MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

HOMEWORK 6 (DUE MAR 07)

Each part (labeled by letters) of every question is worth 2 points. There are 15 parts, for a total of 30 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Differentiate the following functions.

(a)
$$g(u) = e^{3u+7}\sin(u)$$

(b)
$$f(x) = \left(\frac{2^x - 1}{x^2 + 1}\right)^{100}$$

(c)
$$f(\theta) = \tan^2(n\theta)$$

where n is a constant.

(d)
$$h(x) = \ln(x^2 - 1)$$

(e)
$$h(x) = (\cos x)^{\sqrt{x}} \cdot \sqrt{x^3 + 7}$$

(2) Find y'' by implicit differentiation.

(a)
$$x^2 + 2y^3 = 7$$

(b)
$$\sin x + \cos y = 4$$

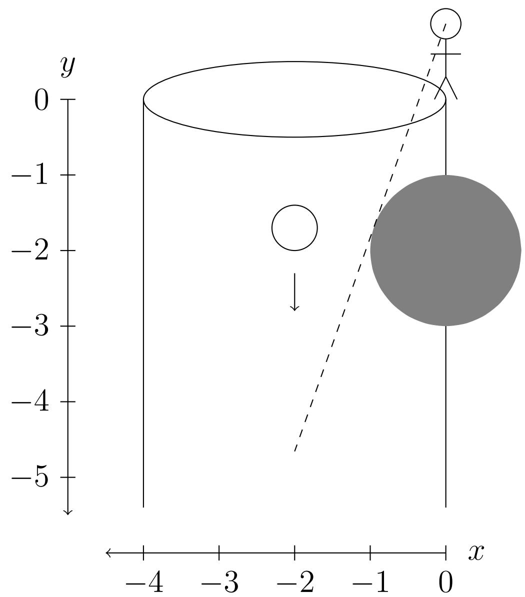
(3) Suppose $f(x)$ has an inverse function $f^{-1}(x)$, and both of them are differentiable.

(a) Find a formula for the derivative of $f^{-1}(x)$ by using implicit differentiation and the chain rule on the equation
$$f(f^{-1}(x)) = x$$
and then rearranging. (Hint: it may help, conceptually, to temporarily rename $f^{-1}(x)$ to something like $g(x)$.)

(b) Using your formula from (a), find the derivative of $\arctan(x)$. (Note: no need to simplify anything yet.)

(c) Explain how to simplify your formula from (b) to get the final answer
$$\frac{d}{dx}(\arctan(x)) = \frac{1}{1 + x^2}.$$

(4) Annoyed by your calculus homework, you crumple it into a ball and throw it into an infinitely deep hole. Standing at the edge of the hole, you watch your homework fall. Unfortunately, there is a rock sticking out into the hole, which prevents you from seeing past a certain depth.



Conveniently, the rock is perfectly circular and is described by the equation

$$x^2 + (y+2)^2 = 1.$$

Your homework is falling vertically along the line $x = -2$, and you would like to figure out how deep it can fall before you can no longer see it.

(a) Your deepest line of sight (the dashed line in the diagram) is tangent to the circular rock at some point (a, b). Using implicit differentiation, find the slope of the tangent line at (a, b). (Your answer will be some expression involving the variables a and b.)

(b) Without using your formula from (a), and using your understanding of circles, what should be the slope at $(0, -1)$? At $(-1, -2)$? Do these answers agree with what your formula from (a) produces?

(c) Your line of sight starts at $(0, 1)$, where your head is. Its slope is determined, in terms of variables a and b, by your formula from (a). Using these pieces of information, what is the equation of the line? Write your answer in the form

$$y = (\text{some expression involving } a \text{ and } b) \cdot x + (\text{some constant}).$$

(d) Your line of sight must also pass through the point of tangency (a, b). Plug this fact into (c) to get an equation that a and b must satisfy. Then write down a second equation that a and b must satisfy, coming from the fact that (a, b) is a point on the circular rock. (Hint: you should end up with a system of two quadratic equations for a and b.)

(e) Solve for a and b, and therefore for the equation of your line of sight. How deep does your homework fall before you can't see it anymore?