MATH S3027 (SECTION 2)

ORDINARY DIFFERENTIAL EQUATIONS - SUMMER 2019

HOMEWORK 3 (DUE JUL 18)

Each question is worth 10 points. There are 5 questions, for a total of 50 points. You are encouraged to discuss the homework with other students but you must write your solutions individually, in your own words.

(1) Prove that if $y(x) = u(x) + iv(x)$ is a solution to the equation
$$y'' + p(x)y' + q(x)y = 0$$
then so are $u(x)$ and $v(x)$.

(2) Find a fundamental set y_1, y_2 of *real-valued* solutions for the equation
$$y'' - 2y' + 2y = 0.$$
Check, using the Wronskian, that y_1, y_2 indeed form a fundamental set of solutions.

(3) Use the method of undetermined coefficients to solve the IVP
$$y'' + 3y' + 2y = 7\sin x + \cos x, \quad y(0) = 0, \quad y'(0) = 1.$$
Explain your ansatz.

(4) Let α, β be real constants. The following equation is known as the **Cauchy–Euler equation**:
$$x^2y'' + \alpha xy' + \beta y = 0.$$
Show that the change of variables
$$t = \ln x$$
transforms it into a constant-coefficient equation. Use this to find the general (real-valued) solution to the Cauchy–Euler equation when $\alpha = \beta = 1$.

(5) Find the general solution to
$$y''' - 2y'' - y' + 2y = 0.$$