Yet another duality for $\mathcal{N}$=1 SQCD

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SU(N) with 2N flavors $q, \tilde{q}$

Seiberg duality $\Updownarrow$

SU(N) with 2N flavors $q, \tilde{q}$

and a singlet M

$W = M q \tilde{q}$

SU(N) with 2N flavors $q, \tilde{q}$

Seiberg duality

SU(N) with 2N flavors $q, \tilde{q}$
and a singlet M
$W = Mq\tilde{q}$

Another description
involving T_N theory

- a conventional field theory consists of:
  - Free matter fields
  - Gauge fields coupled to flavor currents
  - Gauge invariant interactions

- X : Free matter fields
- A : Gauge fields
- V : Interactions

- Y : Free matter fields
- B : Gauge fields
- W : Interactions

the Same Infrared Limit

- X : Free matter fields
- A : Gauge fields
- V : Interactions

- Y : Free matter fields
- B : Gauge fields
- W : Interactions

Duality

the Same Infrared Limit

- a conventional field theory consists of:
  - Free matter fields
  - Gauge fields coupled to flavor currents
  - Gauge invariant interactions

- a non-conventional field theory consists of:
  - Nontrivial Conformal Field Theories
  - Gauge fields coupled to flavor currents
  - Gauge invariant interactions

Georgi, "Unparticles"

Meade-Seiberg-Shih, "GGM"

- Nontrivial Conformal Field Theories
    - can be an infrared limit of other conventional theories.
    - can come from 6d construction.
    - can have multiple realizations
- What determines the dynamics of the combined system is the CFT *per se*.

- a non-conventional field theory consists of:
  - Nontrivial Conformal Field Theories
  - Gauge fields coupled to flavor currents
  - Gauge invariant interactions

- X : Free matter fields
- A : Gauge fields
- V : Interactions

- Y : Free matter fields
- B : Gauge fields
- W : Interactions

Conventional-conventional Duality

the Same Infrared Limit

- X : CFTs
- A : Gauge fields
- V : Interactions

- Y : CFTs
- B : Gauge fields
- W : Interactions

Non-conventional-non-conventional

Duality

the Same Infrared Limit

- X : Free matter fields
- A : Gauge fields
- V : Interactions

- Y : CFTs
- B : Gauge fields
- W : Interactions

Conventional-non-conventional

Duality

the Same Infrared Limit

N=2 SU(2) with 4 flavors $q, \tilde{q}$
at coupling τ

N=2 SU(2) with 4 flavors $q, \tilde{q}$
at coupling $\tau'' = 1/(1-\tau)$

N=2 SU(2) with 4 flavors $q, \tilde{q}$
at coupling $\tau' = -1/\tau$

Seiberg-Witten

$\mathcal{N}=2$ SU(N) with 2N flavors $q, \tilde{q}$
at coupling τ

$\mathcal{N}=2$ SU(2) with 1 flavor $q, \tilde{q}$
at coupling $\tau'' = 1/(1-\tau)$
coupled to T_N theory

$\mathcal{N}=2$ SU(N) with 2N flavors $q, \tilde{q}$
at coupling $\tau' = -1/\tau$

Argyres-Seiberg, 2007

- What would be the $\mathcal{N}=1$ version of Argyres-Seiberg duality?
- It turned out to be subtler than I expected.

SU(2) with 4 flavors $q, \tilde{q}$

Seiberg

SU(2) with 4 flavors $q, \tilde{q}$

and 16 singlets M

$W = Mq\tilde{q}$

Sp(1) with 8 doublets Q

Seiberg

$\updownarrow$

SU(2) with 4 flavors $q, \tilde{q}$

and 16 singlets M

$W = Mq\tilde{q}$

Intriligator-Pouliot

Sp(1) with 8 doublets Q $\longleftrightarrow$ Sp(1) with 8 doublets Q and 28 singlets M $W=MQQ$

$\updownarrow$ Seiberg

SU(2) with 4 flavors $q, \tilde{q}$ and 16 singlets M $W=Mq\tilde{q}$

Intriligator-Pouliot

Sp(1) with 8 doublets Q

Sp(1) with 8 doublets Q
and 28 singlets M
$W=MQQ$

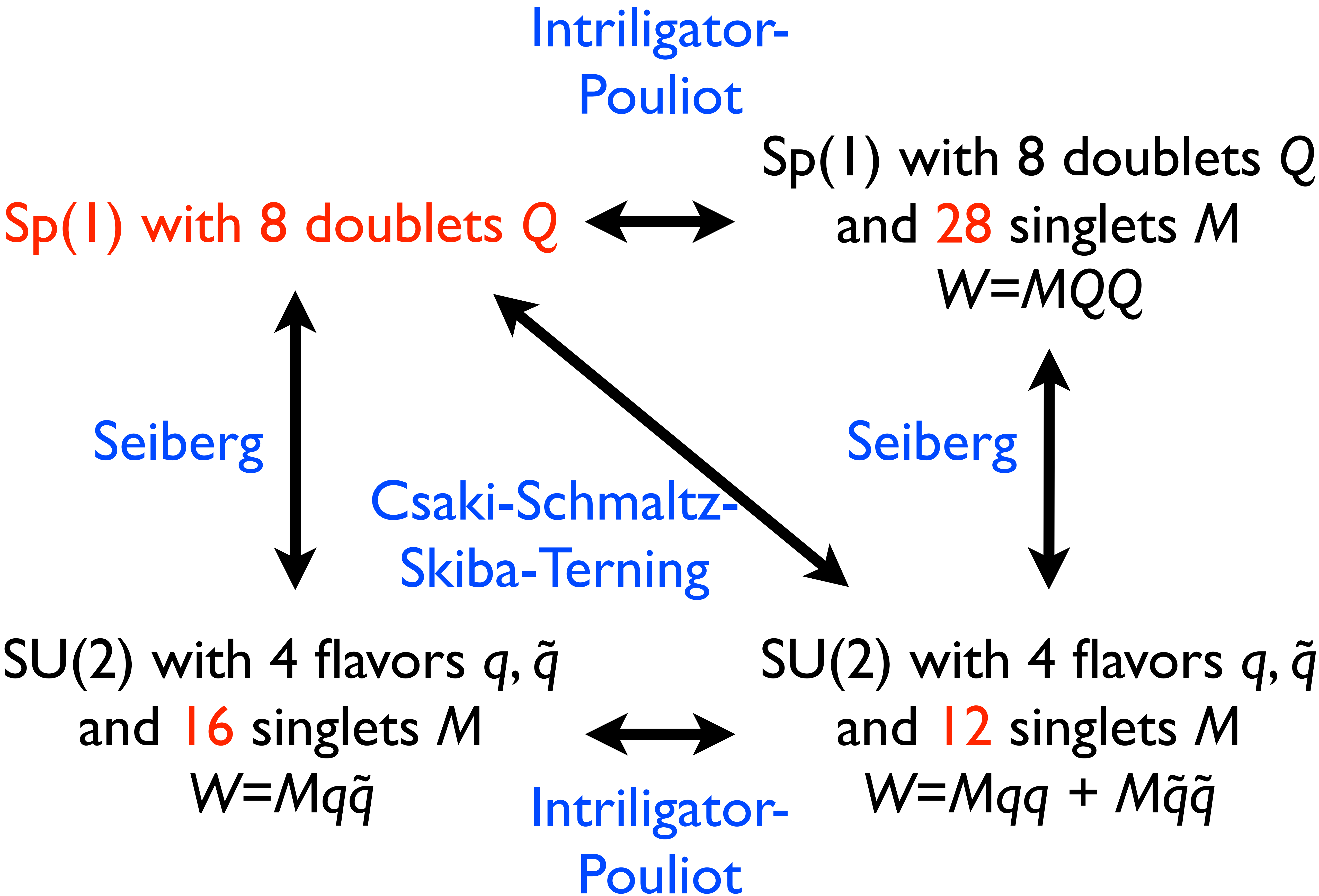
Seiberg

Seiberg

SU(2) with 4 flavors $q, \tilde{q}$
and 16 singlets M
$W=Mq\tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W=Mqq + M\tilde{q}\tilde{q}$

Intriligator-Pouliot


Intriligator-Pouliot
Sp(1) with 8 doublets Q
Sp(1) with 8 doublets Q and 28 singlets M $W=MQQ$
Seiberg
Csaki-Schmaltz-Skiba-Terning
Seiberg
SU(2) with 4 flavors $q, \tilde{q}$ and 16 singlets M $W=Mq\tilde{q}$
SU(2) with 4 flavors $q, \tilde{q}$ and 12 singlets M $W=Mqq + M\tilde{q}\tilde{q}$
Intriligator-Pouliot

SU(2) with 4 flavors $q, \tilde{q}$

Csaki-Schmaltz-
Skiba-Terning

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W = Mqq + M\tilde{q}\tilde{q}$

We're going to generalize this conventional-conventional duality to...

SU(N) with 2N flavors $q, \tilde{q}$

Something non-conventional.

To a conventional-non-conventional duality.

1. Trifundamentals and T_N theory

2. Argyres-Seiberg duality

3. Another dual for $\mathcal{N}$=1 SQCD

Tri-fundamentals and T_N theory

SU(2) with 4 flavors $q, \tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W = Mqq + M\tilde{q}\tilde{q}$

SU(2) with 4 doublets q
and 4 doublets $\tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W = Mqq + M\tilde{q}\tilde{q}$

4 doublets q of SU(2)

4 doublets q of SU(2)

q_{ax} : a=1,2 ; x=1,2,3,4

4 doublets q of SU(2)

q_{ax} : a=1,2 ; x=1,2,3,4

q_{aui} : a=1,2 ; u=1,2 ; i=1,2

makes only $SU(2)^3$ manifest

but generalizable to $SU(N)^3$

SU(2) with 4 flavors $q, \tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M

$W = Mqq + M\tilde{q}\tilde{q}$

SU(2) with q_{aiu} and $\tilde{q}_{akx}$

SU(2) with q_{aiu} and $\tilde{q}_{akx}$
and gauge singlets $M_{(ij)}$, $M_{(uv)}$, $M_{(kl)}$, $M_{(xy)}$

$$W = M_{(ij)}(qq)_{(ij)} + M_{(uv)}(qq)_{(uv)} + M_{(kl)}(\tilde{q}\tilde{q})_{(kl)} + M_{(xy)}(\tilde{q}\tilde{q})_{(xy)}$$

SU(N) with T_N and T_N'

SU(N) with T_N and T_N'
and gauge singlets M_A , M_B , M_A' , M_B'

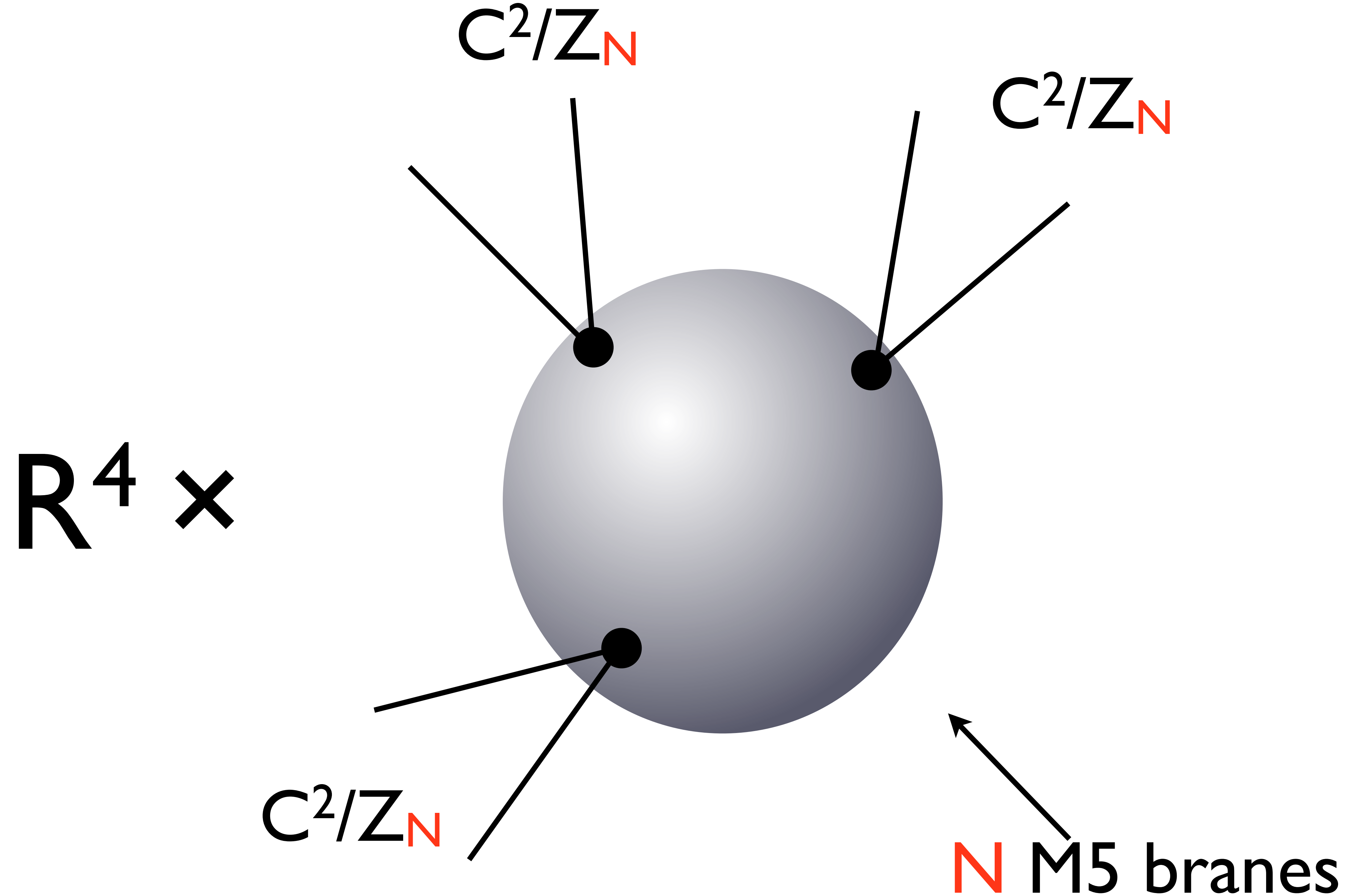
$$W = \mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B$$
$$+ \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$$

The T_N theory

- $T_2 = q_{a,i,u}$ with $SU(2)^3$ symmetry
- T_3 = Minahan&Nemeschansky's E_6 theory
  $E_6 \supset SU(3)^3$
- T_N = an N=2 supersymmetric
  non-Lagrangian theory
  with $SU(N)^3$ symmetry

Gaiotto, 2009

The T_N theory = the infrared limit of



- C^2/Z_N singularity in M-theory has SU(N) gauge symmetry on it.
- So, T_N has $SU(N)^3$ flavor symmetry.
- The Seiberg-Witten curve is known, from this M-theory construction.

- Many other properties are now known.
  - Chiral operators and their relations
  - Central charges
  - Superconformal indices

- T_N is an $\mathcal{N}$=2 supersymmetric theory with $SU(N)_A \times SU(N)_B \times SU(N)_C$ flavor symmetry.

- As such, there are dimension-2 scalar chiral primary operators

$$\mu_A,\ \mu_B,\ \mu_C$$

that are adjoints of $SU(N)_A \times SU(N)_B \times SU(N)_C$

- For example, T_2 is just q_{aiu} with $SU(2)_A \times SU(2)_B \times SU(2)_C$ flavor symmetry.

- Then

$$(\mu_A)_{ab} = q_{aiu}\, q_{bjv}\, \varepsilon^{ij}\, \varepsilon^{uv},$$
$$(\mu_B)_{ij} = q_{aiu}\, q_{bjv}\, \varepsilon^{uv}\, \varepsilon^{ab},$$
$$(\mu_C)_{uv} = q_{aiu}\, q_{bjv}\, \varepsilon^{ab}\, \varepsilon^{ij}.$$

- We can then check

$$\mathrm{tr}\, \mu_A{}^2 = \mathrm{tr}\, \mu_B{}^2 = \mathrm{tr}\, \mu_C{}^2$$

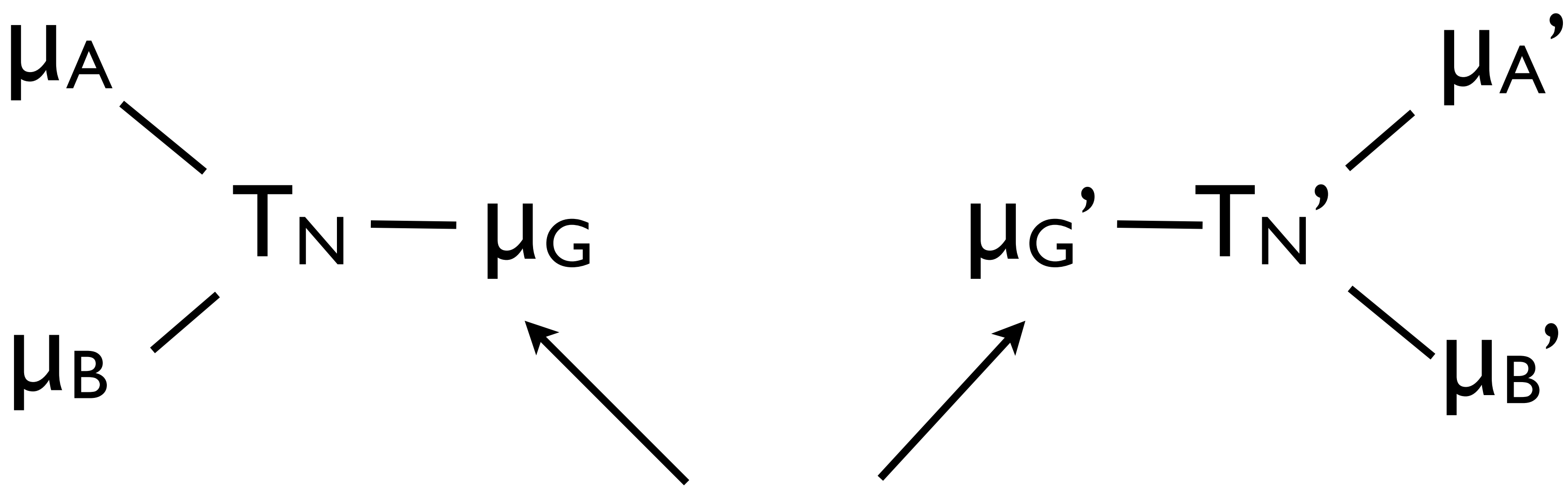
- This is known as Cauchy's hyperdeterminant.

- The same identity holds for T_N :

$$\mathrm{tr}\ \mu_A{}^2 = \mathrm{tr}\ \mu_B{}^2 = \mathrm{tr}\ \mu_C{}^2$$

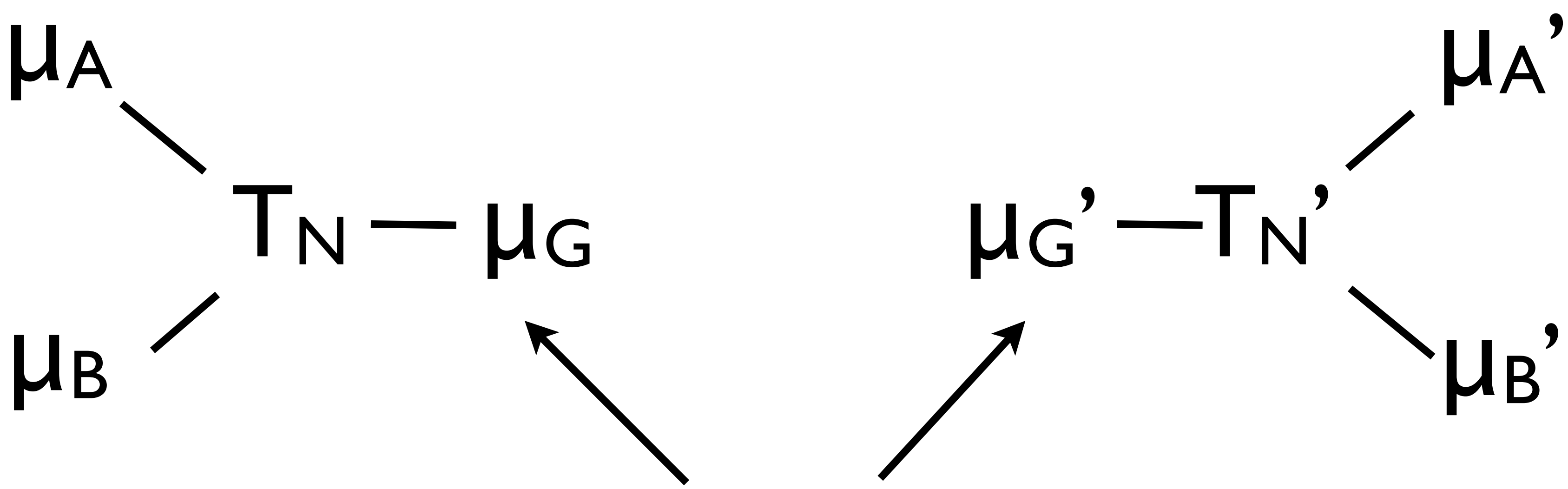
- But there's no known way to write $\mu_{A,B,C}$ in terms of something simpler.

