Supersymmetry

An idea connecting Physics and Mathematics

Yuji Tachikawa

Kavli IPMU

- There are many Kavli Institutes around the world;
- The Kavli Institute I belong to has a special menu to offer: **mathematics**!
- Kavli I**PMU** =
Kavli Institute for **Physics** and **Mathematics** of the **Universe**.
- So I decided to talk about something a bit **mathematical**.

Spectrum of researchers at Kavli IPMU

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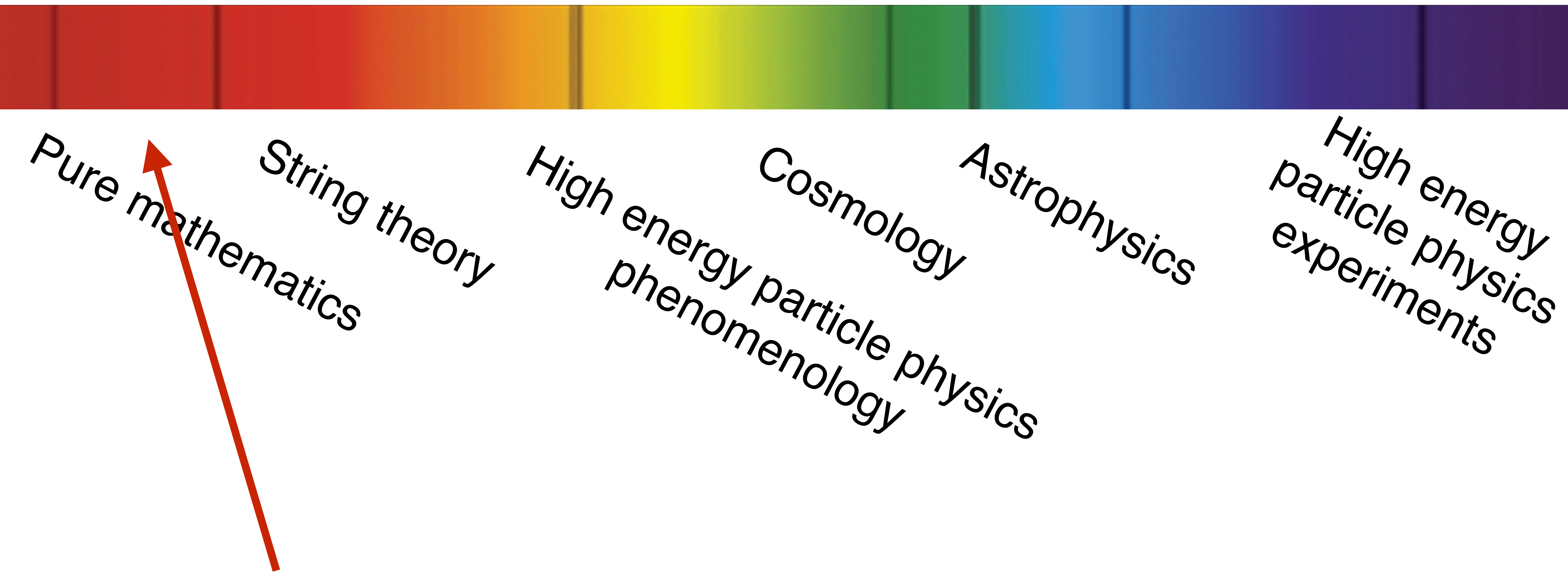


I'm around here, so I'm not very representative ...

Supersymmetry

Symmetry

Supersymmetry

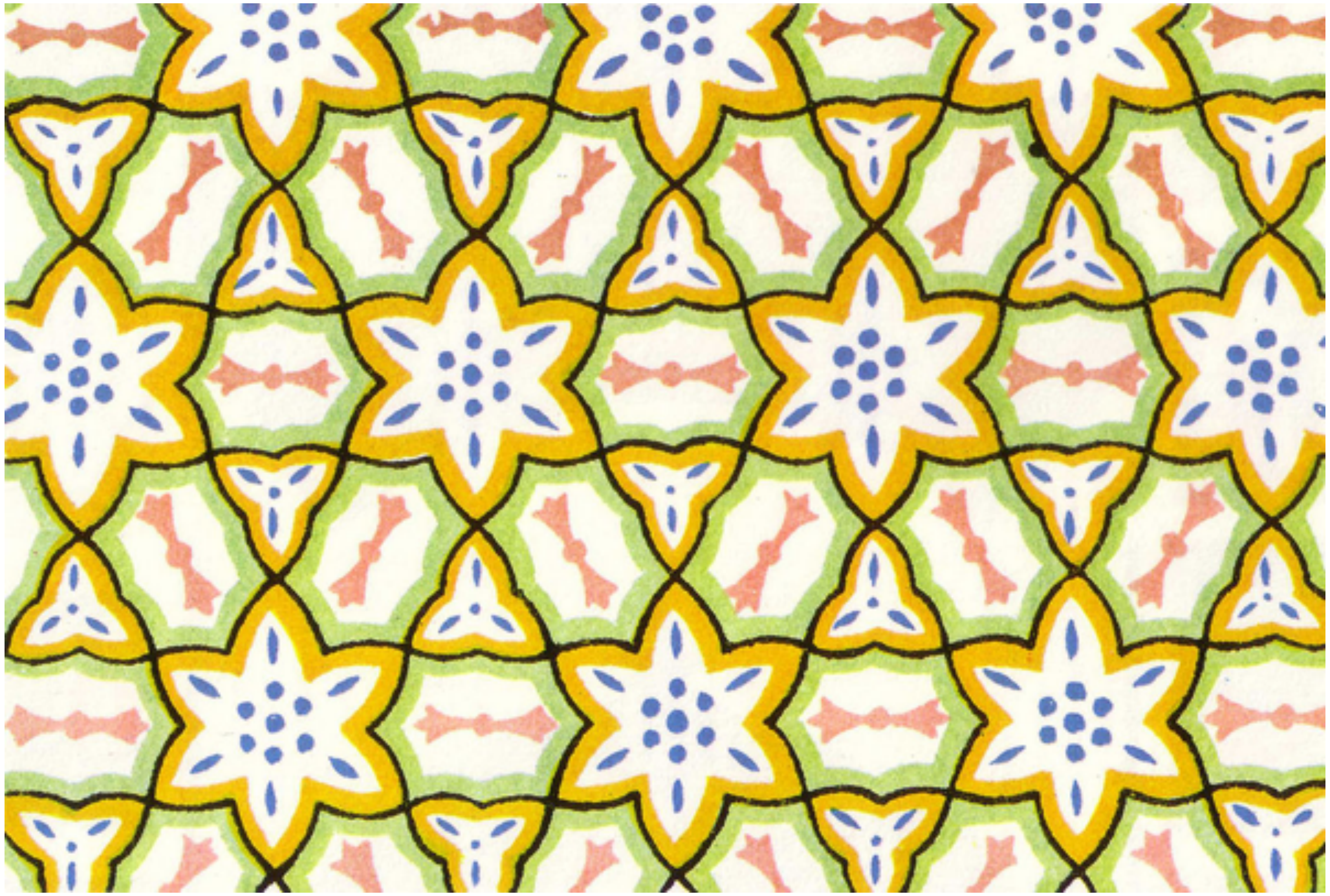
It's an ***Extension*** *of the concept of* ***Symmetry***

First, let me talk about a ***different*** ***Extension*** *of*

the concept of

Symmetry

- There are many periodic tilings



From *the Grammar of Ornament,* (O. Jones, 1856)

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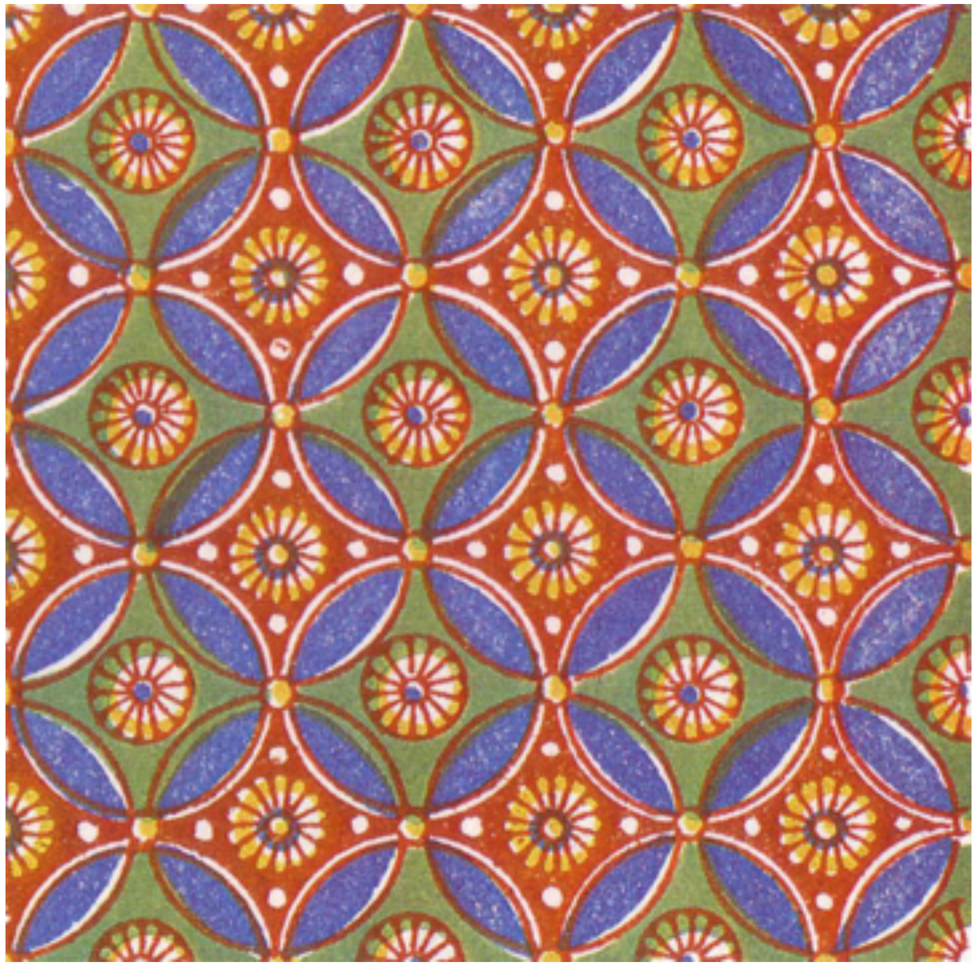

