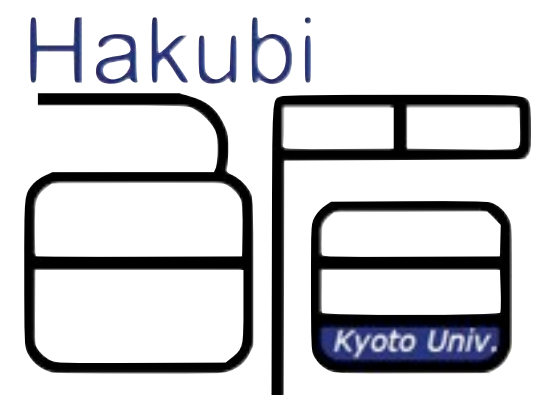


Vainshtein screening in a cosmological background in the most general second-order scalar-tensor theory



Tsutomu Kobayashi
Hakubi Center, Kyoto University



Based on:

work with **Rampei Kimura** and **Kazuhiro Yamamoto** (Hiroshima University)
Phys. Rev. D 85, 024023 (2012) [[arXiv:1111.6749](#)]

Talk Plan

1. Motivation

2. Brief review

- Vainshtein mechanism
- The most general second-order scalar-tensor (ST) theory

3. Vainshtein screening in the most general ST theory

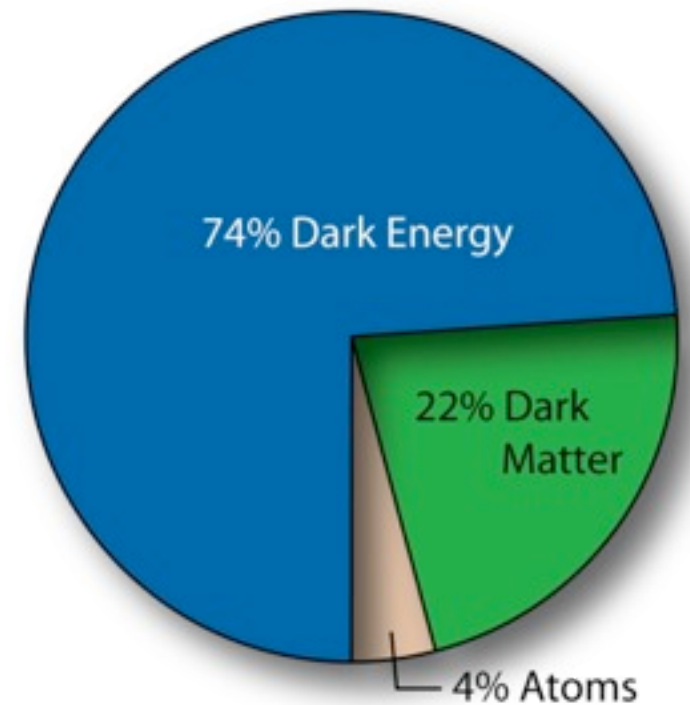
4. Summary

1. *Motivation*

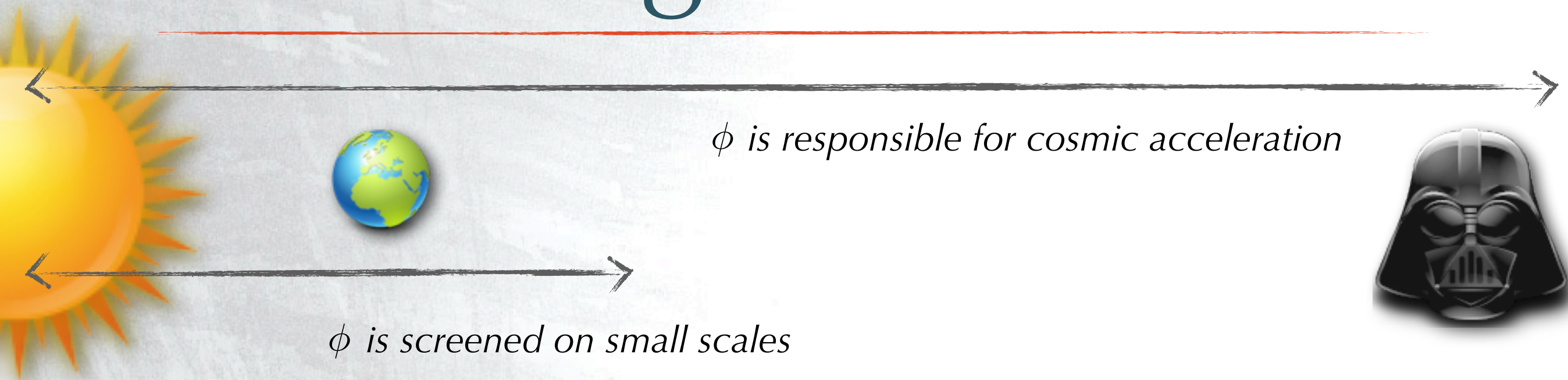
Modified gravity

Modified gravity as an alternative to dark energy

- A new scalar degree of freedom, ϕ , participates in long-range gravitational interaction
- Modification persists down to small scales...
- Need **screening mechanism** in order to evade solar-system tests



Screening mechanisms



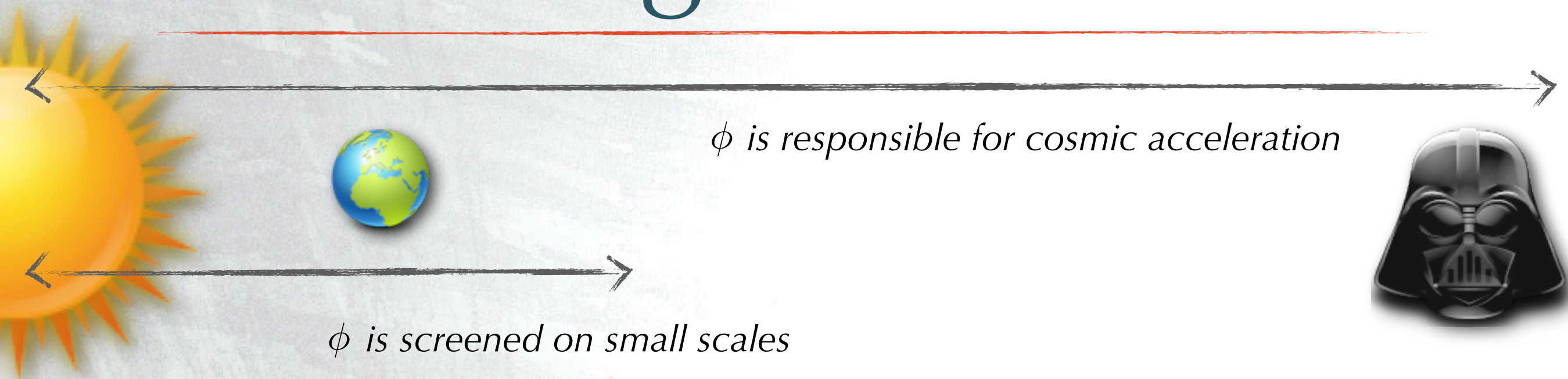
How to screen scalar-mediated force in the vicinity of matter:

