MATH S3027 (SECTION 2)
ORDINARY DIFFERENTIAL EQUATIONS - SUMMER 2019

HOMEWORK 1 (DUE JUL 11)

Each question is worth 10 points. There are 5 questions, for a total of 50 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Roughly sketch the slope field associated to the first-order ODE

$$\frac{dy}{dx} = \frac{x}{y}.$$

(Note that along the line $y = 0$ the slope is infinite, so you should draw a vertical line there.) Sketch the integral curve for each of the following initial conditions: $y(0) = 2$, $y(2) = 0$, and $y(-1) = -1$.

(2) Solve the general non-homogeneous first-order linear ODE with constant coefficients

$$\frac{dy}{dx} + ay = b,$$

using the same method we used in class to solve the homogeneous version $y' + ay = 0$.

(3) Solve the IVP

$$\frac{dy}{dx} = -3x^2y^2, \quad y(0) = 1.$$

If we wanted $y(0) = 0$ instead, what would be the solution?

(4) Consider the ODE

$$\frac{dy}{dx} = \frac{\sin(x)^2 xy}{2 - e^x}.$$

Is it linear? Does there exist a solution with $y(0) = 0$? What is the interval of validity for the solution?

(5) Consider the *partial* differential equation

$$\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = f$$

for the two-variable function $f(x, y)$. Check that both

$$f(x, y) = e^x, \quad f(x, y) = e^x + y$$

are solutions satisfying the initial condition that $f(x, 0) = e^x$ for all x. So we see that IVPs for PDEs do *not* have unique solutions, and that something special happens for ODEs to guarantee uniqueness.