MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 10 (DUE APR 15)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Find the most general f. (Make sure to check your answer via differentiation.)
   (a) $f'(x) = \sqrt{2} + \sqrt{x}$ with $f(2) = 0$.
   (b) $f''(x) = 4x^3 + 1/x^2$ with $f(1) = 0$ and $f'(1) = 1$.
   (c)
   $$f'(t) = (t+2)^9 + \frac{1}{t+2}.$$
   (d)
   $$f'(\theta) = 4\sec^2(\theta) + \frac{3}{\sqrt{1-\theta^2}}.$$
   (e) $f'(x) = f(x)$ with $f(0) = 1$. (Hint: just make an educated guess and check it.)

(2) Consider the function $f(x) = \sqrt{x}$ on the interval $[0, 1]$.
   (a) Numerically approximate the area under $f(x)$ using four rectangles of equal width. Draw a diagram showing your four rectangles in relation to $f(x)$. Explain whether your approximation is smaller or larger than the actual area.

   (b) Write down an expression approximating the area under $f(x)$ using n rectangles of equal width. Do *not* compute the actual value of the expression; that would be hard.

   (c) To get an approximation which is easier to compute, use n rectangles of unequal widths. More precisely, let the base of the first rectangle be $[0, 1^2/n^2]$, the base of the second rectangle be $[1^2/n^2, 2^2/n^2]$, and so on, up to the n-th rectangle, whose base is $[(n-1)^2/n^2, 1]$.
      (i) What is the width of the k-th rectangle? What about its height?
      (ii) Write down an expression approximating the area under $f(x)$. Do compute the actual value of the expression this time. (It will depend on n.)
      (iii) Using (ii), what is the actual area under $f(x)$?

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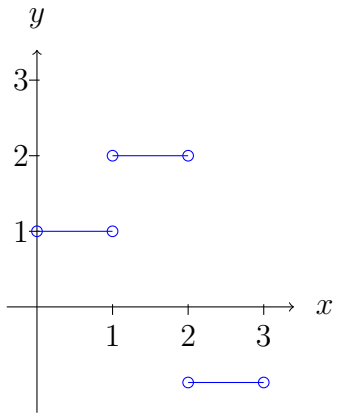
(3) The following is the graph of f'.



(a) Draw the graph of f, assuming $f(0) = 0$.
(b) Let $F(x)$ be the area under the graph of f' on the interval $[0, x]$. For example, $F(1) = 1$ and $F(2) = 1 + 2 = 3$ and $F(3) = 1 + 2 + (-1) = 2$. Draw the graph of F.

(4) Annoyed by your calculus homework, you crumple it into a ball and throw it into an infinitely deep hole.
(a) Acceleration due to gravity is $a(t) = 9.8$ m/s^2. What is the velocity of your homework as a function of t if its initial velocity when you threw it was 2 m/s? (Hint: remember that acceleration is the derivative of velocity.)
(b) How far has your homework fallen after three seconds? (Hint: remember that velocity is the derivative of distance.)
(c) What is the velocity of your homework after ten minutes? Does this make sense? Why or why not? (If you are more used to units of km/hr instead of m/s, convert to km/hr before judging whether your answer makes sense or not.)