1. ϕ is effectively massive – *Chameleon mechanism*
2. ϕ is effectively weakly coupled to matter – *Vainshtein mechanism*

Mota, Barrow 2004;
Khoury, Weltman 2004

Vainshtein 1972

Screening mechanisms



How to screen scalar-mediated force in the vicinity of matter:

1. ϕ is effectively massive – *Chameleon mechanism*
2. ϕ is effectively weakly coupled to matter – *Vainshtein mechanism*

Mota, Barrow 2004;
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Vainshtein 1972

Let's look at how the Vainshtein mechanism operates based on a generic class of modified gravity theories!

2. Brief review

— Vainshtein mechanism

Example

Scalar-field theory non-minimally coupled to matter:

$$\mathcal{L} = \frac{1}{8\pi G} \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{r_c^2}{3}(\partial\varphi)^2\Box\varphi \right] + \varphi T_\mu^\mu$$

(φ : dimensionless)

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Scalar-field theory non-minimally coupled to matter:

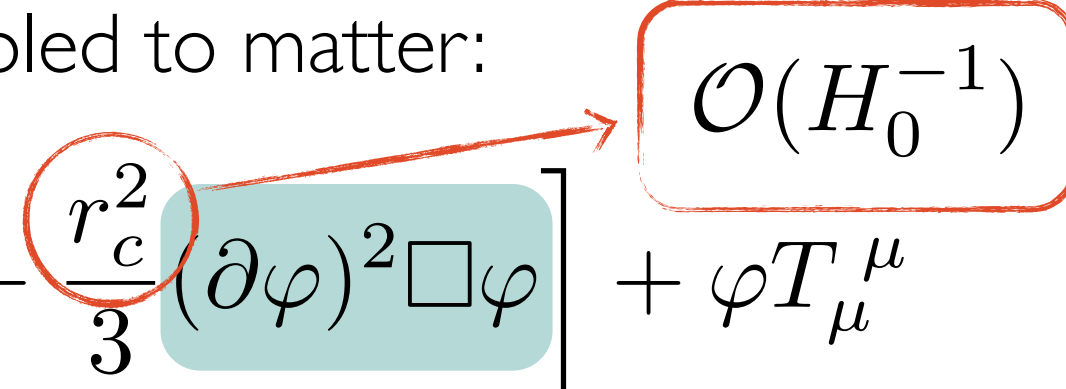
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Non-linear derivative interaction
(cubic Galileon)

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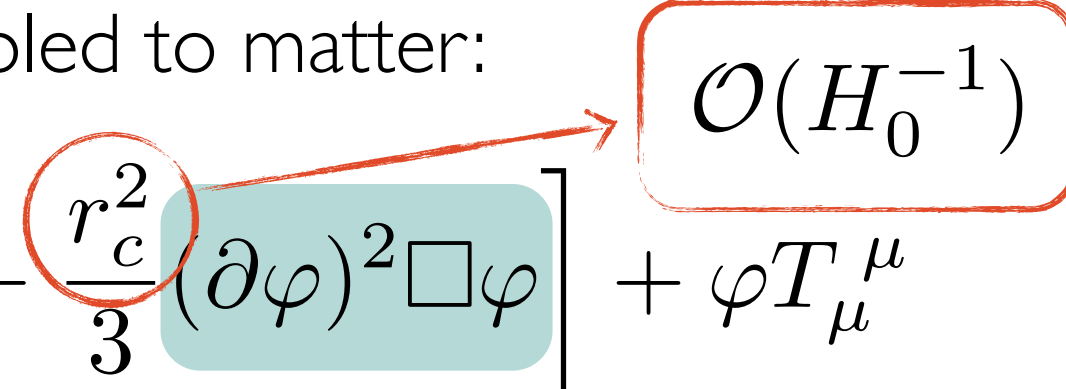
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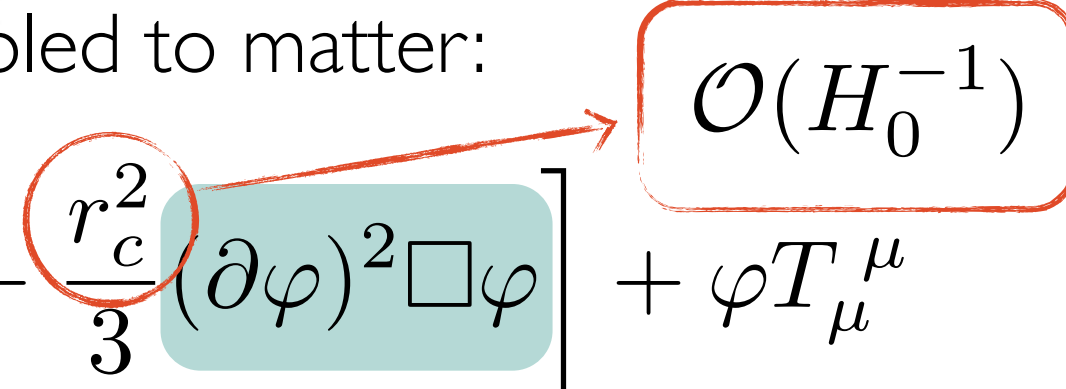
Non-linear derivative interaction
(cubic Galileon)

Key non-linearity

$r_c^2\Box\varphi$ can be large even if $\varphi \ll 1$

Example

Scalar-field theory non-minimally coupled to matter:

$$\mathcal{L} = \frac{1}{8\pi G} \left[-\frac{1}{2}(\partial\varphi)^2 - \frac{r_c^2}{3}(\partial\varphi)^2\Box\varphi \right] + \varphi T_\mu^\mu$$


(φ : dimensionless)

Non-linear derivative interaction
(cubic Galileon)

