

## New counter terms at order $\hbar^N$

$$\begin{aligned} F_{\mu\nu}^0 &= \partial_\mu A_\nu^0 - \partial_\nu A_\mu^0 + [A_\mu^0, A_\nu^0] \\ &= \sqrt{Z_A} (\partial_\mu A_\nu - \partial_\nu A_\mu + \sqrt{Z_A} [A_\mu, A_\nu]) \\ &= \sqrt{Z_A} (F_{\mu\nu} + (\sqrt{Z_A} - 1) [A_\mu, A_\nu]) \end{aligned}$$

$$\bullet \frac{1}{e_0^2} (F_{\mu\nu}^0)^2 = \frac{Z_A}{e_0^2} (F_{\mu\nu} + (\sqrt{Z_A} - 1) [A_\mu, A_\nu])^2$$

$$\left\{ \begin{aligned} \sqrt{Z_A} &= \sqrt{Z_{A \text{ old}}} + \hbar^N \sqrt{Z_{A \text{ new}}^{(N)}} + O(\hbar^{N+1}), \\ \frac{Z_A}{e_0^2} &= \left( \frac{Z_A}{e_0^2} \right)_{\text{old}} + \hbar^N \left( \frac{Z_A}{e_0^2} \right)_{\text{new}}^{(N)} + O(\hbar^{N+1}), \end{aligned} \right.$$

= old

$$+ \hbar^N \left( \frac{Z_A}{e_0^2} \right)_{\text{new}}^{(N)} F_{\mu\nu}^2 + \hbar^N \frac{1}{e_0^2} \sqrt{Z_{A \text{ new}}^{(N)}} F_{\mu\nu} \cdot [A_\mu, A_\nu]$$

$$+ O(\hbar^{N+1}).$$

$$\cdot \bar{\Psi}_0 \not{D}_{A_0} \Psi_0 = z_\Psi \bar{\Psi} (\not{\partial} + \sqrt{z_A} A) \Psi$$

$$= z_\Psi \bar{\Psi} \not{\partial} \Psi + z_\Psi (\sqrt{z_A} - 1) \bar{\Psi} A \Psi$$

$$\left\{ \begin{array}{l} z_\Psi = z_{\Psi \text{ old}} + \hbar^N z_{\Psi \text{ new}}^{(N)}, \\ \sqrt{z_A} = \sqrt{z_A \text{ old}} + \hbar^N \sqrt{z_A \text{ new}}^{(N)} + O(\hbar^{N+1}), \end{array} \right.$$

$$= \text{old} + \hbar^N z_{\Psi \text{ new}}^{(N)} \bar{\Psi} \not{\partial} \Psi + \hbar^N \sqrt{z_A \text{ new}}^{(N)} \bar{\Psi} A \Psi + O(\hbar^{N+1}).$$

$$\cdot \bar{\Psi}_0 m_0 \Psi_0 = \bar{\Psi} z_\Psi m \Psi$$

$$\left[ z_\Psi m_0 = (z_\Psi m_0)_{\text{old}} + \hbar^N (z_\Psi m_0)_{\text{new}}^{(N)} + O(\hbar^{N+1}) \right]$$

$$= \text{old} + \hbar^N \bar{\Psi} (z_\Psi m_0)_{\text{new}}^{(N)} \Psi + O(\hbar^{N+1}).$$

$$\cdot \bar{C}_0 \not{\partial}^\mu D_\mu^0 C_0 = z_C \bar{C} \not{\partial}^\mu (\partial_\mu C + \sqrt{z_A} [A_\mu, C])$$

$$\left\{ \begin{array}{l} z_C = z_C \text{ old} + \hbar^N z_C \text{ new}^{(N)}, \\ \sqrt{z_A} = \sqrt{z_A \text{ old}} + \hbar^N \sqrt{z_A \text{ new}}^{(N)} + O(\hbar^{N+1}), \end{array} \right.$$

= old

$$+ \hbar^N \bar{C} \not{\partial}^\mu (z_C \text{ new}^{(N)} \partial_\mu C + (z_C \text{ new}^{(N)} + \sqrt{z_A \text{ new}}^{(N)}) [A_\mu, C]) + O(\hbar^{N+1}).$$

$$Z_{K^*} = Z_c$$

$$\downarrow$$

$$\cdot K_0^M D_\mu C_0 = Z_c K^M (\partial_\mu C + \sqrt{Z_A} [A_\mu, C])$$

Same as above

$\downarrow$   
= old

$$+ \hbar^N K^M (Z_c^{(N)} \partial_\mu C + (Z_c^{(N)} + \sqrt{Z_A^{(N)}}) [A_\mu, C]) + O(\hbar^{N+1})$$

$$\cdot K_0^\psi C_0 \psi_0 = \underbrace{\sqrt{Z_{K^*}} \sqrt{Z_c} \sqrt{Z_\psi}}_{Z_{K^*} Z_\psi = \# = Z_A Z_c} K^\psi C \psi$$

$$Z_{K^*} Z_\psi = \# = Z_A Z_c \quad \rightarrow \quad \sqrt{Z_A} Z_c$$

Same as above

$$\downarrow$$

$$= \text{old} + \hbar^N (\sqrt{Z_A^{(N)}} + Z_c^{(N)}) K^\psi C \psi + O(\hbar^{N+1})$$

$$\cdot K_0^c [C_0, C_0] = \underbrace{\sqrt{Z_{K^c}} Z_c}_{Z_{K^c} Z_c = \# = Z_A Z_c} K^c \cdot [C, C]$$

$$Z_{K^c} Z_c = \# = Z_A Z_c \quad \rightarrow \quad \sqrt{Z_A} Z_c$$

Same as above

$$\downarrow$$

$$= \text{old} + \hbar^N (\sqrt{Z_A^{(N)}} + Z_c^{(N)}) K^c \cdot [C, C] + O(\hbar^{N+1})$$