Argyres-Seiberg duality



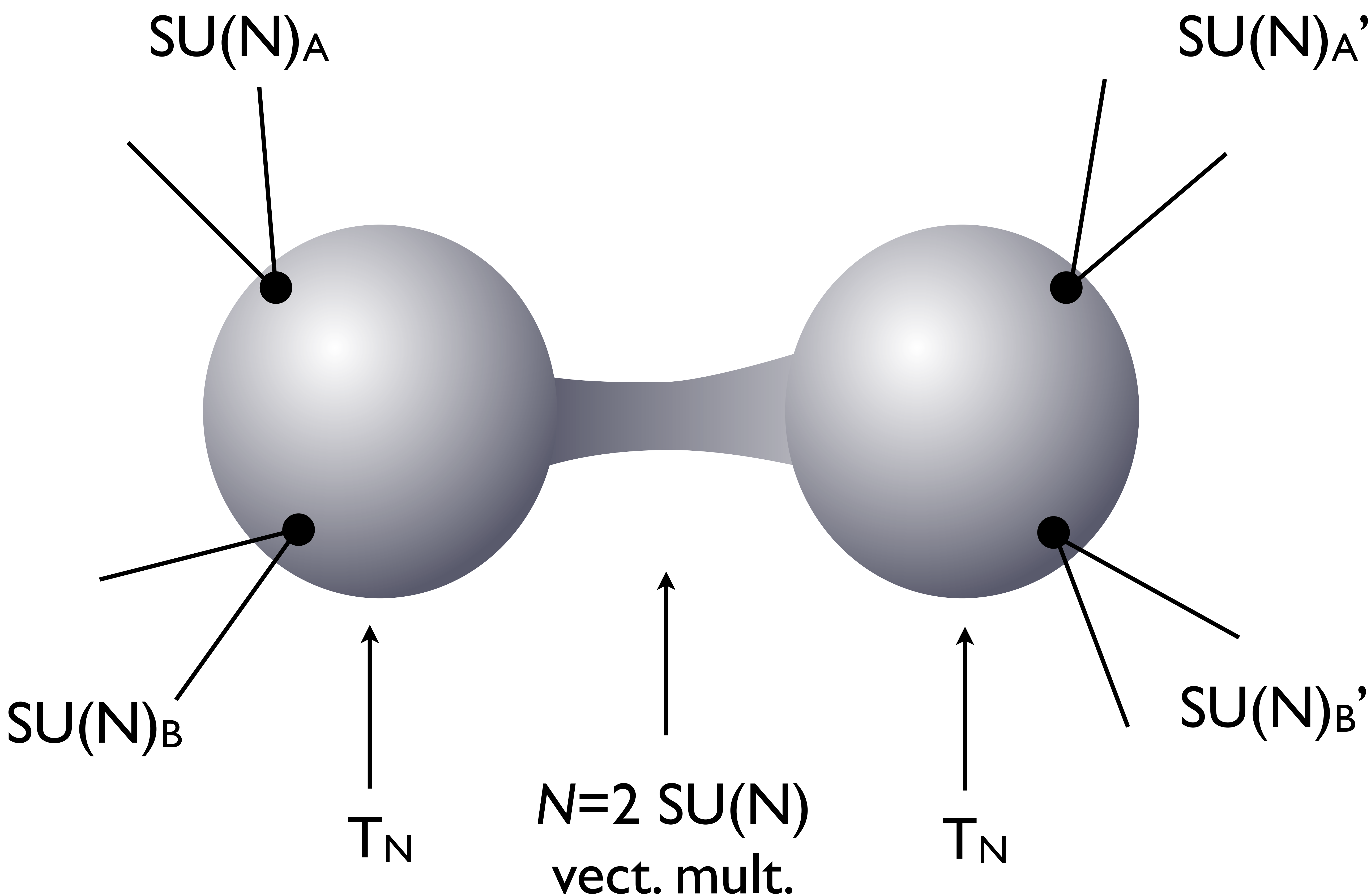
You can couple them to either

1. an *N*=1 SU(N) vector multiplet
2. an *N*=2 SU(N) vector multiplet

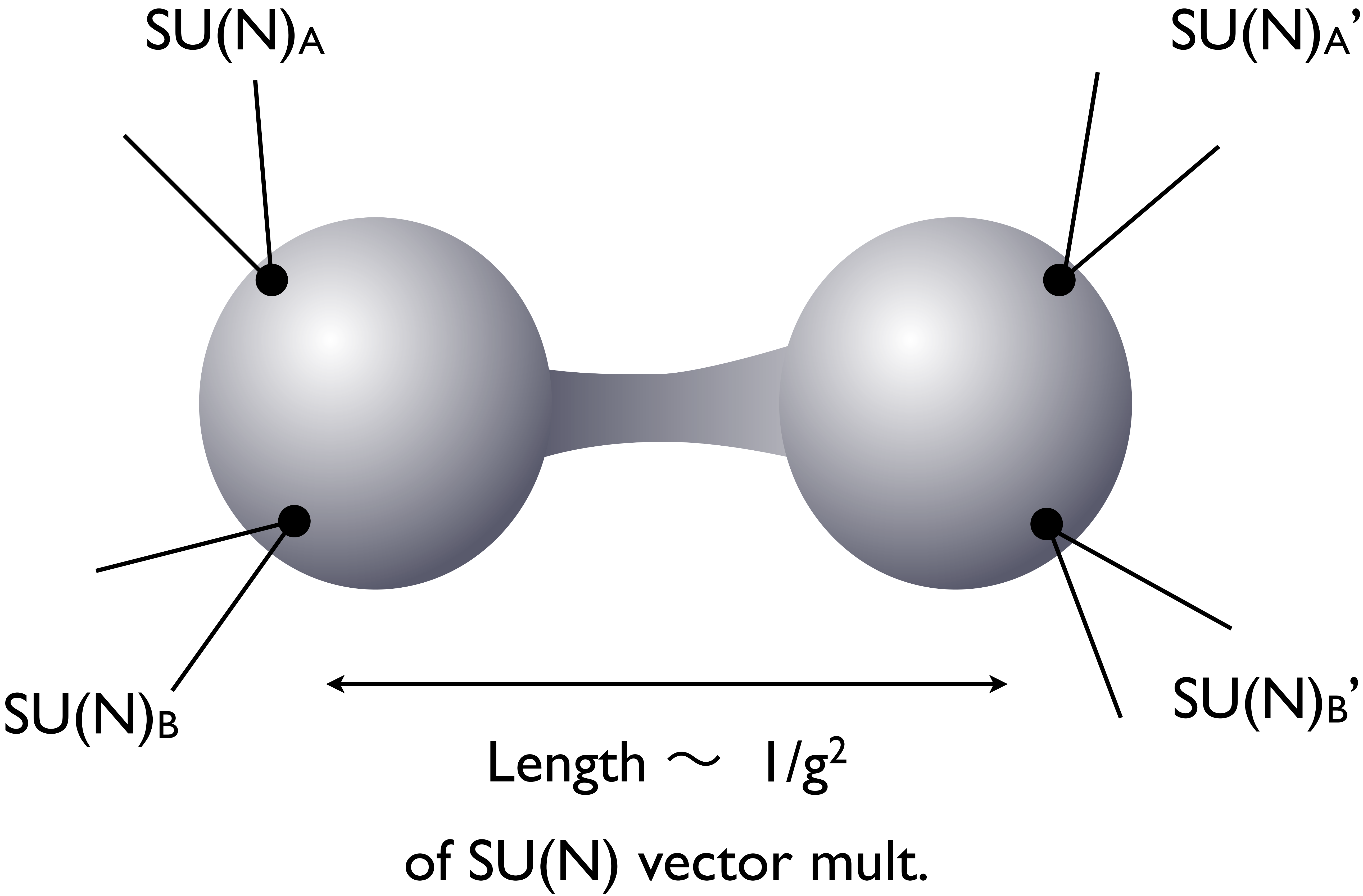


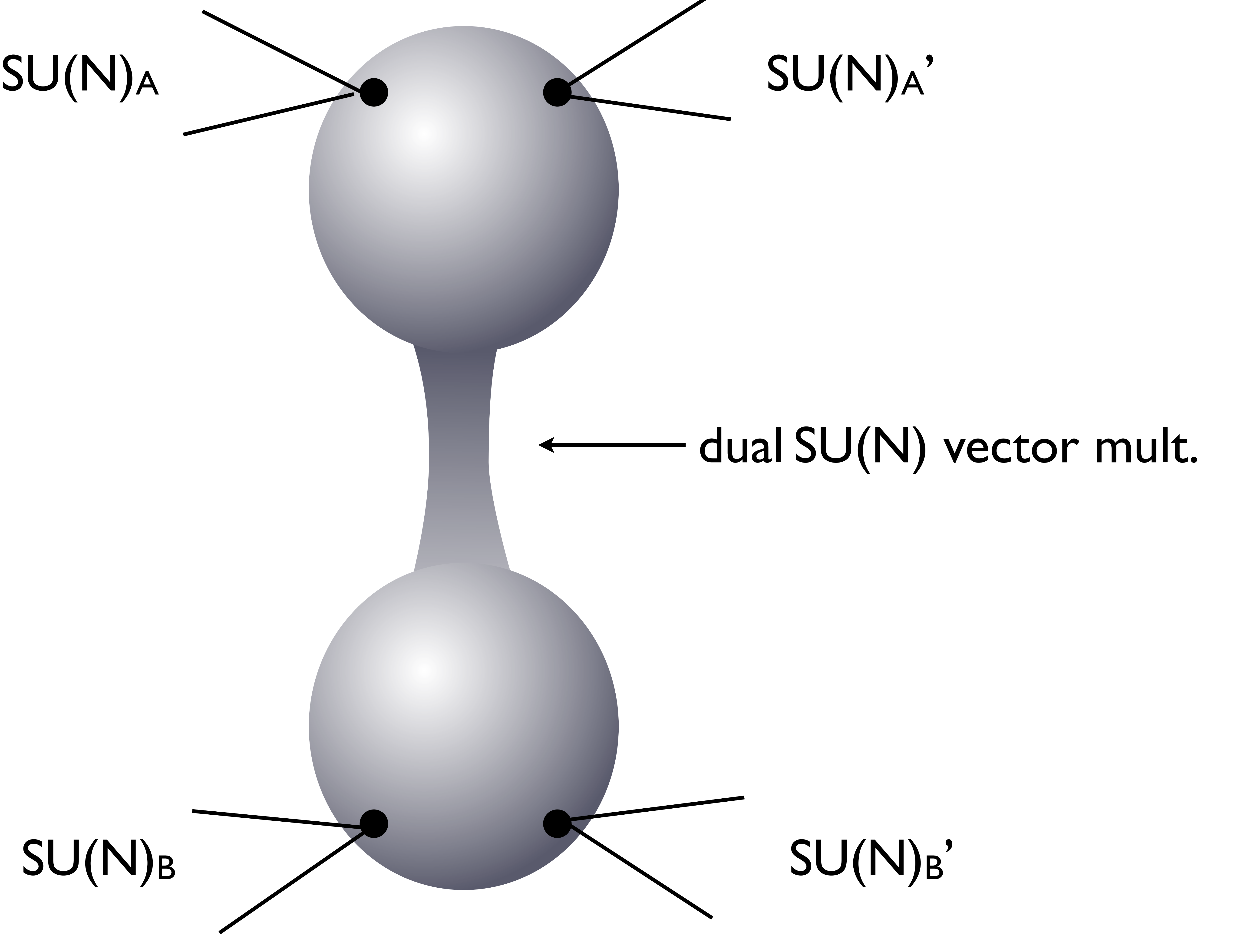
Let's couple them to

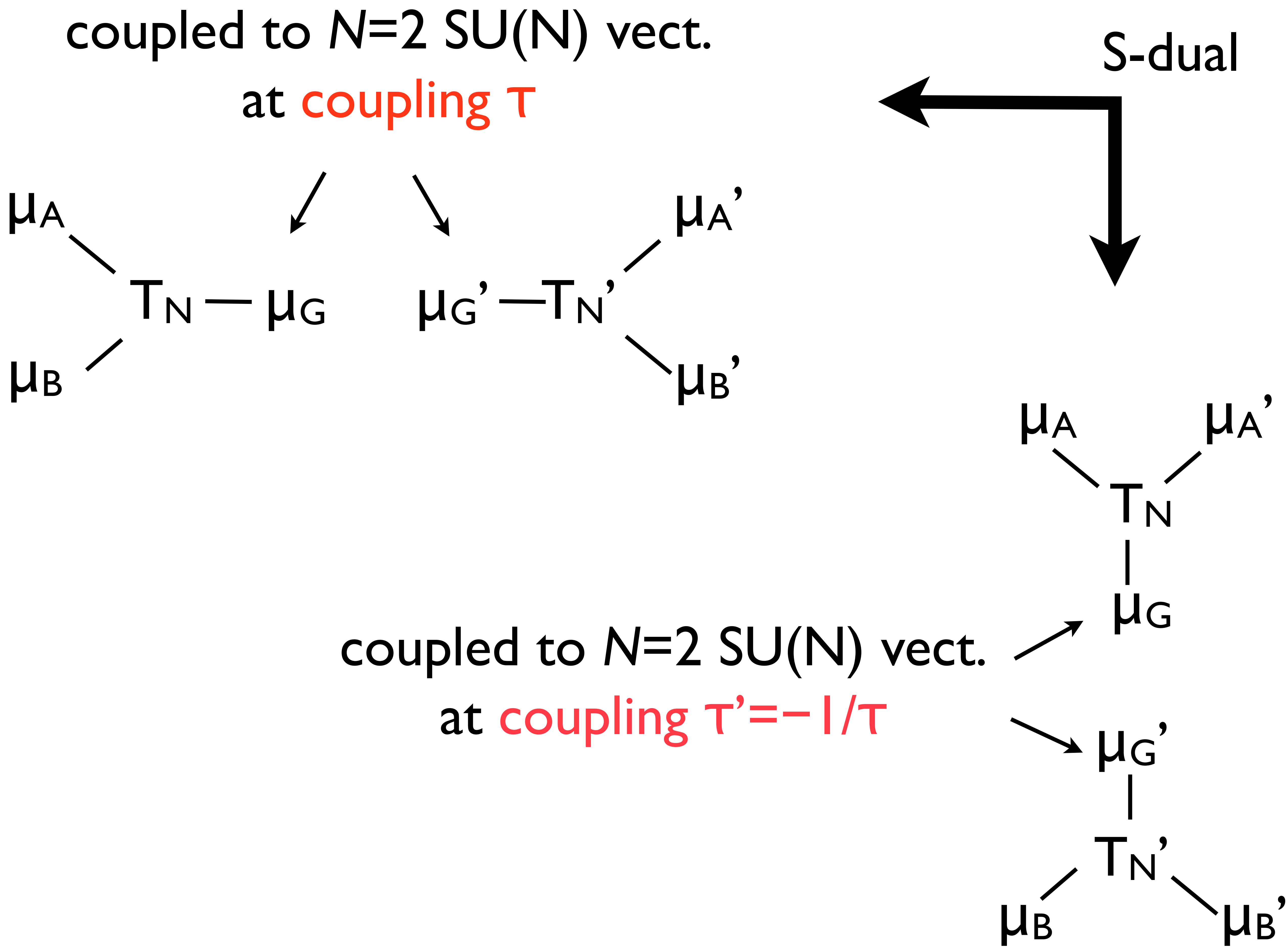
an *N*=2 SU(N) vector multiplet.



SU(N)A
SU(N)A'
SU(N)B
SU(N)B'
TN
N=2 SU(N)
vect. mult.
TN


$SU(N)_A$
$SU(N)_A'$
$SU(N)_B$
$SU(N)_B'$
Length $\sim$ $1/g^2$
of SU(N) vector mult.


SU(N)A
SU(N)A'
dual SU(N) vector mult.
SU(N)B
SU(N)B'


coupled to N=2 SU(N) vect.
at coupling τ
S-dual
μA
μB
TN
μG
μG'
TN'
μA'
μB'
μA
μA'
TN
μG
coupled to N=2 SU(N) vect.
at coupling τ'=−1/τ
μG'
TN'
μB
μB'

- For SU(2), this is the standard S-duality of $\mathcal{N}$=2 SU(2) with 4 flavors.

- For SU(N), this is the S-duality of $\mathcal{N}$=2 SU(N) coupled to two copies of non-Lagrangian theory T_N .

Non-conventional theory

$$\updownarrow \quad \text{S-dual}$$

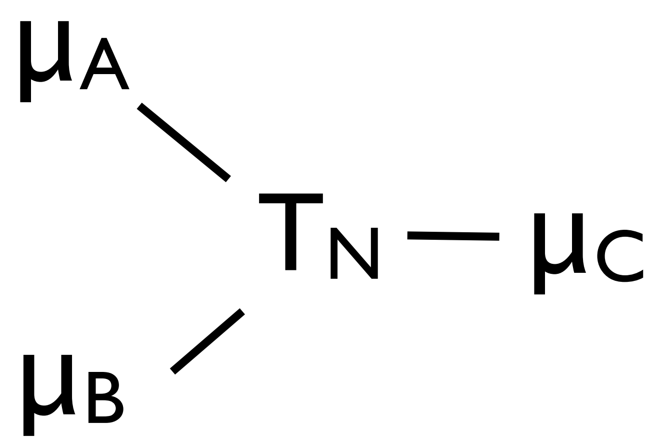
Non-conventional theory

There's an easy way to modify it to

Conventional theory

$\updownarrow$ S-dual

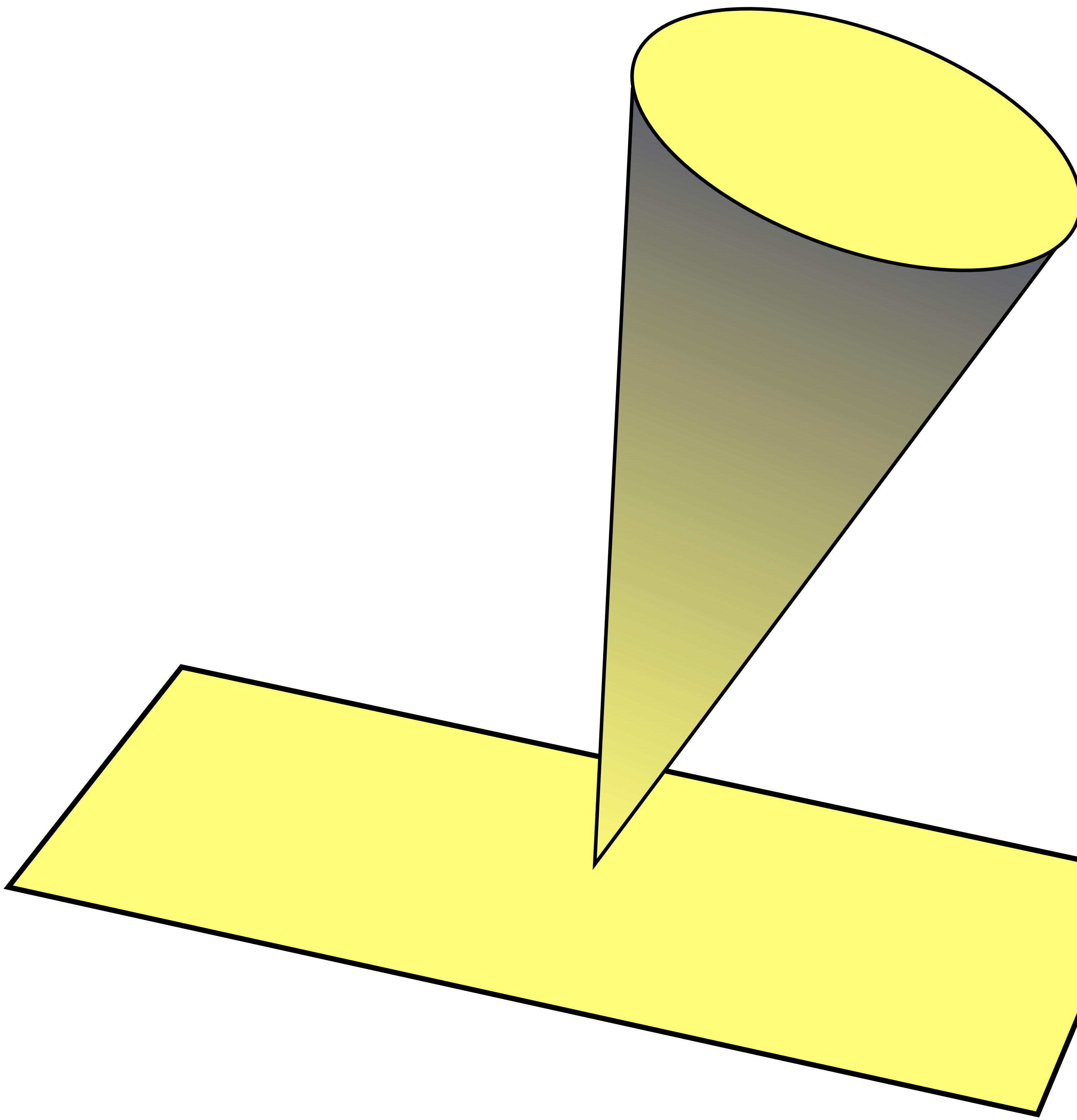
Non-conventional theory

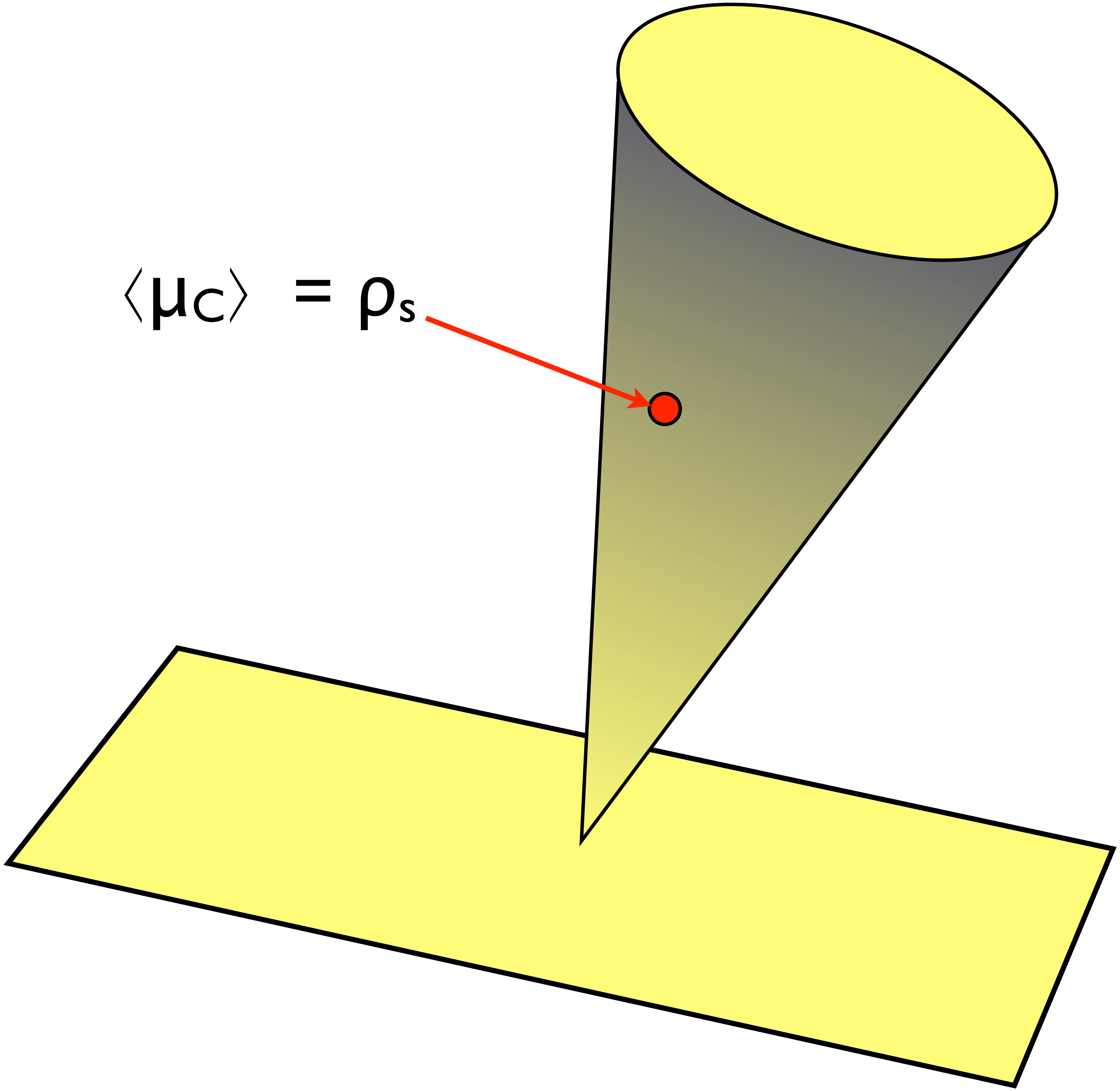


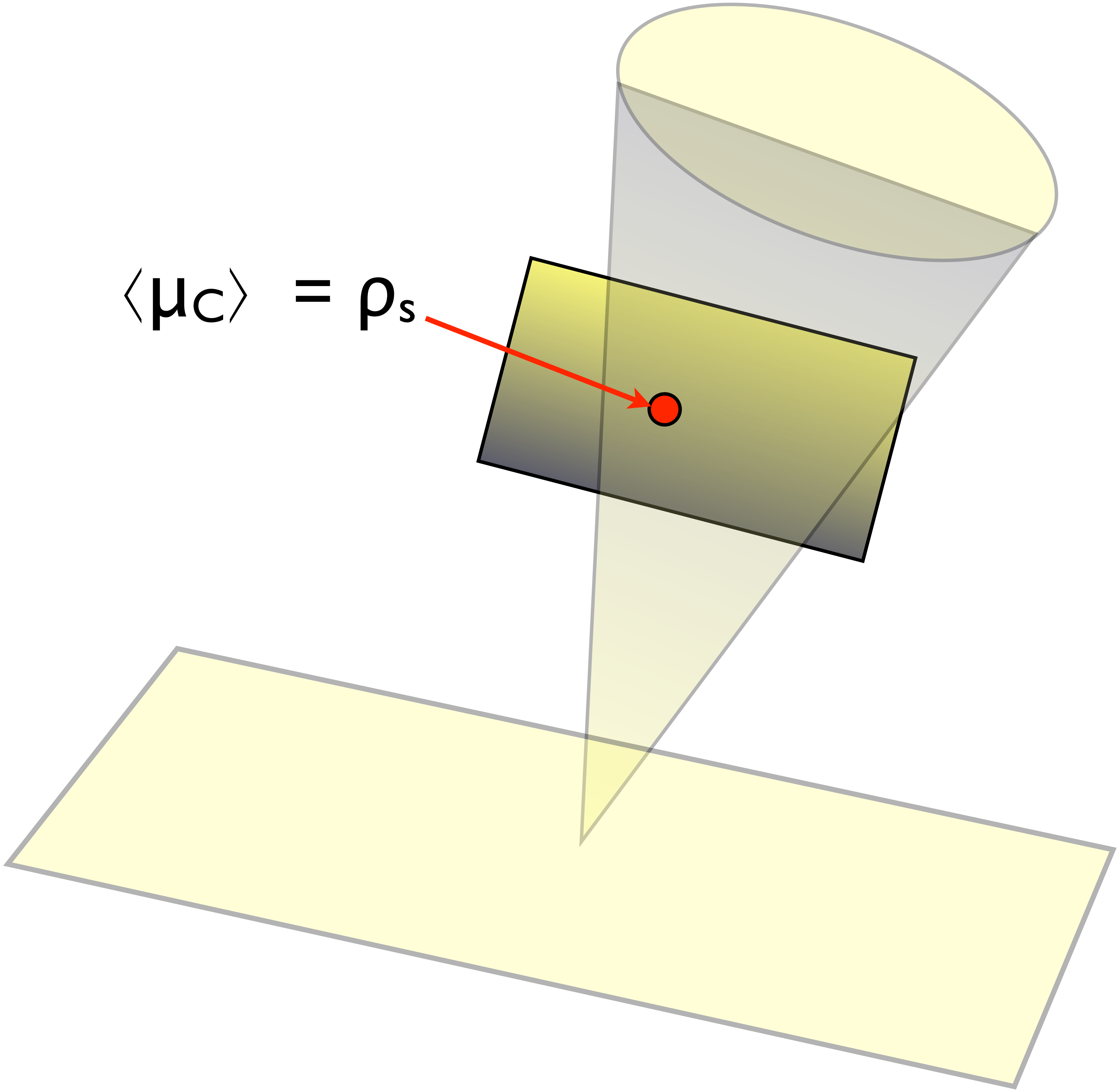
- Recall $\mu_{A,\,B,C}$ are SU(N) adjoints.