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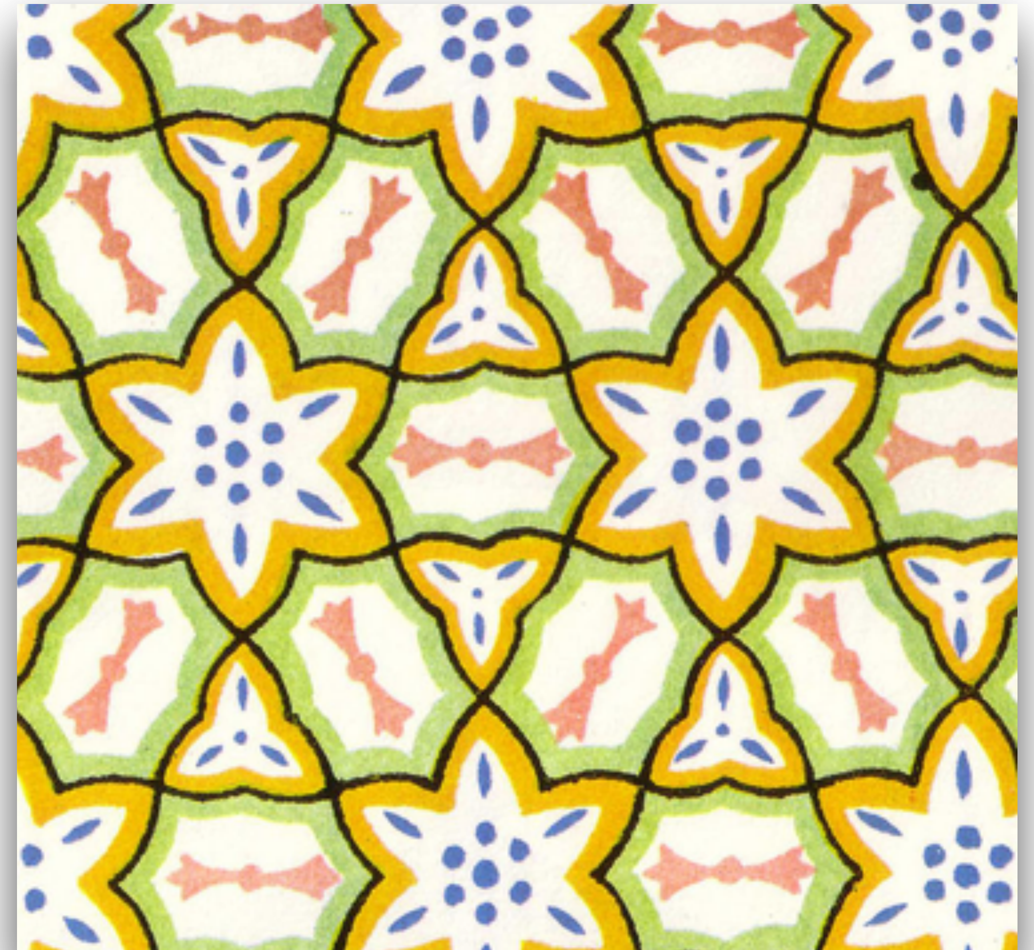
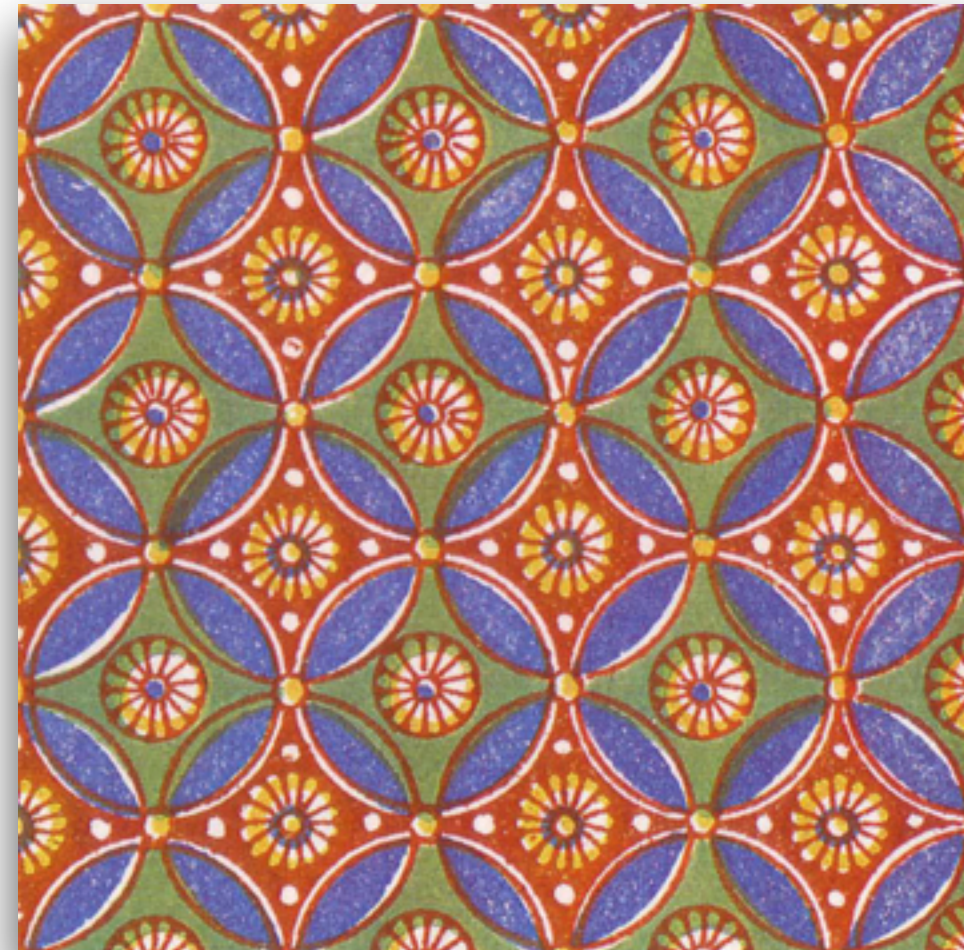
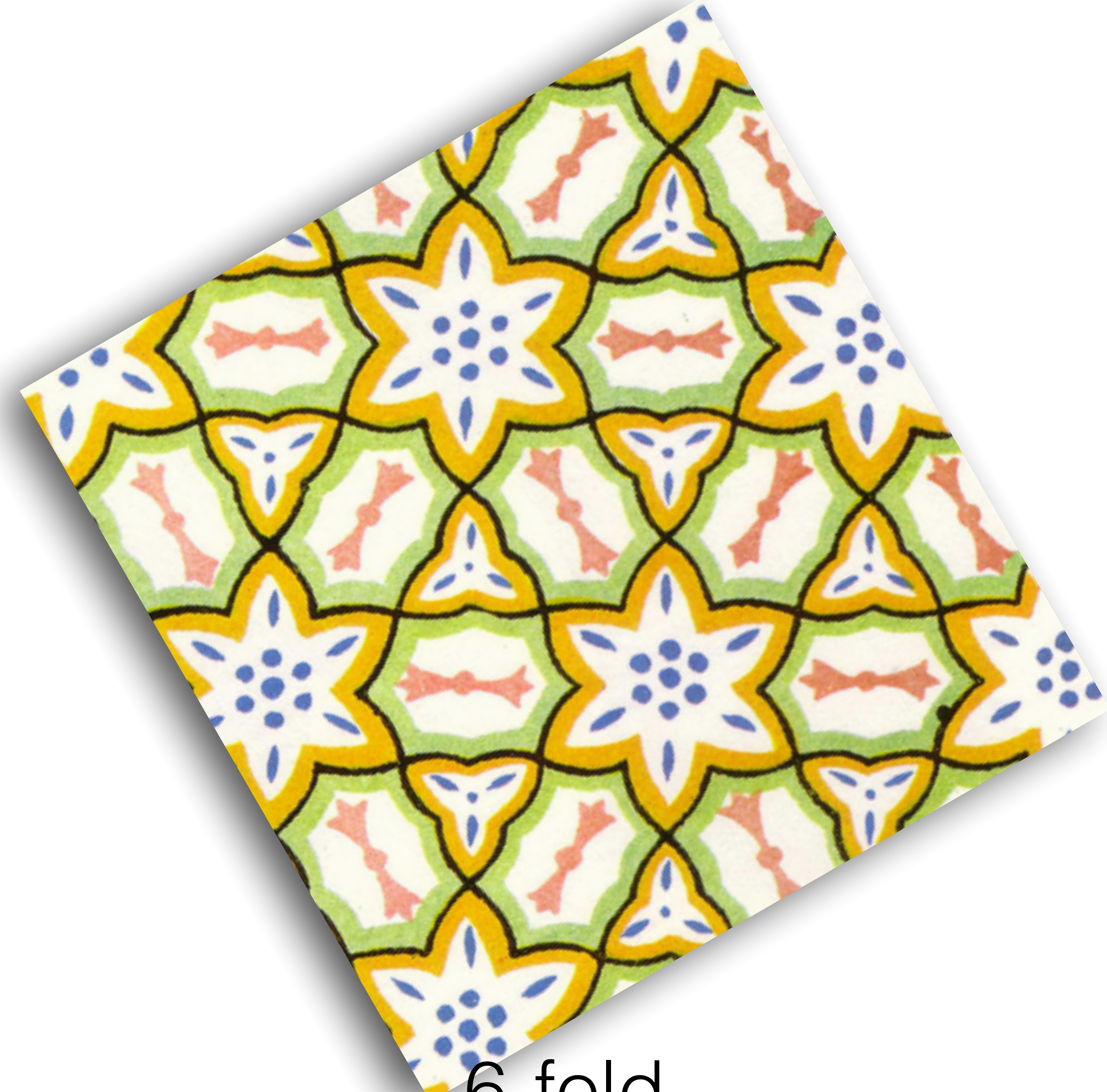


6-fold
symmetric



4-fold
symmetric



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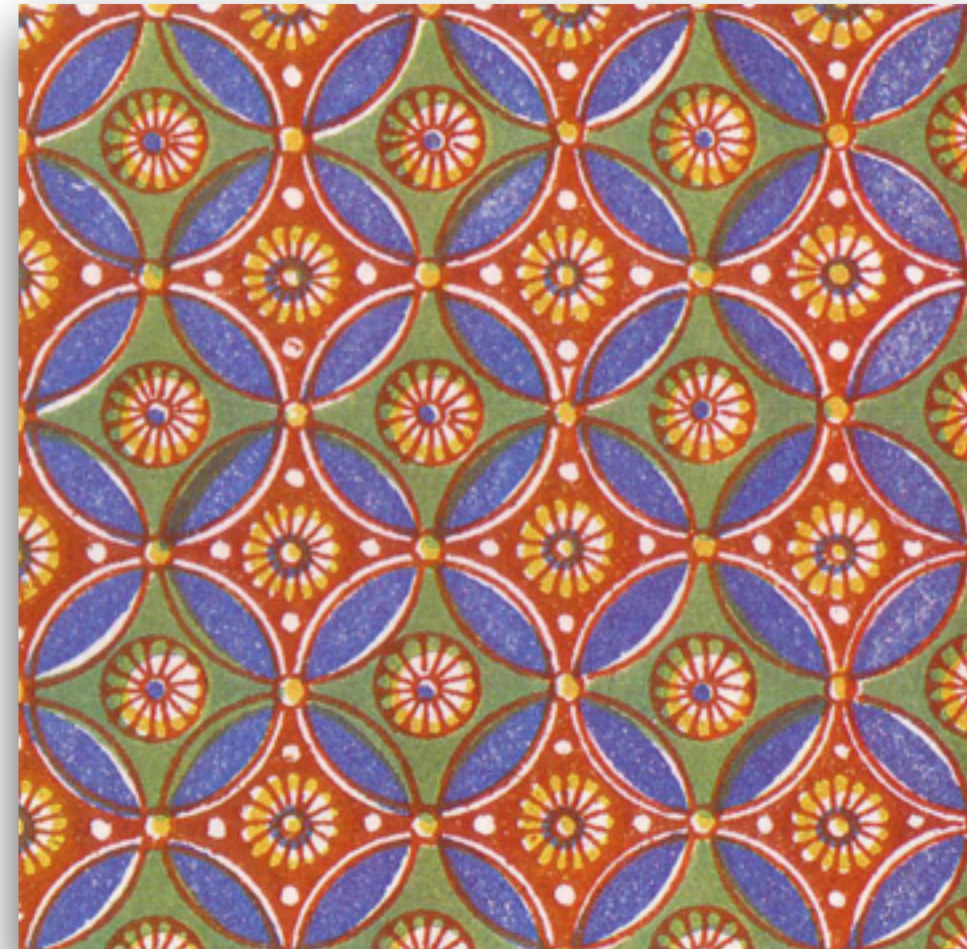

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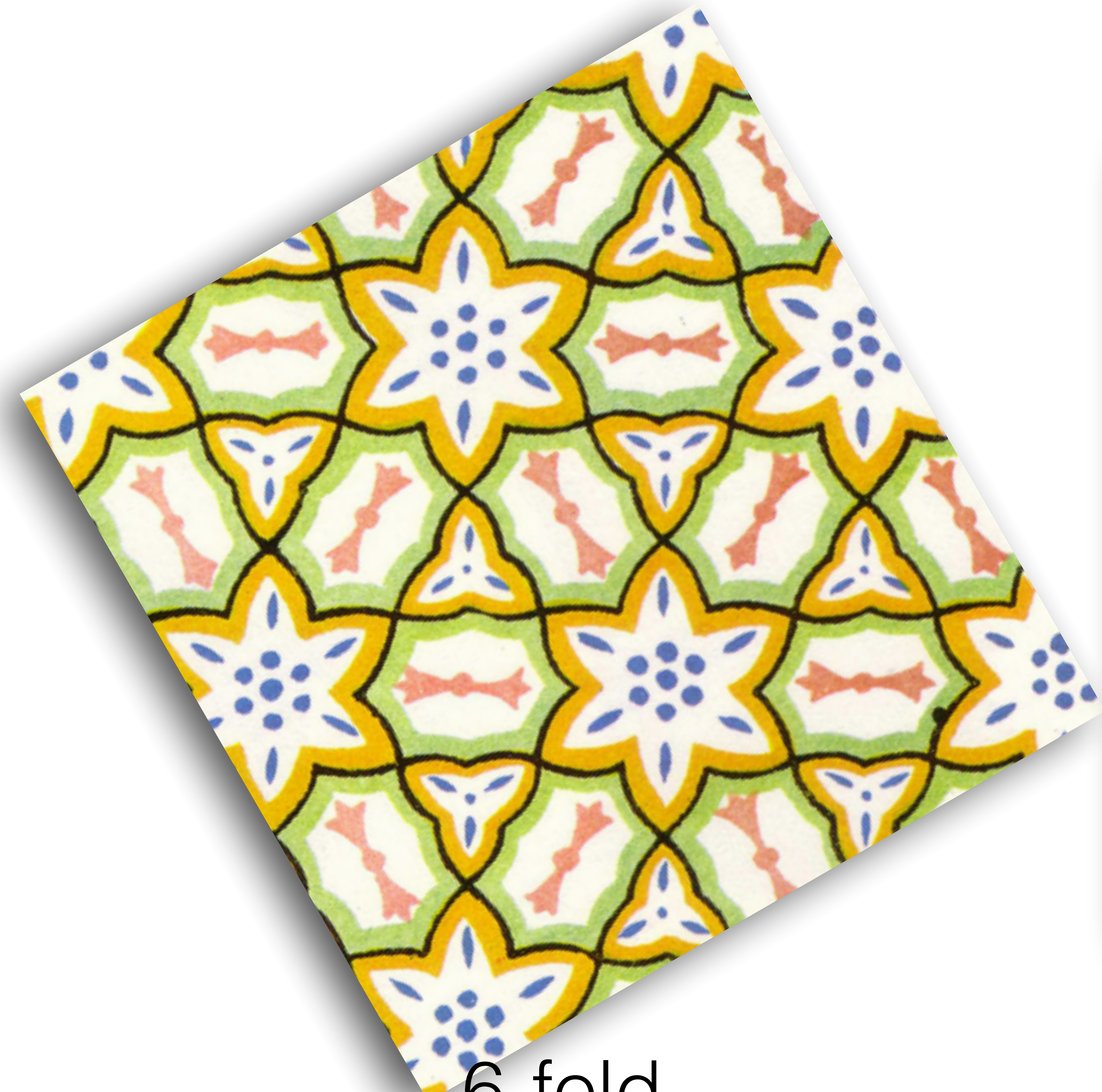

