What is the correct framework for Quantum Field Theories?

Yuji Tachikawa

- My research area

  theoretical high energy physics / string theory / mathematics

- My obsession these days

  what is the **correct framework** for **quantum field theories**?

Disclaimer:

I don't yet have an answer!

But I'll try to explain why it's an important and/or interesting question.

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But I'll try to explain why it's an important and/or interesting question.

I don't have any cute figures in the slides either.

And I don't have any excuse for it. I'm sorry.

What is a **quantum** **field** **theory**?

- It describes **quantum** properties of **fields**,
- where a field is **anything that is extended along the spacetime**, e.g. electromagnetic field, electron field, ...

A prototypical **quantum field theory** (QFT) is
the **Quantum Electrodynamics**

- which describes the quantized electromagnetic field interacting with electrons etc., and
- was established around 1950.

Since then there has been a steady progress.

By now we know that **our world**, at the most microscopic level that is experimentally accessible, is described by a **quantum field theory** called the **Standard Model**, established theoretically in the 1970s.

Its final piece, the Higgs boson, was confirmed experimentally in 2012, and both high energy theorists and experimentalists are looking for **physics beyond the Standard Model**.

Quantum field theories also appear ubiquitously in **condensed matter physics**.

For example, physics at the second-order phase transition is often described by **conformal field theories**,
which form a certain subclass of quantum field theories.

This ubiquity is not surprising, since **quantum fields** are just the **quantum version of anything that are extended along the space**.

Due to its ubiquity, many people have worked on quantum field theories over its history of more than half a century.

Theoretical predictions agree well with experimental results. One extreme example is the **anomalous magnetic moment of electron**:

$$a_e = 1\,159\,652\,181.7... \times 10^{-12}$$

In this case the agreement is to **12 decimal places**.

Agreements in the case of the Standard Model vs. the Large Hadron Collider experiments are quite impressive too.

Even with this impressive agreement, I say **we don't yet know what's the correct framework to study quantum field theories.**

This is in contrast to the situation for **quantum mechanics or general relativity, for which** I say **we know the correct frameworks**.

Why do I say so?

In the case of quantum mechanics or general relativity, I can explain at least one framework to mathematicians in a few sentences.

Quantum mechanics

It's a study of unitary operators acting on a Hilbert space.

General relativity

It's a study of a differential equation satisfied by
the Riemann tensor of a Lorentzian metric on a manifold.

Not bad.

But then, what is a quantum field theory?

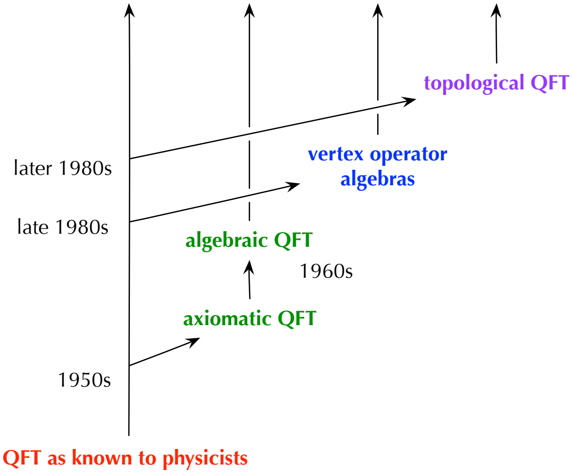
Quantum Field Theory

???

As a physicist, I think I know,
but I don't have a way to explain it to mathematicians.

I believe it's not just a problem about me.

Many people tried to formulate mathematically some small part of what they knew about quantum field theories:



However, in none of the frameworks we can express e.g. the computations that led to the impressive prediction

$$a_e = 1\,159\,652\,181.7... \times 10^{-12}$$

for the anomalous magnetic moment of the electron.

- The framework **axiomatic QFT** is **too broad and generic** to carry out this particular computation
- The frameworks **vertex operator algebras** and **topological QFT** are **too narrow** and exclude the real Quantum Electrodynamics

That said, the method to compute this value

$$\boldsymbol{a_e = 1\,159\,652\,181.7... \times 10^{-12}}$$

using quantum electrodynamics is **explained in quantum field theory textbooks for physicists**.

Doesn't it suffice to mathematically formulate what are written there?

I don't think so.

What are explained in QFT textbooks for physicists are **how to compute things** in the **quantum electrodynamics** or its extension, the **Standard Model**.

But they are both rather **mundane** from a more modern point of view of quantum field theories. After all, although experimentally verified only a few years ago, the Standard Model was theoretically established in the early 70s.

In a sense, the problem is that quantum field theories as known to physicists **always move on,** and that the previous attempts to formulate them were always too early.