Key non-linearity

$r_c^2\Box\varphi$ can be large even if $\varphi \ll 1$

→ $\mathcal{L} \sim \frac{1}{8\pi G_{\text{eff}}} \left[-\frac{1}{2}(\partial\varphi)^2 \right] + \varphi T_\mu^\mu, \quad G_{\text{eff}} \ll G$

Effectively weakly coupled to matter

Spherically symmetric solution

$$\text{EOM: } \square\varphi + \frac{2r_c^2}{3} [(\square\varphi)^2 - (\partial_\mu\partial_\nu\varphi)^2] = -8\pi GT_\mu^\mu \approx 8\pi G\rho$$

Spherically symmetric solution:

$$\partial_r\varphi = \frac{3r}{8r_c^2} \left(-1 + \sqrt{1 + \frac{16}{3} \frac{r_s r_c^2}{r^3}} \right)$$

(r_s : Schwarzschild radius)

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$r_V := (r_s r_c^2)^{1/3}$: **Vainshtein radius**  (r_s : Schwarzschild radius)

Spherically symmetric solution

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$$r_V := (r_s r_c^2)^{1/3} \quad \text{: Vainshtein radius} \quad (r_s : \text{Schwarzschild radius})$$

✓ $r \gg r_V \longrightarrow \partial_r\varphi \sim \frac{r_s}{r^2} \sim \partial_r\Phi \quad (\Phi : \text{gravitational potential})$

✓ $r \ll r_V \longrightarrow \partial_r\varphi \sim \frac{r_s}{r^2} \left(\frac{r}{r_V} \right)^{3/2} \ll \partial_r\Phi \quad \text{— screened!}$

2. Brief review

— The most general second-order
scalar-tensor theory

Horndeski's theory (or generalized Galileon)

In 1974, Horndeski determined **the most general** Lagrangian of the form

$$\mathcal{L} = \mathcal{L}(\phi, \partial\phi, \partial^2\phi, \partial^3\phi, \dots ; g_{\mu\nu}, \partial g_{\mu\nu}, \partial^2 g_{\mu\nu}, \partial^3 g_{\mu\nu}, \dots)$$

having **second-order** field equations both for ϕ and $g_{\mu\nu}$

Horndeski, Int. J. Theor. Phys. 10,363 (1974)

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Horndeski, Int. J. Theor. Phys. 10,363 (1974)

Horndeski's theory is equivalent to **the generalized Galileons** [Deffayet *et al.* 2011] **in 4D**

TK, Yamaguchi, Yokoyama, Prog. Theor. Phys. 126, 511 (2011)

Horndeski's theory (or generalized Galileon)

Lagrangian for *the most general second-order ST theory*:

$$\begin{aligned}\mathcal{L} = & K(\phi, X) - G_3(\phi, X)\square\phi \\ & + G_4(\phi, X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X} [(\square\phi)^3 \\ & \quad - 3\square\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]\end{aligned}$$

where $X := -\frac{1}{2}(\partial\phi)^2, \quad G_{iX} := \partial G_i / \partial X$

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- **4** arbitrary functions of ϕ and X

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where $X := -\frac{1}{2}(\partial\phi)^2$, $G_{iX} := \partial G_i / \partial X$

- **4** arbitrary functions of ϕ and X
- Non-minimal coupling to gravity

Special cases

$$G_4 = \frac{M_{\text{Pl}}^2}{2} \quad \longrightarrow \quad \mathcal{L} \supset \frac{M_{\text{Pl}}^2}{2} R \quad \text{Einstein-Hilbert}$$

$$G_4 = f(\phi) \quad \longrightarrow \quad \mathcal{L} \supset f(\phi) R \quad \text{Familiar non-minimal coupling}$$

$$-G_3(\phi, X) \square \phi \quad \supset \quad (\partial \phi)^2 \square \phi$$

Kinetic gravity braiding

Deffayet *et al.* 2010

DGP (brane bending mode)

Luty, Porrati, Rattazzi 2003;
Nicolis, Rattazzi 2004

$$G_5 = -\phi \quad \longrightarrow \quad \mathcal{L} \supset G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi$$

Gravitationally enhanced friction/purely kinetic coupled gravity

Germani *et al.* 2011; Gubitosi, Linder 2011

$$\begin{aligned}
\mathcal{L} = & K(\phi, X) - G_3(\phi, X)\square\phi \\
& + G_4(\phi, X)R + G_{4X} [(\square\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\
& + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X} [(\square\phi)^3 \\
& \quad - 3\square\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]
\end{aligned}$$

Galileon-like non-linear derivative interactions

$$\begin{aligned}\mathcal{L} = & K(\phi, X) - G_3(\phi, X)\Box\phi \\ & + G_4(\phi, X)R + G_{4X} [(\Box\phi)^2 - (\nabla_\mu\nabla_\nu\phi)^2] \\ & + G_5(\phi, X)G_{\mu\nu}\nabla^\mu\nabla^\nu\phi - \frac{1}{6}G_{5X} [(\Box\phi)^3 \\ & - 3\Box\phi(\nabla_\mu\nabla_\nu\phi)^2 + 2(\nabla_\mu\nabla_\nu\phi)^3]\end{aligned}$$

Vainshtein mechanism operates generically?

3. Vainshtein screening in the most general ST theory

Setup

- The most general ST theory, **minimally coupled to matter**

$$S = \int d^4x \sqrt{-g} [\mathcal{L} + \mathcal{L}_m(\psi, g_{\mu\nu})]$$

Horndeski's Lagrangian

- **Cosmological background**

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

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Non-relativistic matter

Setup

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Horndeski's Lagrangian

Non-relativistic matter

- Cosmological background**

$$ds^2 = -dt^2 + a^2(t)d\mathbf{x}^2$$

Perturbations – Consistent treatment for scalar field and **metric** perturbations

$$ds^2 = -(1 + 2\Phi)dt^2 + a^2(1 - 2\Psi)d\mathbf{x}^2$$

$$\phi \rightarrow \phi(t) + \delta\phi(t, \mathbf{x}), \quad \rho_m \rightarrow \rho_m(t)[1 + \delta(t, \mathbf{x})]$$

Approximations

Weak gravitational field on **subhorizon** scales
— Useful e.g. for the study of structure formation

De Felice, TK, Tsujikawa (2011)

☑ $\Phi, \Psi, Q := H \frac{\delta\phi}{\dot{\phi}} \ll 1$

☑ $\partial_t \ll \partial_i$ (Quasi-static approximation)

Keep relevant **non-linear** terms written schematically as

$$(\nabla^2 \epsilon)^2, (\nabla^2 \epsilon)^3, \dots \quad \text{where} \quad \epsilon = \Phi, \Psi, Q$$

