MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 1 (DUE JAN 28)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) For each of the following functions, state its domain, range and inverse. If the inverse does not exist, state why.
   (a) $f(x) = \dfrac{2}{1-x^3}$
   (b) $f(x) = \ln(x^2 - 2)$
   (c) $f(x) = 4 + \arctan(x)$

(2) The step function $H(x)$ is defined by

$$H(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0. \end{cases}$$

   (a) Sketch the function $x^2 H(4 - 2^x)$.
   (b) Sketch the function $1 + H(\sin x)/2$.

(3) Write $2 + \sin(2 + \ln(\ln x))$ as some composition of *only* the following functions:

$$f(x) = 2 + x, \quad g(x) = \sin(x), \quad h(x) = \ln(x).$$

(4) Compute the exact value for each of the following expressions. Show and explain all your steps briefly.
   (a) $e^{2 \ln 3}$
   (b) $\sin(\tan^{-1}(1/3))$

(5) Annoyed by your calculus homework, you crumple it into a ball and throw it into an infinitely deep hole. You observe that its speed in meters per second is given by the function

$$v(t) = 3 - \frac{1}{t+1}$$

where t is the time in seconds since you threw it. (Note: this is *not* a physically realistic model, which would be more complicated.)
   (a) How fast is your homework initially traveling, right when you threw it?
   (b) What should the domain of $v(t)$ be? Explain why. (Hint: many values of t do not make sense as inputs.) What is the range of $v(t)$?
   (c) At what time t does the speed of your homework reach v meters per second?

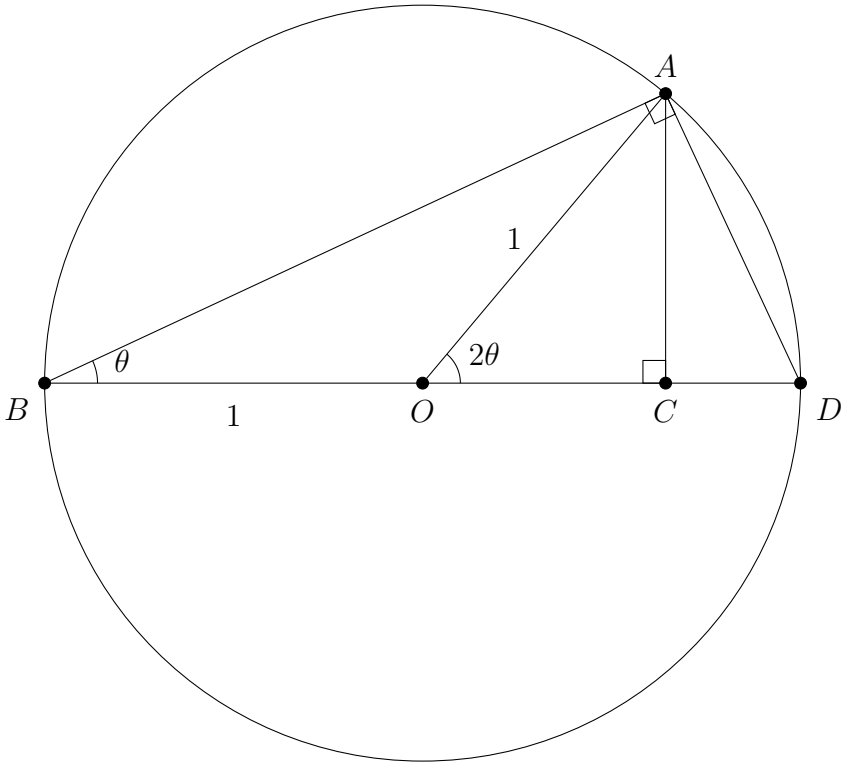
(6) Use the following steps and diagram in the unit circle to compute a formula for $\sin(2\theta)$ in terms of $\sin(\theta)$ and $\cos(\theta)$. Briefly explain each of your answers. A correct answer with no explanation is worth zero points.

(a) Using triangle OCA, what is the length of AC in terms of 2θ?

(b) Using triangle BAD, what is the length of AB in terms of θ?

(c) Using triangle BCA, write $\sin\theta$ in terms of the answers of the previous two questions. Rearrange it to get the *double angle formula* for $\sin(2\theta)$. What is the formula?

(d) What is the length of OC in terms of 2θ? Use it and triangle BCA to compute $\cos(\pi/12)$ in terms of $\sin(\pi/6)$ and $\cos(\pi/6)$.



(The circle is a unit circle, with origin O.)