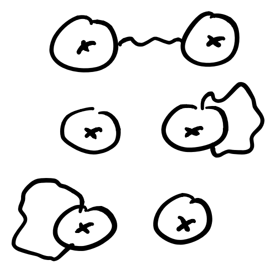
(2) next-to-leading order $\sim e^{-\frac{1}{g_s}} \cdot g_s$

four diagrams:

(i) 

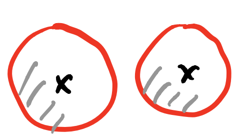
(ii) 

cancel log-divergence due to open string coll. mode ∂X^0



(iii) 

Polchinski '94

(iv)  

leaving a finite ambiguity

$\mathcal{O}(g_s)$ correction to S_{zz}

2 constant ambiguities

(A) Sen: fix using open + closed SFT

rigorous but takes work!

agree!

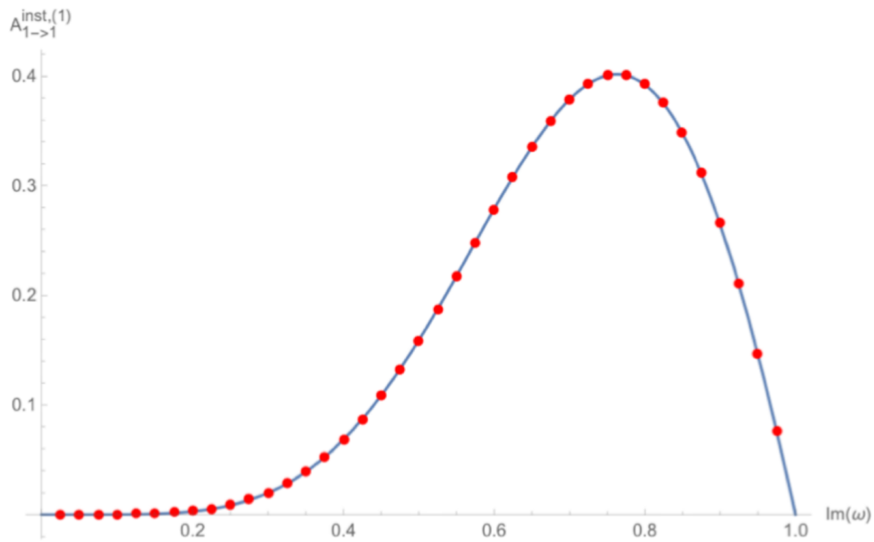
[modulo numerical error]

(B) BRY: fix by matching with MQM (numerically)

not 1st principle from world-sheet

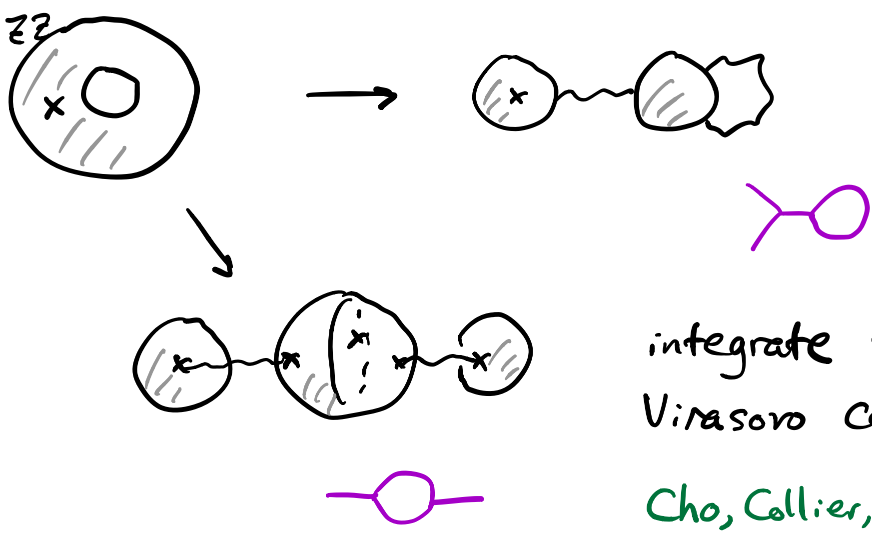
MQM prediction for 1-instanton contribution to $1 \to 1$ closed string amplitude at order $e^{-\frac{1}{g_s}} \cdot g_s$

$$\propto \omega \left(\frac{\pi\omega}{\tanh \pi\omega} - 1 \right) \sinh^2(\pi\omega)$$



• numerics for imaginary ω

- most numerical error comes from evaluating



integrate torus 2-pt Virasoro conf. blocks

Cho, Collier, XY '18

Multi - D -instanton contribution

- May need to include (m,n) ZZ -instantons

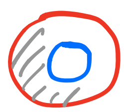
  turns out only $(m,1)$

  by matching with MQM !