- Give a vev

$$\langle \mu_C \rangle = \rho_s = \left[\begin{array}{ccc|c} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \end{array}\right]$$

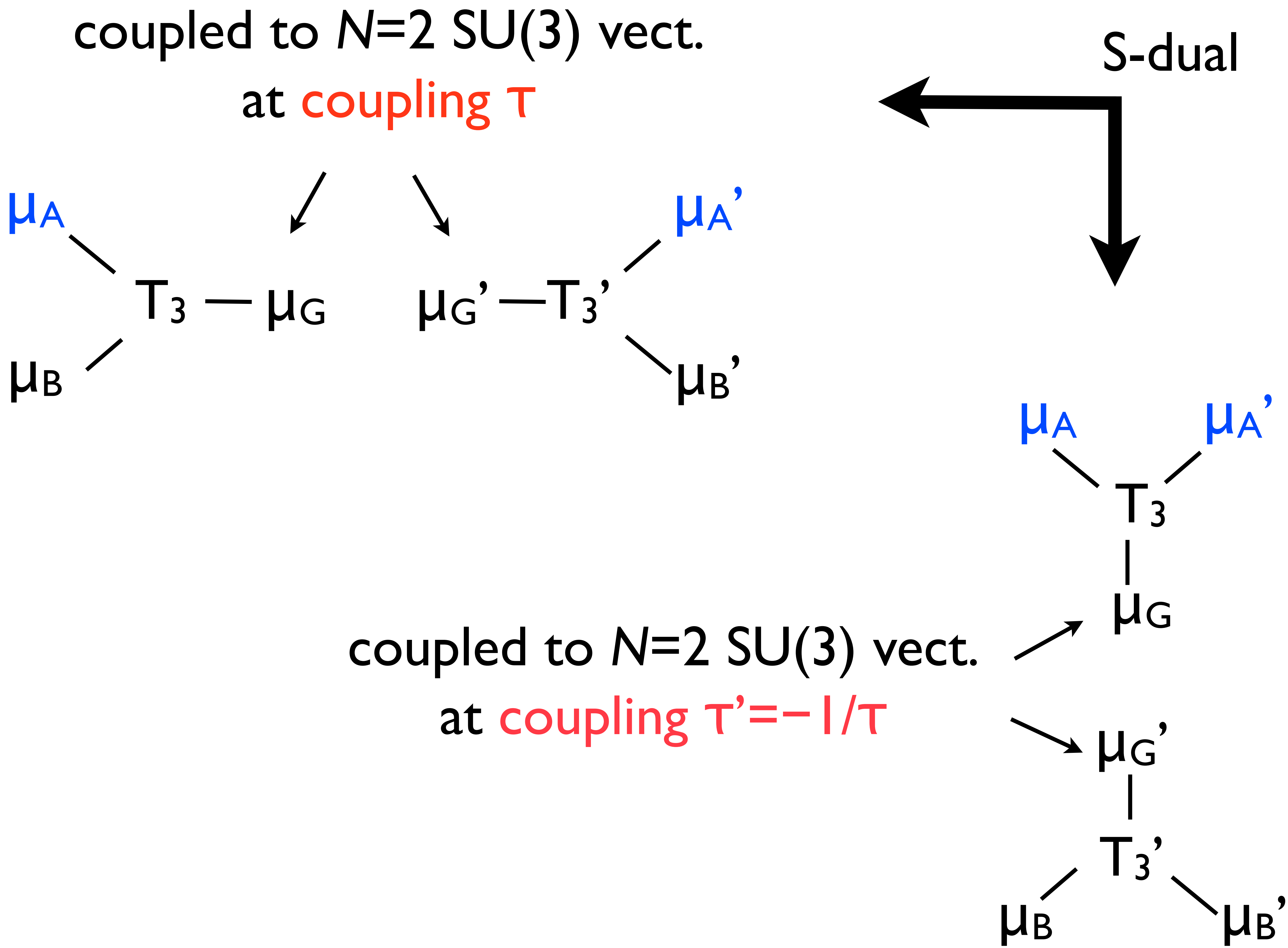



$\langle \mu_C \rangle = \rho_s$


$\langle \mu_C \rangle = \rho_s$

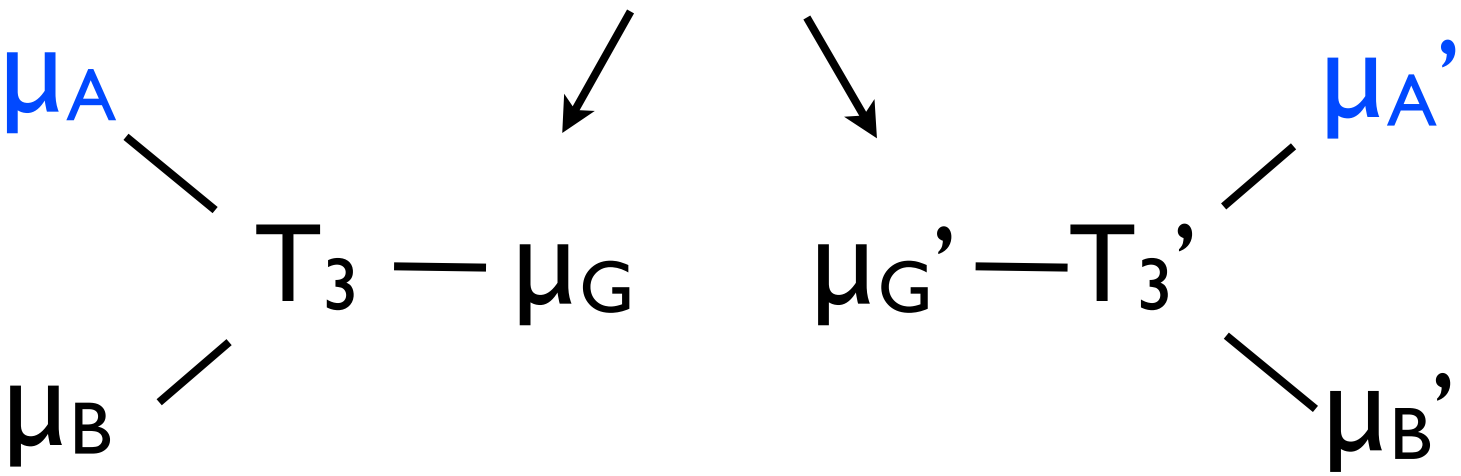
$$\begin{array}{ll} \mu_A & = q_a{}^i \, \tilde{q}_i{}^b \\ \uparrow\downarrow & \\ \mu_B & = \tilde{q}_i{}^a \, q_a{}^j \end{array}$$

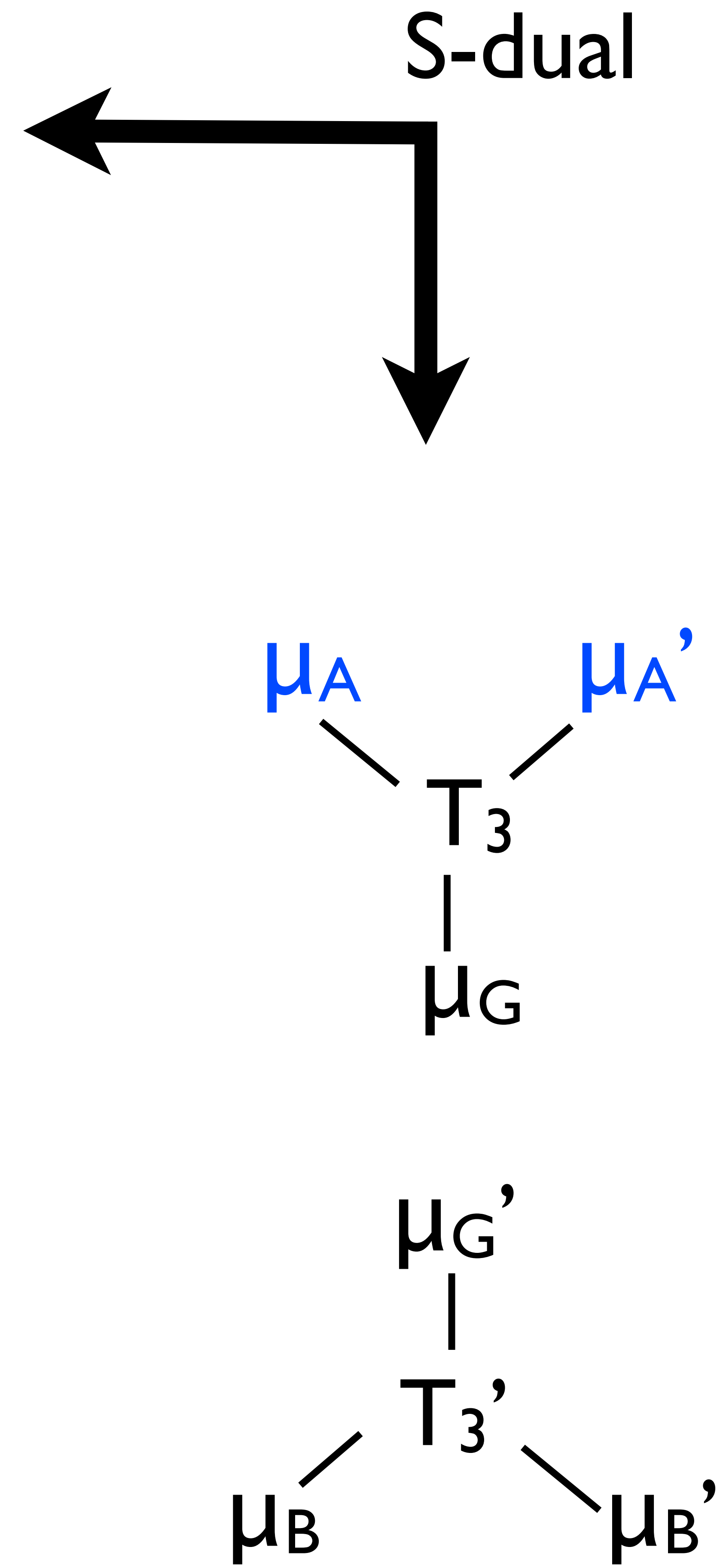
- It becomes an $SU(N)_A \times SU(N)_B$ bifundamental,
  with free neutral hypermultiplets.



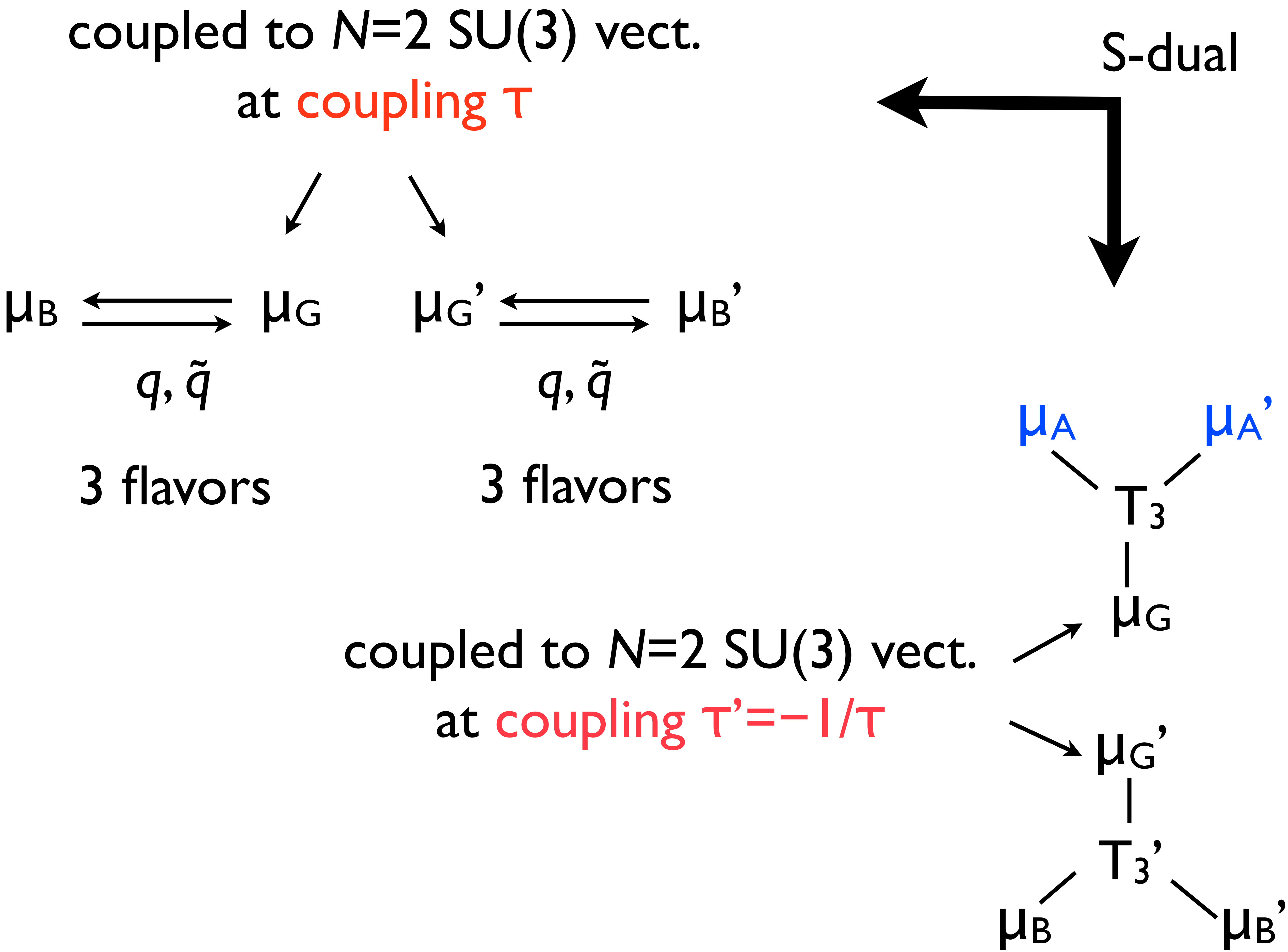
coupled to N=2 SU(3) vect.
at coupling τ
S-dual
μA
μB
T3
μG
μG'
T3'
μA'
μB'
μA
μA'
T3
μG
coupled to N=2 SU(3) vect.
at coupling τ'=−1/τ
μG'
T3'
μB
μB'

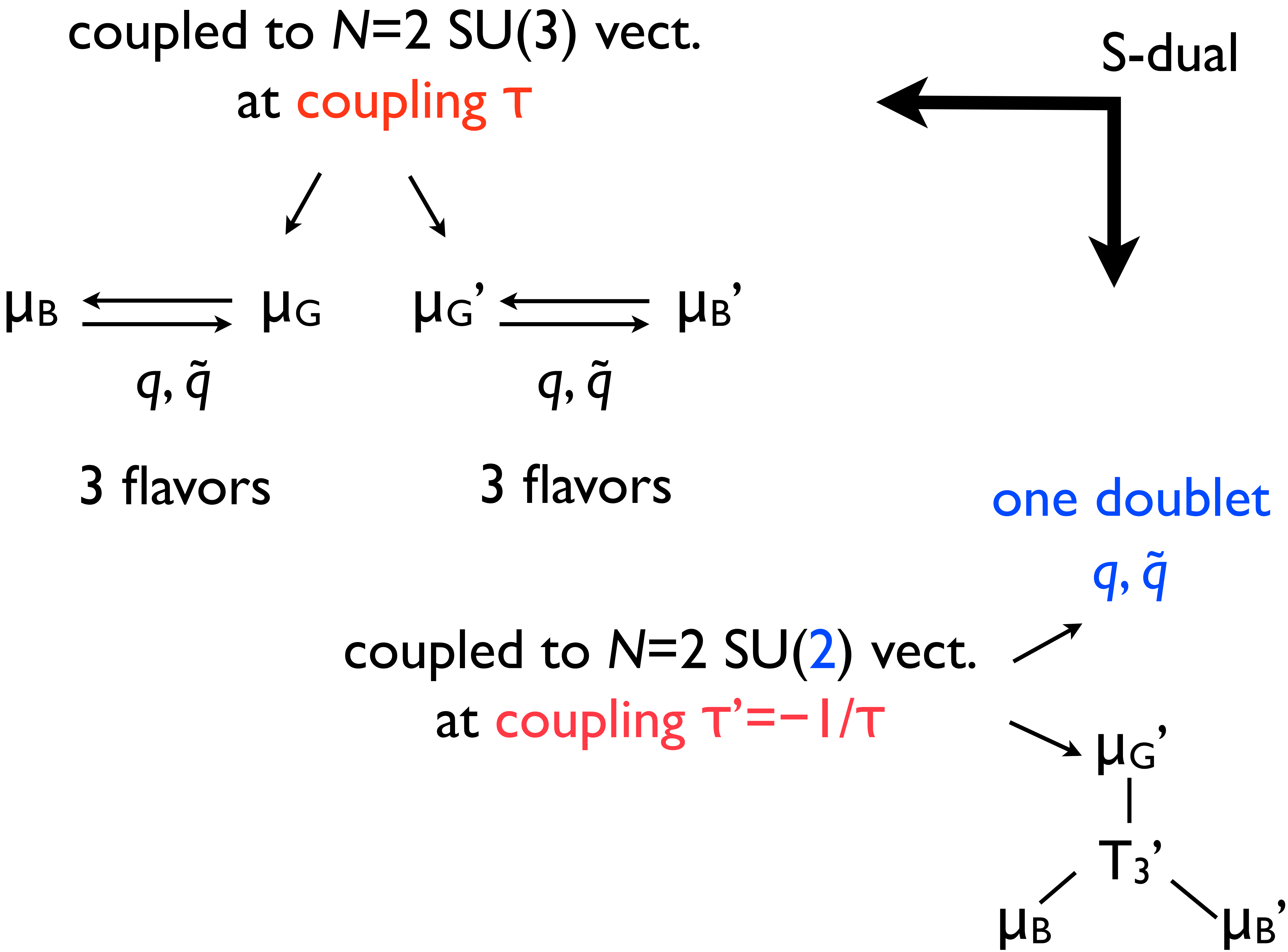
coupled to $\mathcal{N}$=2 SU(3) vect.
at coupling τ





Give vevs
$\mu_A = \mu_A' = \rho_s$!


coupled to $\mathcal{N}$=2 SU(3) vect.
at coupling τ
μ_B
μ_G
μ_G'
μ_B'
$q, \tilde{q}$
$q, \tilde{q}$
3 flavors
3 flavors
S-dual
μ_A
μ_A'
T_3
μ_G
coupled to $\mathcal{N}$=2 SU(3) vect.
at coupling $\tau' = -1/\tau$
μ_G'
T_3'
μ_B
μ_B'



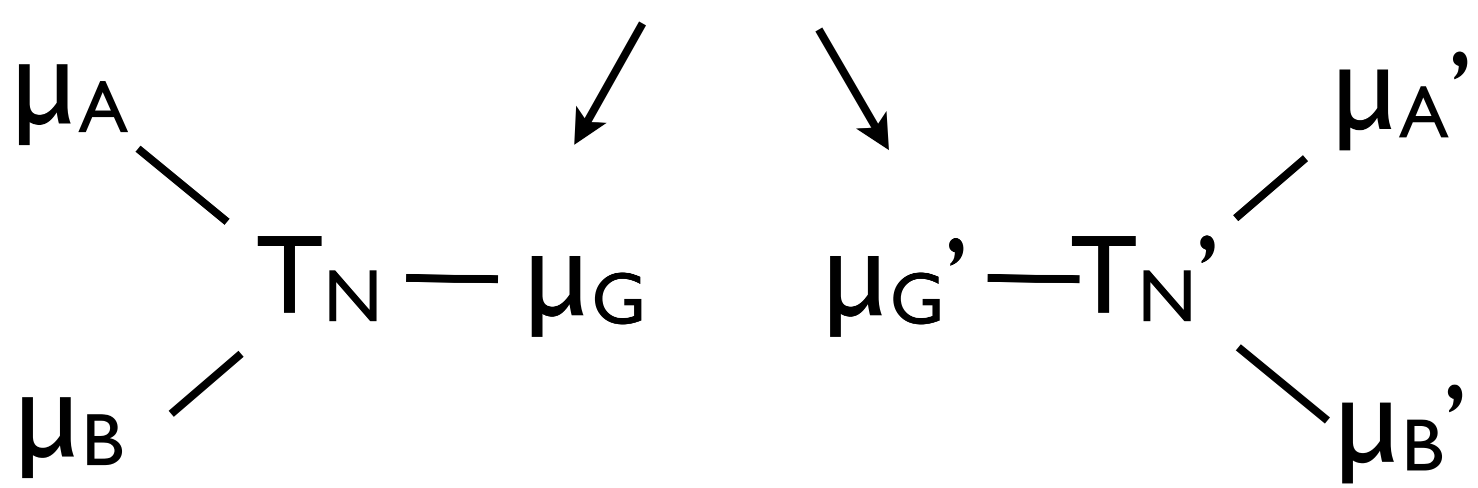
coupled to $\mathcal{N}$=2 SU(3) vect.
at coupling τ
S-dual
μ_B
μ_G
μ_G'
μ_B'
$q, \tilde{q}$
$q, \tilde{q}$
3 flavors
3 flavors
one doublet
$q, \tilde{q}$
coupled to $\mathcal{N}$=2 SU(2) vect.
at coupling $\tau' = -1/\tau$
μ_G'
T_3'
μ_B
μ_B'

- $\mathcal{N}=2$ SU(3) with 6 flavors at coupling τ
- $\mathcal{N}=2$ SU(2) with one flavor at coupling $-1/\tau$ coupled to one T_3 theory
- This is the original Argyres-Seiberg duality.
- We'd like an $\mathcal{N}=1$ supersymmetric version.

Another dual for $\mathcal{N}=1$ SQCD

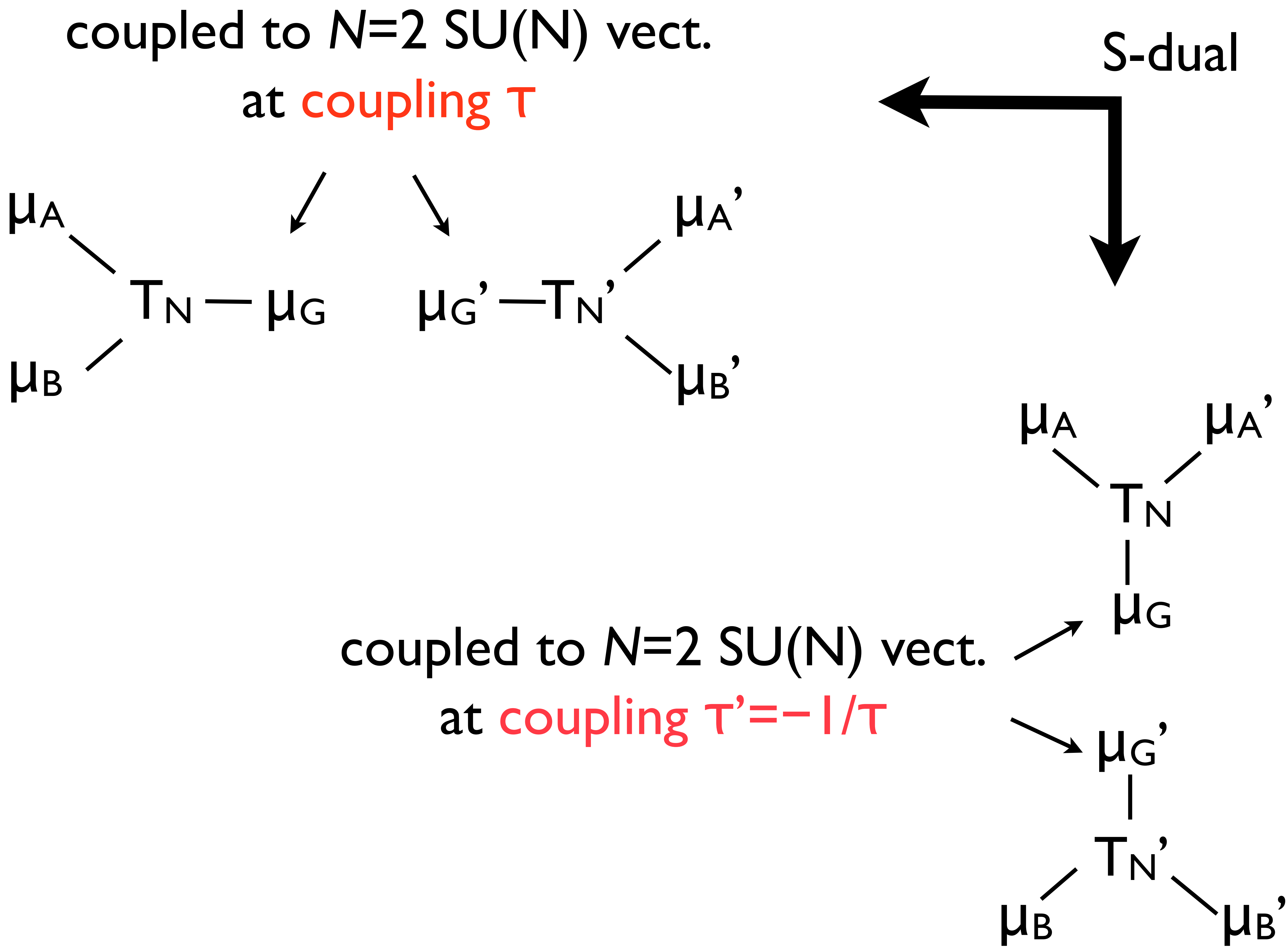
coupled to $\mathcal{N}$=1 SU(N) vect.

