MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 4 (DUE FEB 18)

Each part (labeled by letters) of every question is worth 2 points. There are 10 parts, for a total of 20 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Differentiate the following functions. State the differentiation rule used at each step.

(a) $f(x) = x^{30} - x\sqrt{x} - 2^{10}$.

(b)
$$g(t) = \frac{t+1}{t^2-2}.$$

(c)
$$h(z) = \frac{Az^5 + Bz^3 + C/z}{z^{10}}$$
where A, B, C are constants.

(d)
$$F(x) = (x - 5x^3)\left(\frac{2}{x^2} + \frac{1}{x^4}\right).$$

(e)
$$f(x) = \frac{1 + xg(x)}{\sqrt{x}}$$
where $g(x)$ is a differentiable function.

(2) We want a formula for derivatives of functions like $f(x) = (g(x))^n$ for integers $n \geq 0$.

(a) Let f_1, f_2, f_3 be differentiable functions. Use the product rule twice to show that the derivative of their product is
$$(f_1 f_2 f_3)' = f_1' f_2 f_3 + f_1 f_2' f_3 + f_1 f_2 f_3'.$$

(b) Let $f_1, f_2, \ldots, f_n$ be differentiable functions. Guess a formula for
$$(f_1 f_2 \cdots f_n)',$$
and briefly explain the reasoning behind your guess.

(c) In the special case where $f_1(x) = f_2(x) = \cdots = f_n(x) = x$, we know from the power rule that
$$(f_1(x) f_2(x) \cdots f_n(x))' = (x^n)' = nx^{n-1}.$$
Check that your guessed formula indeed produces this answer.

(d) In the special case where $f_1(x) = f_2(x) = \cdots = f_n(x) = g(x)$, i.e. are all equal to the same differentiable function $g(x)$, what does your formula produce? In other words, what is the formula for the derivative of $g(x)^n$?

(e) Use the formula from (d) to calculate the derivative of $f(x) = (3x^2 + x - 7)^{100}$.