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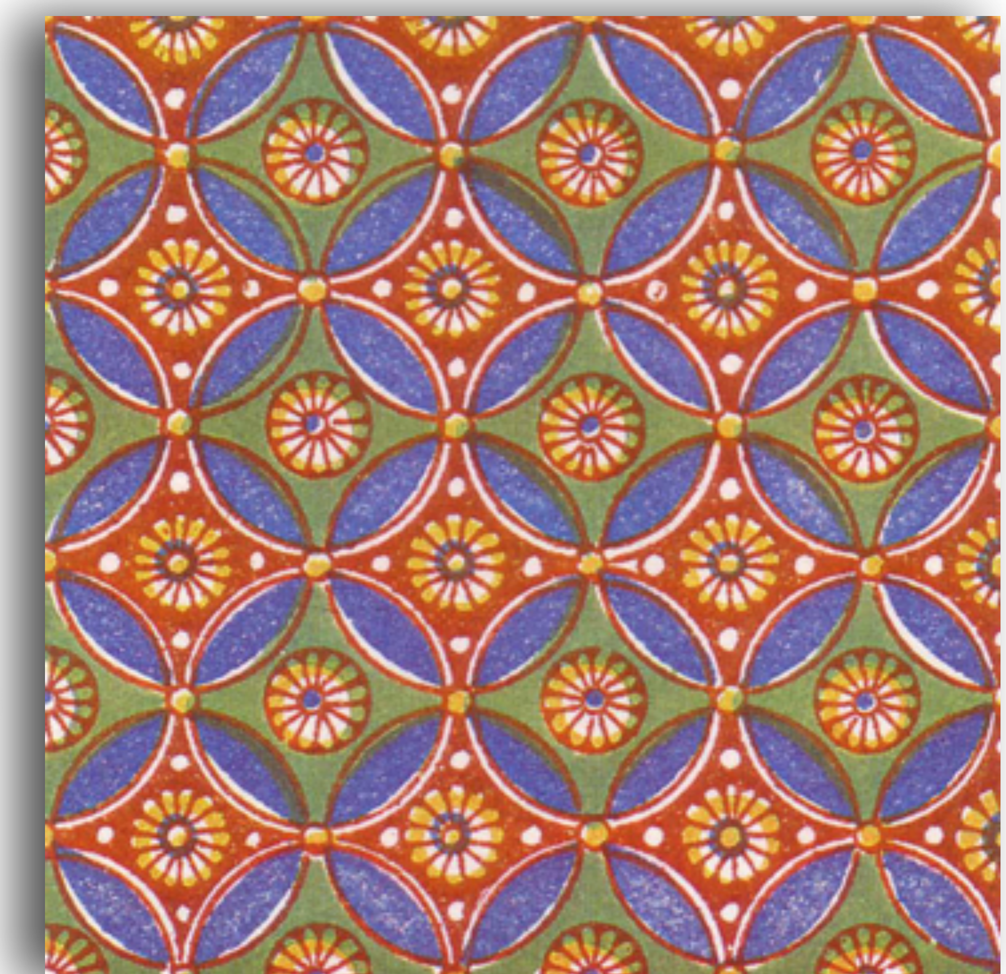


4-fold
symmetric

- It's a simple mathematical fact that **periodic tilings** can only have **2, 3, 4, or 6**-fold rotational symmetry.

- But there are crystalline materials that are **5**-fold symmetric.

- Crystals are thought to be periodic arrays of atoms.

- But this cannot be completely periodic, since it is 5-fold symmetric.

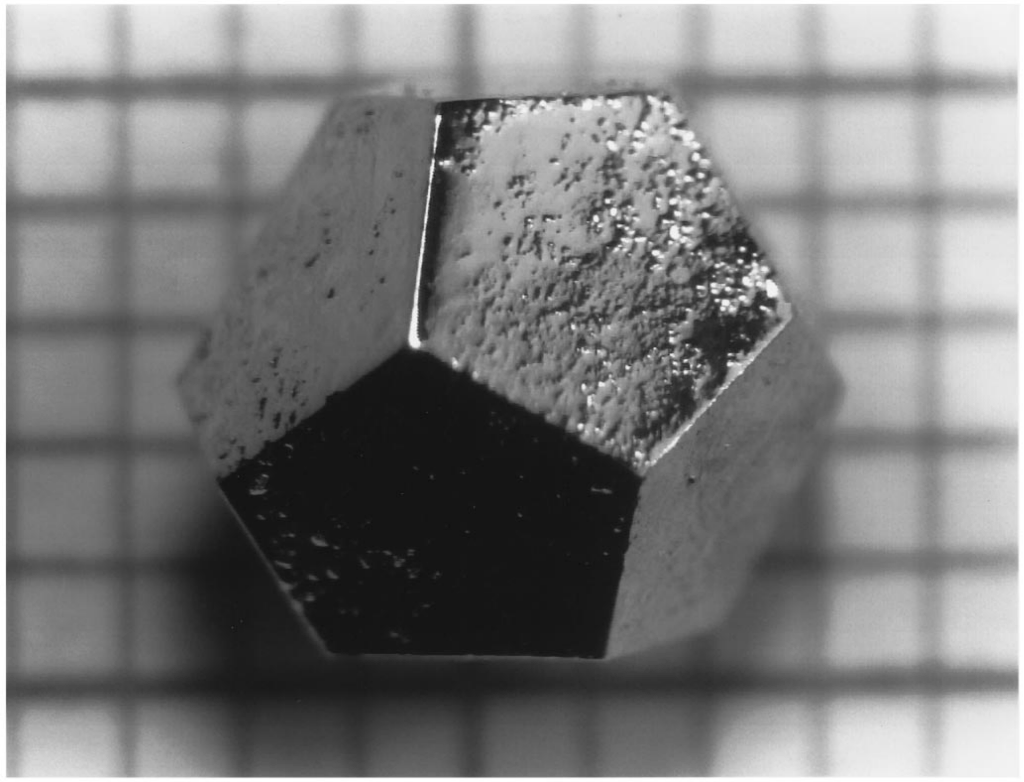


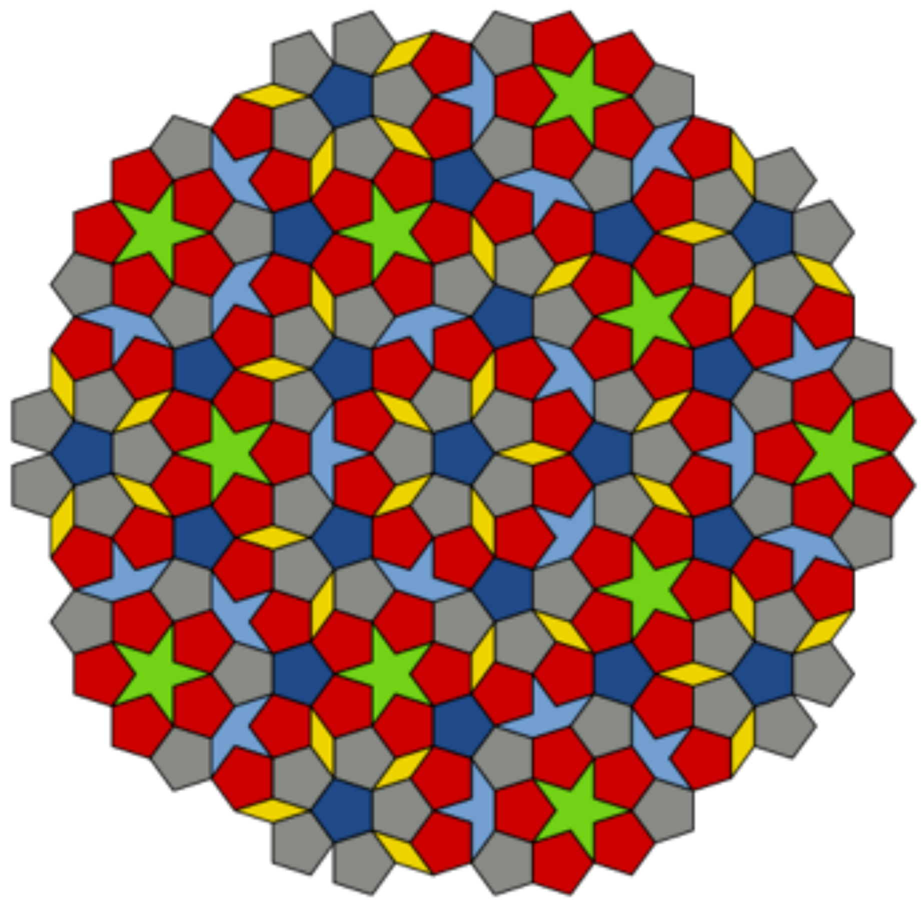
FIG. 1. Photograph of a single-grain icosahedral Ho-Mg-Zn quasicrystal grown from the ternary melt. Shown over a mm scale, the edges are 2.2 mm long. Note the clearly defined pentagonal facets, and the dodecahedral morphology.

from Fischer *et al.*, Phys. Rev. B 59 (1999) 308

- The way out: ***Quasi-periodicity***.
- It's infinitely close to being periodic, but not quite periodic.

- This quasi-periodic extension of the concept of symmetry was known to mathematicians, including Roger Penrose, since around **1970s**.

- This was before real materials were found.



An example of Penrose tiling, from Wikipedia

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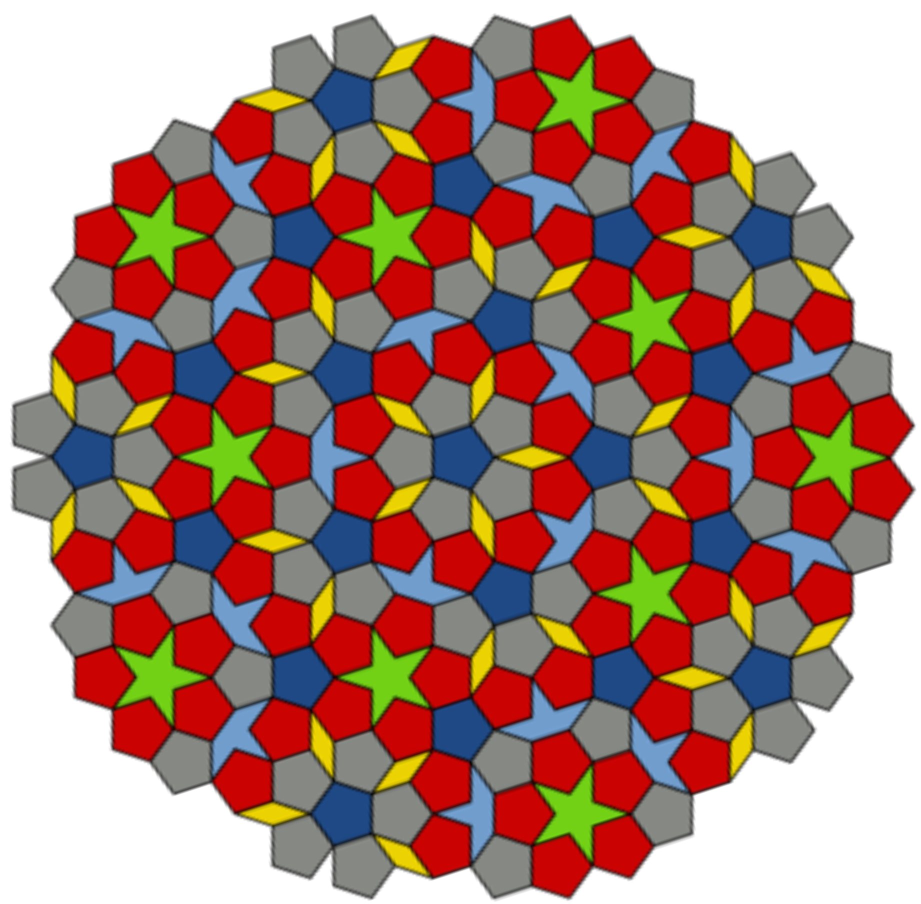


An example of Penrose tiling, from Wikipedia

- In fact it was **known to Muslim artists in medieval times**, as rediscovered in an article from **2007**.

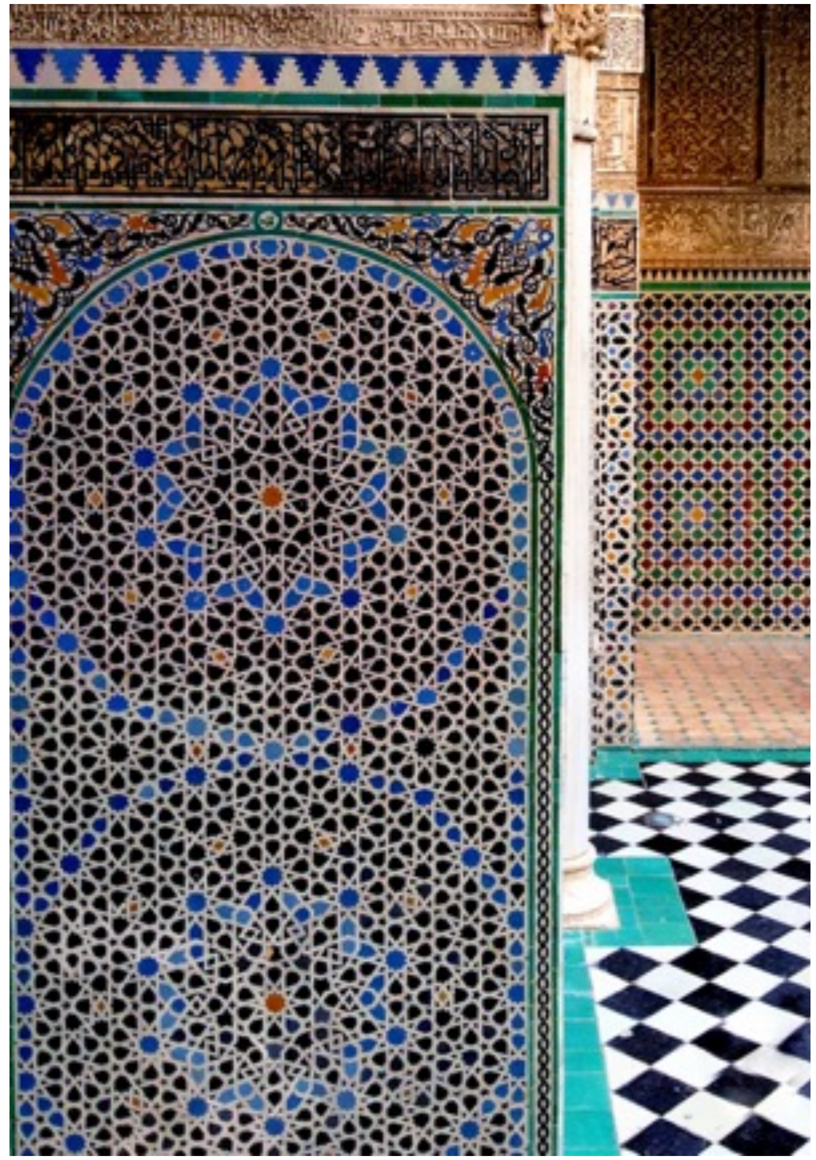
Decagonal and Quasi-Crystalline Tilings in Medieval Islamic Architecture

Peter J. Lu[1]* and Paul J. Steinhardt[2]

The conventional view holds that girih (geometric star-and-polygon, or strapwork) patterns in medieval Islamic architecture were conceived by their designers as a network of zigzagging lines, where the lines were drafted directly with a straightedge and a compass. We show that by 1200 C.E. a conceptual breakthrough occurred in which girih patterns were reconceived as tessellations of a special set of equilateral polygons ("girih tiles") decorated with lines. These tiles enabled the creation of increasingly complex periodic girih patterns, and by the 15th century, the tessellation approach was combined with self-similar transformations to construct nearly perfect quasi-crystalline Penrose patterns, five centuries before their discovery in the West.