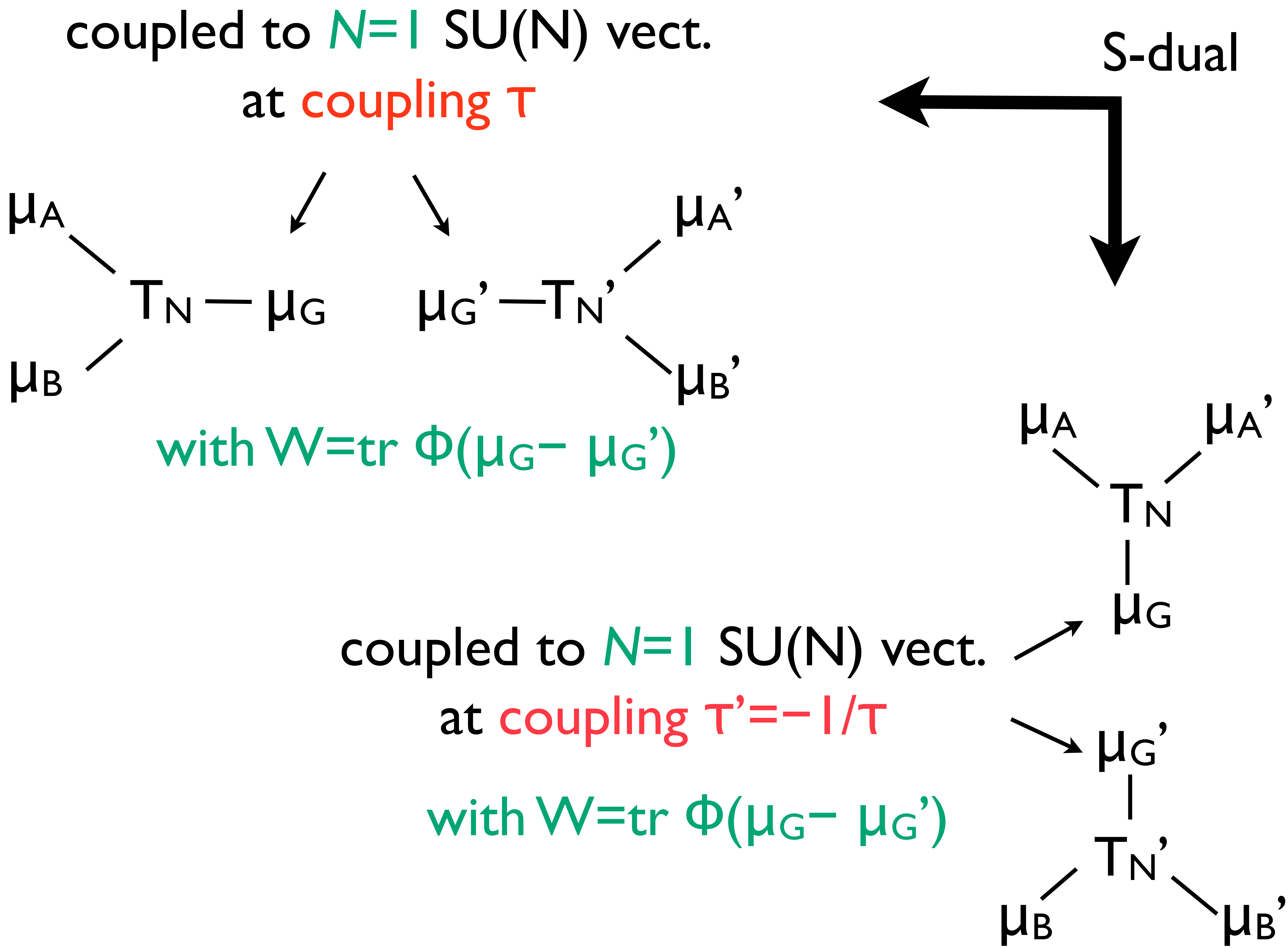
dual



with W=0

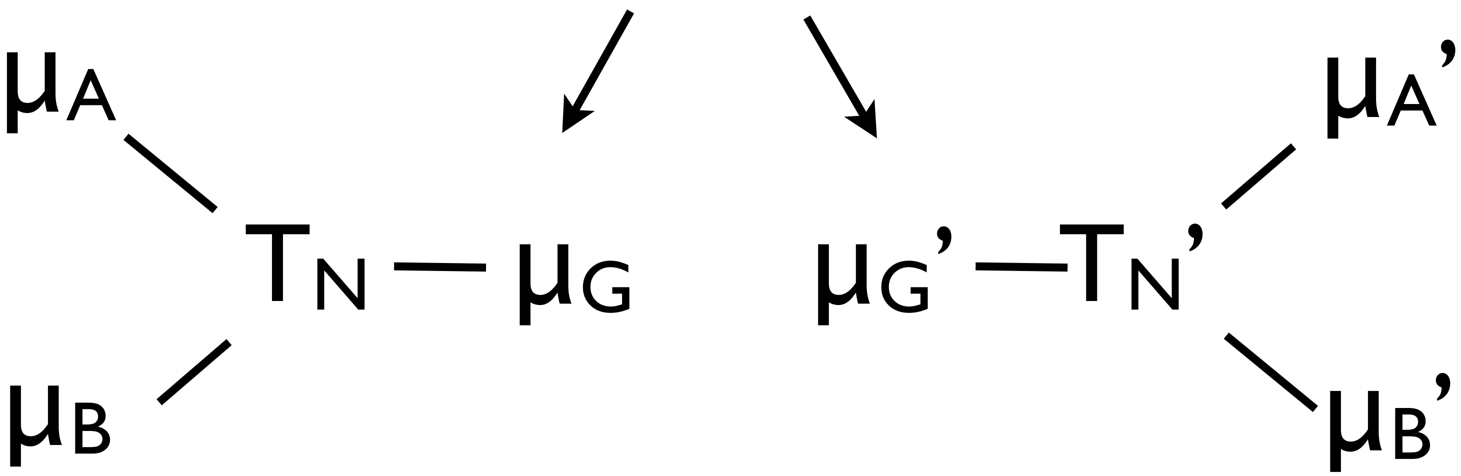
???


coupled to N=2 SU(N) vect.
at coupling τ
S-dual
μA
TN
μG
μB
μG’
TN’
μA’
μB’
μA
μA’
TN
μG
coupled to N=2 SU(N) vect.
at coupling τ’=−1/τ
μG’
TN’
μB
μB’


coupled to $\mathcal{N}=1$ SU(N) vect.
at coupling τ
S-dual
μ_A
μ_B
T_N
μ_G
μ_G'
T_N'
μ_A'
μ_B'
with $W=\mathrm{tr}\,\Phi(\mu_G-\mu_G')$
μ_A
μ_A'
T_N
μ_G
coupled to $\mathcal{N}=1$ SU(N) vect.
at coupling $\tau'=-1/\tau$
μ_G'
with $W=\mathrm{tr}\,\Phi(\mu_G-\mu_G')$
T_N'
μ_B
μ_B'

coupled to $\mathcal{N}=1$ SU(N) vect.

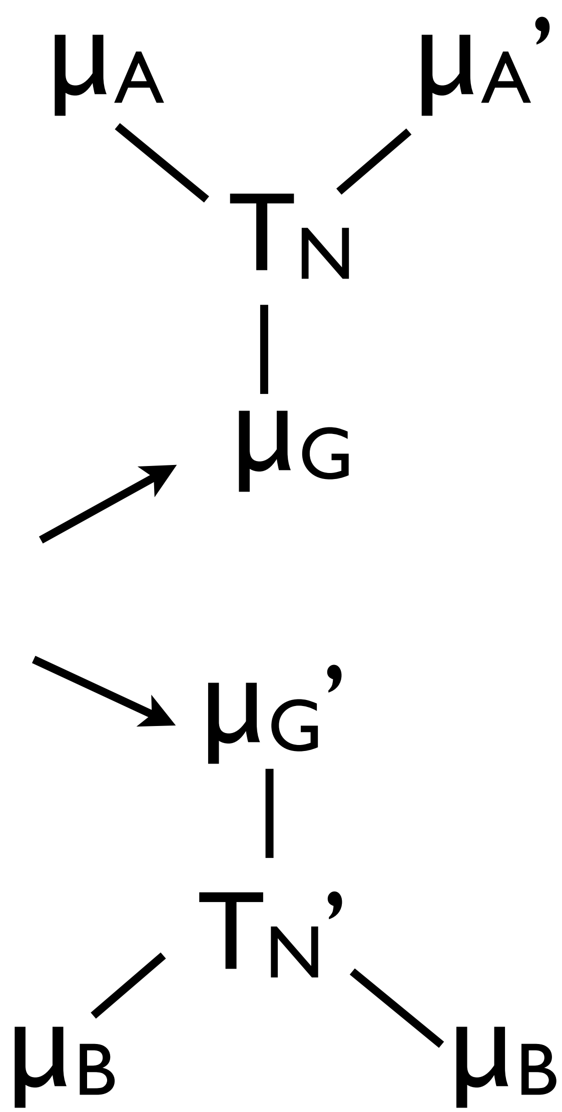
at coupling τ



with $W=\mathrm{tr}\,\Phi(\mu_G-\mu_G')$

$+m\,\mathrm{tr}\Phi^2$

dual



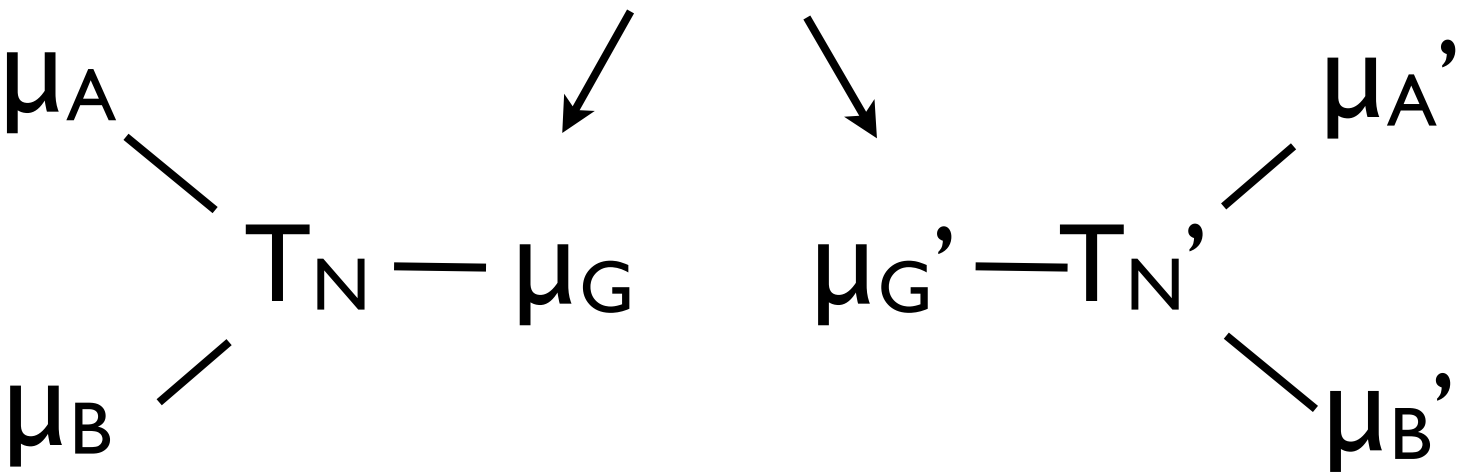
coupled to $\mathcal{N}=1$ SU(N) vect.

at coupling $\tau'=-1/\tau$

with $W=\mathrm{tr}\,\Phi(\mu_G-\mu_G')$

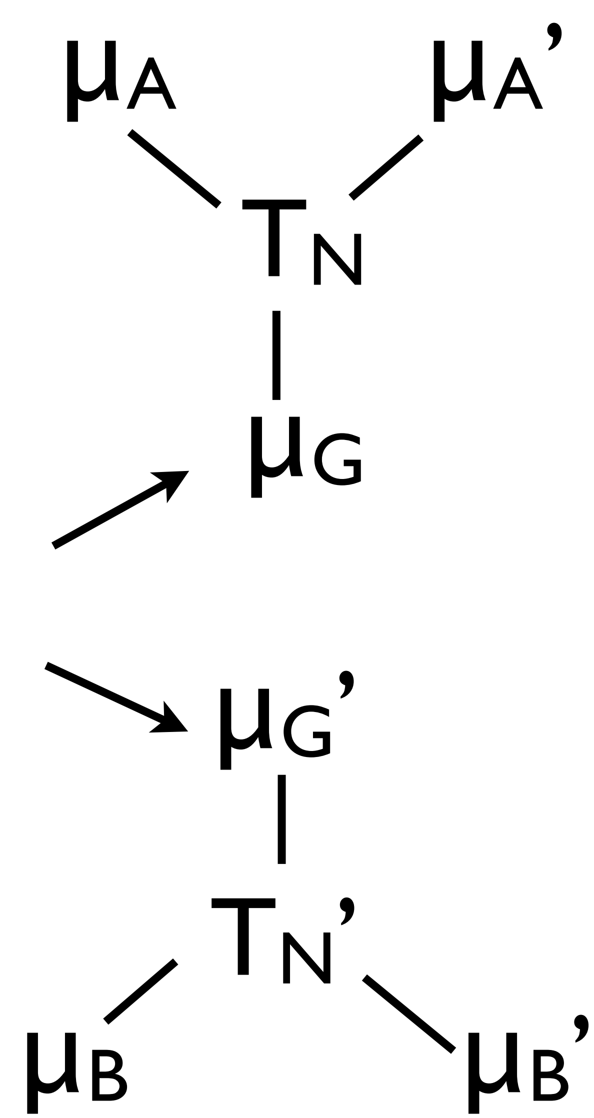
$+m\,\mathrm{tr}\Phi^2$

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W = c\ \mathrm{tr}\,(\mu_G - \mu_G')^2$

dual

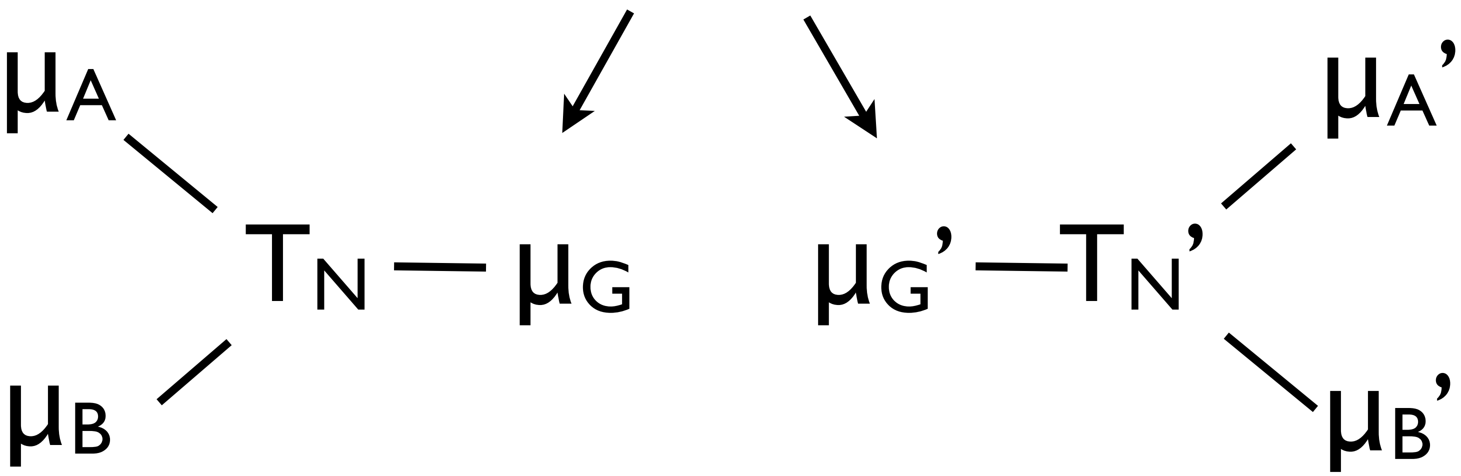


coupled to $\mathcal{N}=1$ SU(N) vect.

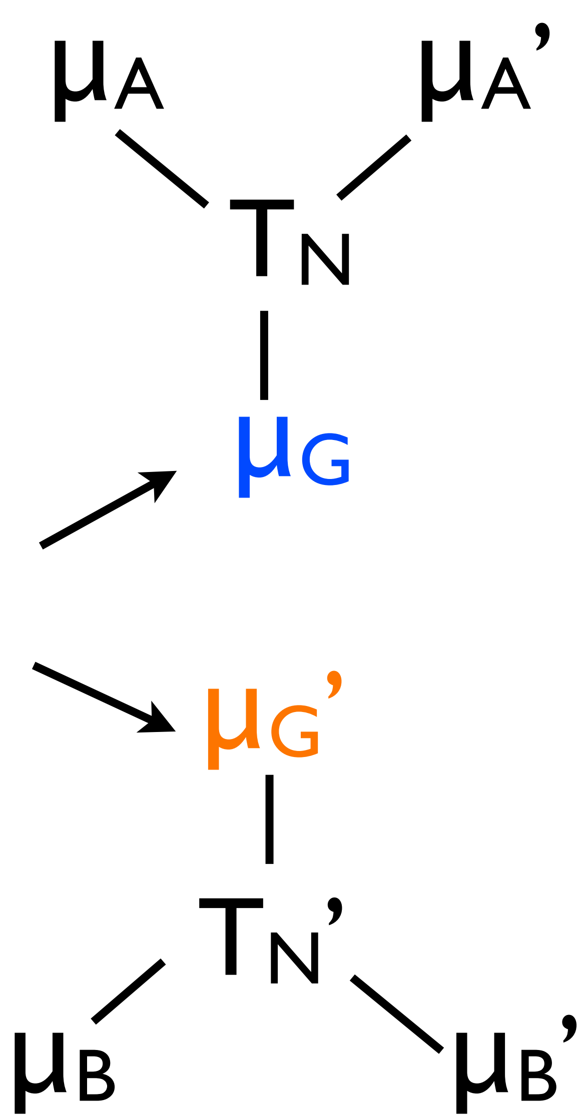
with $W = c^{-1}\ \mathrm{tr}\,(\mu_G - \mu_G')^2$

coupled to $\mathcal{N}=1$ SU(N) vect.

dual



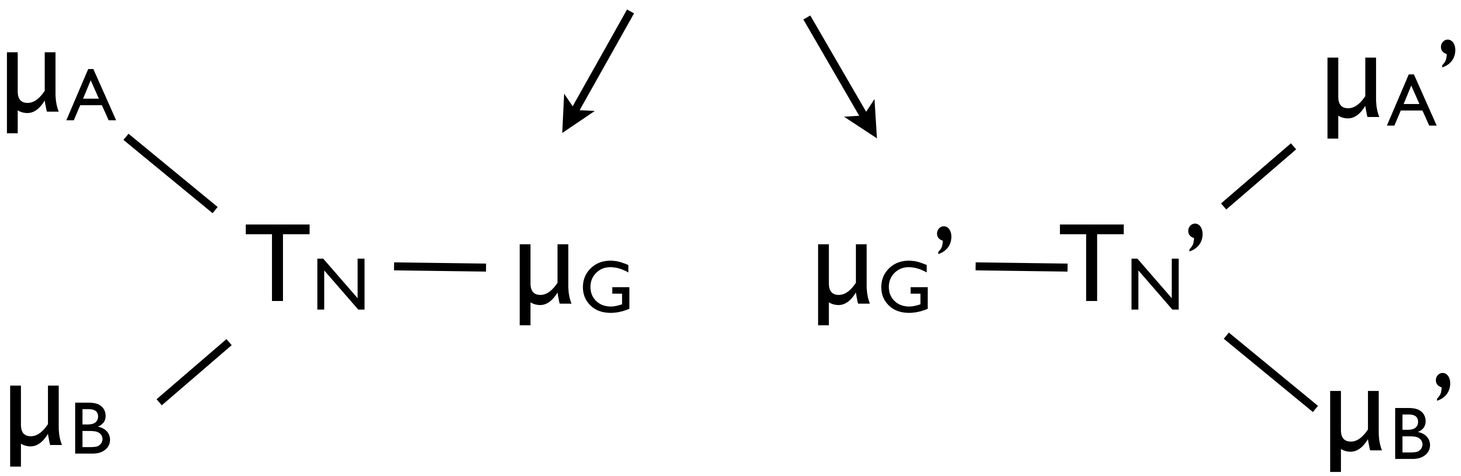
with $W = c\ \mathrm{tr}\ \mu_G^2 - 2\ c\ \mathrm{tr}\ \mu_G\, \mu_G' + c\ \mathrm{tr}\ \mu_G'^{\,2}$



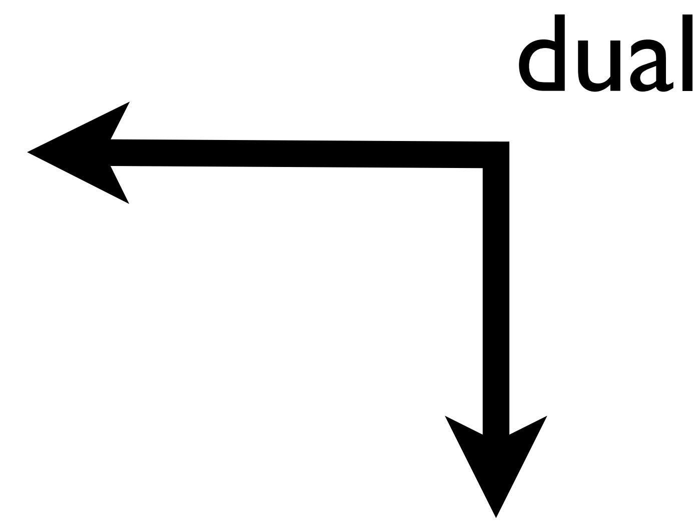
coupled to $\mathcal{N}=1$ SU(N) vect.

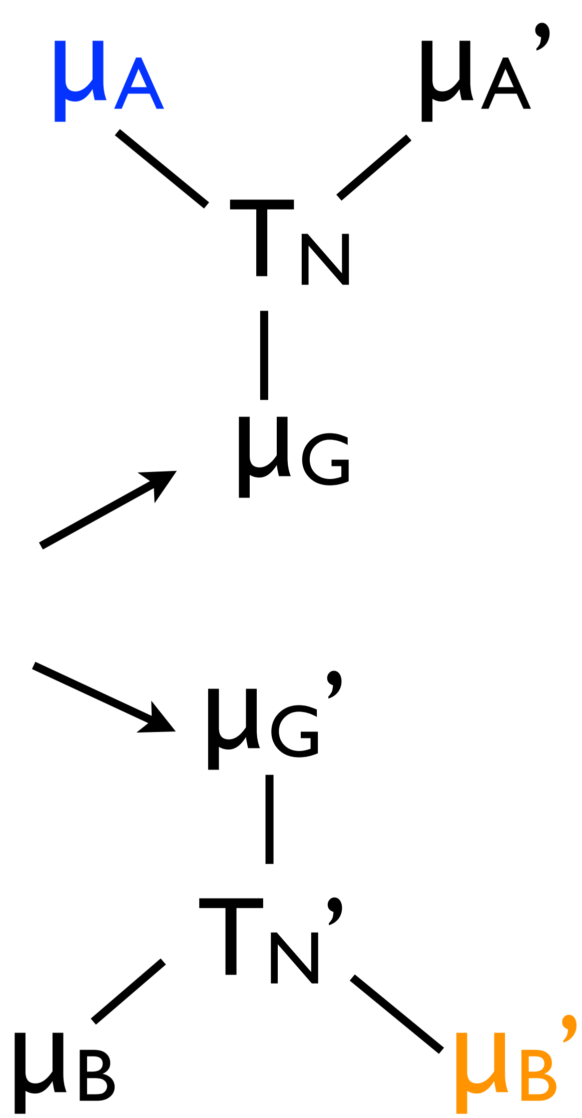
with $W = c^{-1}\ \mathrm{tr}\ \mu_G^2 - 2\ c^{-1}\ \mathrm{tr}\ \mu_G\, \mu_G' + c^{-1}\ \mathrm{tr}\ \mu_G'^{\,2}$

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W = c\ \mathrm{tr}\ \mu_G^2 - 2\ c\ \mathrm{tr}\ \mu_G\,\mu_G'$
$+ \ c\ \mathrm{tr}\ \mu_G'^{\,2}$



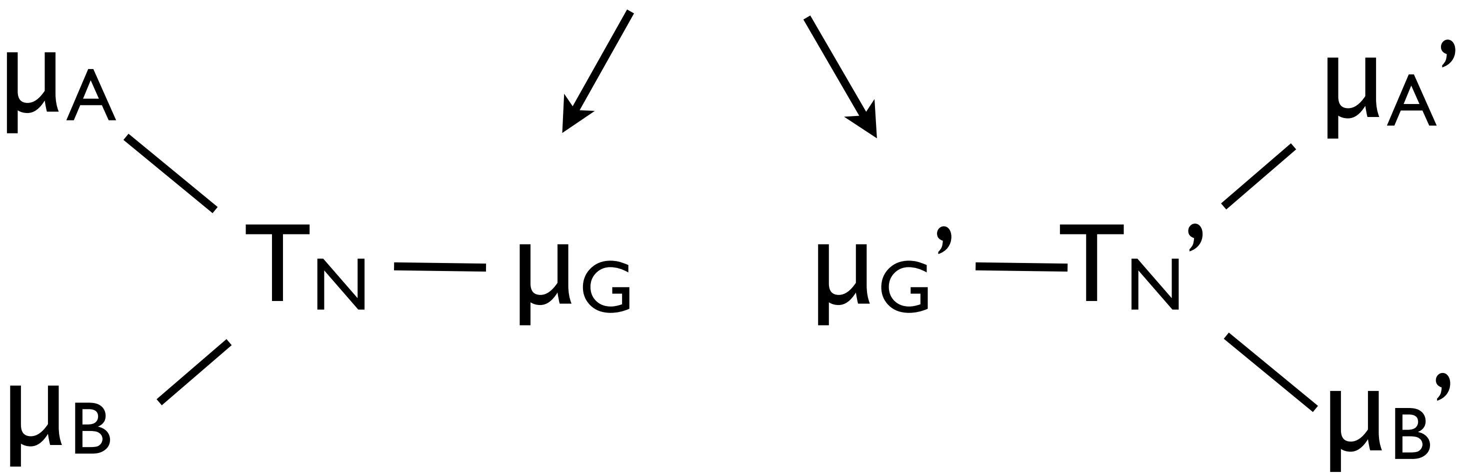


coupled to $\mathcal{N}=1$ SU(N) vect.

