MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 12 (DUE APR 30)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Compute the following indefinite integrals.

(a)
$$\int \left(\sqrt[3]{x} + \frac{1}{\sqrt{x}} \right) dx$$

(b)
$$\int \tan^2 \theta \, d\theta$$

(Hint: $\tan = \sin / \cos$, and then use a trig identity for $\sin^2$.)

(c)
$$\int \frac{1}{\sqrt{1-4x^2}} \, dx$$

(d)
$$\int \sqrt{2 \tan t} \sec^2 t \, dt$$

(e)
$$\int \frac{\sin(\ln x)}{x} \, dx$$

(f)
$$\int x^5 \sqrt{1+x^3} \, dx$$

(2) Compute the following definite integrals.

(a)
$$\int_{-2}^{2} (x+3)\sqrt{4-x^2} \, dx$$

(Hint: split it into two pieces, which are computed using different methods.)

(b)
$$\int_0^2 x f(x^2) \, dx$$

if $\int_0^4 f(x) \, dx = 3$.

(c)

$$\int_{-5}^{5} \frac{\sin(x)\cos^2(x)}{1+x^4}\,dx$$

(d)

$$\int_{0}^{1} xe^{-x^2}\,dx$$

(3) Water is flowing into a container at a rate of $f(t)$ liters per minute at time t. Explain in words what $\int_2^5 f(t)\,dt$ represents.

(4) A bacteria colony starts with 200 bacteria and grows at a rate of 2^t bacteria per hour after t hours. What is the population after 1 hour?

(5) Let $f(x)$ be a continuous function on $[0,1]$.

(a) (Warmup) Use a substitution to show that

$$\int_0^1 f(x)\,dx = \int_0^1 f(1-x)\,dx.$$

(b) Use substitution to show that

$$\int_0^{\pi/2} f(\cos x)\,dx = \int_0^{\pi/2} f(\sin x)\,dx.$$

(Hint: the obvious substitution *will* work out in the end. Use the trig identity $\sin^2 x + \cos^2 x = 1$ and don't give up.)

(c) Using (b), compute the two integrals

$$\int_0^{\pi/2} \cos^2 x\,dx, \quad \int_0^{\pi/2} \sin^2 x\,dx.$$

(Hint: their sum is really easy to compute.)