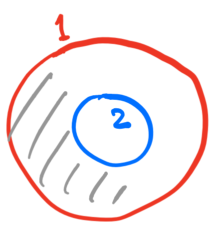
- Multi- instanton moduli space parameterized by coll. coord.

  $x_1^0, \; x_2^0, \; \ldots$

  correction to measure factor

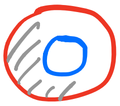
  e 

  e.g. a pair of ZZ -instantons at x_1^0, x_2^0



$$= \int_0^\infty \frac{dt}{2t} \underbrace{\frac{e^{2\pi t} - 1}{\eta(it)}}_{\text{Liouville}} \underbrace{\frac{e^{-t\frac{(\Delta X^E)^2}{2\pi}}}{\eta(it)}}_{X^0} \underbrace{\eta(it)^2}_{bc}$$

$$= \frac{1}{2} \log \frac{(\Delta X^E)^2}{(\Delta X^E)^2 - (2\pi)^2}$$

e^{2}  $= \dfrac{(\Delta x^E)^2}{(\Delta x^E)^2 - (2\pi)^2}$

"Vandermonde"

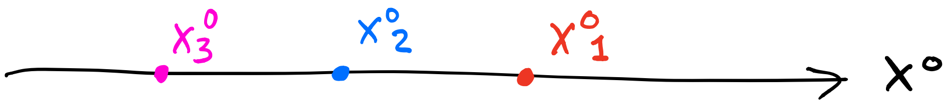
"open string tachyon pole"

$$\longrightarrow \frac{(\Delta x)^2}{(\Delta x)^2 + (2\pi)^2} \qquad \text{Lorentzian } \Delta x$$

prescription:

integrate D-instanton moduli along Lorentzian contour in x_i^0

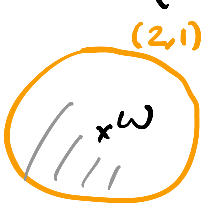
[equivalent to a suitable $i\varepsilon$- prescription for Euclidean contour]



Example: 2-instanton contribution to $\mathcal{A}_{1\to 1}$ at order $e^{-\frac{2}{g_s}}$

$\mathcal{N}_1^2 \ \tfrac{1}{2} \int dx_1\, dx_2 \Big[\Big($   $+ 1 \leftrightarrow 2\Big)\, e^{2}$ 

$+\, 2$   $\Big(e^{2}$  $- 1\Big)\Big]$

$+\ \mathcal{N}_2 \int dx$  

Result for D-instanton contribution to $\mathcal{A}_{1\to 1}$ at order $e^{-\frac{n}{g_s}}$

$$\sum_{\{m_1,\cdots,m_\ell\}} \frac{\mathcal{N}_{m_1}\cdots\mathcal{N}_{m_\ell}}{S} (-)^\ell\, 2^{2\ell}\,\pi^{2\ell-1}\,\frac{m_1\cdots m_\ell}{n}$$

$$\times\, (\ell-1)!\; e^{-\pi\omega n}\,\sinh(\pi\omega n)\,\left(\ell-\sum_{i=1}^{\ell} e^{2\pi\omega m_i}\right)$$

MQM result

$$\pi^{-\frac{3}{2}}\,\frac{(-)^n}{n}\,\frac{\Gamma(n+\frac{1}{2})}{n!}\,e^{\pi\omega n}\,\sinh(\pi\omega n)$$

$$\times\, {}_2F_1\left(-\tfrac{1}{2}, -n; \tfrac{1}{2}-n; e^{-2\pi\omega}\right)$$

By some combinatoric magic ...

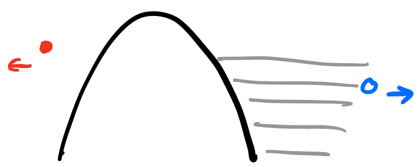
they agree ! provided fixing

$$\mathcal{N}_m = \frac{(-)^m}{4\pi^2 m}\,\frac{(2m-1)!!}{(2m)!!}$$

Conclusion

- $c=1$ string theory defined by world-sheet genus expansion + D-instanton effects is non-perturbatively "complete"

- So far, restricted to exclusive closed string amplitudes, which do not saturate unitarity

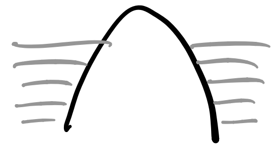


restore unitarity by including ZZ-brane w/ OS tachyon rolling to "wrong side"?

- D-instanton contribution unambiguous thanks to Borel summability of pert. series, and absence of $O(e^{-\frac{1}{g_s}})$ effects

  - special to $c=1$ string

    [may not hold w/ FZZT-branes, ⇝ non-singlet sector, black hole ...]

- generalization
  - finite temperature
  - 2D 0B string / MQM 

W.I.P.

- Lessons for critical superstrings

[need O+C SFT @ order $e^{-\frac{1}{g_s}} g_s^{n>1}$]