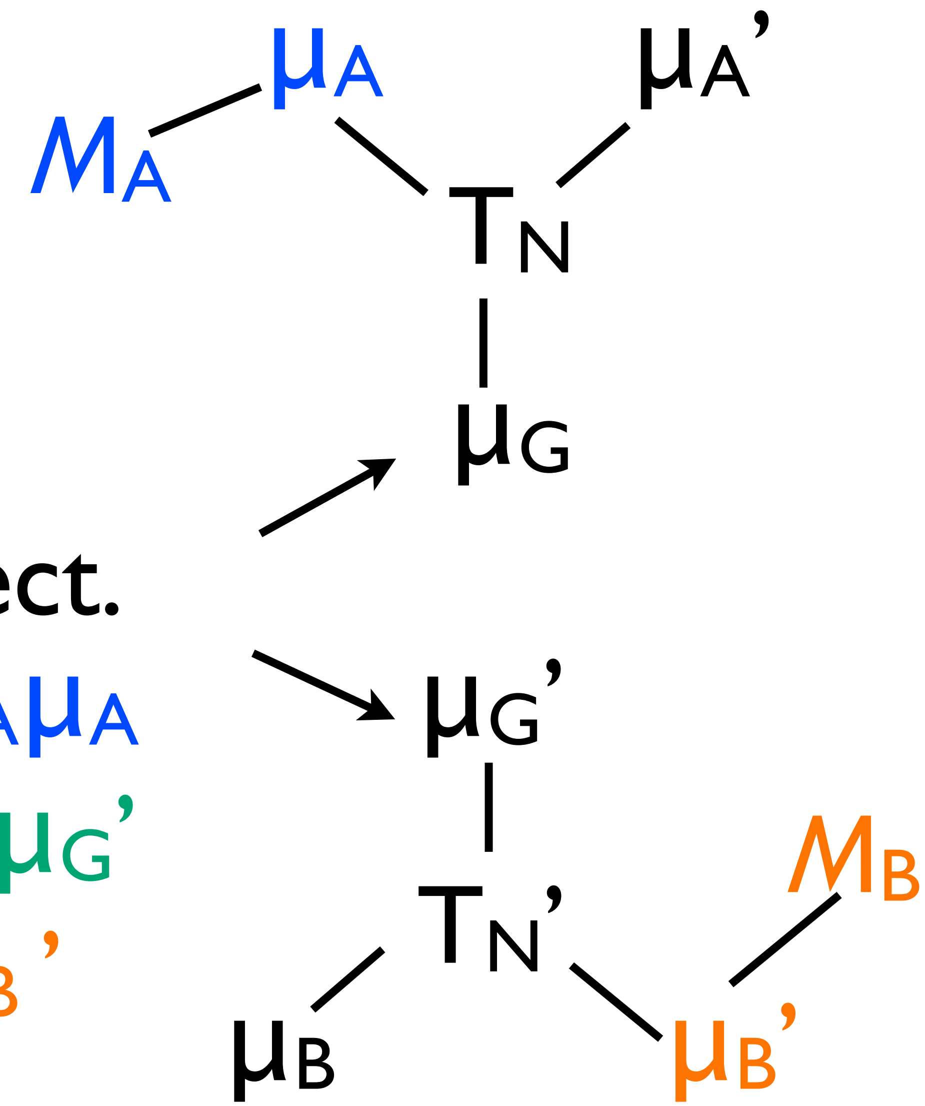
with $W = c^{-1}\ \mathrm{tr}\ \mu_A^2 - 2\ c^{-1}\ \mathrm{tr}\ \mu_G\,\mu_G'$
$+ \ c^{-1}\ \mathrm{tr}\ \mu_B'^{\,2}$

coupled to $\mathcal{N}=1$ SU(N) vect.

dual



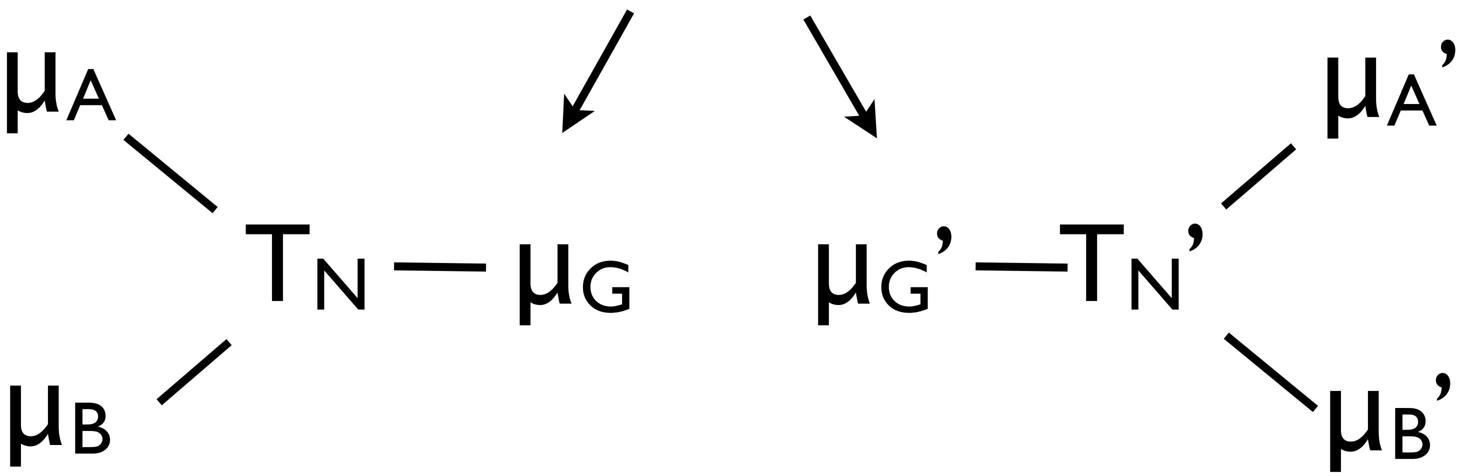
with $W = c\,\mathrm{tr}\,\mu_G^2 - 2c\,\mathrm{tr}\,\mu_G\,\mu_G' + c\,\mathrm{tr}\,\mu_G'^{\,2}$



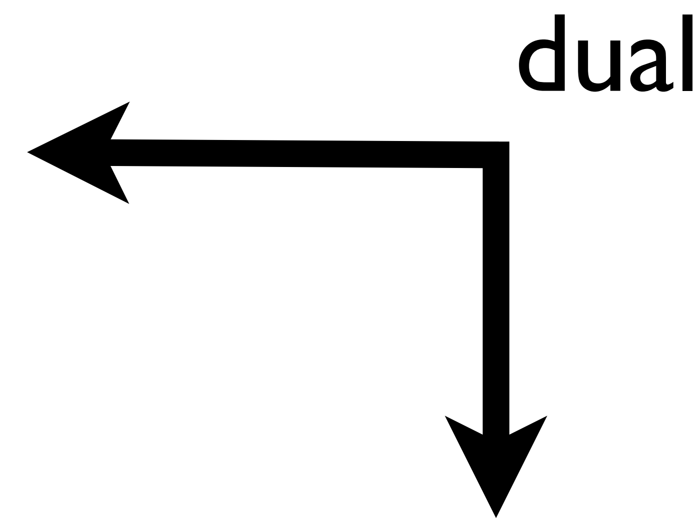
coupled to $\mathcal{N}=1$ SU(N) vect.

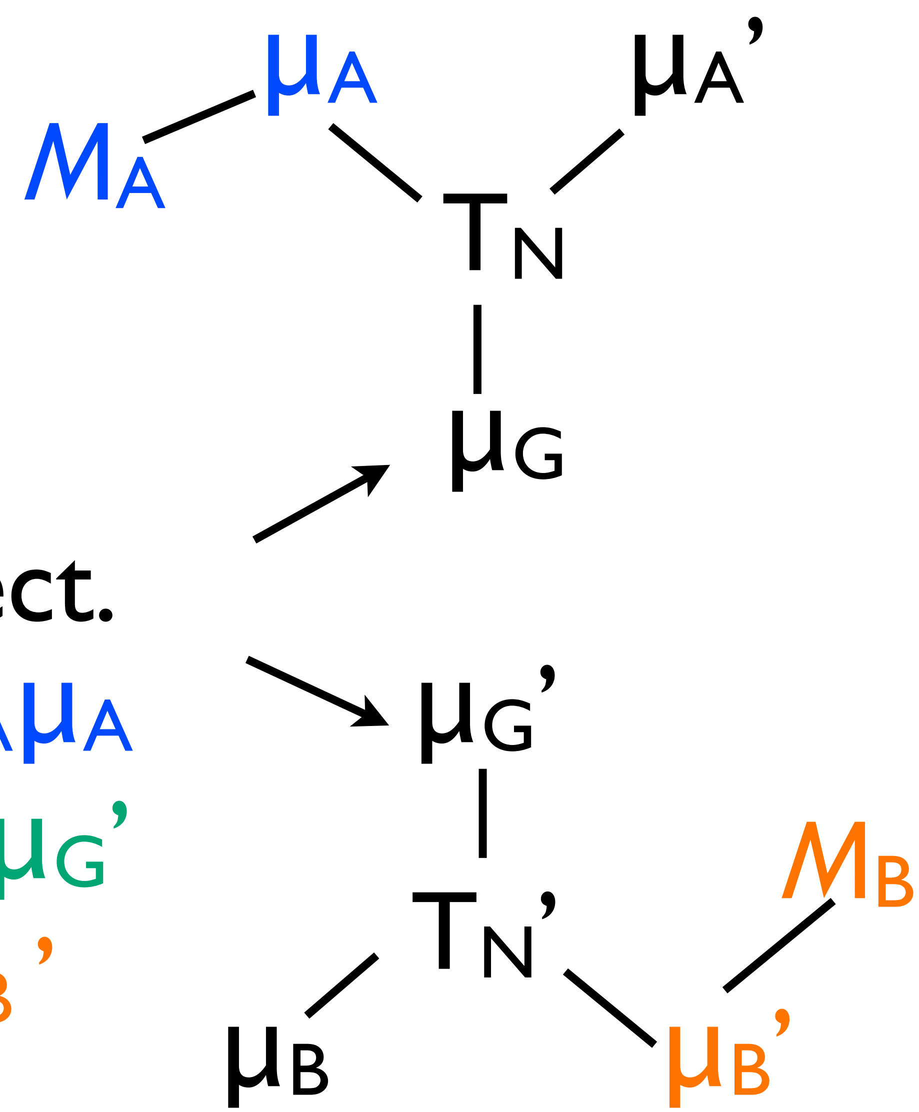
with $W = c\,\mathrm{tr}\,M_A^2 + \mathrm{tr}\,M_A\mu_A - 2c^{-1}\,\mathrm{tr}\,\mu_G\,\mu_G' + c\,\mathrm{tr}\,M_B^2 + \mathrm{tr}\,M_B\mu_B'$

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W= \qquad -2\,c\;\mathrm{tr}\,\mu_G\,\mu_G'$





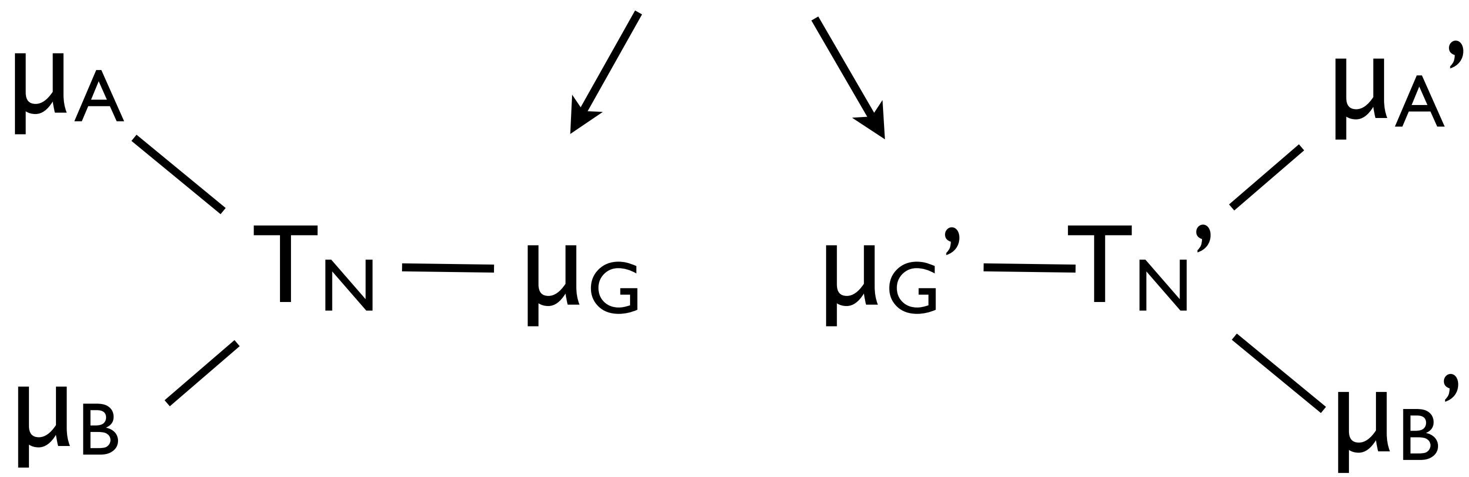
coupled to $\mathcal{N}=1$ SU(N) vect.

with $W= \qquad +\,\mathrm{tr}\,M_A\mu_A$

$-2\,c^{-1}\;\mathrm{tr}\,\mu_G\,\mu_G'$

$+\,\mathrm{tr}\,M_B\mu_B'$

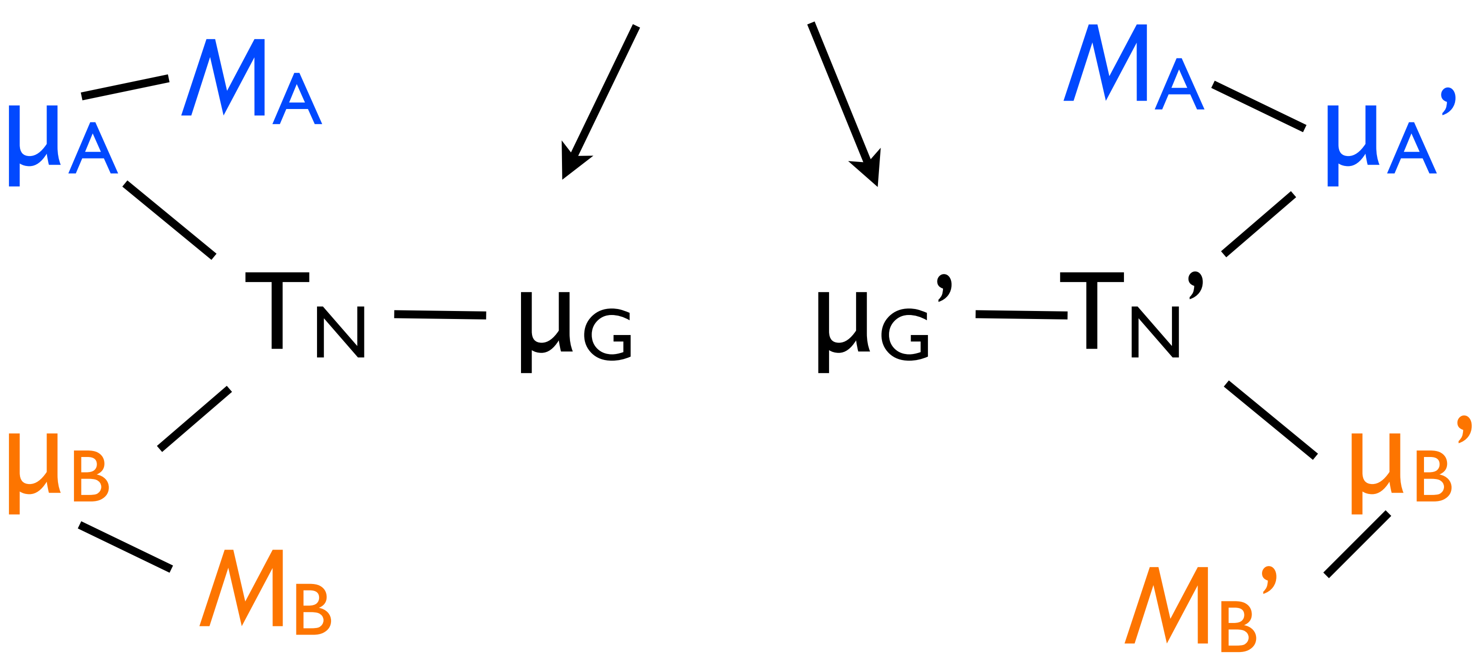
coupled to $\mathcal{N}=1$ SU(N) vect.



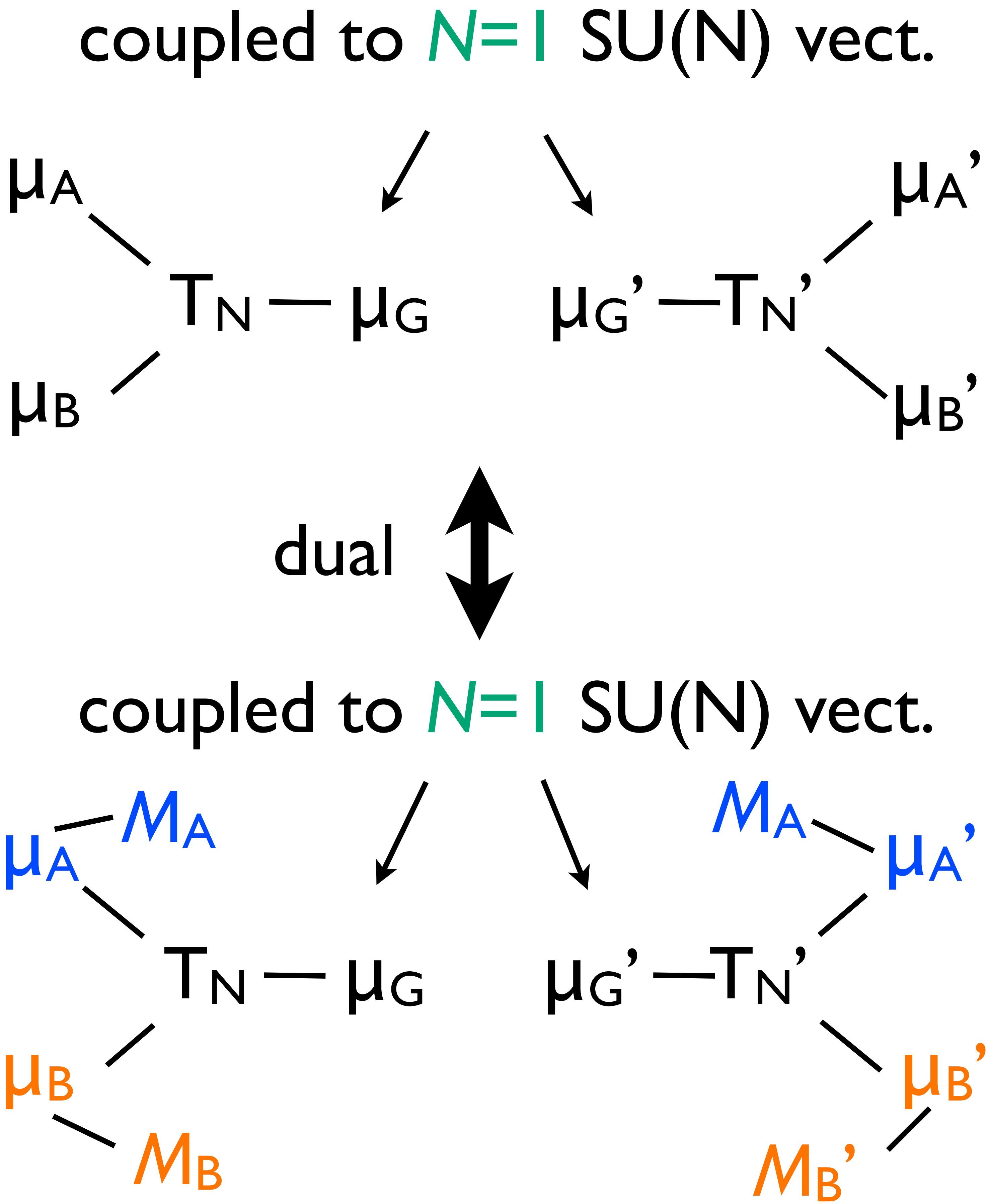
with $W = -2\,c\,\mathrm{tr}\,\mu_G\,\mu_G'$

dual

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W =$
$\mathrm{tr}\,M_A\mu_A + \mathrm{tr}\,M_B\mu_B$
$+ \mathrm{tr}\,M_A'\mu_A' + \mathrm{tr}\,M_B'\mu_B'$
$-2\,c\,\mathrm{tr}\,\mu_G\mu_G'$



with $W = 0$

with $W =$
$\mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B$
$+\, \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$

For SU(2), we have

SU(2) with q_{aiu} and $\tilde{q}_{akx}$

SU(2) with q_{aiu} and $\tilde{q}_{akx}$
and gauge singlets $M_{(ij)}$, $M_{(uv)}$, $M_{(kl)}$, $M_{(xy)}$

$$W = M_{(ij)}(qq)_{(ij)} + M_{(uv)}(qq)_{(uv)} + M_{(kl)}(\tilde{q}\tilde{q})_{(kl)} + M_{(xy)}(\tilde{q}\tilde{q})_{(xy)}$$

SU(2) with 4 flavors $q, \tilde{q}$

Csaki-Schmaltz-
Skiba-Terning, 1997

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W = Mqq + M\tilde{q}\tilde{q}$

SU(N) with T_N and T_N'

SU(N) with T_N and T_N'
and gauge singlets M_A , M_B , M_A' , M_B'

$$W = \mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B + \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$$

Non-conventional N=1 theory

$\updownarrow$ “Seiberg” dual

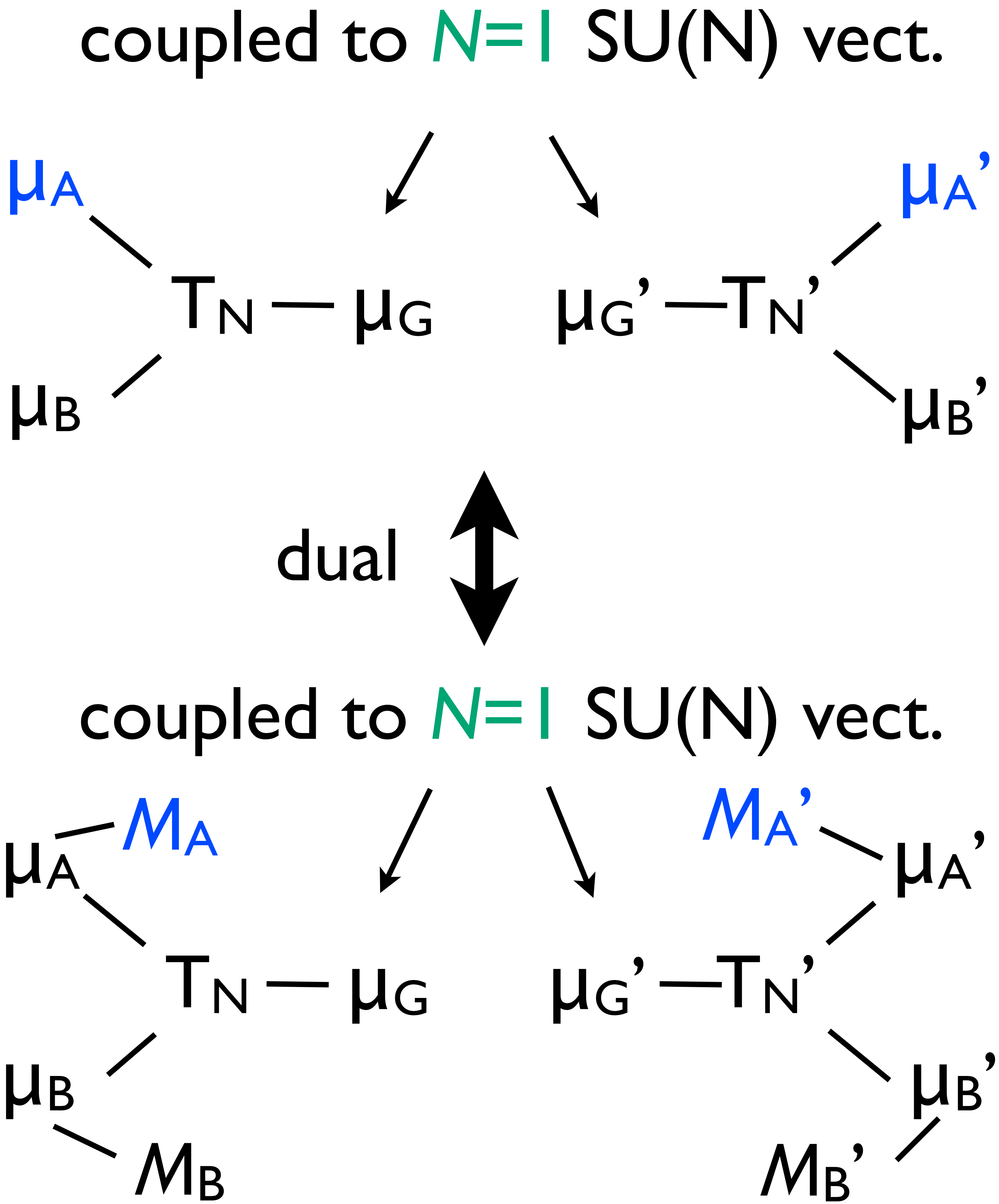
Non-conventional N=1 theory

There's an easy way to modify it to

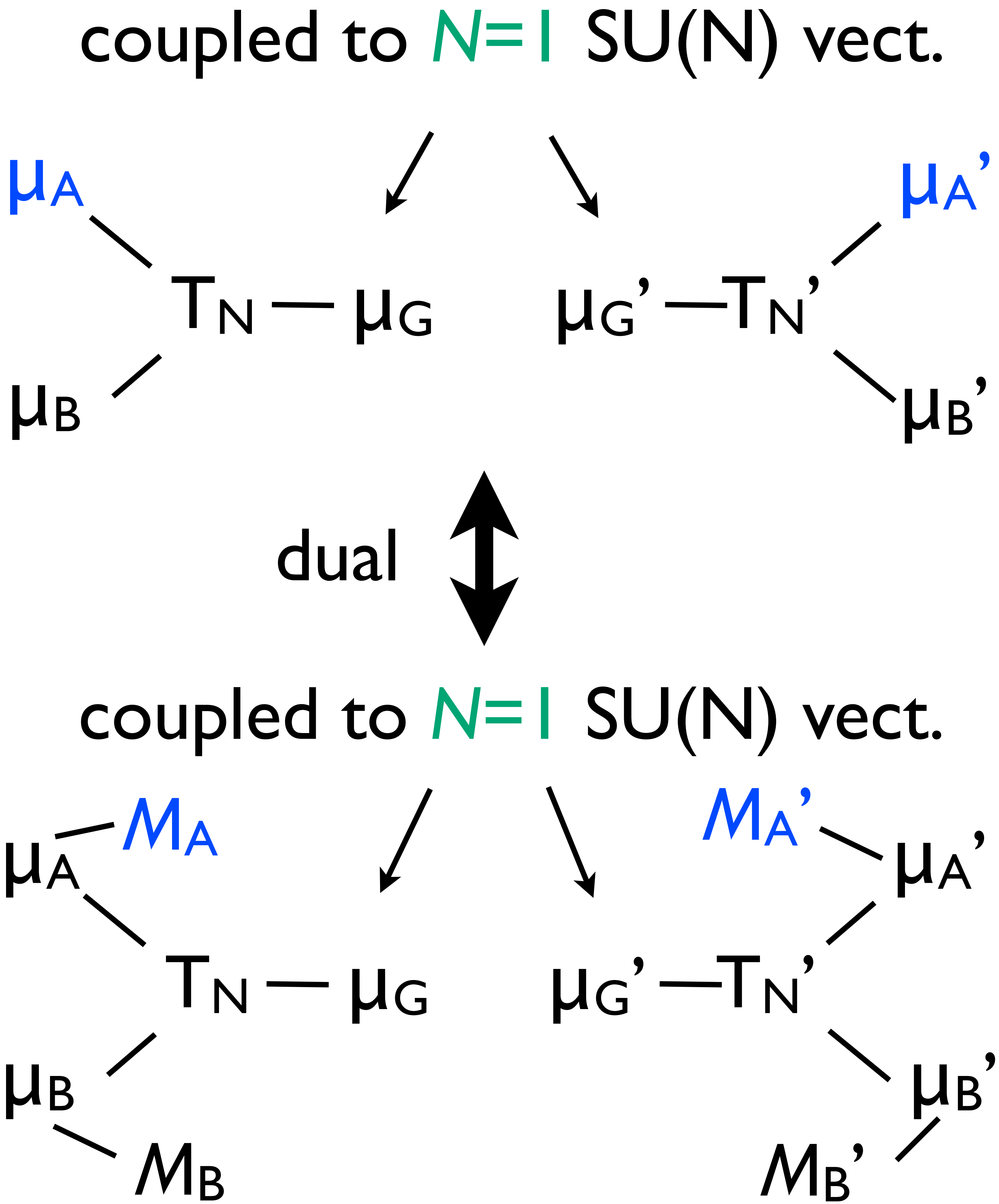
Conventional N=1 theory

"Seiberg" dual

Non-conventional N=1 theory



with $W=$
$\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B$
$+\, \mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$