Science 315(2007)1106.

- e.g. from Al Attarine Madrasa, Fez, Morocco, 14c.



as pointed out in R. A. Al Ajlouni, Acta Crystalographica 2012 A68.
Photo taken from http://toeuropeandbeyond.com/5-things-to-do-in-fes-morocco/

- It's a famous landmark in Morocco, visited by many, but nobody realized it's there until recently.

- e.g. from

as pointed out
Photo taken fro

- It's a famo
many, but

Quasi-periodicity: *an* ***Extension*** *of the concept of* ***Symmetry***

Supersymmetry*:*
an ***Extension*** *of*
the concept of
Symmetry

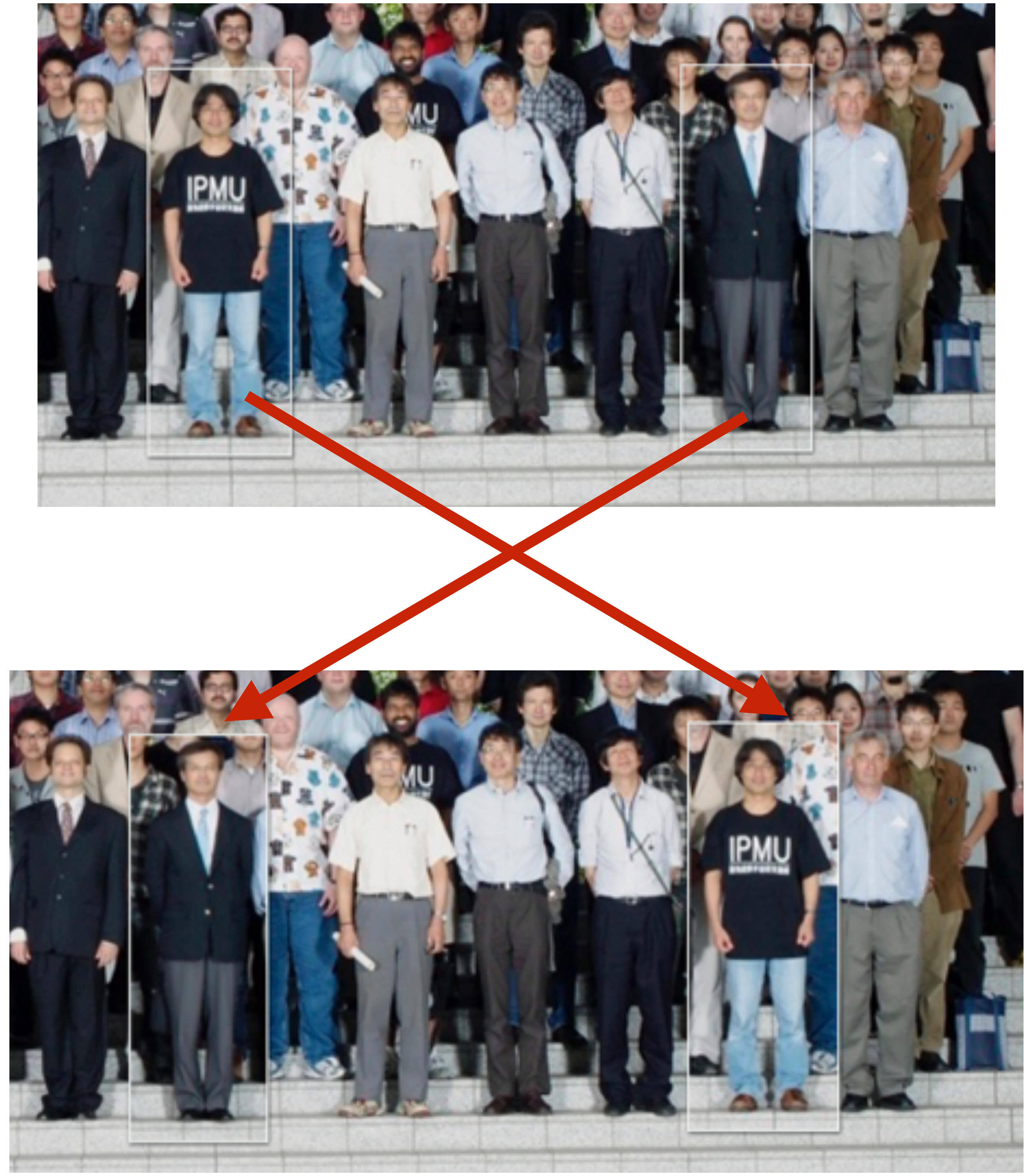
- It's again an extension of the concept of symmetry.
- It exchanges **bosons** and **fermions**.

This is an anniversary photo of
Kavli IPMU from a few years ago.


IPMU


IPMU


IPMU



If you exchange humans,
you of course get different configurations!

$\psi($ 

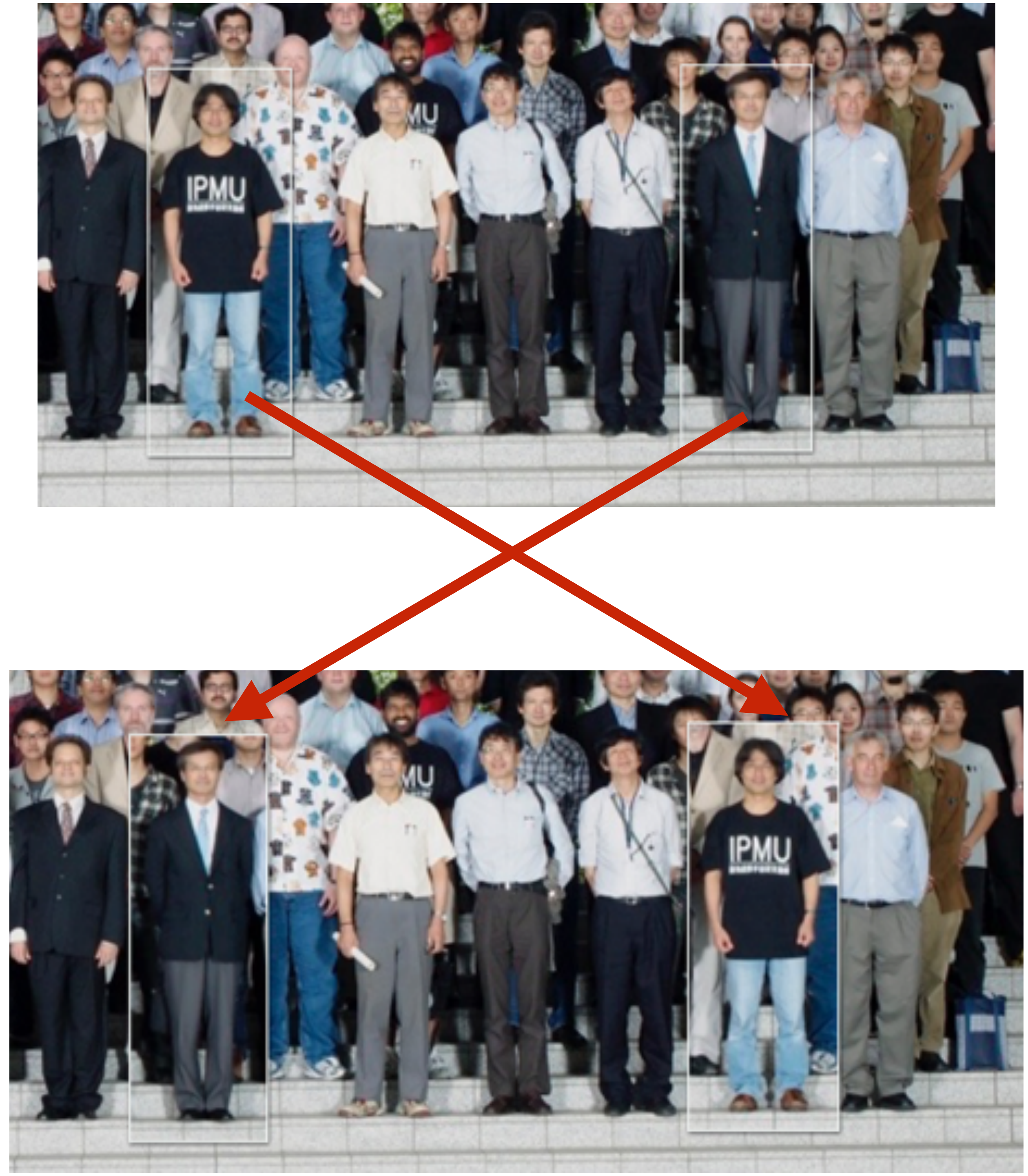
$)$

$= \pm\psi($ 

$)$

If they were elementary particles,
things are different.