(Quartic terms $(\nabla^2 \epsilon)^4$ do not appear)

Neglect “mass” terms: $K_{\phi\phi} \ll \nabla^2, \dots$

[1] Scalar-field EOM

$$\begin{aligned} & A_0 \nabla^2 Q - A_1 \nabla^2 \Psi - A_2 \nabla^2 \Phi + \frac{B_0}{a^2 H^2} \mathcal{Q}^{(2)} \\ & - \frac{B_1}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q) \\ & - \frac{B_2}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q) \\ & - \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi) \\ & - \frac{C_0}{a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] \\ & - \frac{C_1}{a^4 H^4} \left[\mathcal{Q}^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi + 2 \partial_i \partial_j Q \partial^j \partial^k Q \partial_k \partial^i \Phi \right] = 0, \end{aligned}$$

$$\text{where } \mathcal{Q}^{(2)} := (\nabla^2 Q)^2 - (\partial_i \partial_j Q)^2$$

[1] Scalar-field EOM

Linear terms

$$\begin{aligned} & \underbrace{A_0 \nabla^2 Q - A_1 \nabla^2 \Psi - A_2 \nabla^2 \Phi + \frac{B_0}{a^2 H^2} Q^{(2)}}_{\text{Linear terms}} \\ & - \frac{B_1}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q) \\ & - \frac{B_2}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q) \\ & - \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi) \\ & - \frac{C_0}{a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] \\ & - \frac{C_1}{a^4 H^4} \left[Q^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi + 2 \partial_i \partial_j Q \partial^j \partial^k Q \partial_k \partial^i \Phi \right] = 0, \end{aligned}$$

where $Q^{(2)} := (\nabla^2 Q)^2 - (\partial_i \partial_j Q)^2$

[1] Scalar-field EOM

Linear terms

$$\begin{aligned}
 & \frac{A_0 \nabla^2 Q - A_1 \nabla^2 \Psi - A_2 \nabla^2 \Phi + \frac{B_0}{a^2 H^2} \mathcal{Q}^{(2)}}{(\nabla^2 \epsilon)^2} \text{ terms} \\
 & - \frac{B_1}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q) \\
 & - \frac{B_2}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q) \\
 & - \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi) \\
 & - \frac{C_0}{a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] \\
 & - \frac{C_1}{a^4 H^4} \left[\mathcal{Q}^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi + 2 \partial_i \partial_j Q \partial^j \partial^k Q \partial_k \partial^i \Phi \right] = 0,
 \end{aligned}$$

where $\mathcal{Q}^{(2)} := (\nabla^2 Q)^2 - (\partial_i \partial_j Q)^2$

[1] Scalar-field EOM

Linear terms

$$A_0 \nabla^2 Q - A_1 \nabla^2 \Psi - A_2 \nabla^2 \Phi + \frac{B_0}{a^2 H^2} Q^{(2)}$$

$$- \frac{B_1}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q) \quad (\nabla^2 \epsilon)^2 \text{ terms}$$

$$- \frac{B_2}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q)$$

$$- \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi) \quad (\nabla^2 \epsilon)^3 \text{ terms}$$

$$- \frac{C_0}{a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right]$$

$$- \frac{C_1}{a^4 H^4} \left[Q^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi + 2 \partial_i \partial_j Q \partial^j \partial^k Q \partial_k \partial^i \Phi \right] = 0,$$

where $Q^{(2)} := (\nabla^2 Q)^2 - (\partial_i \partial_j Q)^2$

[1] Scalar-field EOM

Linear terms

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 & - \frac{B_1}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial^i \partial^j Q) \\
 & - \frac{B_2}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q) \\
 & - \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 \Psi - \partial_i \partial_j \Phi \partial^i \partial^j \Psi) \\
 & - \frac{C_0}{a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + \right. \\
 & \left. - \frac{C_1}{a^4 H^4} \left[Q^{(2)} \nabla^2 \Phi - 2 \nabla^2 Q \partial_i \partial_j Q \partial^i \partial^j \Phi \right] \right]
 \end{aligned}$$

where $Q^{(2)} := (\nabla^2 Q)^2 - (\partial_i \partial_j Q)^2$

Coefficients are written in terms of K, G_3, G_4, G_5 . (messy!)

$$\begin{aligned}
 A_0 &:= \frac{\dot{\Theta}}{H^2} + \frac{\Theta}{H} + \mathcal{F}_T - 2\mathcal{G}_T - 2\frac{\dot{\mathcal{G}}_T}{H} - \frac{\mathcal{E} + \mathcal{P}}{2H^2}, \\
 A_1 &:= \frac{1}{H} \frac{d\mathcal{G}_T}{dt} + \mathcal{G}_T - \mathcal{F}_T, \\
 A_2 &:= \mathcal{G}_T - \frac{\Theta}{H}, \\
 B_0 &:= \frac{X}{H} \left\{ \dot{\phi} G_{3X} + 3(\dot{X} + 2HX) G_{4XX} + 2X\dot{X} G_{4XXX} - \right. \\
 &\quad \left. + (\dot{H} + H^2) \dot{\phi} G_{5X} + \dot{\phi} [2H\dot{X} + (\dot{H} + H^2)X] G_{5XX} - \right. \\
 &\quad \left. - \dot{\phi} X G_{5\phi\phi X} - X(\dot{X} - 2HX) G_{5\phi XX} \right\}, \\
 B_1 &:= 2X [G_{4X} + \ddot{\phi} (G_{5X} + X G_{5XX}) - G_{5\phi} + X G_{5\phi X}], \\
 B_2 &:= -2X (G_{4X} + 2X G_{4XX} + H\dot{\phi} G_{5X} + H\dot{\phi} X G_{5XX} - \\
 B_3 &:= H\dot{\phi} X G_{5X}, \\
 C_0 &:= 2X^2 G_{4XX} + \frac{2X^2}{3} (2\ddot{\phi} G_{5XX} + \ddot{\phi} X G_{5XXX} - 2G_{5\phi XX}), \\
 C_1 &:= H\dot{\phi} X (G_{5X} + X G_{5XX}).
 \end{aligned}$$

[2] (00) equation

$$\begin{aligned}
 & \mathcal{G}_T \nabla^2 \Psi + A_2 \nabla^2 Q && \longleftarrow \nabla^2 \epsilon \\
 & + \frac{B_2}{2a^2 H^2} Q^{(2)} + \frac{B_3}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial_i \partial_j Q) && \longleftarrow (\nabla^2 \epsilon)^2 \\
 & + \frac{C_1}{3a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] && \longleftarrow (\nabla^2 \epsilon)^3 \\
 = & \frac{a^2}{2} \rho_m \delta
 \end{aligned}$$

