MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 3 (DUE FEB 11)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) For each of the following functions, state where it fails to be continuous and where it fails to be differentiable. At the points where it fails to be continuous or differentiable, briefly explain why it fails.

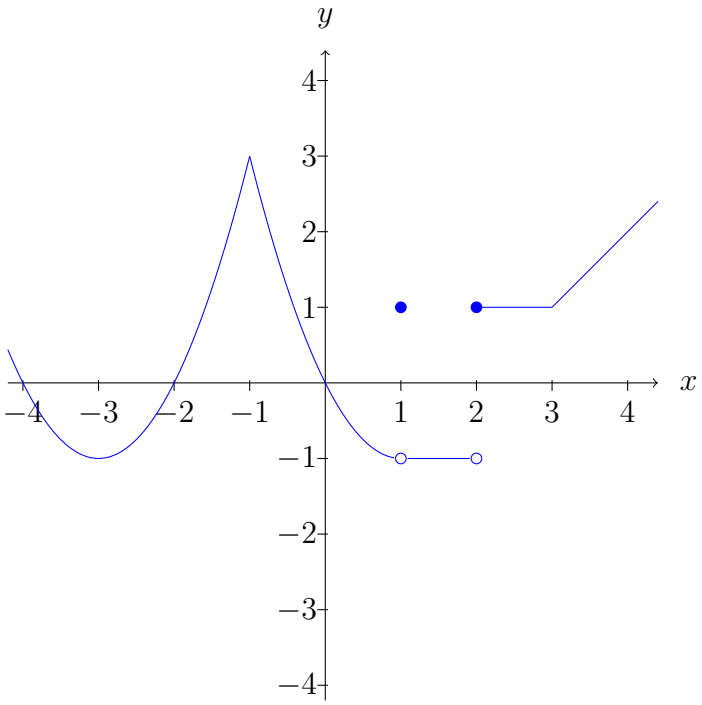
(a)

$$f(t) = \frac{2+t}{2-t}$$

(b)

$$f(x) = \begin{cases} -x^2 & x \leq 0 \\ 0 & 0 < x < 1 \\ x - 1 & x \geq 1 \end{cases}$$

(c) The function $f(x)$ given by the graph:



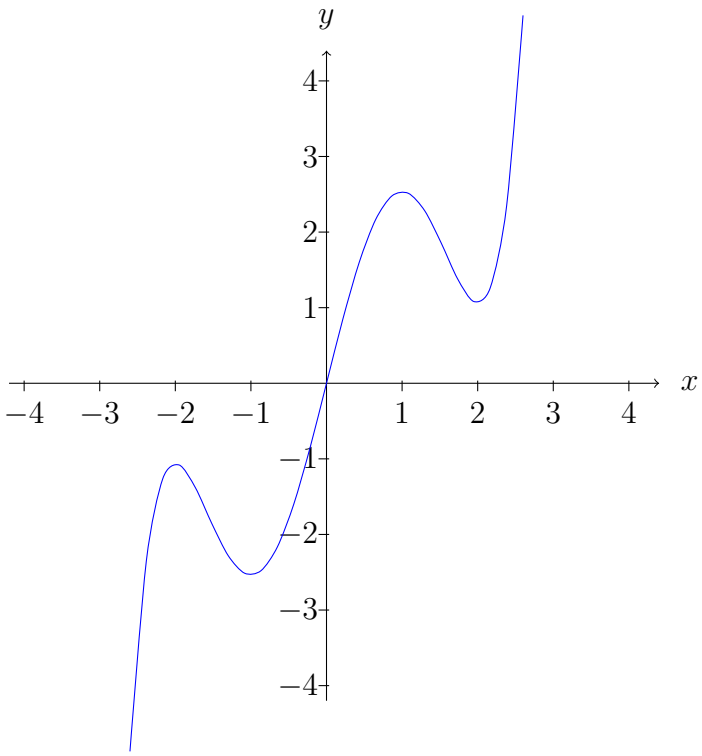
(2) (**Updated**) Compute the limit

$$\lim_{x\to 4} \ln\left(\frac{2-\sqrt{x}}{4-x}\right)$$

using that $\ln(x)$ is a continuous function. Briefly explain the step in the computation which requires this fact.

(3) You want to examine solutions to the equation $\sin(x) = \cos(x)$.
   (a) Use the intermediate value theorem to show that there must exist a solution in the interval $(0, \pi/2)$.
   (b) Explain why this implies the equation has infinitely many solutions.

(4) Use the following graph of the function $f(x)$ to roughly sketch the graph of its derivative $f'(x)$.



(5) Find the equation of the tangent line to $y = 1/x + 1$ at the point $(1, 2)$.

(6) (**Updated**) Consider the function

$$f(x) = \begin{cases} 3x - 1 & x \neq 2 \\ 6 & x = 2. \end{cases}$$

   (a) Find a number δ such that if $0 < |x - 2| < \delta$, then $|3x - 6| < \epsilon$ where $\epsilon = 0.1$.
   (b) Find a formula for δ (in terms of ϵ) such that if $0 < |x-2| < \delta$, then $|3x-6| < \epsilon$. (This formula should give the answer you got in (a) when you plug in $\epsilon = 0.1$.)
   (c) From (b), what can you conclude about $\lim_{x\to 2} f(x)$? Briefly explain why.

(7) Annoyed by your calculus homework, you crumple it into a ball and throw it into an infinitely deep hole. As you watch it fall, a physicist passing by says to you "the depth $x(t)$ that your homework is at, as a function of the time t since you threw it, is given by

$$x(t) = t^{3/2},$$

now stop staring at your hole and go do the next question". You decide to show that their proposed model is wrong.

(a) Using the proposed model, compute the velocity $v(t)$ of your homework at time t. (Hint: remember that velocity is the rate of change of position over time.)

(b) Using the proposed model, compute the acceleration $a(t)$ of your homework at time t. (Hint: remember that acceleration is the rate of change of velocity over time.)

(c) What should the domains of $x(t)$, $v(t)$ and $a(t)$ be?

(d) As $t \to 0^+$, what happens to the acceleration of your homework? Explain why this makes no sense physically, and therefore invalidates the proposed model.