And **the modern view itself is in flux** in the last few years.

I'd like to illustrate this point in the rest of my talk,
using my own recent research as an example.

A big topic in our area is **supersymmetric** quantum field theories.

Supersymmetry allows us to relate
the **bosons** (photons, etc.) and the **fermions** (electrons, etc.):

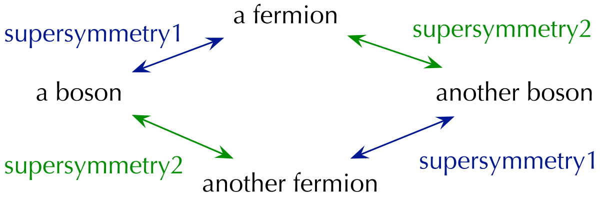
$$\text{a boson} \xleftrightarrow{\text{supersymmetry}} \text{a fermion}$$

If it is physically realized, it can solve various unsettling questions in the current Standard Model.

There are (theoretical) condensed-matter realizations too.

But I'm more interested in its theoretical structure.

Then there can be quantum field theories that can have **multiple supersymmetries**:



If there are two, it is called an $\mathcal{N}=2$ supersymmetry, for historical reasons.

How many supersymmetries can a quantum field theory have?

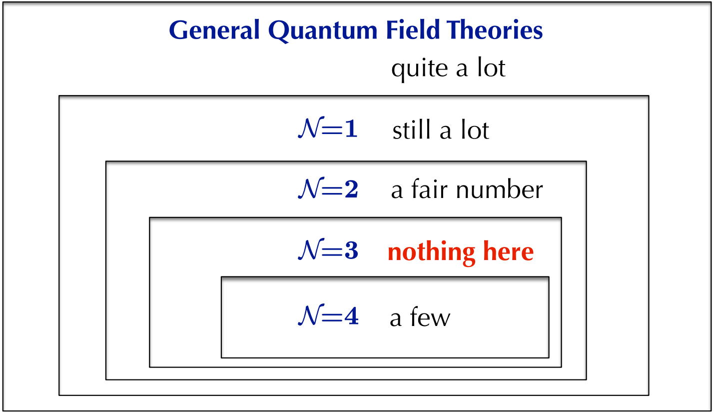
That's a topic covered often in the **first chapter** of a textbook on supersymmetry, and the standard answer **has been** as follows:

$$\mathcal{N} = 1, \qquad \mathcal{N} = 2, \qquad \mathcal{N} = 4$$

and that's it.

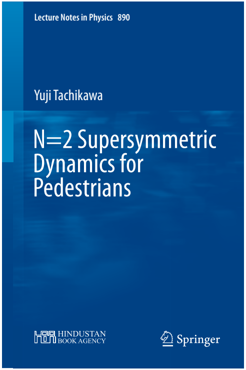
I don't have time to explain why there can't be more than four.

You might ask, why not $\mathcal{N} = 3$?

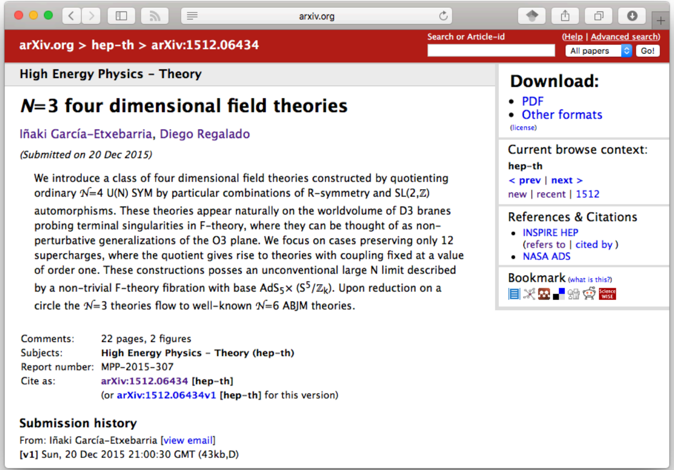


Well, if a quantum field theory has $\mathcal{N}=3$,
you can easily check it has in fact $\mathcal{N}=4$, or so people said.

I myself explained thus in a lecture note published two years ago from Springer:



And still, this paper appeared on **December 20, 2015** !



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High Energy Physics – Theory

$\mathcal{N}$=3 four dimensional field theories

Iñaki García-Etxebarria, Diego Regalado

(Submitted on 20 Dec 2015)

We introduce a class of four dimensional field theories constructed by quotienting ordinary $\mathcal{N}$=4 U(N) SYM by particular combinations of R-symmetry and SL(2,$\mathbb{Z}$) automorphisms. These theories appear naturally on the worldvolume of D3 branes probing terminal singularities in F-theory, where they can be thought of as non-perturbative generalizations of the O3 plane. We focus on cases preserving only 12 supercharges, where the quotient gives rise to theories with coupling fixed at a value of order one. These constructions posses an unconventional large N limit described by a non-trivial F-theory fibration with base $AdS_5 \times (S^5/\mathbb{Z}_k)$. Upon reduction on a circle the $\mathcal{N}$=3 theories flow to well-known $\mathcal{N}$=6 ABJM theories.