[2] (00) equation

$$\mathcal{G}_T := 2 \left[G_4 - 2X G_{4X} - X \left(H \dot{\phi} G_{5X} - G_{5\phi} \right) \right]$$

$$\begin{aligned}
 & \mathcal{G}_T \nabla^2 \Psi + A_2 \nabla^2 Q && \longleftarrow \nabla^2 \epsilon \\
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 & + \frac{C_1}{3a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] && \longleftarrow (\nabla^2 \epsilon)^3 \\
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 & \rightarrow \mathcal{G}_T := 2 \left[G_4 - 2X G_{4X} - X \left(H \dot{\phi} G_{5X} - G_{5\phi} \right) \right] \\
 & \mathcal{G}_T \nabla^2 \Psi + A_2 \nabla^2 Q \quad \leftarrow \nabla^2 \epsilon \\
 & + \frac{B_2}{2a^2 H^2} Q^{(2)} + \frac{B_3}{a^2 H^2} (\nabla^2 \Psi \nabla^2 Q - \partial_i \partial_j \Psi \partial_i \partial_j Q) \quad \leftarrow (\nabla^2 \epsilon)^2 \\
 & + \frac{C_1}{3a^4 H^4} \left[(\nabla^2 Q)^3 - 3 \nabla^2 Q (\partial_i \partial_j Q)^2 + 2 (\partial_i \partial_j Q)^3 \right] \quad \leftarrow (\nabla^2 \epsilon)^3 \\
 & = \frac{a^2}{2} \rho_m \delta
 \end{aligned}$$

[3] Traceless part

$$\begin{aligned}\nabla^2 (\mathcal{F}_T \Psi - \mathcal{G}_T \Phi - A_1 Q) &= \frac{B_1}{2a^2 H^2} Q^{(2)} \\ &+ \frac{B_3}{a^2 H^2} (\nabla^2 \Phi \nabla^2 Q - \partial_i \partial_j \Phi \partial^i \partial^j Q)\end{aligned}$$

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 $\mathcal{F}_T := 2 \left[G_4 - X \left(\ddot{\phi} G_{5X} + G_{5\phi} \right) \right]$

Related to **propagation speed of gravitational waves**:

$$c_h^2 := \frac{\mathcal{F}_T}{\mathcal{G}_T}$$

Spherically symmetric configurations

$$r = a\sqrt{\delta_{ij}x^ix^j}, \quad rH \ll 1$$

The 3 equations can be integrated once to give
algebraic equations for Φ', Ψ', Q'

$$\begin{aligned} c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} &= \frac{\beta_1}{H^2} \left(\frac{Q'}{r} \right)^2 + 2 \frac{\beta_3}{H^2} \frac{\Phi'}{r} \frac{Q'}{r}, \\ \frac{\Psi'}{r} + \alpha_2 \frac{Q'}{r} &= \frac{1}{8\pi\mathcal{G}_T} \frac{\delta M(t, r)}{r^3} - \frac{\beta_2}{H^2} \left(\frac{Q'}{r} \right)^2 - 2 \frac{\beta_3}{H^2} \frac{\Psi'}{r} \frac{Q'}{r} - \frac{2}{3} \frac{\gamma_1}{H^4} \left(\frac{Q'}{r} \right)^3, \\ \alpha_0 \frac{Q'}{r} - \alpha_1 \frac{\Psi'}{r} - \alpha_2 \frac{\Phi'}{r} &= 2 \left[-\frac{\beta_0}{H^2} \left(\frac{Q'}{r} \right)^2 + \frac{\beta_1}{H^2} \frac{\Psi'}{r} \frac{Q'}{r} + \frac{\beta_2}{H^2} \frac{\Phi'}{r} \frac{Q'}{r} + \frac{\beta_3}{H^2} \frac{\Phi'}{r} \frac{\Psi'}{r} \right. \\ &\quad \left. + \frac{\gamma_0}{H^4} \left(\frac{Q'}{r} \right)^3 + \frac{\gamma_1}{H^4} \frac{\Phi'}{r} \left(\frac{Q'}{r} \right)^2 \right] \end{aligned}$$

Coefficients are dimensionless and written in terms of K, G_3, G_4, G_5

Spherically symmetric configurations

$$r = a\sqrt{\delta_{ij}x^i x^j}, \quad rH \ll 1$$

The 3 equations can be integrated once to give
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Enclosed mass:

$$\delta M(t, r) = 4\pi\rho_m(t) \int^r \delta(t, r') r'^2 dr'$$

$$\begin{aligned} c_h^2 \frac{\Psi'}{r} - \frac{\Phi'}{r} - \alpha_1 \frac{Q'}{r} &= \frac{\beta_1}{H^2} \left(\frac{Q'}{r} \right)^2 + 2 \frac{\beta_3}{H^2} \frac{\Phi' Q'}{r}, \\ \frac{\Psi'}{r} + \alpha_2 \frac{Q'}{r} &= \frac{1}{8\pi\mathcal{G}_T} \frac{\delta M(t, r)}{r^3} - \frac{\beta_2}{H^2} \left(\frac{Q'}{r} \right)^2 - 2 \frac{\beta_3}{H^2} \frac{\Psi' Q'}{r} - \frac{2}{3} \frac{\gamma_1}{H^4} \left(\frac{Q'}{r} \right)^3, \\ \alpha_0 \frac{Q'}{r} - \alpha_1 \frac{\Psi'}{r} - \alpha_2 \frac{\Phi'}{r} &= 2 \left[-\frac{\beta_0}{H^2} \left(\frac{Q'}{r} \right)^2 + \frac{\beta_1}{H^2} \frac{\Psi' Q'}{r} + \frac{\beta_2}{H^2} \frac{\Phi' Q'}{r} + \frac{\beta_3}{H^2} \frac{\Phi' \Psi'}{r} \right. \\ &\quad \left. + \frac{\gamma_0}{H^4} \left(\frac{Q'}{r} \right)^3 + \frac{\gamma_1}{H^4} \frac{\Phi'}{r} \left(\frac{Q'}{r} \right)^2 \right] \end{aligned}$$

Coefficients are dimensionless and written in terms of K, G_3, G_4, G_5

Let's see whether or not usual gravity is reproduced in the vicinity of the source:

$$\Phi' \simeq \Psi' \simeq \frac{G_N \delta M}{r^2} \quad \text{---} \quad ???$$

