MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

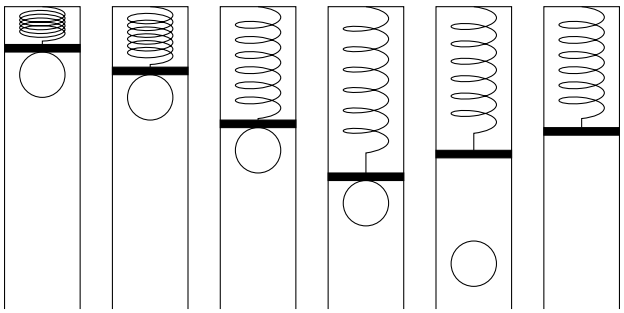
HOMEWORK 5 (DUE FEB 26)

Each part (labeled by letters) of every question is worth 2 points. There are 5 parts, for a total of 10 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Compute the derivatives of the following functions. You no longer need to state differentiation rules, but do write each step clearly.
(a)
$$h(\theta) = \cot(\theta).$$

(2) Compute the following limits. Briefly explain your steps.
(a)
$$\lim_{h\to 0} \frac{\sin(\pi/2 + h) - 1}{h}.$$
(Hint: this should involve almost no calculation. It's secretly the definition of the derivative $f'(\pi/2)$ for some function $f(x)$...)

(3) Annoyed by your calculus homework, you crumple it into a ball and launch it into an infinitely deep hole using your new Spring Launcher Technology™.



At time t (in milliseconds), the end of the spring is at depth (in centimeters)
$$x(t) = \frac{10\sin(t)}{t} - 15.$$
(a) Compute the function $v(t)$ which describes the instantaneous velocity of the end of the spring at time t.
(b) There are infinitely many times t where the spring will be fully extended (and about to retract back). Briefly explain why at such times, $v(t) = 0$.
(c) Let $t_0 > 0$ be the smallest positive time such that $v(t_0) = 0$. Find an interval (a, b) that must contain t_0, and prove that it does.