MATH UN1101

CALCULUS I (SECTION 5) - SPRING 2019

PRACTICE MIDTERM 2

The exam is **75 minutes**. No additional material or calculators are allowed.

- Write your **name and UNI** clearly on your exam booklet.
- **Show your work** and reasoning, not just the final answer. Partial credit will be given for correct reasoning, even if the final answer is completely wrong.
- **Don't cheat**!
- Don't panic!

(1) (10 points) State whether the following are true/false. No explanations necessary.

(a) Since $\sec x = (\cos x)^{-1}$, the derivative of $\sec x$ is $-(\cos x)^{-2}$.

(b) There exists a differentiable function $f(x)$ such that $f'(x) < 1$ and $f(0) = 0$ and $f(2) = 2$.

(c) The function $\tan(x)$ has an absolute maximum on $[0, \pi/2)$.

(d) The function $\tan(x)$ has an absolute minimum on $[0, \pi/2)$.

(e) If a function f is continuous on an interval $[a, b]$, it must have a critical value in (a, b).

(f) There exists c in the interval $(1, 2)$ such that the function

$$f(x) = x^3 - x + \cos(\pi/x)$$

has derivative $f'(c) = 7$.

(g) Let $g(x)$ be the inverse function of $f(x) = xe^x$ (e.g. $f(g(x)) = x$). Then $g'(0) = 2$.

(h) $\lim_{x\to 0} x^{\sqrt{x}}$ does not exist.

(i) If $f'(c) = 0$, then $f(c)$ is either a local maximum or a local minimum.

(j) There exists a function f such that $f(x) > 0$ and $f'(x) < 0$ and $f''(x) > 0$ for all x.

(2) Compute the derivative dy/dx. Write it as a function of just x if possible.

(a) (5 points)

$$y = \frac{(x+1)^5(x-2)^6}{\sqrt{2x-5}}$$

(Hint: take ln of both sides.)

(b) (5 points)

$$e^{xy} - y = x.$$

(3) (5 points) Use linear approximation to give an estimate for $\tan(\pi/180)$. (Leave $\pi/180$ alone; no need to calculate its actual value.) If you repeated the same procedure to estimate $\tan(\pi/90)$, would it be more or less accurate than your estimate for $\tan(\pi/90)$? Briefly explain.

(4) Let $f(x) = x^4 - 4x^3 + 4x^2 - 1$.
   (a) (5 points) Find the critical points. For each, determine whether it is a local minimum, local maximum, or neither.
   (b) (5 points) Find the inflection points, and the intervals where f is concave up and concave down.
   (c) (3 points) What is the absolute maximum and absolute minimum on the interval $[-1, 4]$?
   (d) (2 points) Roughly sketch the graph.

(5) (5 points) The area of an equilateral triangle is growing at 30 cm^2/min. How fast are the sides growing when they are exactly $\sqrt{3}$ cm?

(6) (5 points) Prove that among all rectangles with the same area A, the one with smallest perimeter is a square. (Hint: let the side lengths be x and y, and minimize the perimeter.)