Linear solution at large r

At sufficiently large r all the non-linear terms may be neglected

$$\Phi' = \frac{c_h^2 \alpha_0 - \alpha_1^2}{\alpha_0 + (2\alpha_1 + c_h^2 \alpha_2) \alpha_2} \frac{\mu}{r^2}$$

$$\Psi' = \frac{\alpha_0 + \alpha_1 \alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2 \alpha_2) \alpha_2} \frac{\mu}{r^2}$$

$$Q' = \frac{\alpha_1 + c_h^2 \alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2 \alpha_2) \alpha_2} \frac{\mu}{r^2}$$

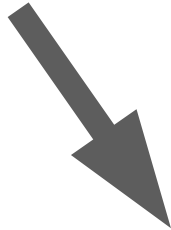
where $\mu := \frac{\delta M}{8\pi \mathcal{G}_T}$

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$$Q' = \frac{\alpha_1 + c_h^2 \alpha_2}{\alpha_0 + (2\alpha_1 + c_h^2 \alpha_2) \alpha_2} \frac{\mu}{r^2}$$


where $\mu := \frac{\delta M}{8\pi \mathcal{G}_T}$

In general, $\Phi' \neq \Psi'$
(as expected)

Linear solution at large r
(but $rH \ll 1$)



Large non-linearity
(even for weak field)

Linear solution at large r
(but $rH \ll 1$)



Large non-linearity
(even for weak field)

Linear solution at large r
(but $rH \ll 1$)



Solve the 3 algebraic equations for Φ' , Ψ' , Q' :

Case 1 $G_{4X} = 0, G_5 = 0 \longrightarrow$ Single quadratic equation for Q'

$$\Rightarrow \mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\square\phi$$

Case 2 $G_{5X} = 0 \longrightarrow$ Single cubic equations for Q'

Case 3 the most general case, $G_{5X} \neq 0$

$\longrightarrow$ Difficult to solve, but can draw some conclusion

Case I: $G_{4X} = 0 = G_5$

(Non-minimally coupled version of)
Kinetic gravity braiding

Deffayet, Pujolas, Sawicki, Vikman 2010

$$\mathcal{L} = G_4(\phi)R + K(\phi, X) - G_3(\phi, X)\Box\phi$$

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In order for the solution to be real, $G_{3X} (XG_{3X} + G_{4\phi}) > 0$

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In order for the solution to be real, $G_{3X} (XG_{3X} + G_{4\phi}) > 0$

Short-distance solution: $\Phi' \simeq \Psi' \simeq \frac{G_N \delta M}{r^2}$ **Two potentials coincide!**

where
$$8\pi G_N = \frac{1}{2G_4} = \frac{1}{2G_4(\phi(t))}$$

Time-dependent G in cosmological background

(Consequences of time dependence will be discussed later)

Case 2: $G_{5X} = 0$

The problem reduces to solving the following cubic equation:

$$(Q')^3 + \mathcal{C}_2 H^2 r (Q')^2 + \left(\frac{\mathcal{C}_1}{2} H^4 r^2 - H^2 \mathcal{C}_\beta \frac{\mu}{r} \right) Q' - \frac{H^4 \mathcal{C}_\alpha \mu}{2} = 0$$

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Written in terms of $K, G_3, G_4, G_5 = G_5(\phi)$

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$$\mu \propto \delta M$$

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Written in terms of $K, G_3, G_4, G_5 = G_5(\phi)$ $\mu \propto \delta M$

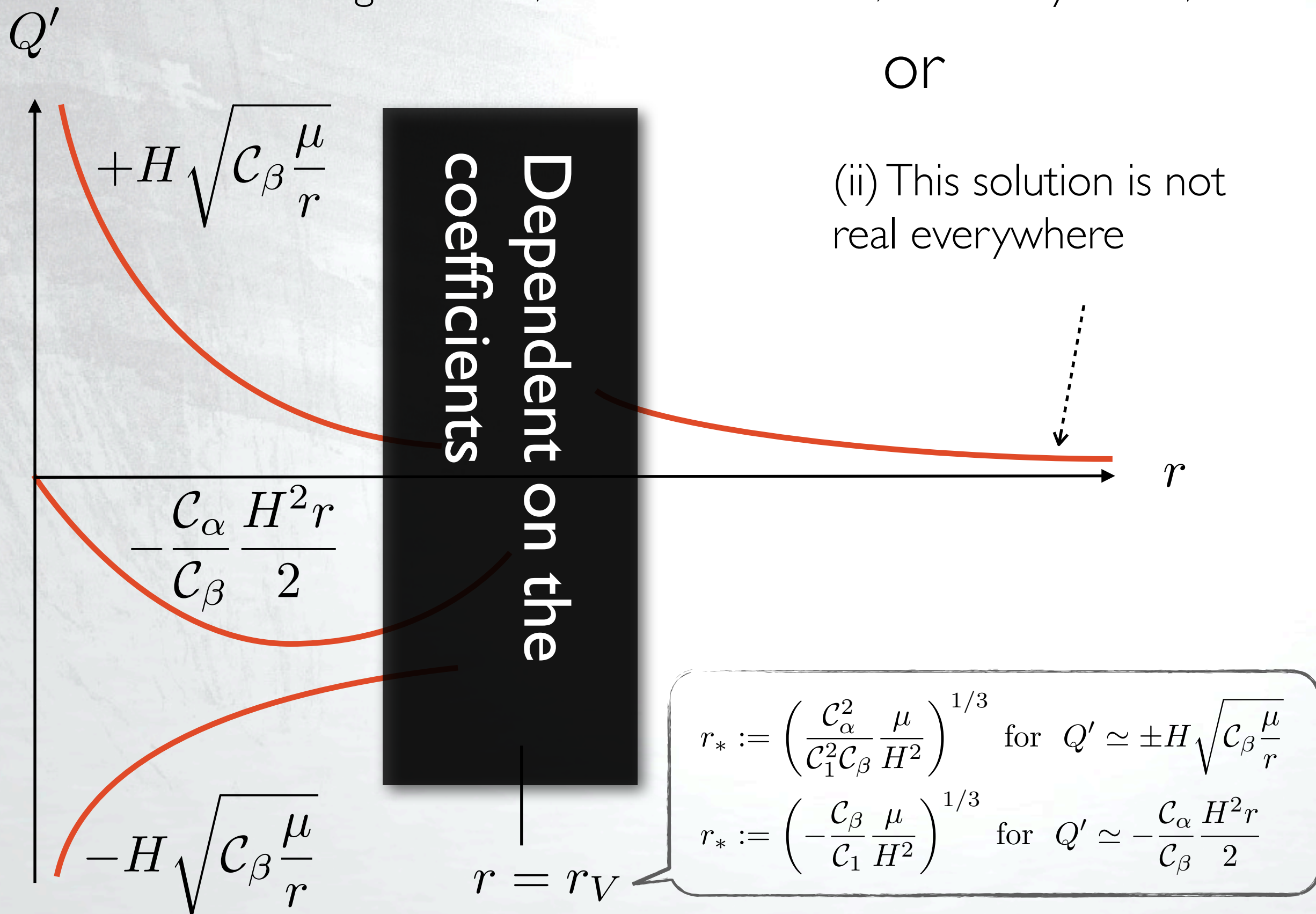
3 possible solutions at short distances:

