MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 11 (DUE APR 23)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Evaluate the following definite integrals using any method.

(a)
$$\int_0^2 (2x - x^2)\,dx$$

(b)
$$\int_{-2}^2 (1 + \sqrt{4 - x^2})\,dx$$

(c)
$$\int_0^\pi \cos(\theta)\,d\theta$$

(d)
$$\int_3^3 \sin(x)^3 \sqrt{x^7 + 1}\,dx$$

(e)
$$\int_1^6 (3f(x) - 4g(x))\,dx$$

if $\int_1^8 f(x)\,dx = 2$ and $\int_6^8 f(x)\,dx = 1$ and $\int_6^1 g(x)\,dx = 3$.

(f)
$$\int_0^1 (u+2)(u-1)\sqrt{u}\,du$$

(g)
$$\int_{-\pi}^\pi |\sin(\theta)|\,d\theta$$

(2) Express the limit as a definite integral, and then evaluate it.

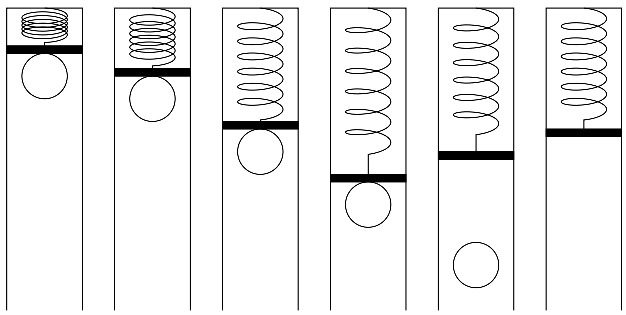
(a)
$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^n \frac{1}{1 + (k/n)^2}$$

(b)
$$\lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^n e^{1+k/n}$$

(3) Let $f(x) = \sqrt{1+x^4}$.
   (a) Show that $1 \leq f(x) \leq 1 + x^4$ for $x \geq 0$.
   (b) Show that $1 \leq \int_0^1 f(x)\,dx \leq 1.2$. (Hint: use (a).)

(4) Find the derivative $f'(x)$.
   (a)
$$f(x) = \int_1^x \sin^3(\theta)\cos^4(\theta)\,d\theta$$
   (b)
$$f(x) = \int_0^{x^2+3} (u-1)^{u-1}\,du.$$

(5) Annoyed by your calculus homework, you crumple it into a ball and launch it into an infinitely deep hole using the Spring Launcher Technology$^{\text{TM}}$ from Homework 5.



Your new and improved measurements show that at time t (in milliseconds), the end of the spring is at depth (in centimeters)
$$x(t) = -5 - \int_0^t \frac{10\sin x}{x}\,dx.$$
(The integral is a special function called the sine integral. It is important in electrical engineering.)
   (a) There are infinitely many times t where the spring will be fully extended (and about to retract back). Find all such t.
   (b) When is the first time t that the end of the spring changes from accelerating downward (i.e. extending) to accelerating upward (i.e. retracting)? You do not need to find an exact value for t; just give an equation that t must satisfy. For example: "t is the only solution to $e^{-t} = \sin(t)$ in the interval $(3, 4)$". (Hint: look back at Homework 5.)