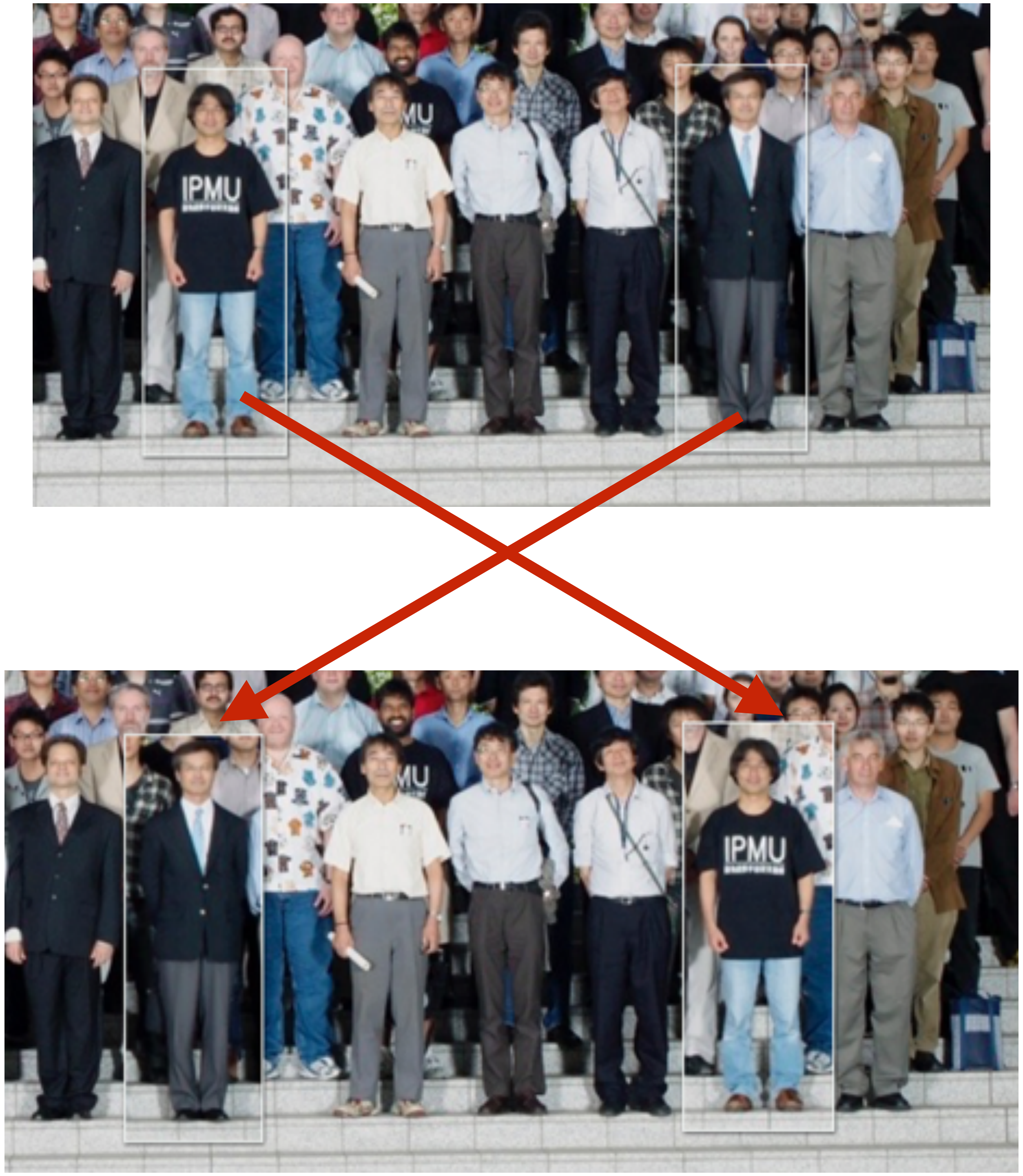
$\psi($ $)$



$= \pm\psi($ $)$

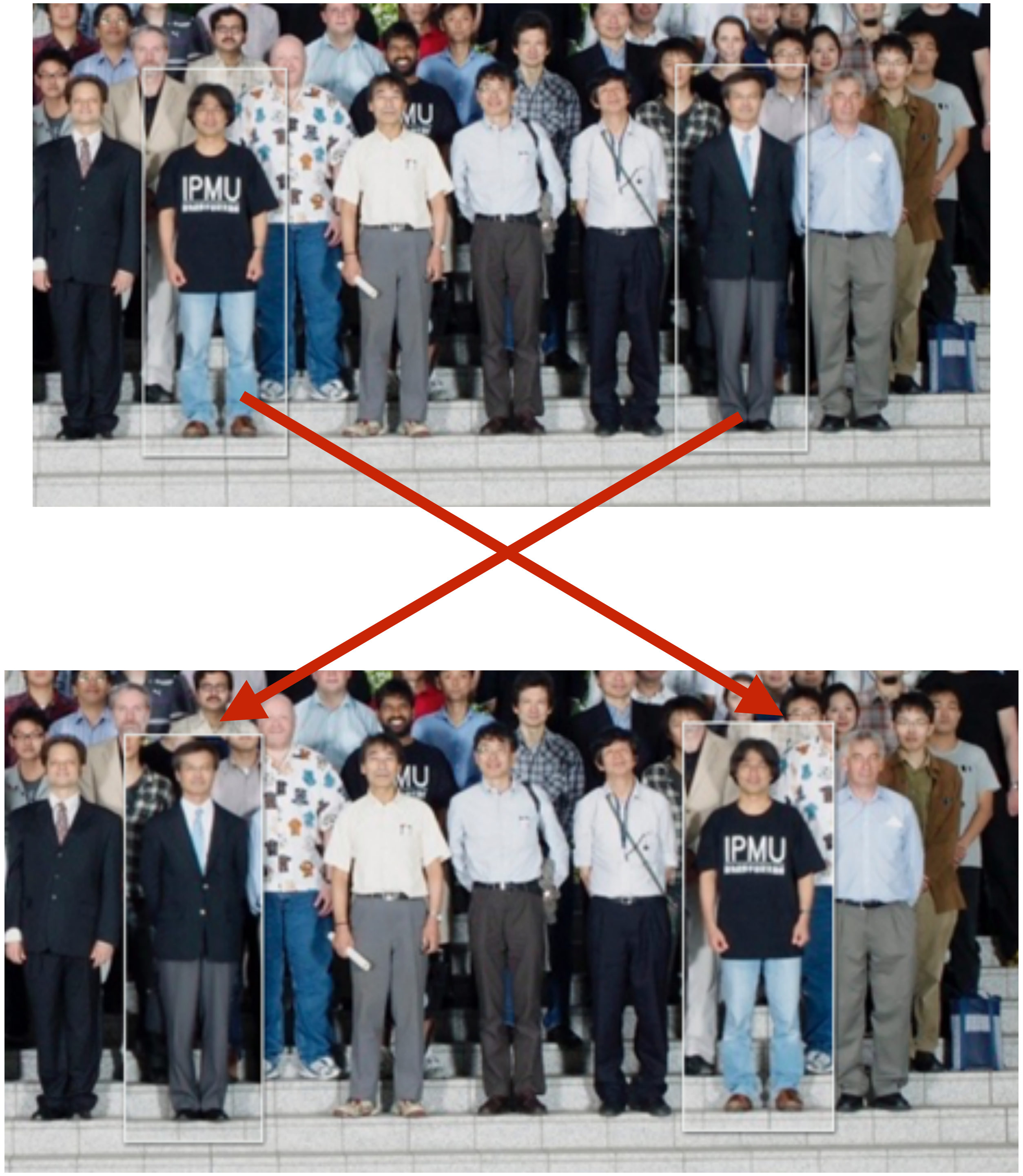
The quantum state remains the same,
up to an overall plus or minus sign.

$\psi(\quad)$



$= +\psi(\quad)$

They are **bosons**
if the sign is **plus**.

$\psi($  $)$

$= -\psi($ $)$

They are **fermions**
if the sign is **minus**.

- **Bosons**:
  photons, Higgs bosons ...

- **Fermions**:
  electrons, quarks, protons ...

- They are quite distinct !

- Ordinary symmetries can only map
  bosons to **bosons**
  and
  fermions to **fermions**.

- **Supersymmetry** maps

  fermions to **bosons**

  and

  bosons to **fermions**.

- Quite unusual, but not very super.
- Somehow the extravagant terminology used by the original authors stuck.

- Zero point energy of
  a **bosonic** harmonic oscillator
  = + $\hbar\omega/2$

- Quantum **bosonic** fields (such as photons)
  ~ infinite number of **bosonic** harmonic oscillators

ω

~ **infinite positive** energy
~ infinite gravitational attraction

- **Not observed!**

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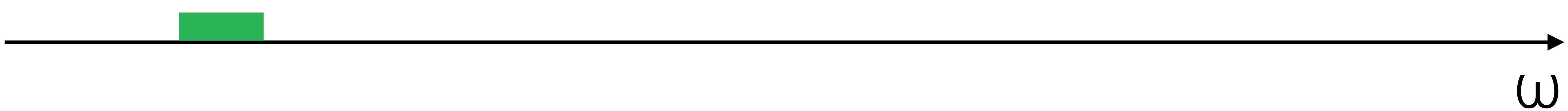

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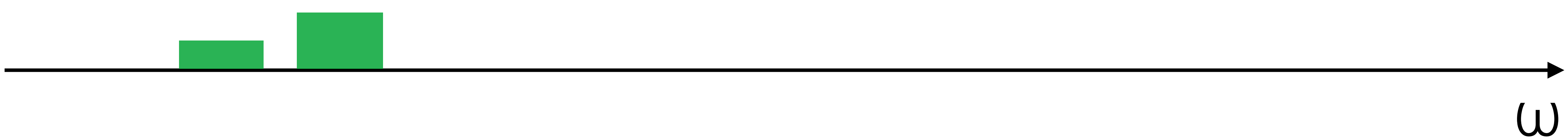

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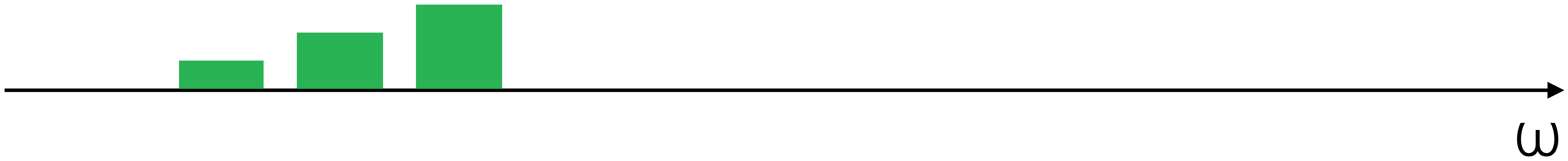

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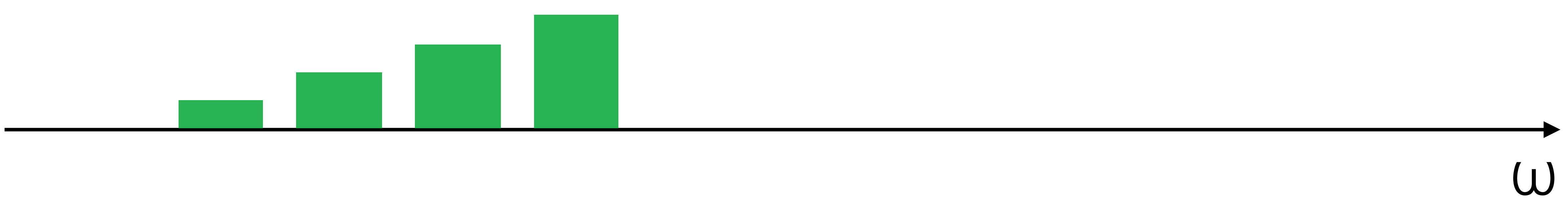

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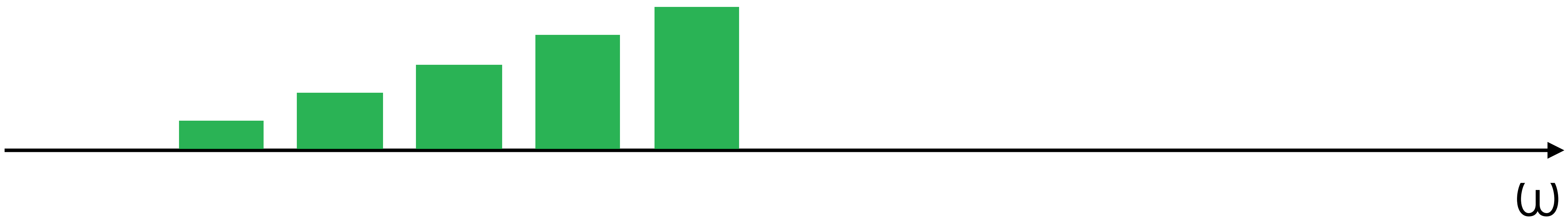

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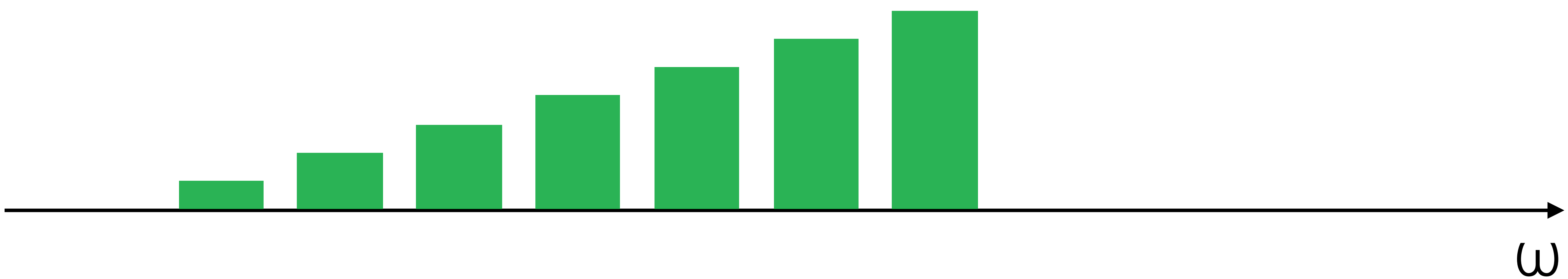

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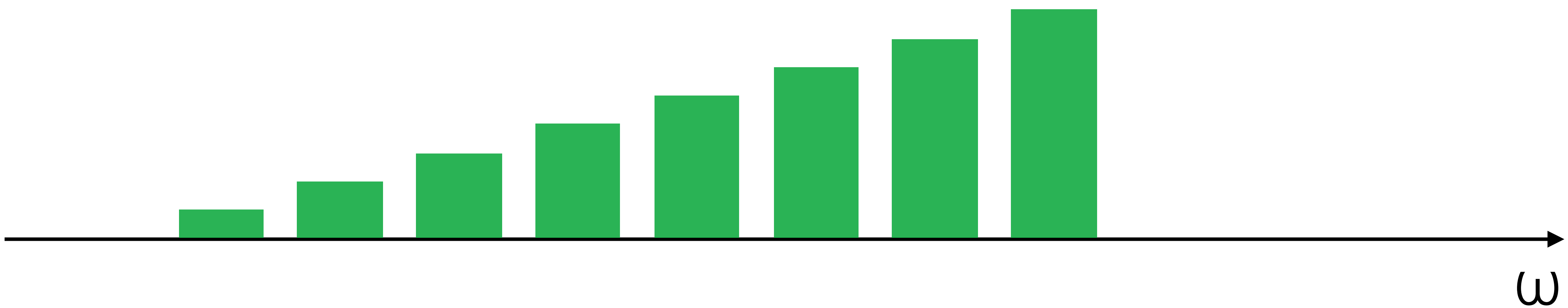

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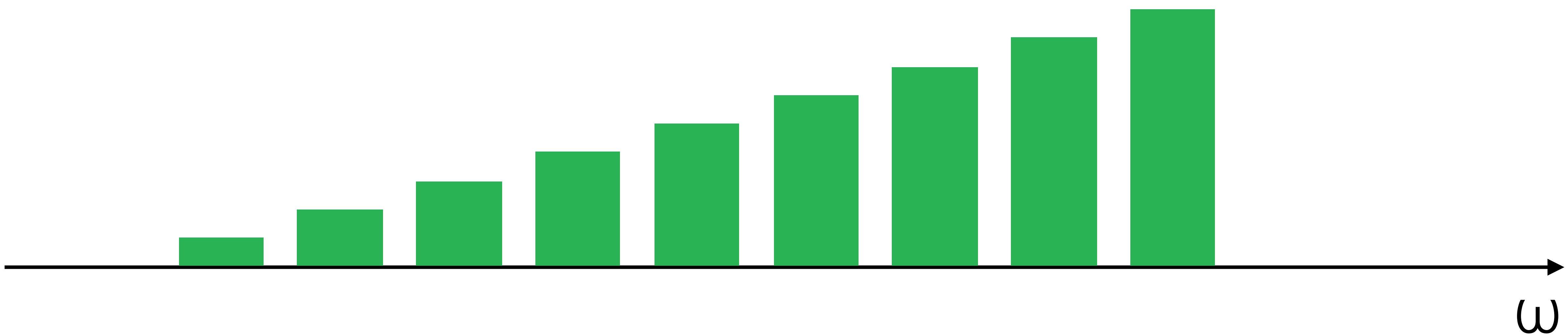