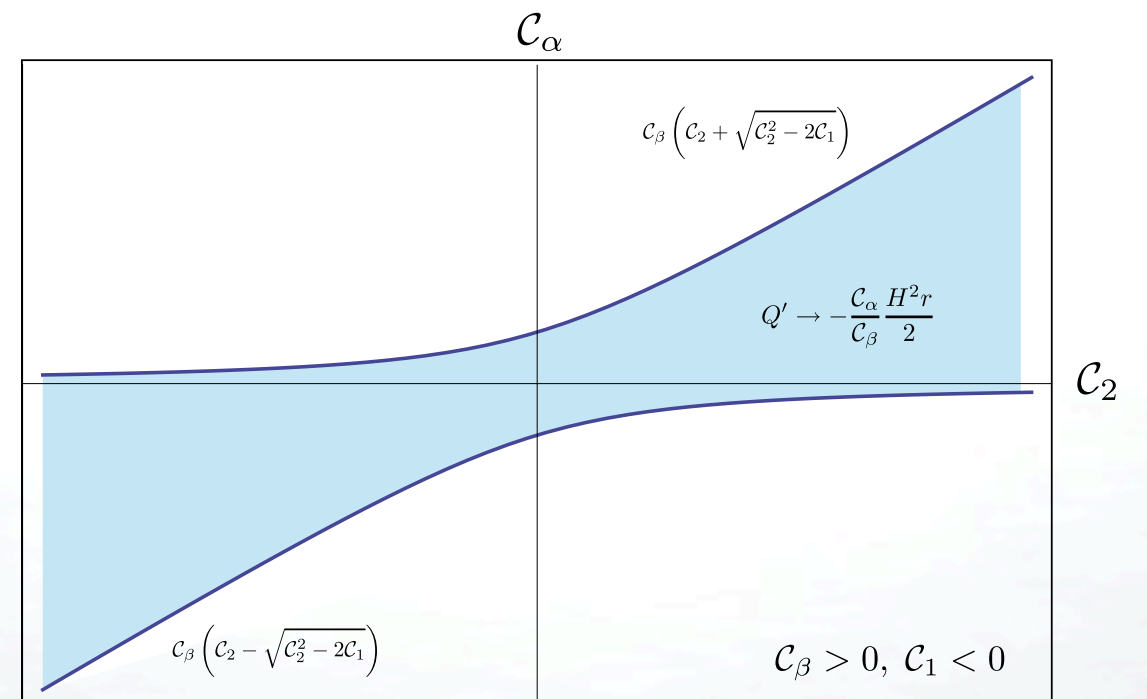
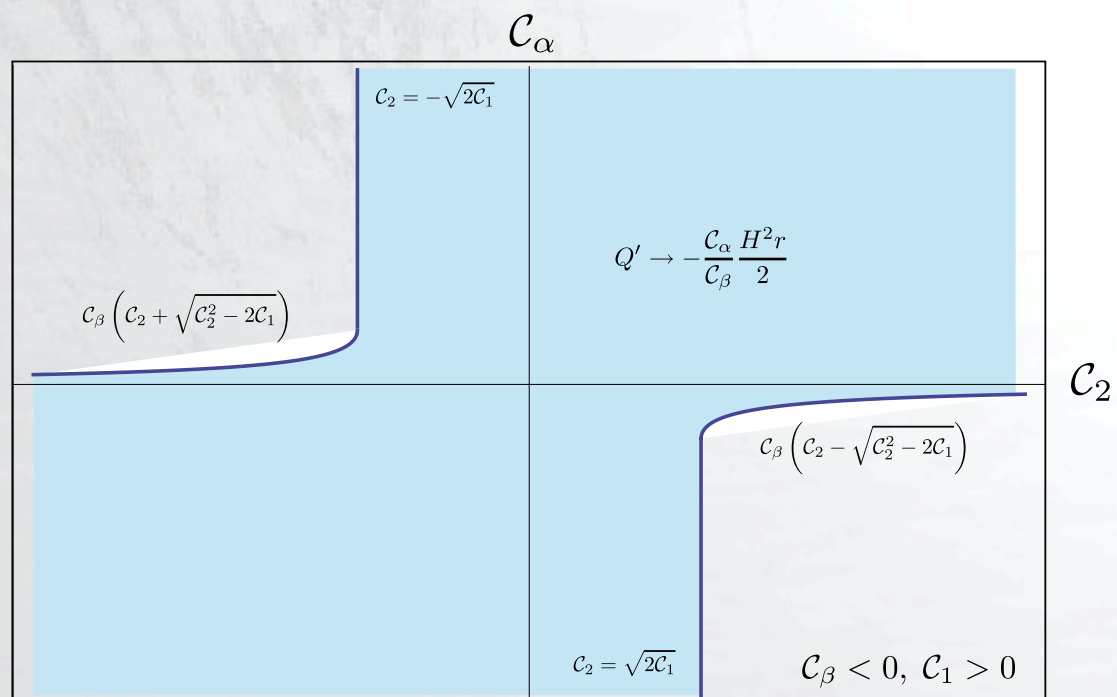
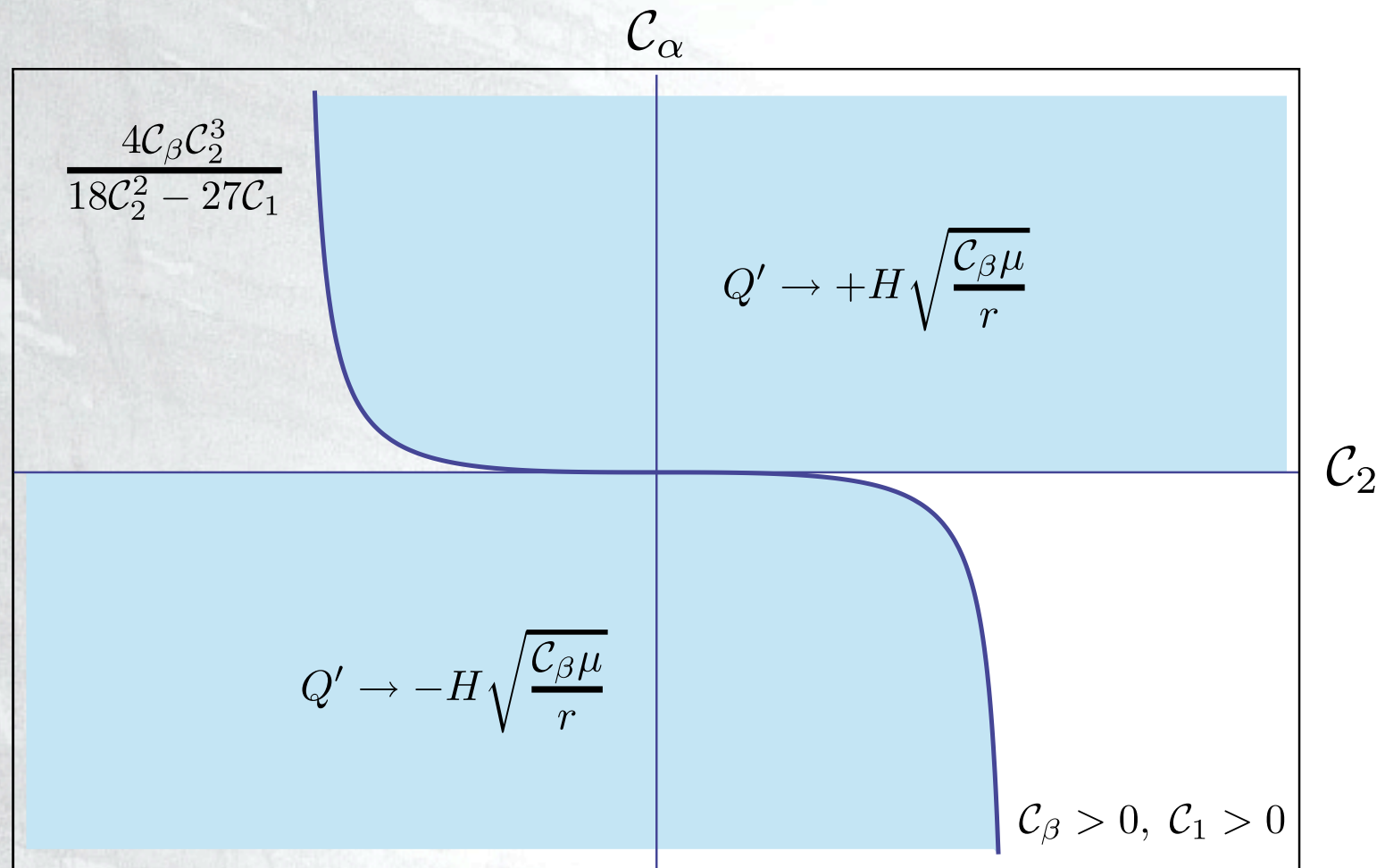
$$Q' \simeq +H \sqrt{C_\beta \frac{\mu}{r}}, \quad -H \sqrt{C_\beta \frac{\mu}{r}}, \quad -\frac{C_\alpha}{C_\beta} \frac{H^2 r}{2}$$