Give vevs

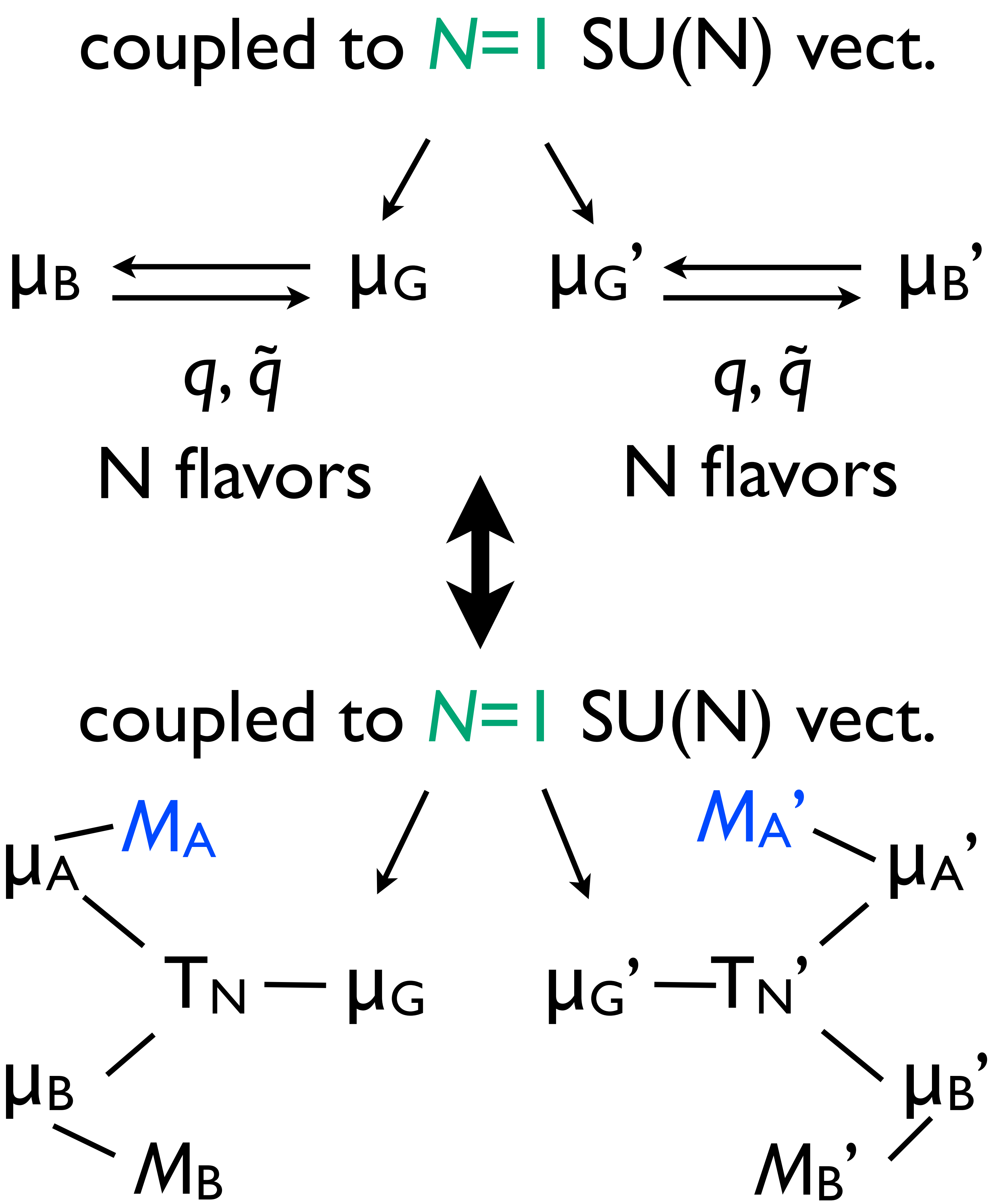
$$\mu_A=\mu_A'=\rho_s \ !$$

Give vevs

$$M_A=M_A'=\rho_s \ !$$

with W=

$$\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B + \mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$$



+ neutral chiral multiplets

Give vevs

$M_A = M_A' = \rho_s$!

with $W =$

$\mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B$

$+ \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$

$\mathcal{N}$=1 SU(N) with 2N flavors

\+ neutral chiral multiplets

dual

coupled to $\mathcal{N}$=1 SU(N) vect.

μ_A — M_A $\quad$ M_A' — μ_A'

T_N — μ_G $\quad$ μ_G' — T_N'

μ_B — M_B $\quad$ μ_B' — M_B'

Give vevs

$M_A = M_A' = \rho_s$!

with W=

tr $M_A \mu_A$ +tr $M_B \mu_B$

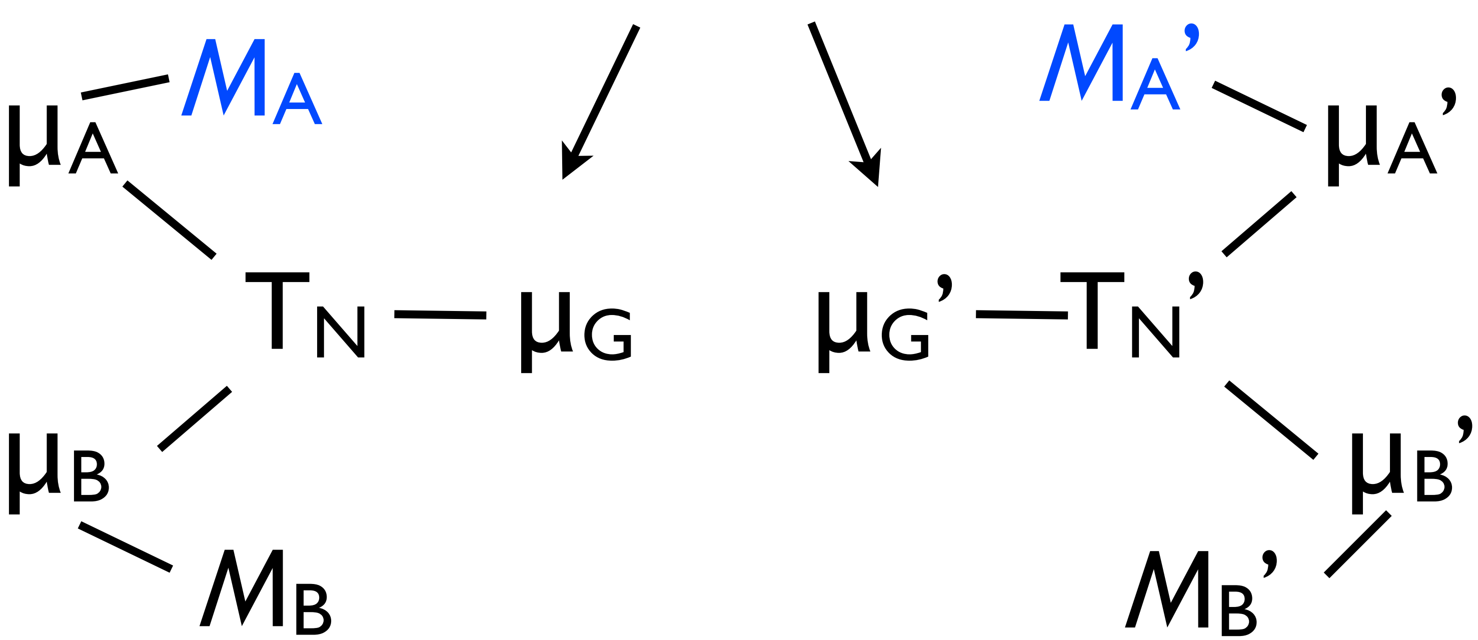
\+ tr $M_A' \mu_A'$ +tr $M_B' \mu_B'$

$\mathcal{N}=1$ SU(N) with 2N flavors

+ neutral chiral multiplets

dual

coupled to $\mathcal{N}=1$ SU(N) vect.



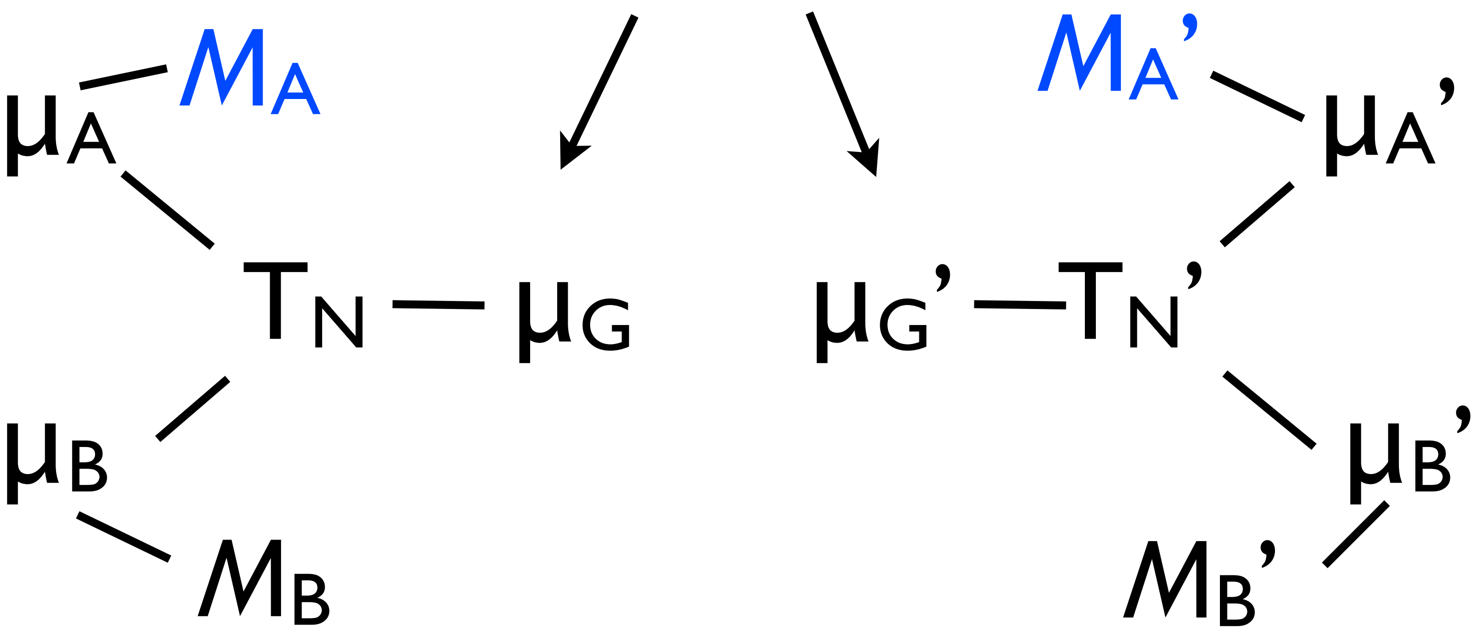
with $W=$

$\mathrm{tr}\,\rho_s\mu_A + \mathrm{tr}\,\rho_s\mu_A'$

$+\mathrm{tr}\,M_A\mu_A + \mathrm{tr}\,M_B\mu_B$

$+\,\mathrm{tr}\,M_A'\mu_A' + \mathrm{tr}\,M_B'\mu_B'$

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W=$

$\mathrm{tr}\, \rho_s \mu_A + \mathrm{tr}\, \rho_s \mu_{A'}$

$+ \mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B$

$+ \mathrm{tr}\, M_{A'} \mu_{A'} + \mathrm{tr}\, M_{B'} \mu_{B'}$

$$\rho_s = \left[\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right]$$

$$\rho_s = \begin{array}{|c|c|c|}\hline 0 & 1 & 0 \\ \hline 0 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline +1 & 0 & 0 \\ \hline 0 & -1 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 1 & 0 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array}$$

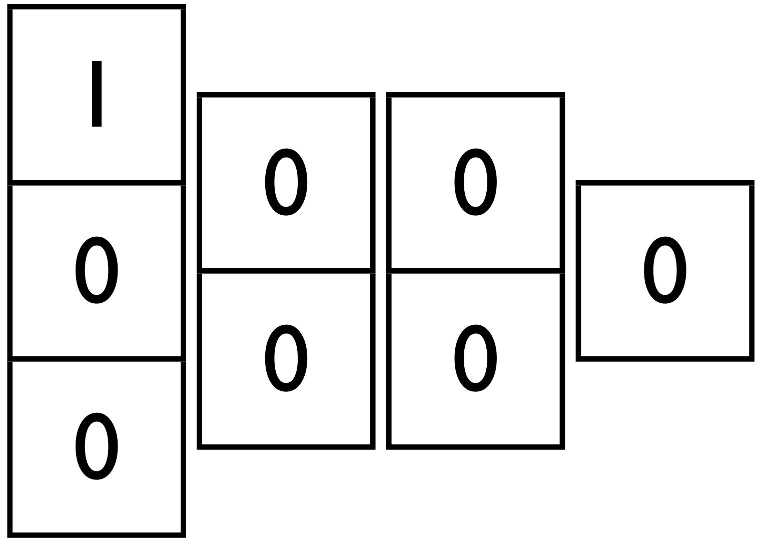
$$\rho_s = \left(\begin{array}{cc|c} 0 & 1 & 0 \\ 0 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad F^+$$

$$\left(\begin{array}{cc|c} +1 & 0 & 0 \\ 0 & -1 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad F^3$$

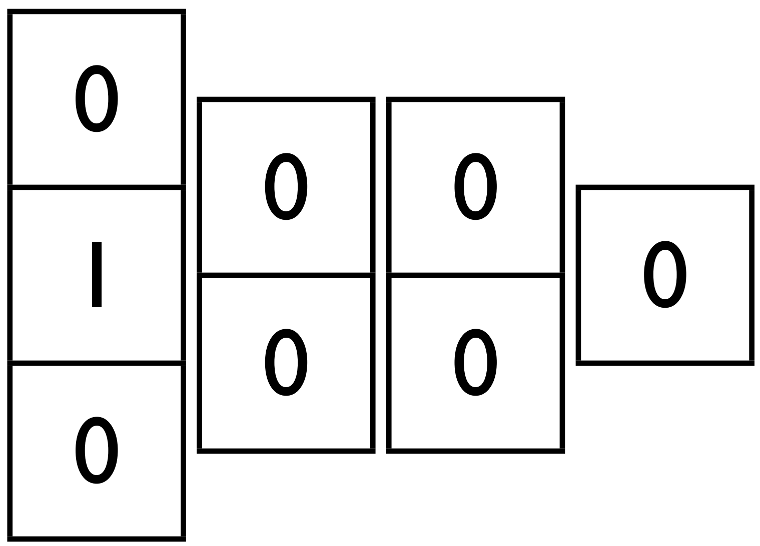
They generate an SU(2) subalgebra of SU(N) symmetry.

$$\left(\begin{array}{cc|c} 0 & 0 & 0 \\ 1 & 0 & 0 \\ \hline 0 & 0 & 0 \end{array}\right) \qquad F^-$$

$\rho_s=$

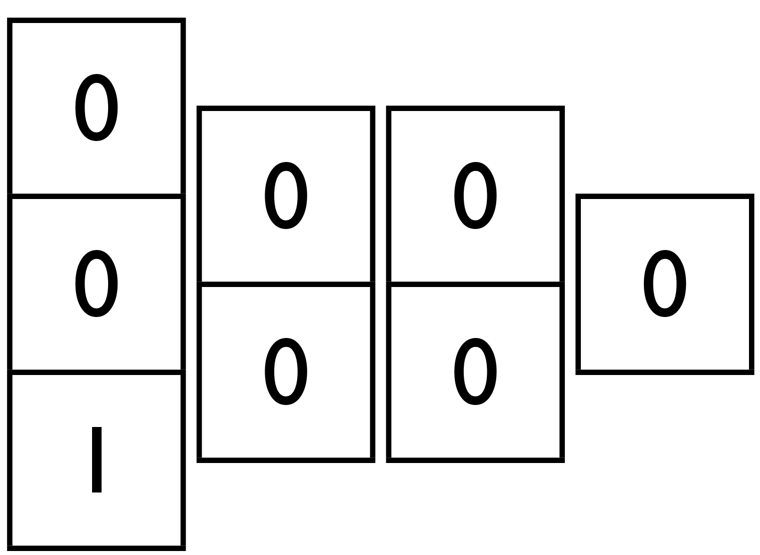


F^+

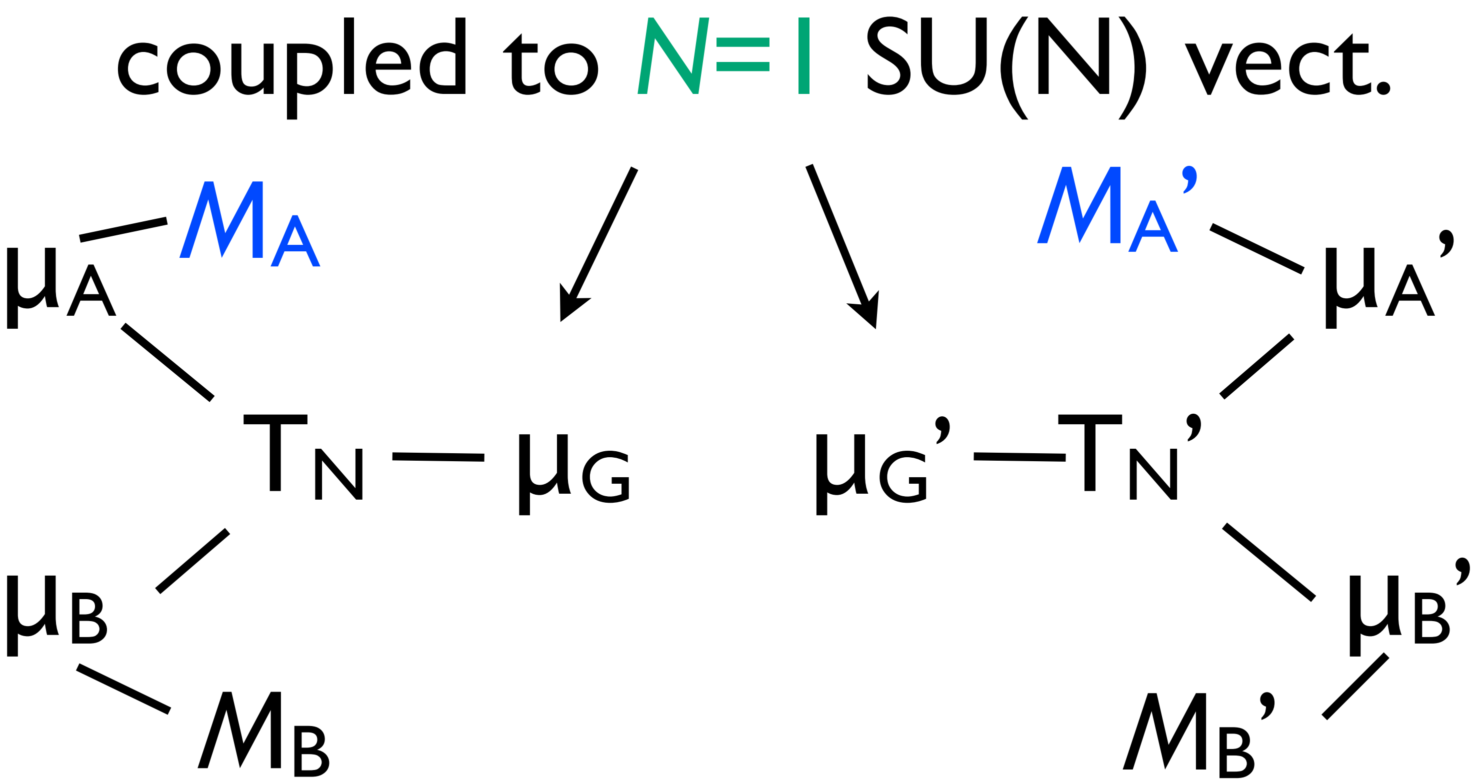


F^3

They generate
an SU(2) subalgebra of
SU(N) symmetry.



