MATH S3027 (SECTION 2)
ORDINARY DIFFERENTIAL EQUATIONS - SUMMER 2019

HOMEWORK 2 (DUE JUL 15)

Each question is worth 10 points. There are 5 questions, for a total of 50 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Roughly sketch the slope field of
$$\frac{dy}{dx} = -\frac{y}{x}.$$
For each choice of $M(x,y)$ and $N(x,y)$
$$(M,N) = (-y,x), \quad (M,N) = (-2y,2x), \quad (M,N) = (-xy,x^2),$$
roughly sketch the vector field corresponding to the autonomous system
$$\frac{dy}{dt} = M(x,y), \quad \frac{dx}{dt} = N(x,y).$$
Briefly describe the relationship between the integral curves in these three cases.

(2) Consider the first-order equation
$$(xy^2 + bx^2y) + x^2(x+y)\frac{dy}{dx} = 0.$$
Find the value of b so that it is exact. For that value of b, solve the equation.

(3) Using an integrating factor, solve the linear first-order equation
$$\frac{dy}{dx} - y = e^{2x} - 1.$$

(4) Solve the equation
$$\frac{dy}{dx} = \frac{(x+y)^2}{2x^2}.$$

(5) A first-order ODE of the form
$$\frac{dy}{dx} + p(x)y = q(x)y^n$$
is called a **Bernoulli equation**. For $n \geq 2$, it is a *non-linear* equation. Show how dividing by y^n and using the change of variables
$$u = y^{1-n}$$
transforms it into a *linear* equation. Use this technique to solve
$$\frac{dy}{dx} = ay + by^3$$
where $a, b > 0$ are constants.