(i) Either of 3 short-distance solutions is joined to the long-distance, linearized solution; real everywhere;

or

(ii) This solution is not real everywhere





Gravity at short distances

☑ $Q' \simeq \pm H \sqrt{C_\beta \frac{\mu}{r}}$ $\longrightarrow$

Two potentials coincide!

$$\Phi' \simeq \Psi' \simeq \frac{G_N \delta M}{r^2}$$

Time-dependent G :

$$8\pi G_N = \frac{1}{2(G_4 - 4XG_{4X} - 4X^2G_{4XX} + 3XG_{5\phi})}$$

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⚠ Experimental constraints: $|\dot{G}_N/G_N| < 0.02H_0$ (Lunar Laser Ranging)

— must be much slower than the cosmological time scale

Williams *et al.* 2004; Babichev *et al.* 2011

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! $G_N(t) = G_{\text{cos}}(t)$ (“ G ” in Friedmann equation)

Gravity at short distances

☑ $Q' \simeq -\frac{C_\alpha}{C_\beta} \frac{H^2 r}{2}$



Two potentials do not coincide...

$$\Phi' \simeq \frac{c_h^2}{8\pi\mathcal{G}_T} \frac{\delta M}{r^2}, \quad \Psi' \simeq \frac{1}{8\pi\mathcal{G}_T} \frac{\delta M}{r^2}$$

$$\gamma_{\text{PPN}} = \frac{1}{c_h^2}$$

Case 3: $G_{5X} \neq 0$

Difficult to analyze a variety of possible solutions in detail...

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Difficult to analyze a variety of possible solutions in detail...

But, can show that **inverse-square** potentials,

$$\Phi' \simeq \Psi' \propto \frac{1}{r^2}$$

cannot be a solution on the shortest scales

Evolution of density perturbations

ϕ is minimally coupled to matter
— matter equations are not modified

Poisson equation
is modified

$$\ddot{\delta} + 2H\dot{\delta} - \frac{4}{3} \frac{\dot{\delta}^2}{1 + \delta} = (1 + \delta) \frac{\nabla^2}{a^2} \Phi \quad \longleftrightarrow \quad \frac{\nabla^2}{a^2} \Phi = 4\pi G_{\text{eff}} \rho_m \delta$$

- On large scales (but well inside the horizon)

$$G_{\text{eff}} \rightarrow \dots \quad (\neq G_N)$$

De Felice, TK, Tsujikawa (2011)

(messy expression)

- On small scales

$$G_{\text{eff}} \rightarrow G_N(t)$$

(For an appropriate model choice
with $G_{5X} = 0$)

4. Summary

Summary

- Generic scalar-tensor theory contains Galileon-like non-linear derivative interaction
- Vainshtein screening in the most general ST theory?
 - **Time-dependent G** in cosmological background
... Time dependence is *not* screened
 - constrained from observations and experiments
 - Inverse-square law cannot be reproduced on the smallest scales if $G_{5X} \neq 0$
 - Application to the study of structure formation

Thank you!