F^-

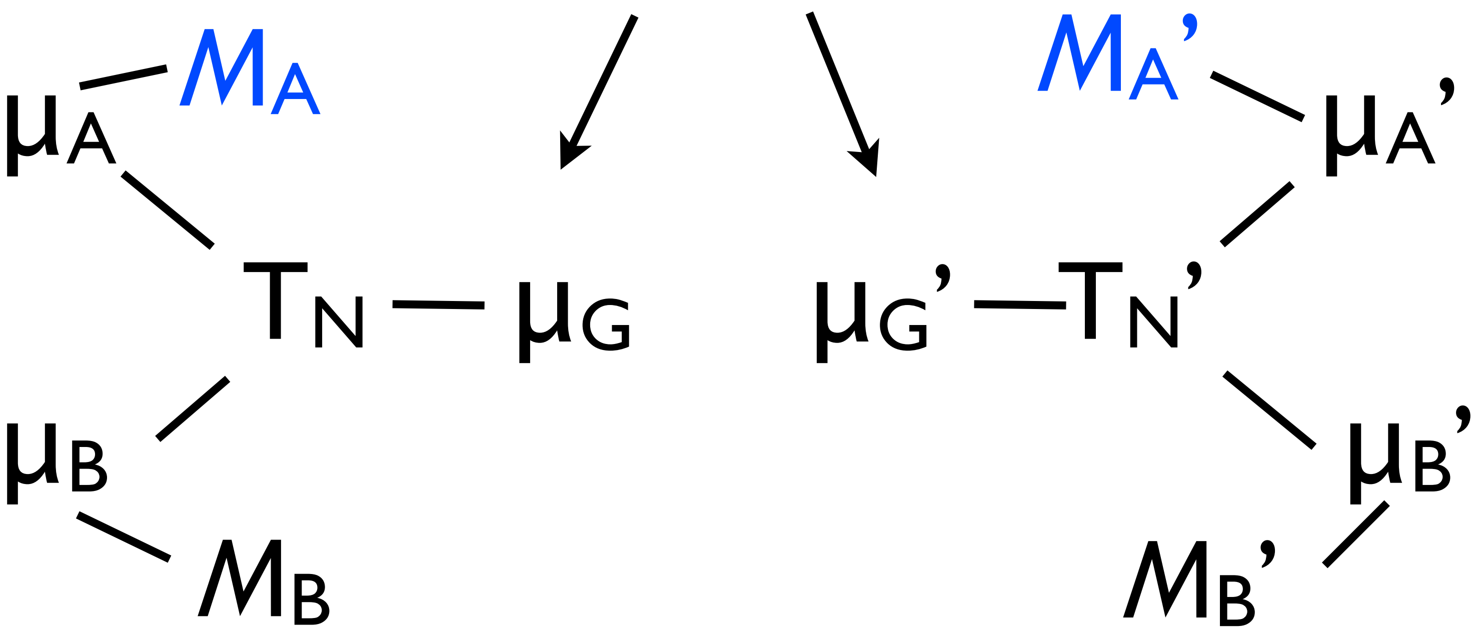


with $W=$
$\mathrm{tr}\ \rho_s\mu_A + \mathrm{tr}\ \rho_s\mu_A'$
$+\mathrm{tr}\ M_A\mu_A + \mathrm{tr}\ M_B\mu_B$
$+\ \mathrm{tr}\ M_A'\mu_A' + \mathrm{tr}\ M_B'\mu_B'$

$$\rho_s = \boxed{\begin{array}{c} 1 \\ 0 \\ 0 \end{array}}\ \boxed{\begin{array}{c} 0 \\ 0 \end{array}}\ \boxed{\begin{array}{c} 0 \\ 0 \end{array}}\ \boxed{0}$$

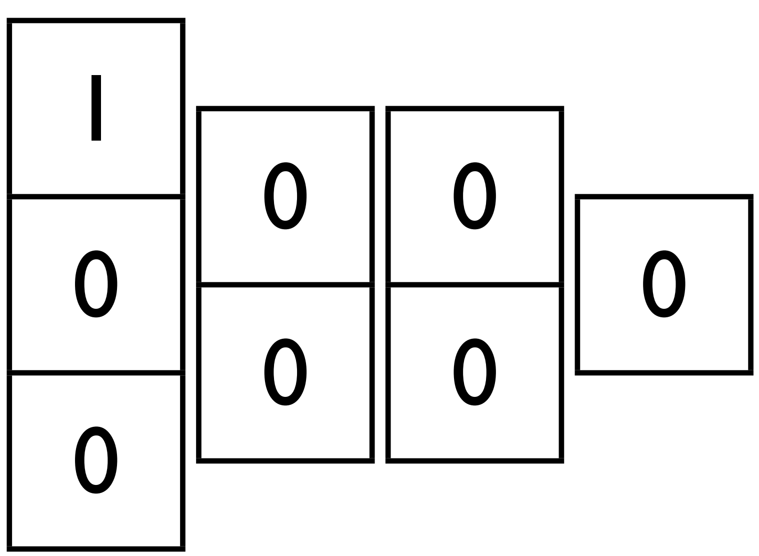
$$\mu_A = \boxed{\begin{array}{c} \mu^+ \\ \mu^3 \\ \mu^- \end{array}}\ \boxed{\begin{array}{c} \mu^{1,+} \\ \mu^{1,-} \end{array}}\ \boxed{\begin{array}{c} \mu^{2,+} \\ \mu^{2,-} \end{array}}\ \boxed{\mu^0}$$

coupled to $\mathcal{N}=1$ SU(N) vect.

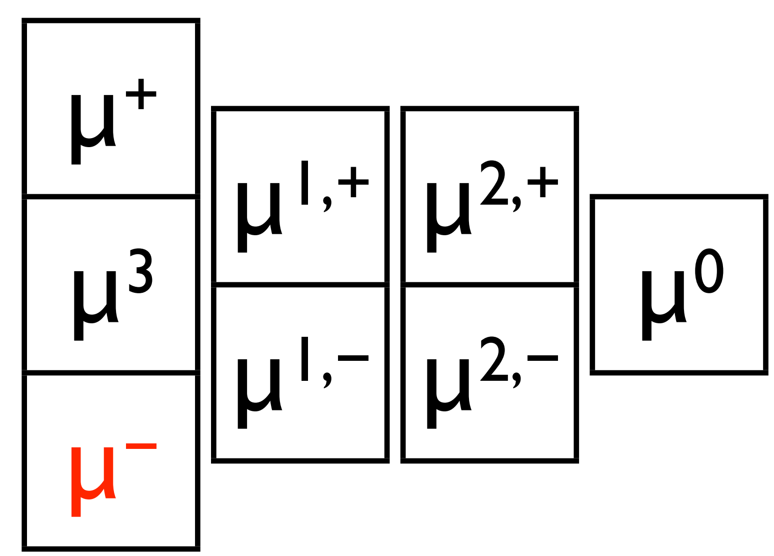


with $W=$
$$\mu_A^- + \mu_A'^- + \mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B + \mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$$

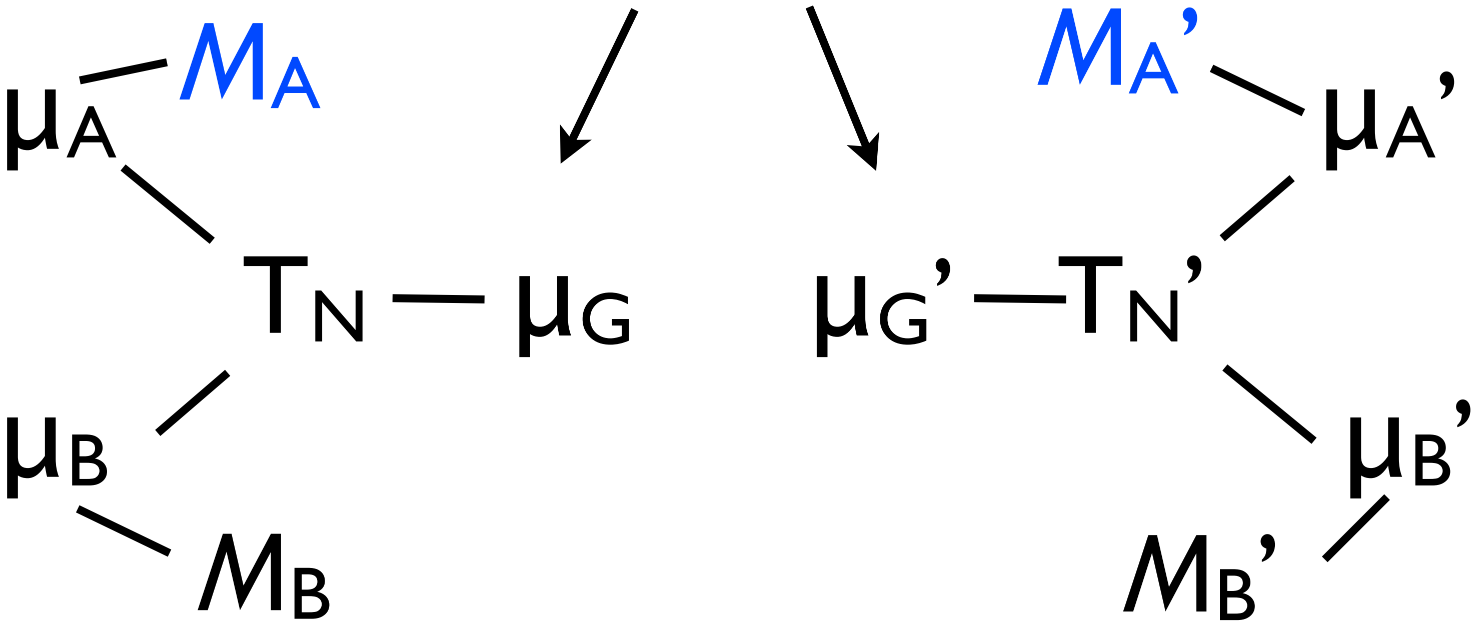
$\rho_s=$



$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.



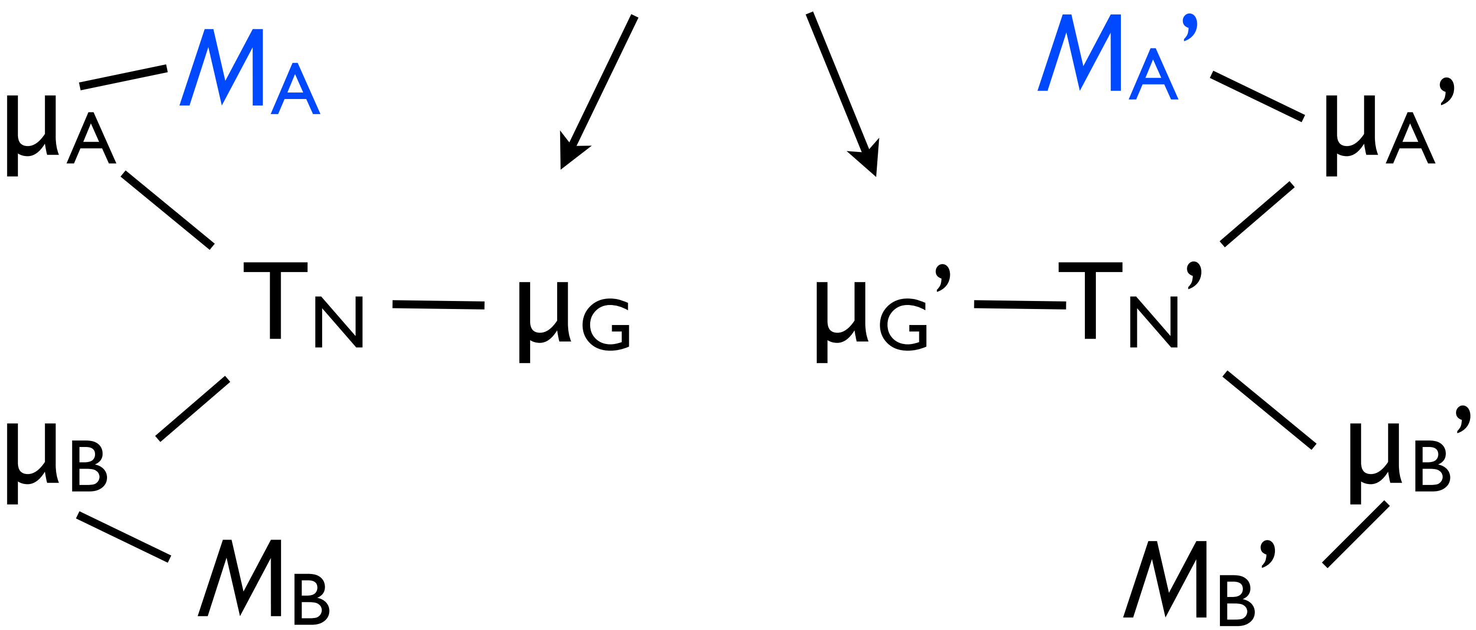
with $W=$

$$+\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B$$
$$+\, \mathrm{tr}\, M_{A'}\mu_{A'} + \mathrm{tr}\, M_{B'}\mu_{B'}$$

R-charges are originally

$$R_0 = \begin{array}{|c|c|c|c|} \hline 1 & 1 & 1 & 1 \\ \hline 1 & 1 & 1 & \\ \hline 1 & & & \\ \hline \end{array} \qquad \mu_A = \begin{array}{|c|c|c|c|} \hline \mu^+ & \mu^{1,+} & \mu^{2,+} & \mu^0 \\ \hline \mu^3 & \mu^{1,-} & \mu^{2,-} & \\ \hline \mu^- & & & \\ \hline \end{array}$$

coupled to $\mathcal{N}=1$ SU(N) vect.

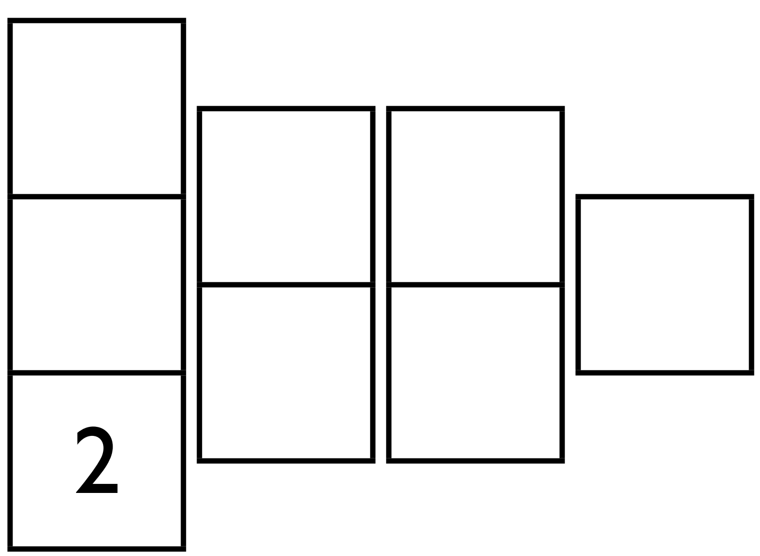


with $W=$

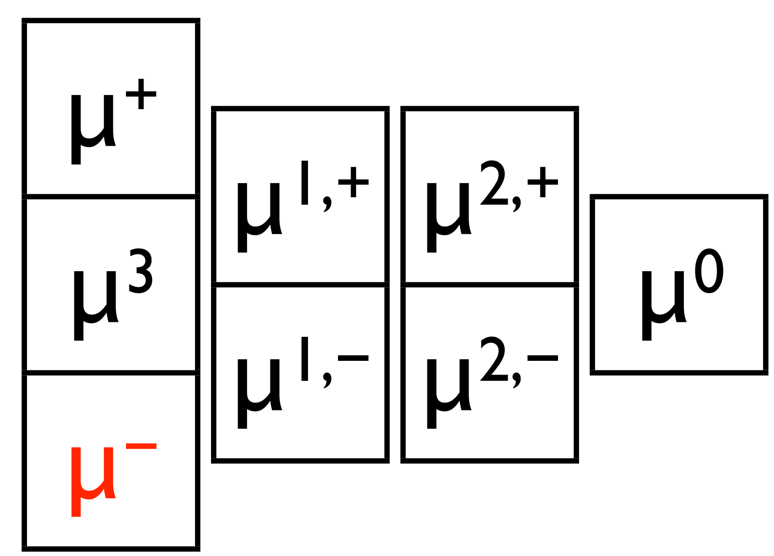
$$\mu_A^- + \mu_A'^- + \mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B + \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$$

R-charges should be now

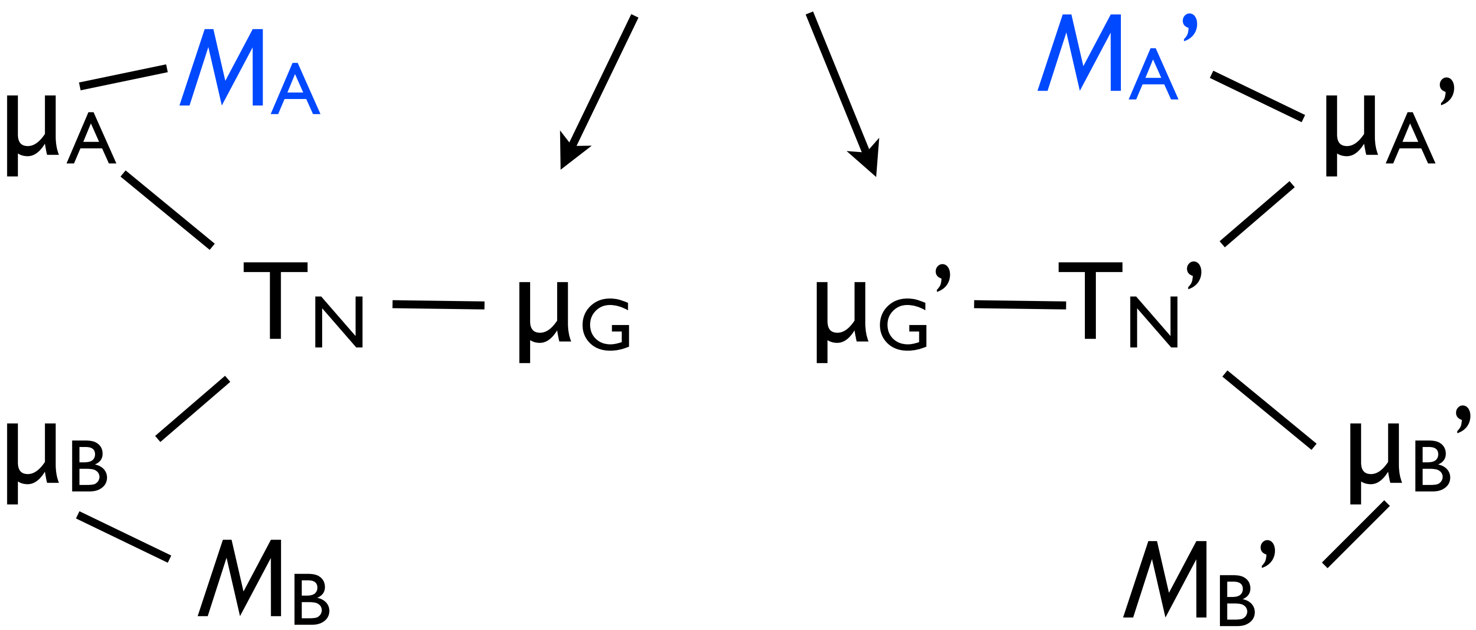
R=



$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.



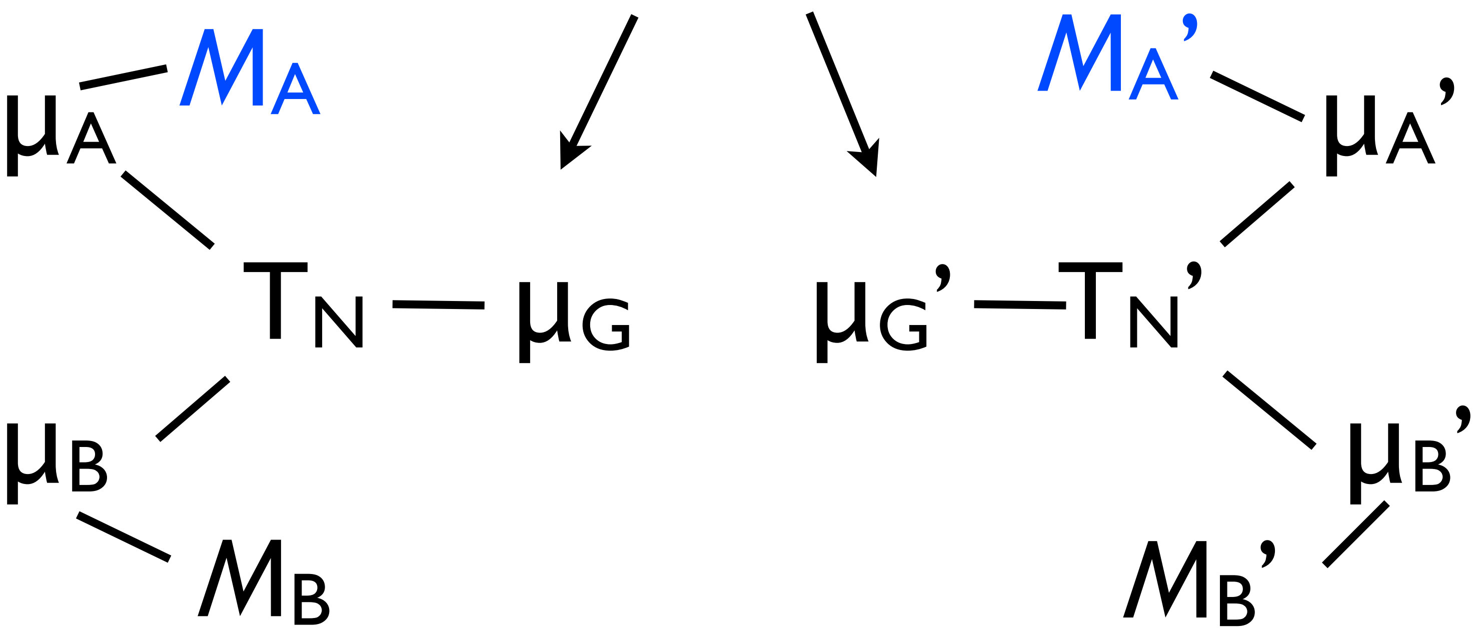
with W=

$$\mu_A^- + \mu_A'^-$$
$$+\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B$$
$$+\mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$$

This can be achieved by

$$R=\left(R_0=\begin{array}{c}\boxed{1}\\\boxed{1}\\\boxed{1}\end{array}\ \begin{array}{c}\boxed{1}\\\boxed{1}\end{array}\ \begin{array}{c}\boxed{1}\\\boxed{1}\end{array}\ \boxed{1}\right)\quad -\left(F^3=\begin{array}{c}\boxed{1}\\\boxed{0}\\\boxed{-1}\end{array}\ \begin{array}{c}\boxed{1/2}\\\boxed{-1/2}\end{array}\ \begin{array}{c}\boxed{1/2}\\\boxed{-1/2}\end{array}\ \boxed{0}\right)$$

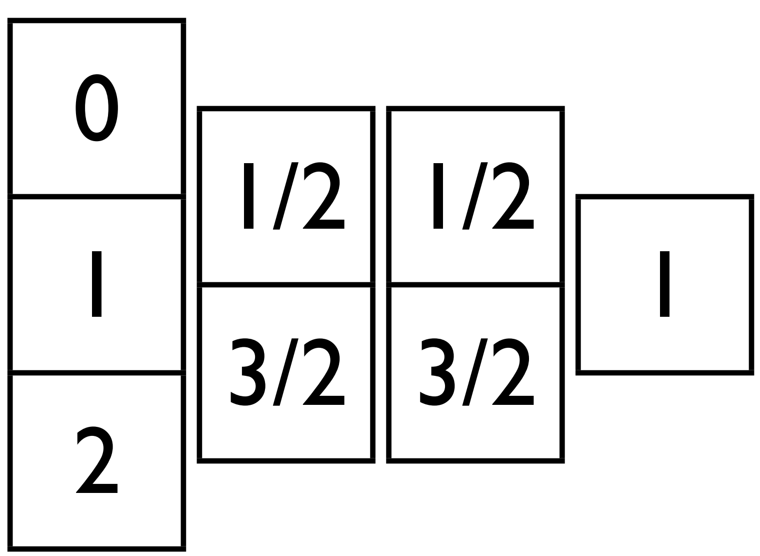
coupled to $\mathcal{N}=1$ SU(N) vect.



