MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

PRACTICE FINAL

The exam is **170 minutes** (1:10pm – 4:00pm). No additional material or calculators are allowed. There are 100 points in total,

- Write your **name and UNI** clearly on your exam booklet.
- **Show your work** and reasoning, not just the final answer. Partial credit will be given for correct reasoning, even if the final answer is completely wrong.
- **Don't cheat**!
- Don't panic!

(1) (10 points) State whether the following are true/false. No explanation necessary.

(a) An anti-derivative of a polynomial is always a polynomial.

(b) Given a function f, there is exactly one function F such that $F'(x) = f(x)$.

(c) If f and g are continuous functions,

$$\int f(x)g(x)\,dx = \left(\int f(x)\,dx\right)\left(\int g(x)\,dx\right).$$

(d) An alternate but equivalent way to define the derivative is

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x-h)}{2h}.$$

(e) It is possible for a continuous function on (a, b) to have a local maximum but not a global maximum.

(2) Compute the limit.

(a) (5 points)

$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} \cos(1 + k/n).$$

(b) (5 points) $\lim_{z\to 0} z^3 \cos^{99}(2/z)$

(c) (5 points)

$$\lim_{h\to 0} \frac{(x+h)^{10} - x^{10}}{h}$$

(d) (5 points) $\lim_{u\to 3} (f(u)g(-u))^{10}$ where

$$f(u) = \begin{cases} 1/(u-3) & u < 4 \\ u^2 & u \geq 4 \end{cases}, \quad g(u) = \begin{cases} -2u-6 & u < 0 \\ \sin(u) & u \geq 0 \end{cases}.$$

(e) (5 points) $\lim_{t\to\infty} t^{3/2} \cos(1/t)$

(3) Compute the derivative dy/dx.

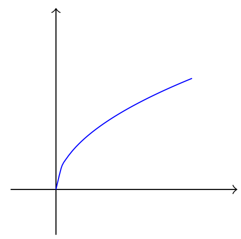
(a) (5 points)

$$y = \int_{-x}^{x^2} \arctan(t^3)\, dt$$

(Hint: $\int_{-x}^{x^2} = \int_{-x}^{0} + \int_{0}^{x^2}$.)

(b) (5 points) $y = (\tan x)^{1/x}$

(4) You are making a bowl by rotating the curve $y = x^{1/4}$ around the x-axis.



(a) (5 points) If your bowl has height h, what is the total volume of water it can hold?

(b) (5 points) You fill the bowl with water at a rate of 4 cm^3/s. Using (a), how fast is the water level increasing when the water level is $h = 1$ cm?

(5) (5 points) Find the equation of the tangent line to $4x^2 + y^2 = 8$ at $(x, y) = (1, 2)$.

(6) (10 points) Use linear approximation to explain why for x close to zero, $\sin x \approx x$ and $\cos x \approx 1$. Use this to estimate

$$\int_{-0.1}^{0.1} e^{(\sin x)^3} (\sin x)^2 (1 + \ln(\cos x))\, dx.$$

(7) Compute the integral.

(a) (5 points)

$$\int_{-2}^{2} e^{x^2} \sin(x)^3\, dx$$

(b) (5 points)

$$\int_{-2}^{2} \sqrt{4 - x^2}\, dx$$

(8) (10 points) Sketch the graph of $y = x^4 - 8x^2 + 8$ by finding its critical points, inflection points, and regions of convexity/concavity.

(9) (10 points) Now that the course is over, you would like to build a box to hold all your course materials for disposal. You have 10 m^2 of wood to make the box. Because you have accumulated so much material, you would like to maximize the volume of the box. For aesthetic reasons, you want the box to have a square base. What is the maximum possible volume?