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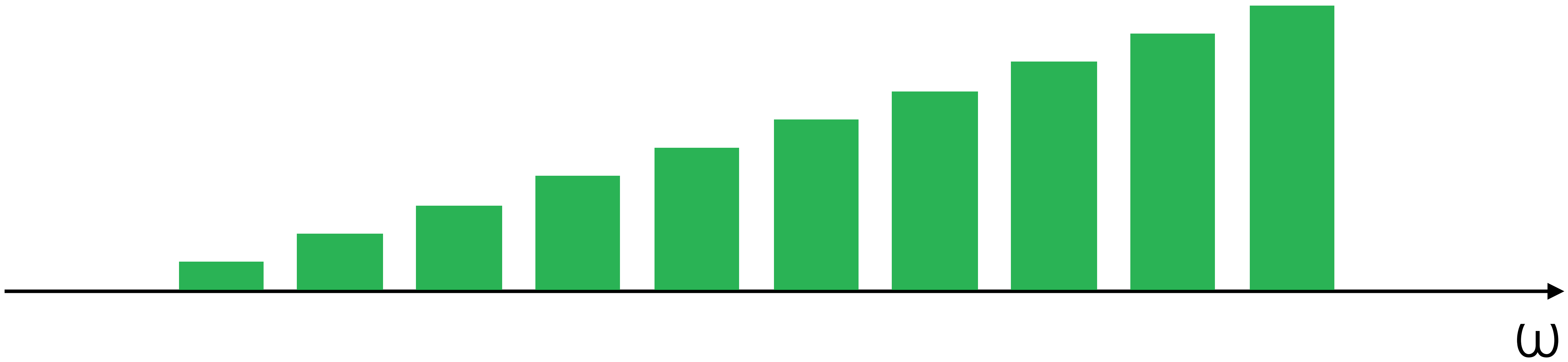

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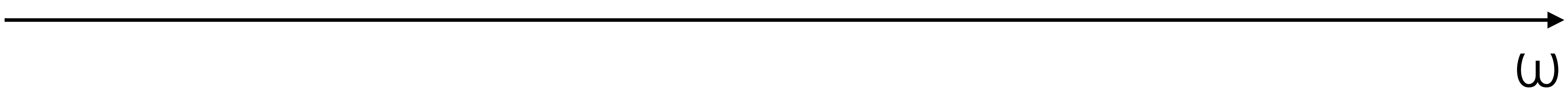

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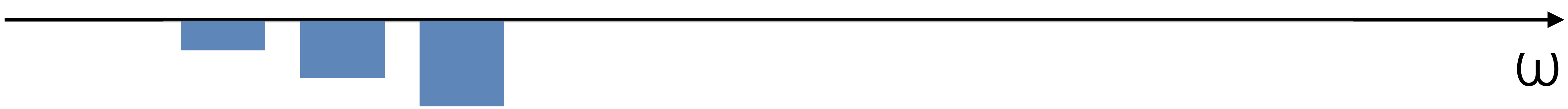

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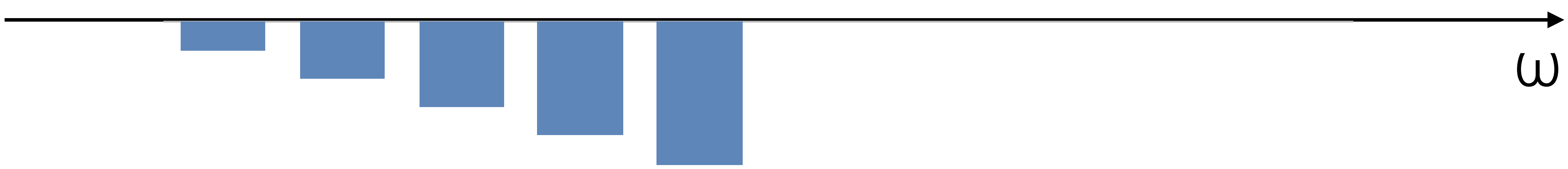

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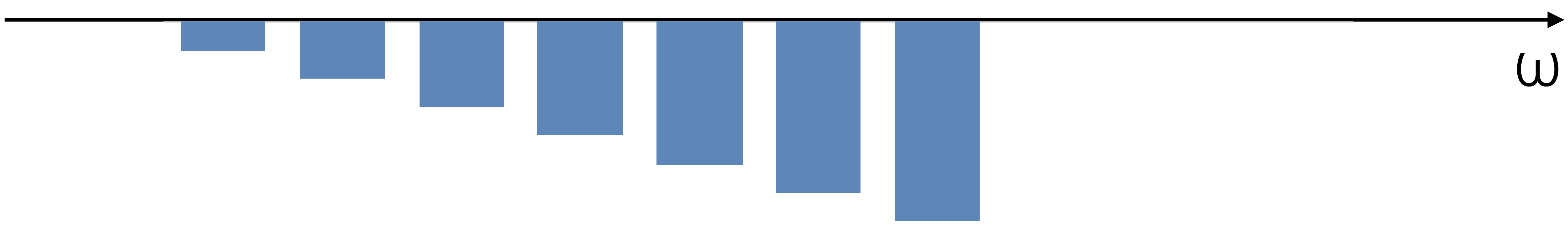

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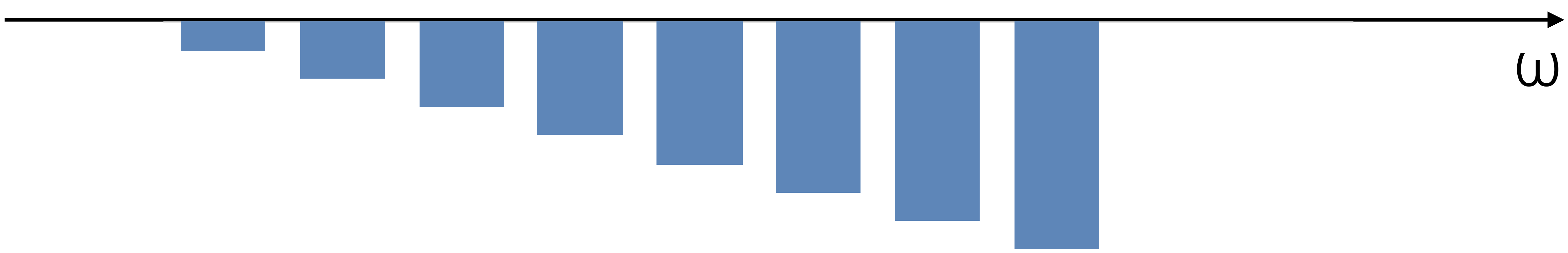

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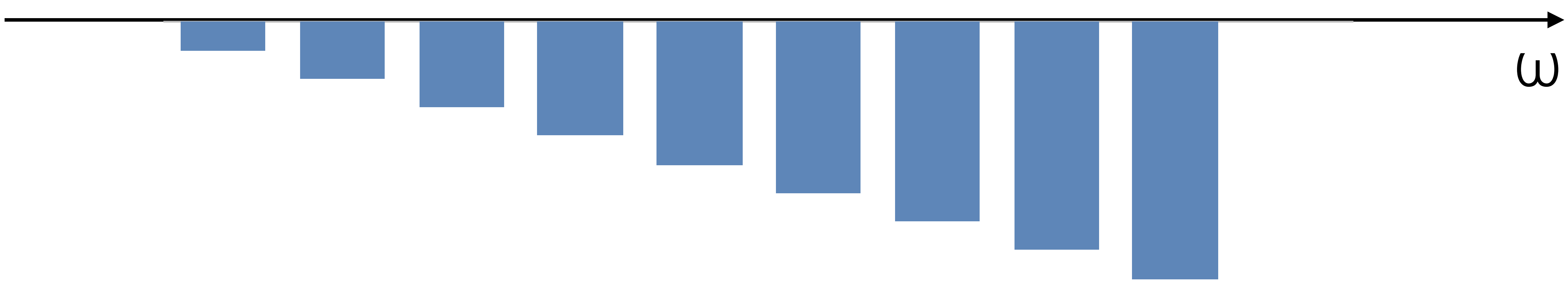

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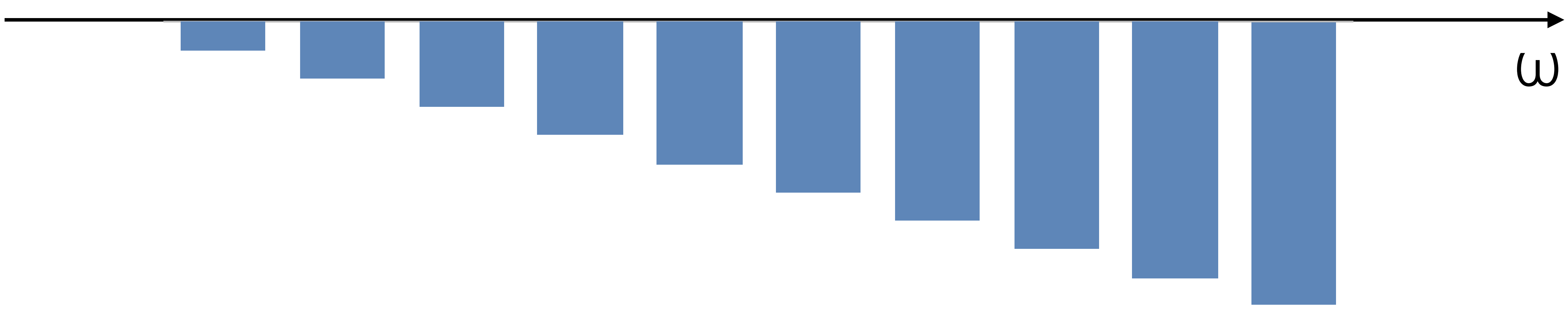

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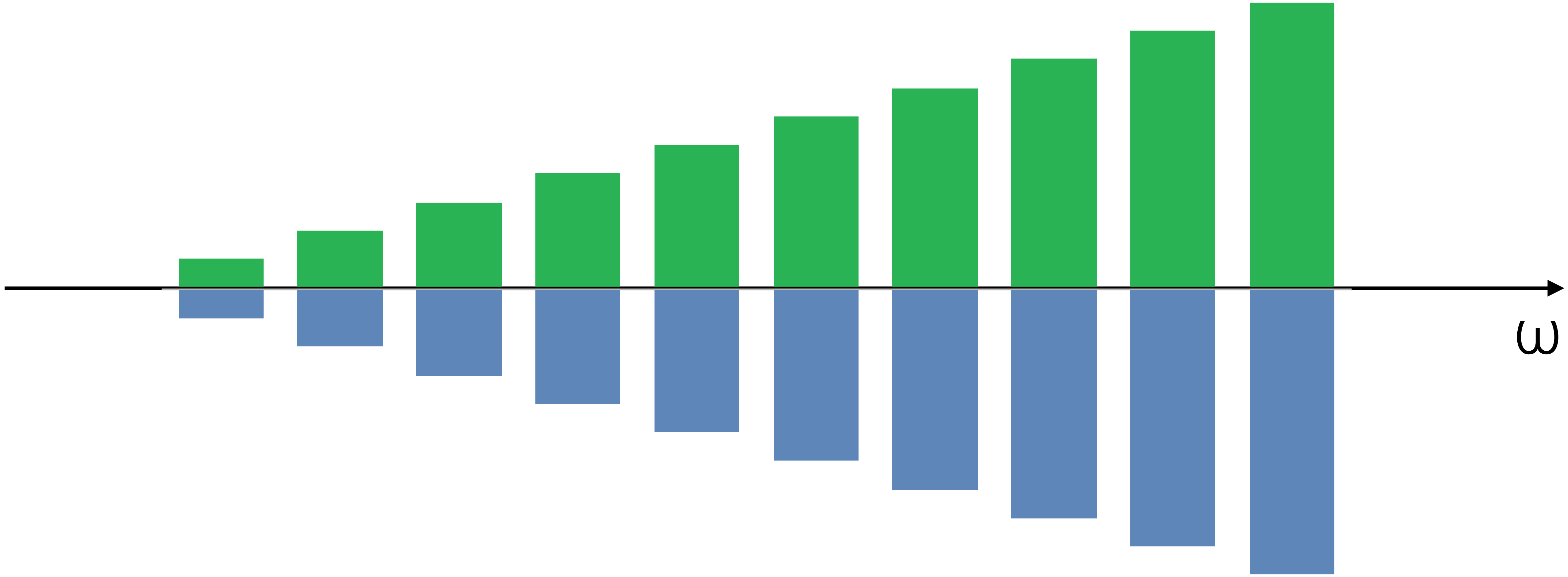