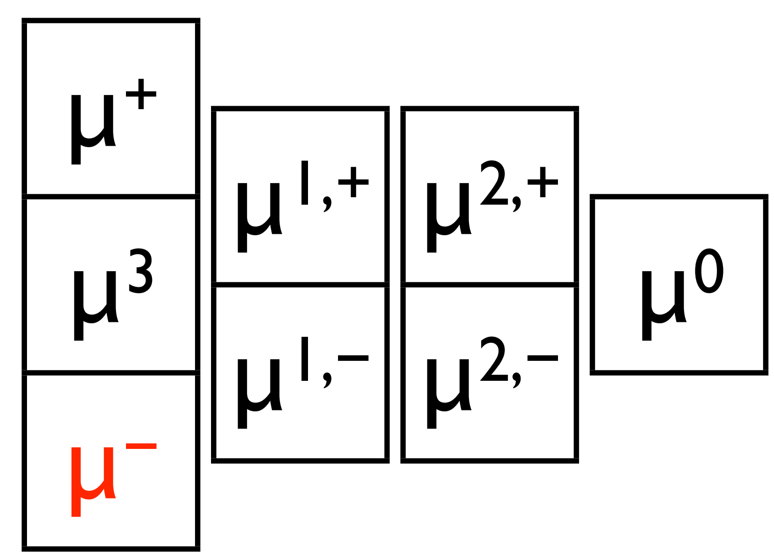
with $W=$
$\mu_A^- + \mu_A'^-$
$+\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B$
$+\mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$

which is

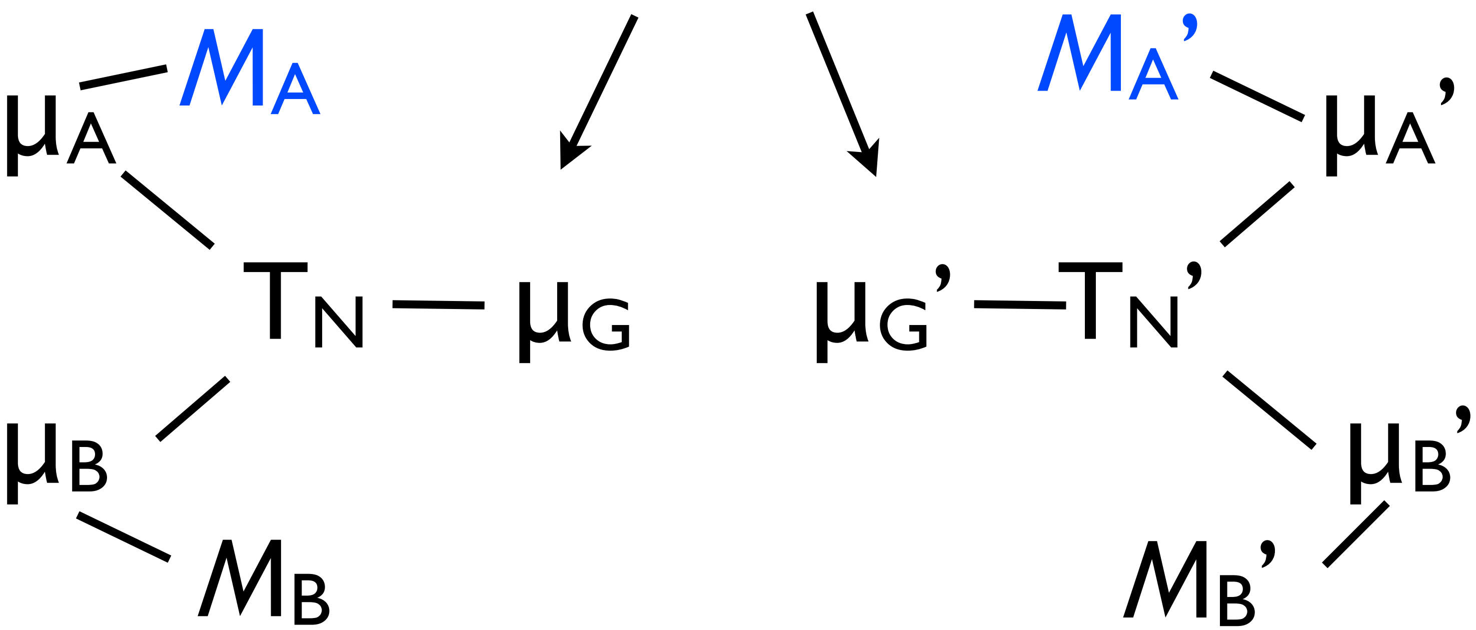
$R=$



$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.



with W=

$\mu_A^- + \mu_A'^-$

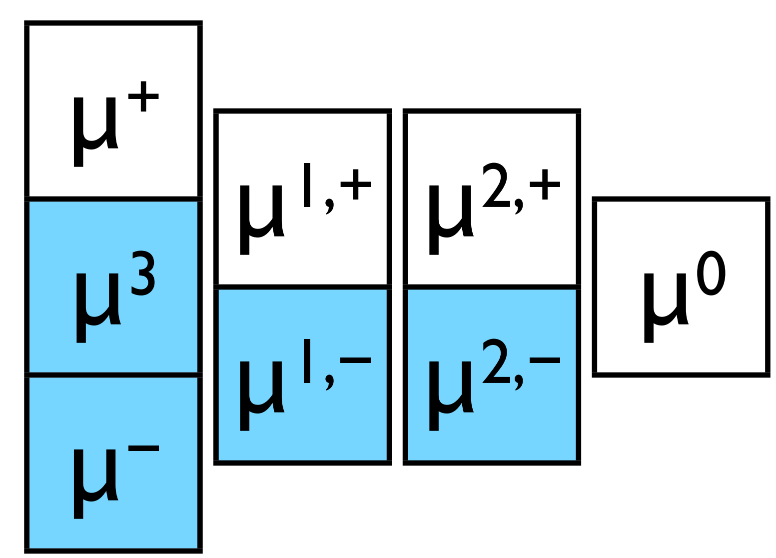
$+\mathrm{tr}\, M_A\mu_A + \mathrm{tr}\, M_B\mu_B$

$+\mathrm{tr}\, M_A'\mu_A' + \mathrm{tr}\, M_B'\mu_B'$

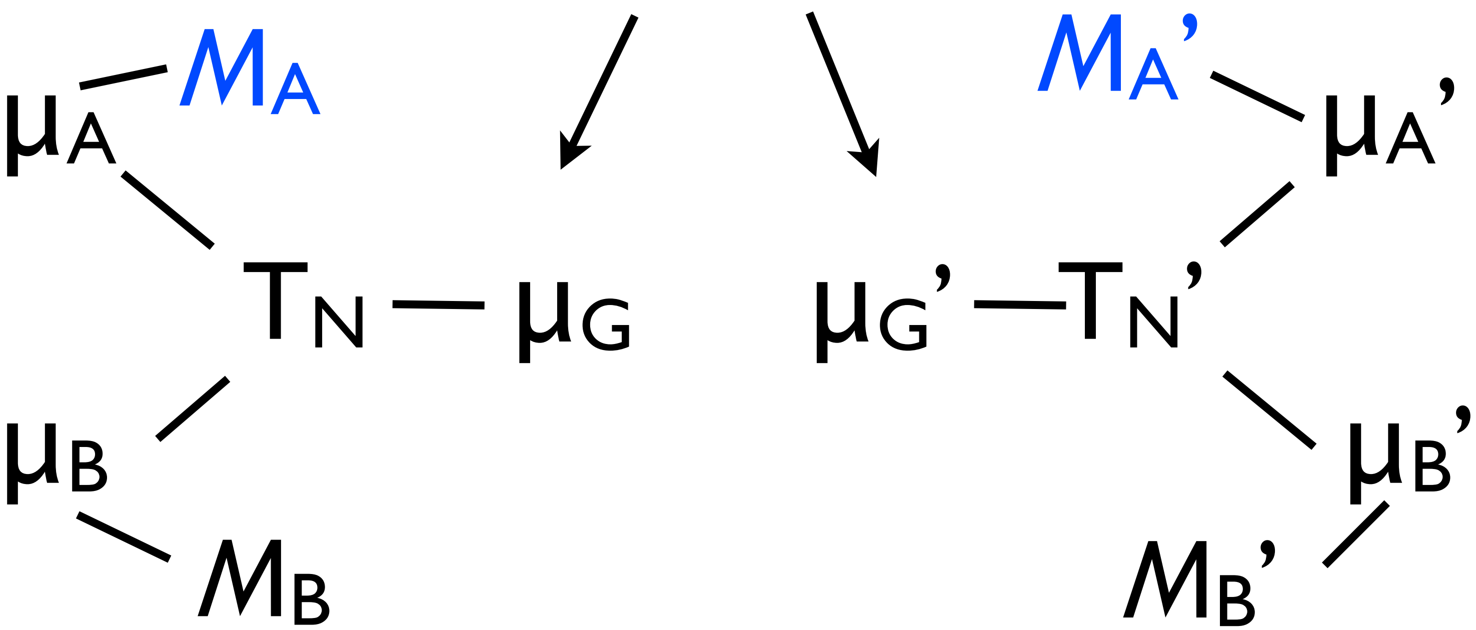
Presence of μ_A^- breaks many of SU(N) currents:

$D^2J = \delta W = \mu^3$ etc.

$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.

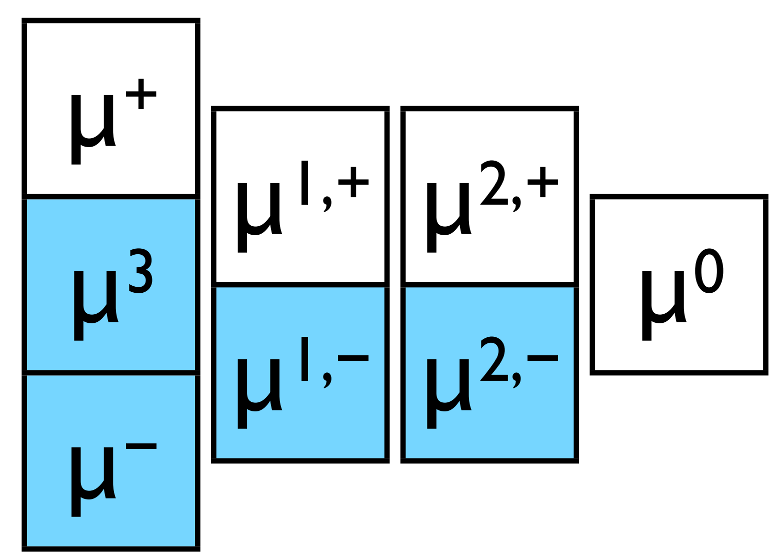


with $W=$

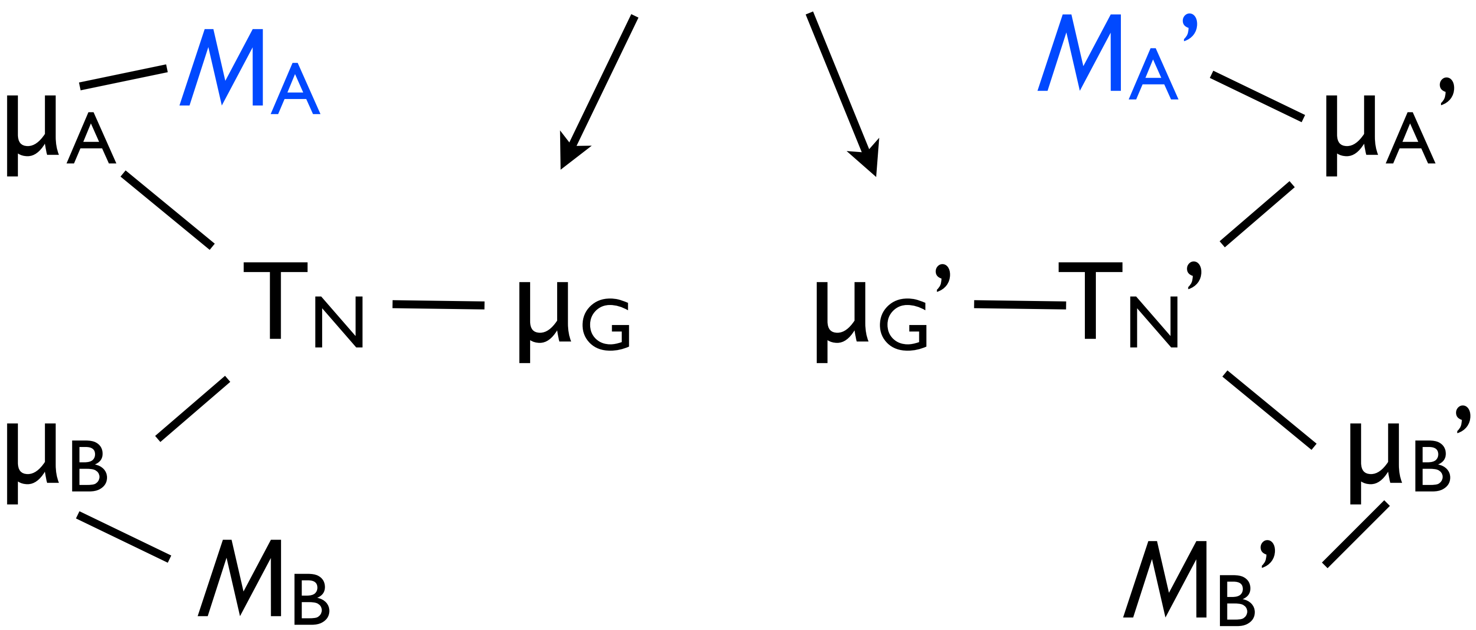
$$\mu_A^- + \mu_A'^- + \mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B + \mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$$

Those are not chiral primary anymore.

$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.



with $W=$

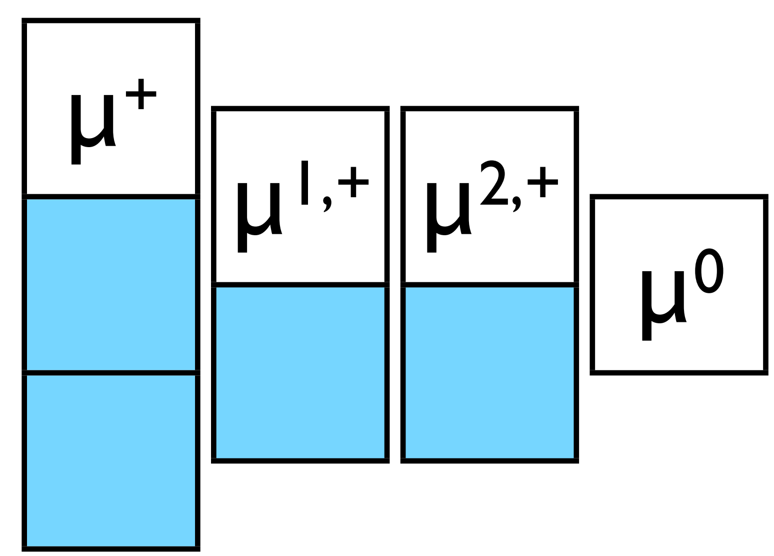
$\mu_A^- + \mu_A'^-$

$+\mathrm{tr}\, M_A \mu_A + \mathrm{tr}\, M_B \mu_B$

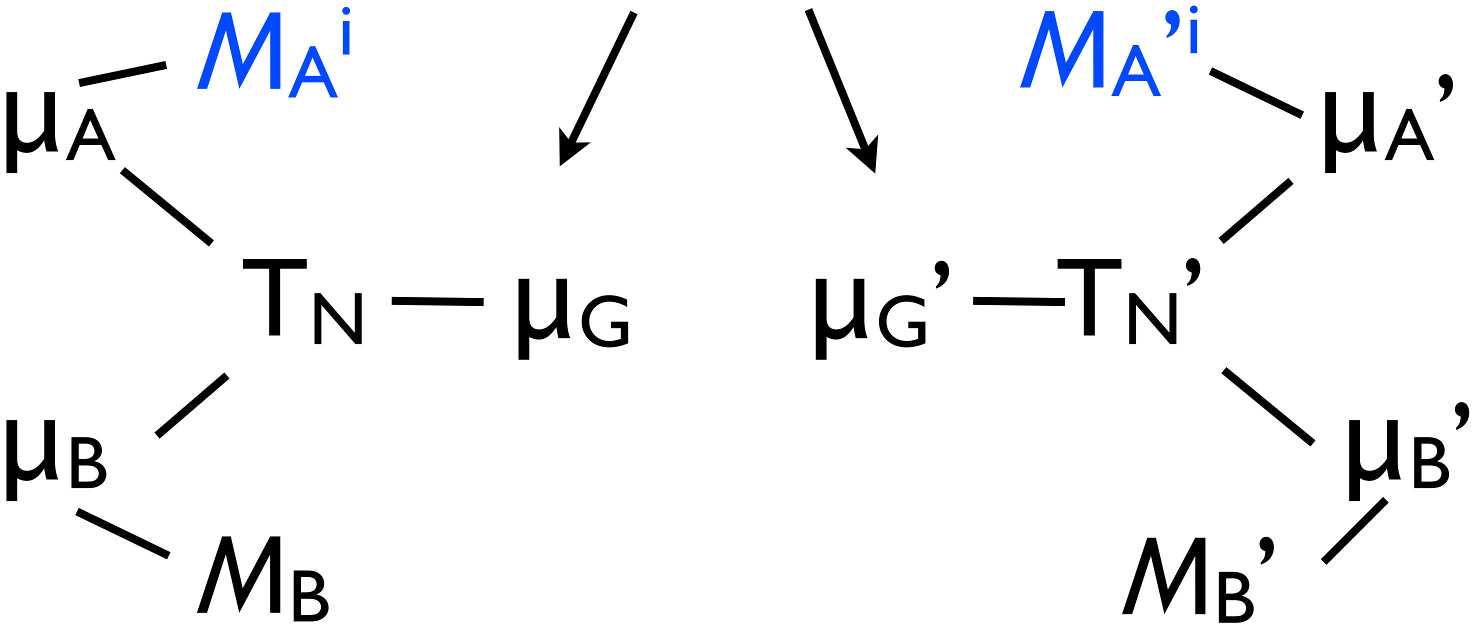
$+\mathrm{tr}\, M_A' \mu_A' + \mathrm{tr}\, M_B' \mu_B'$

plus neutral chiral multiplets.

$\mu_A=$



coupled to $\mathcal{N}=1$ SU(N) vect.

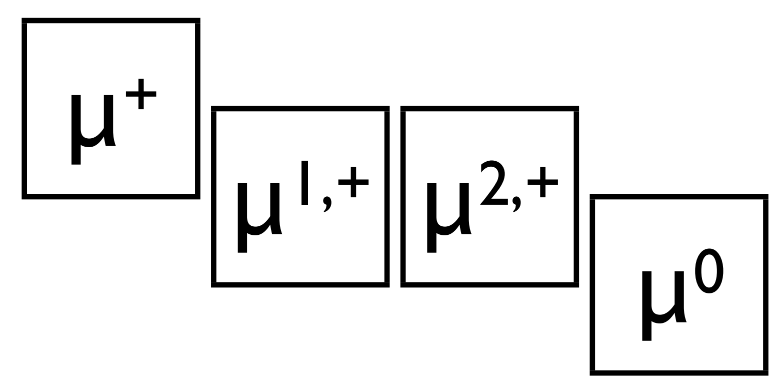


with W=

$$\mu_A{}^- + \mu_A'^-$$
$$+M_A{}^i\mu_A{}^i + \mathrm{tr}\, M_B\mu_B$$
$$+ M_A'^i\mu_A'^i + \mathrm{tr}\, M_B'\mu_B'$$

plus neutral chiral multiplets.

$\mu_A{}^i=$

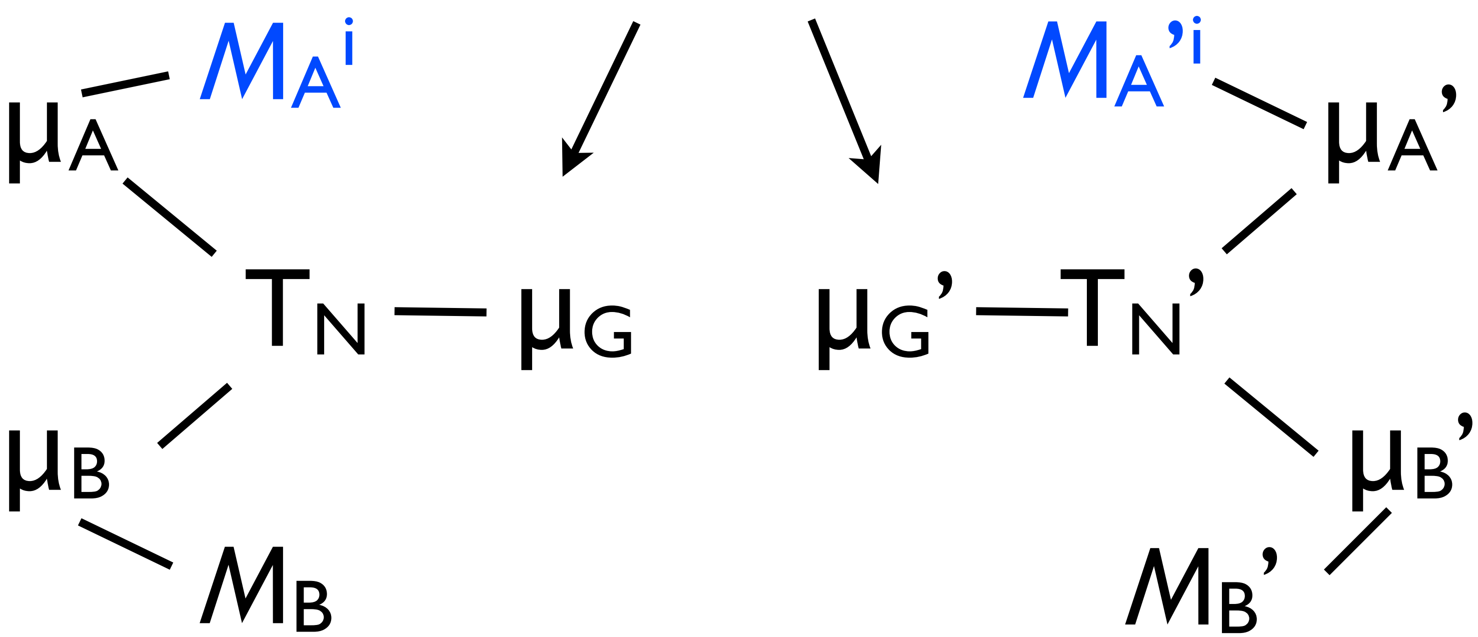


$\mathcal{N}=1$ SU(N) with 2N flavors

plus neutral chiral multiplets

dual

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W=$
$\mu_A{}^- + \mu_A{}'^-$
$+M_A{}^i\mu_A{}^i + \mathrm{tr}\, M_B\mu_B$
$+ M_A{}'^i\mu_A{}'^i + \mathrm{tr}\, M_B{}'\mu_B{}'$

plus neutral chiral multiplets.

- We checked that the central charges a, c and the superconformal indices (in a limit) agree on both sides.

- Of course we can consider a *quiverised version* where we have more than 2 T_Ns

- Explain the dualities of M5-branes on a Riemann surface with only $\mathcal{N}$=1 preserved found by [Beem-Gadde]

Summary

SU(2) with 4 flavors $q, \tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and 12 singlets M
$W = Mqq + M\tilde{q}\tilde{q}$

SU(2) with 4 flavors $q, \tilde{q}$
and a singlet M
$W = Mq\tilde{q}$

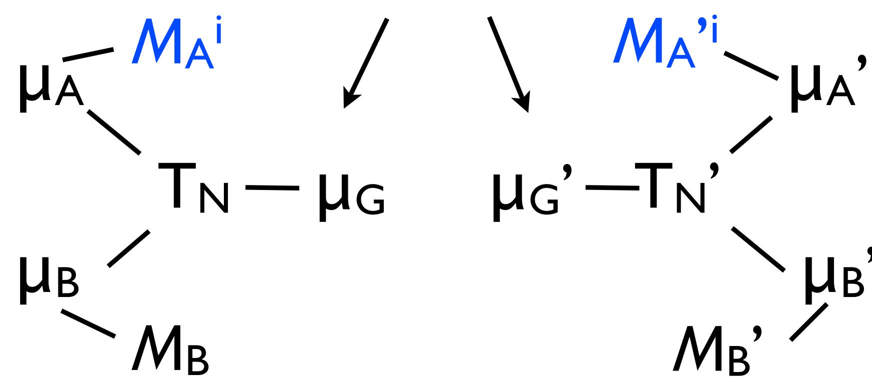
SU(N) with 2N flavors $q, \tilde{q}$

Another description involving T_N theory

SU(N) with 2N flavors $q, \tilde{q}$
and a singlet M
$W = Mq\tilde{q}$

SU(N) with 2N flavors $q, \tilde{q}$

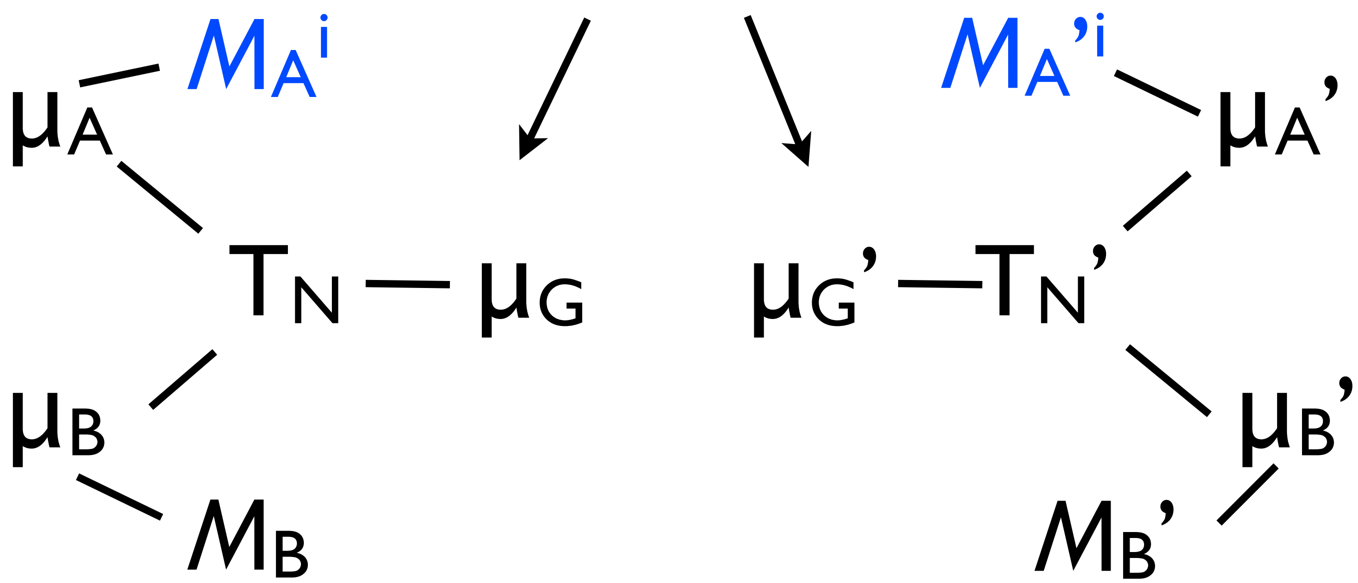
coupled to $\mathcal{N}$=1 SU(N) vect.



with $W=$
$\mu_A{}^- + \mu_A{}'^-$
$+ M_A{}^i \mu_A{}^i + \mathrm{tr}\, M_B \mu_B$
$+ M_A{}'^i \mu_A{}'^i + \mathrm{tr}\, M_B{}' \mu_B{}'$

SU(N) with 2N flavors $q, \tilde{q}$
and a singlet M
$W = M q \tilde{q}$

coupled to $\mathcal{N}=1$ SU(N) vect.



with $W=$

$$\mu_A{}^- + \mu_A{}'^- + M_A{}^i \mu_A{}^i + \mathrm{tr}\, M_B \mu_B + M_A{}'^i \mu_A{}'^i + \mathrm{tr}\, M_B{}' \mu_B{}'$$

Somehow this description is rather complicated for SU(3) and bigger ...

- My main message is that a

  conventional↔conventional duality,

  which is only known

  with a low-rank gauge group,

  often generalize to a

  non-conventional↔conventional duality

  with general gauge groups.

- Phrased differently, I say,
by considering only conventional theories
you’re imposing yourself
artificial, arbitrary restrictions.

- Non-conventional theories are as important as Lagrangian theories once you start talking about dualities.

- Most of the post-1994 techniques are applicable to both conventional and non-conventional theories.