Comments:	22 pages, 2 figures
Subjects:	**High Energy Physics – Theory (hep-th)**
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[v1] Sun, 20 Dec 2015 21:00:30 GMT (43kb,D)

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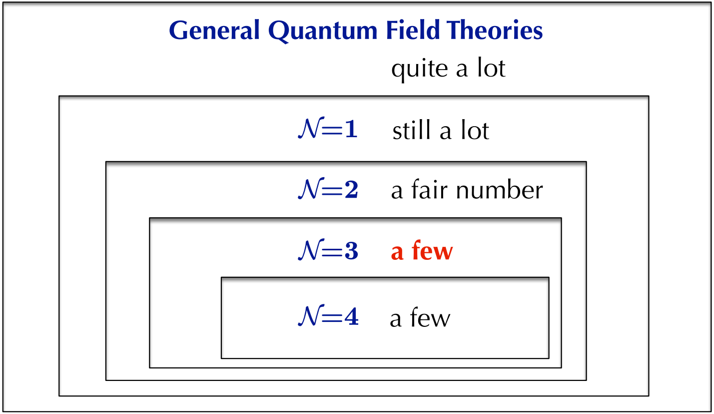
where **genuinely $\mathcal{N}$=3 theories were found, using string theory**.



So, why were all the textbooks (including mine) wrong?

In the case of an object attached to a mechanical spring:

Equation of motion: $$m\frac{d^2}{dt^2}x = -kx$$

In the case of an object attached to a mechanical spring:

Hamiltonian: $$H_t = \frac{1}{2m}p^2 + \frac{k}{2}x^2$$

$$\downarrow$$

Equation of motion: $$m\frac{d^2}{dt^2}x = -kx$$

In the case of an object attached to a mechanical spring:

Lagrangian: $$S = \int dt(\frac{m}{2}(\frac{dx}{dt})^2 - \frac{k}{2}x^2)$$

$\updownarrow$

Hamiltonian: $$H_t = \frac{1}{2m}p^2 + \frac{k}{2}x^2$$

$\downarrow$

Equation of motion: $$m\frac{d^2}{dt^2}x = -kx$$

We are then taught that **Hamiltnian and Lagrangian frameworks are equivalent.**

In textbooks on quantum field theories, it's often said:

Equation**s** of motion: complicated.

In textbooks on quantum field theories, it's often said:

Hamiltonian**s**:		$\boldsymbol{H_t}$, $\boldsymbol{H_x}$, $\boldsymbol{H_y}$, $\boldsymbol{H_z}$, still complicated.
	$\downarrow$	
Equation**s** of motion:		complicated.

In textbooks on quantum field theories, it's often said:

Lagrangian:	$S = \int (\text{something simple}) d^4x$
$\downarrow$	
Hamiltonian**s**:	H_t, H_x, H_y, H_z, still complicated.
$\downarrow$	
Equation**s** of motion:	complicated.

Richard Feynman even got a Nobel Prize for coming up with a Lagrangian framework for quantum field theories.

But in the last several years, we learned that

(Lagrangian: not available)

Hamiltonians: $\boldsymbol{H_t}$, $\boldsymbol{H_x}$, $\boldsymbol{H_y}$, $\boldsymbol{H_z}$, complicated.

$\downarrow$

Equations of motion: complicated.

for many quantum field theories.

So, why did all the textbooks (including mine) mistakenly state that any $\mathcal{N}=3$ quantum field theory automatically has $\mathcal{N}=4$ supersymmetry?

Well, if you assume the existence of a **Lagrangian**,

- you write down all possible $\mathcal{N}=2$ supersymmetric **Lagrangians**
- look for any theory that has more than $\mathcal{N}=2$ in the list
- you only find those with $\mathcal{N}=4$, never with $\mathcal{N}=3$.

But there can be $\mathcal{N}=3$ theories **without Lagrangians**,
and indeed there are.

Let me summarize:

Basically, **all the textbooks on quantum field theories out there** use an old framework that is **simply too narrow**, in that **it assumes the existence of a Lagrangian**.

This is a serious issue, because when you try to come up e.g. with a theory beyond the Standard Model, people habitually start by writing a Lagrangian…but **that might be putting too strong an assumption.**

We need to do something.