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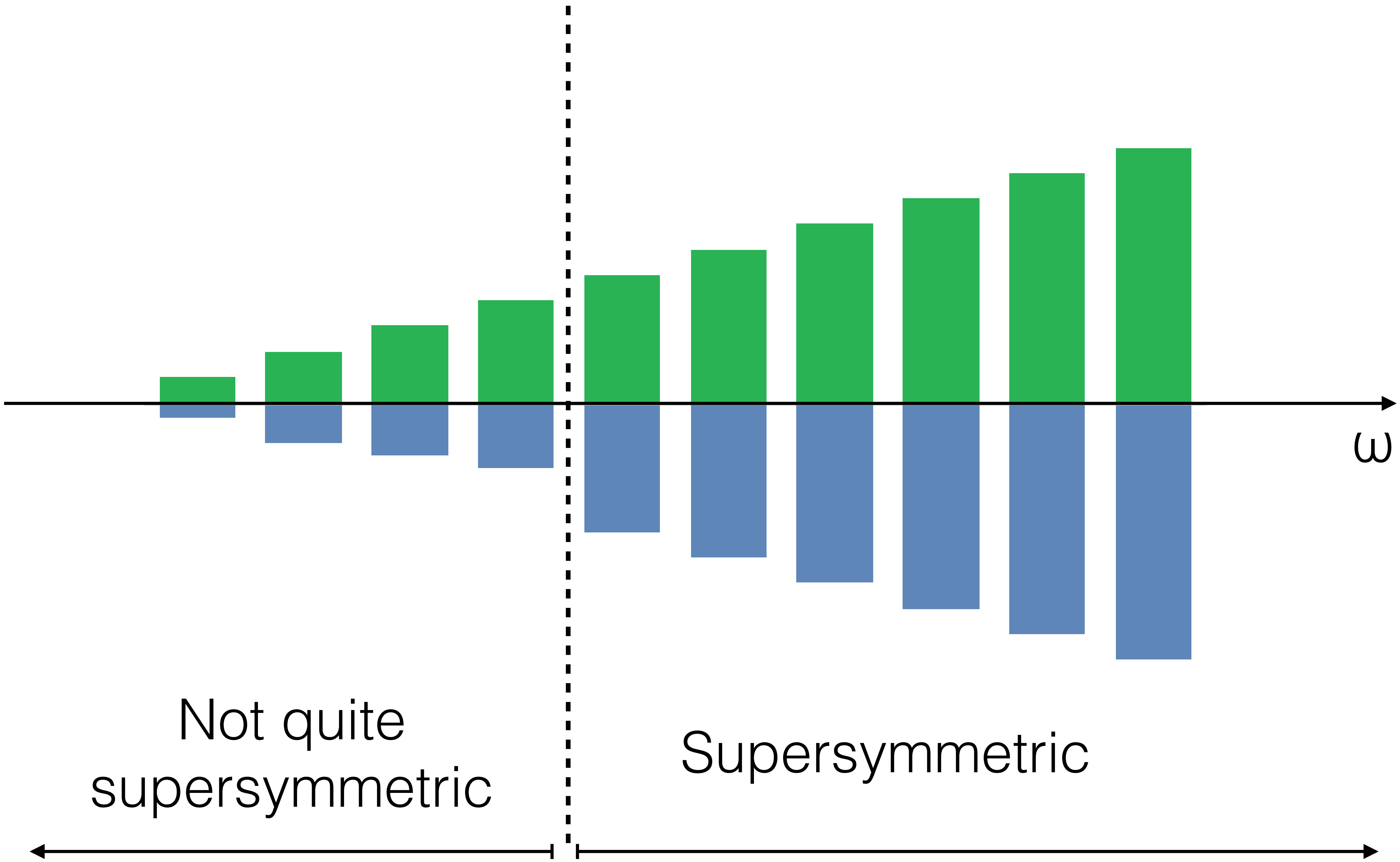
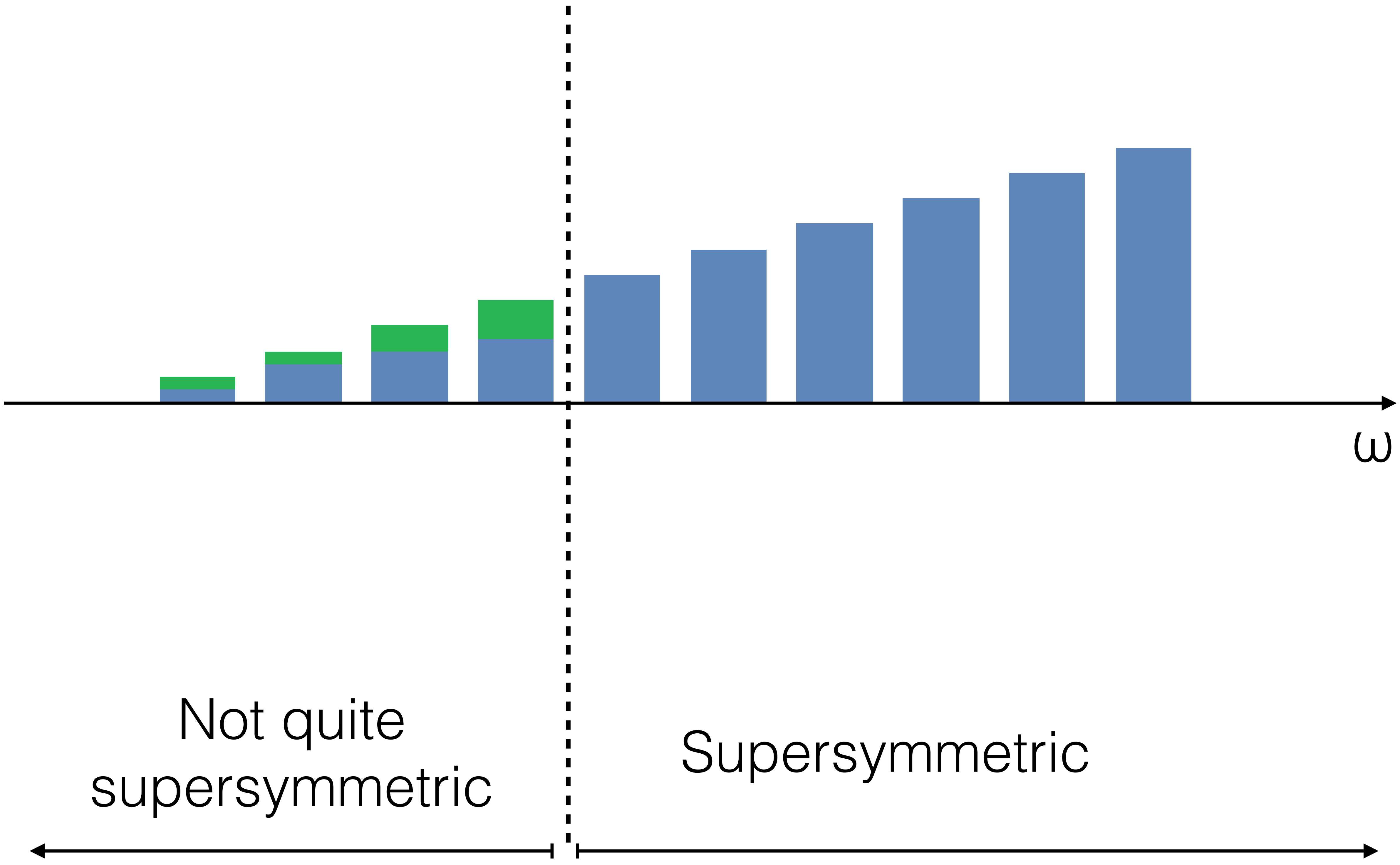
$$\infty - \infty = 0 \ !$$


ω
Not quite supersymmetric
Supersymmetric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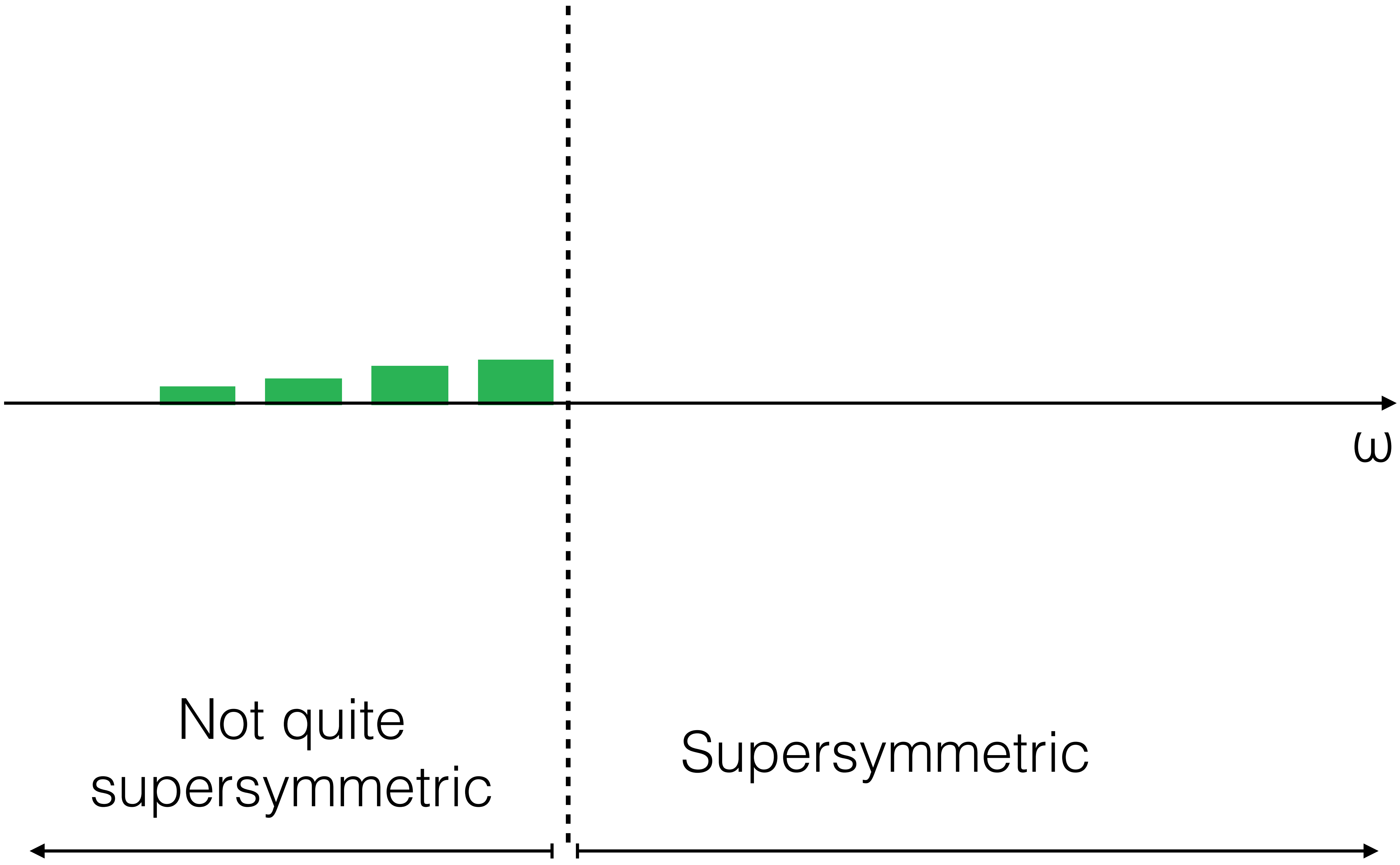


- in this case $\infty - \infty$ is not exactly zero, but finite.

- Does the world work in this way?
- Nobody knows. One big motivation to build LHC was to see if this is the case.

from CERN webpage

- There's a theoretical suggestion that supersymmetry can be realized on the surface of a suitable material

Emergent Space-Time Supersymmetry at the Boundary of a Topological Phase

Tarun Grover,[1] D. N. Sheng,[2] Ashvin Vishwanath[3,4]*

In contrast to ordinary symmetries, supersymmetry (SUSY) interchanges bosons and fermions. Originally proposed as a symmetry of our universe, it still awaits experimental verification. Here, we theoretically show that SUSY emerges naturally in condensed matter systems known as topological superconductors. We argue that the quantum phase transitions at the boundary of topological superconductors in both two and three dimensions display SUSY when probed at long distances and times. Experimental consequences include exact relations between quantities measured in disparate experiments and, in some cases, exact knowledge of the universal critical exponents. The topological surface states themselves may be interpreted as arising from spontaneously broken SUSY, indicating a deep relation between topological phases and SUSY.

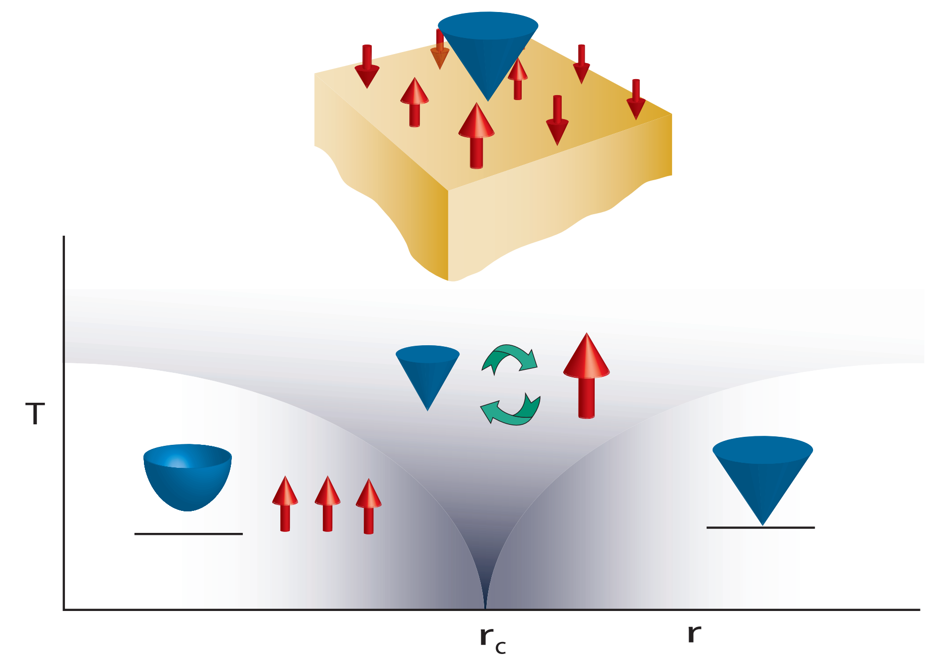


Fig. 1. Supersymmetry in a 3D TSC. Ising magnetic fluctuations (denoted by red arrows) at the boundary couple to the Majorana fermions (blue cone). When the tuning parameter $r < r_c$, the Ising spins are ordered, leading to a gap for the Majorana fermions. The critical point that separates the two sides is supersymmetric, where bosons (Ising order parameter) and Majorana fermions transform into each other.

Grover,Sheng,Vishwanath, Science 344(2014) 281

- No experimental realization yet,
  but I think it is just a matter of time

- So, it's not clear supersymmetry is in nature or not.

- I am often asked,
  "do you think LHC find supersymmetry?"
  or
  "what would you do if LHC would not find supersymmetry?"

- Well, I do not care.

- I'm no Christian nor Buddhist,
  but I like reading books on
  Christian Theology and Buddhist Buddhology.
- Just in the same way,
  I like thinking about supersymmetry.
  It's interesting to me.

- For me, **supersymmetry** is interesting mostly because of its connection with **mathematics**.

1983: **Donaldson** (a mathematician) found that
by studying the **Yang-Mills** equation
describing quantum chromodynamics
very carefully, you can understand
the mathematical properties of
four-dimensional manifolds
in exquisite detail.

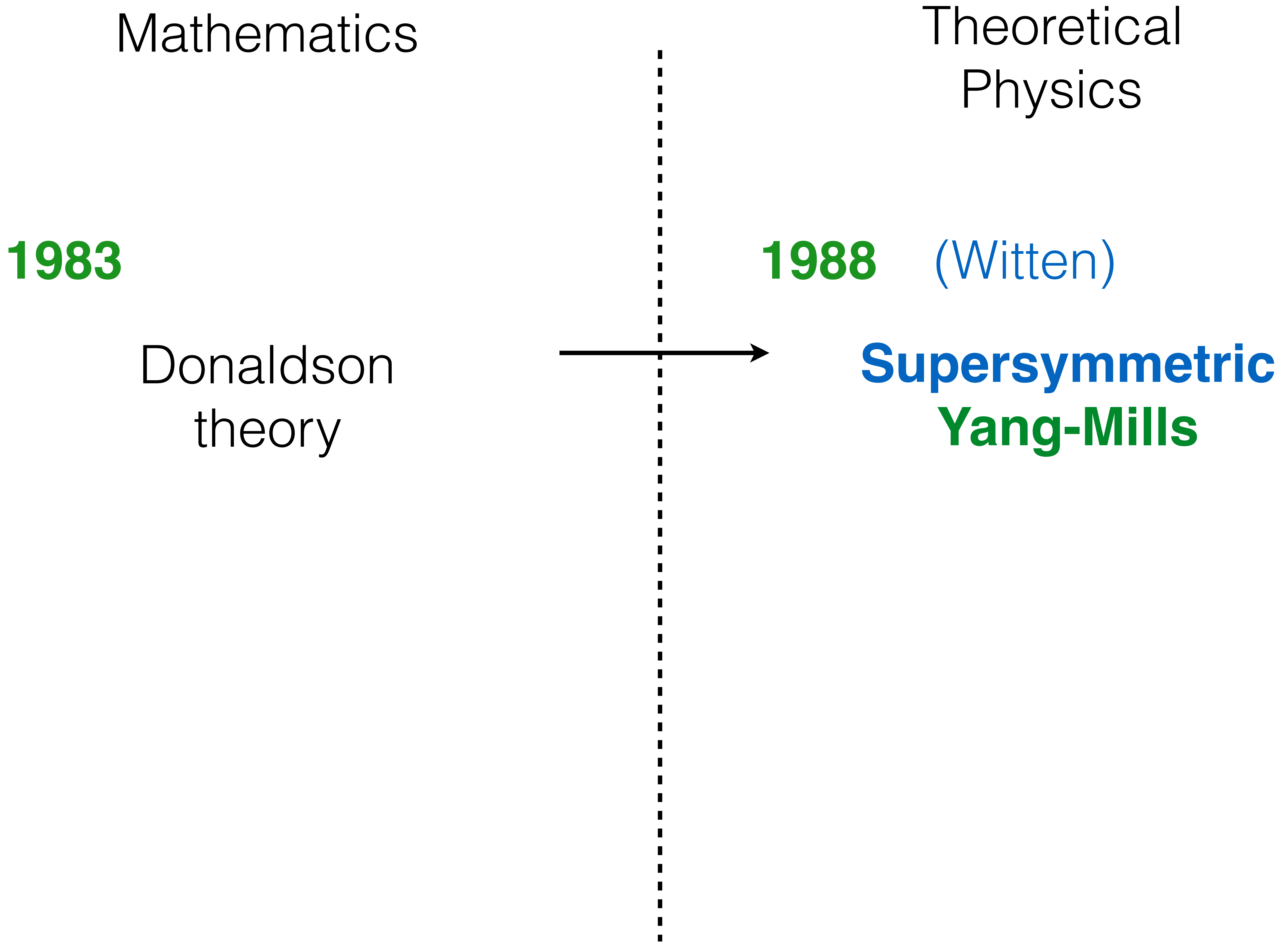
Mathematics

Theoretical Physics

1983

Donaldson theory

1988: **Witten** (a physicist) found that Donaldson's results can be thought of as a statement about **supersymmetric Yang-Mills**.


Mathematics
Theoretical Physics
1983
1988 (Witten)
Donaldson theory
Supersymmetric Yang-Mills

1994: **Seiberg** and **Witten** found physically that **supersymmetric Yang-Mills** reduces to **supersymmetric Maxwell**.

Mathematics

Theoretical Physics

1983

Donaldson theory

1988 (Witten)

Supersymmetric Yang-Mills

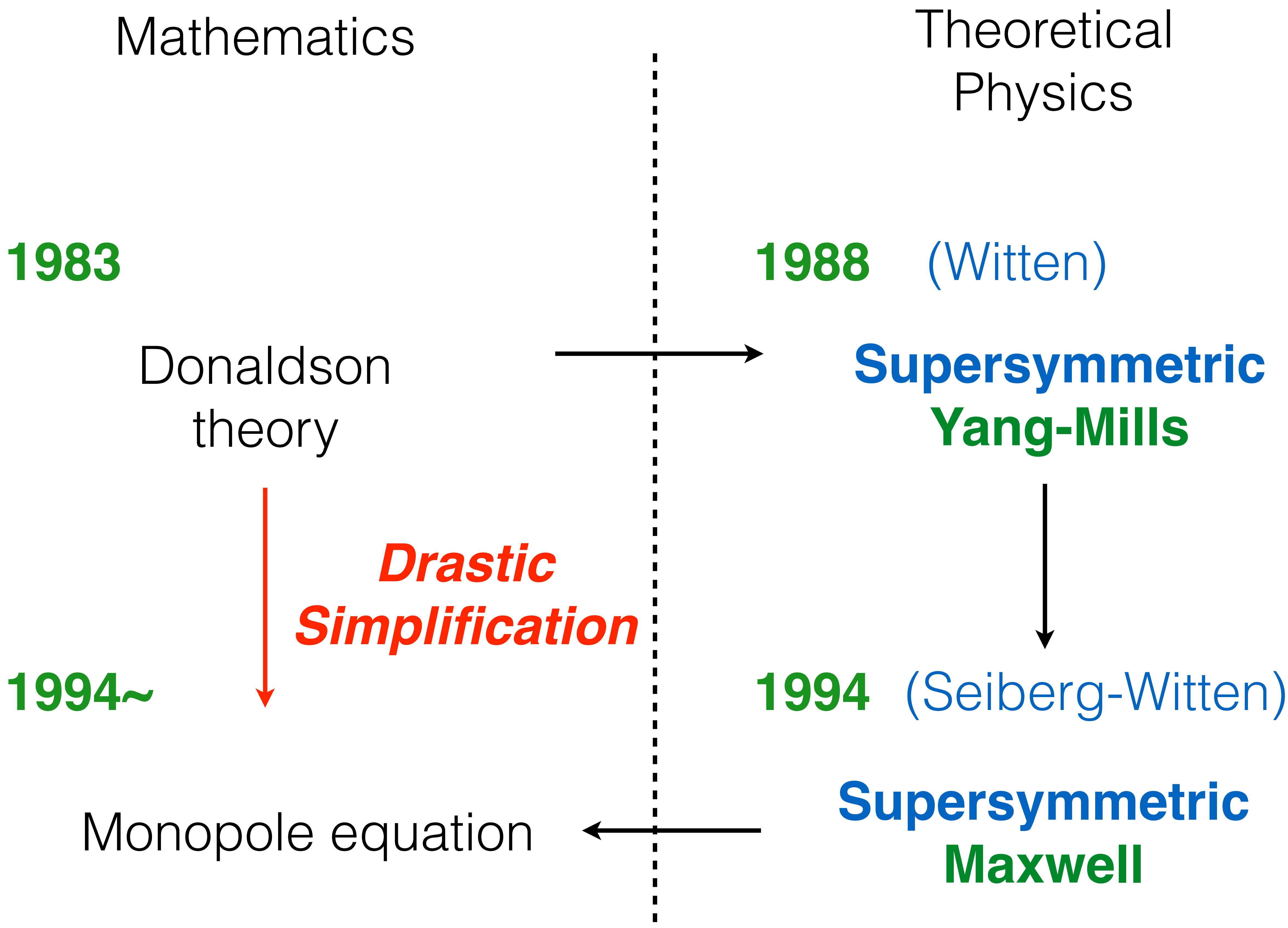
1994 (Seiberg-Witten)

Supersymmetric Maxwell

1994~: Therefore, to study four-dimensional manifolds, you don't have to study
the Yang-Mills equation which is rather difficult;

You just have to study the Maxwell equation.

This brought a sudden revolutionary development in this area of **mathematics**.


Mathematics
Theoretical
Physics
1983
1988 (Witten)
Donaldson
theory
Supersymmetric
Yang-Mills
Drastic
Simplification
1994~
1994 (Seiberg-Witten)
Monopole equation
Supersymmetric
Maxwell

Theoretical
Physics

1988 (Witten)

Supersymmetric
Yang-Mills

↓

1994 (Seiberg-Witten)

Supersymmetric
Maxwell

2002 **Nekrasov** (a physicist) reformulated this derivation in a way **understandable to mathematicians**

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Supersymmetric Maxwell

2003 That reformulation was then **proved by mathematicians**
Nakajima, Yoshioka;
Braverman, Etingof;
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2009 Based on these results, **Alday, Gaiotto** and **I** thought more about physics and found a **mathematical conjecture**

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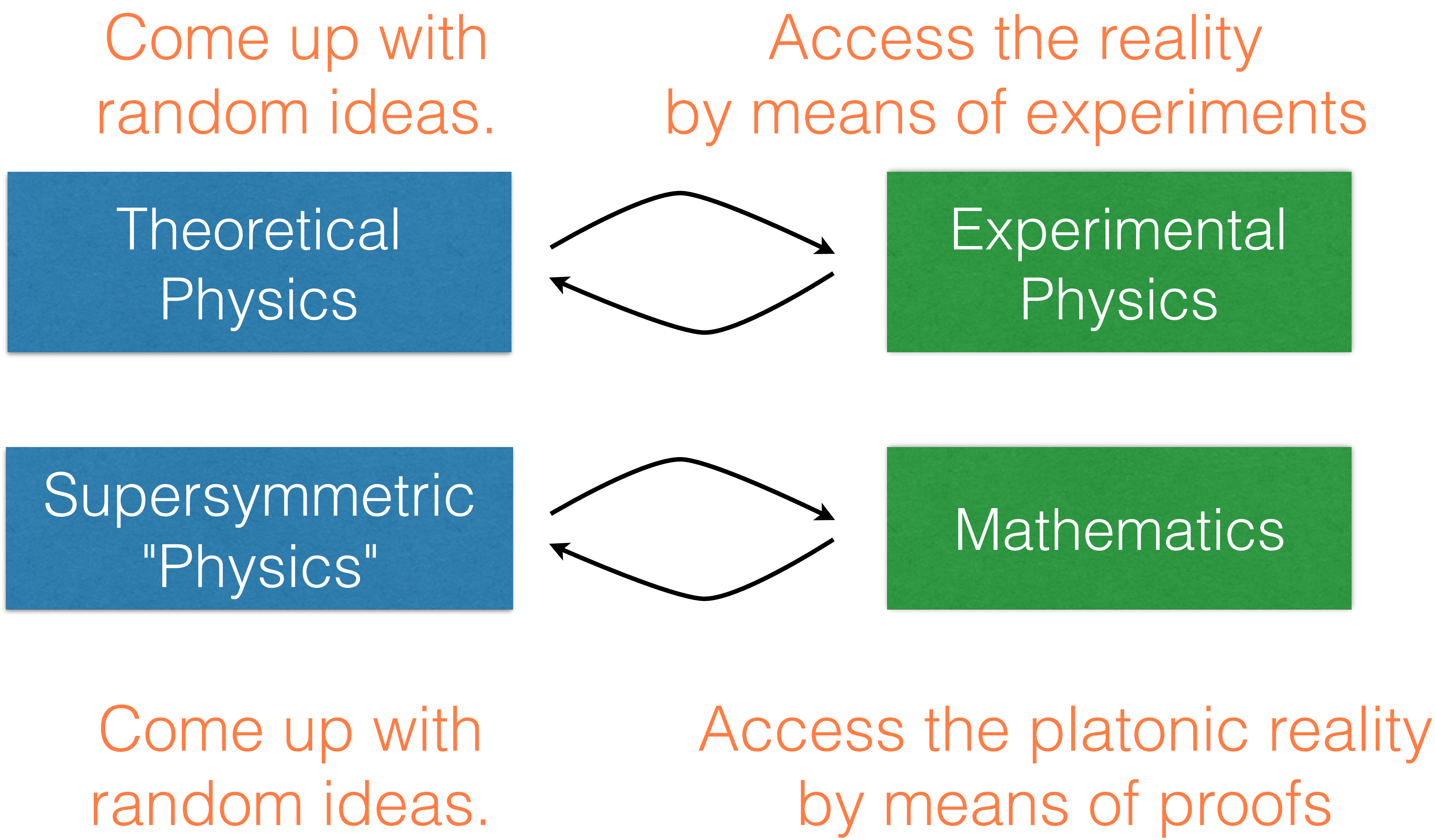
Supersymmetric Maxwell

2012 The conjecture was **proven by mathematicians**, Shiffman and Vasserot; Maulik and Okounkov


Come up with
random ideas.
Access the reality
by means of experiments
Theoretical
Physics
Experimental
Physics
Supersymmetric
"Physics"
Mathematics
Come up with
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Access the platonic reality
by means of proofs