Hermiticity of gauge fixed system

In real time, the Lagrangian of the gauge fixed Yang-Mills theory is

$$\tilde{\mathcal{L}} = -\frac{1}{4e^2} F^{\mu\nu} \cdot F_{\mu\nu} + \frac{e^2 \xi}{2} B^2 - B \cdot \partial^\mu A_\mu - i\bar{c} \cdot \partial^\mu D_\mu c$$

and the BRST transformation is

$$\delta_B A_\mu = D_\mu c, \quad \delta_B B = 0, \quad \delta_B \bar{c} = iB, \quad \delta_B c = -\frac{1}{2}[c, c]$$

We would like to propose that the fields have the following reality/hermiticity

$$(*) \quad A_\mu^* = A_\mu, \quad B^* = B, \quad \bar{c}^* = \bar{c}, \quad c^* = c$$

① Show that $\tilde{\mathcal{L}}$ is real under $(*)$, $\mathcal{L}^* = \mathcal{L}$

② The Noether charge Q_B of the BRST symmetry (= BRST charge) satisfy the Ward identity

$$\delta_B \mathcal{O}_b = i[Q_B, \mathcal{O}_b] \quad \text{if } \mathcal{O}_b \text{ is bosonic}$$

$$\delta_B \mathcal{O}_f = i\{Q_B, \mathcal{O}_f\} \quad \text{if } \mathcal{O}_f \text{ is fermionic}$$

Note that $(i[A,B])^\dagger = i[A^\dagger, B^\dagger]$

$$(i\{A,B\})^\dagger = -i\{A^\dagger, B^\dagger\}.$$

Therefore $(\delta_B \mathcal{O}_b)^\dagger = i[Q_B^\dagger, \mathcal{O}_b]$

$$(\delta_B \mathcal{O}_f)^\dagger = -i\{Q_B^\dagger, \mathcal{O}_f\}$$

Check that under (*), or

$$A_\mu^\dagger = A_\mu, \quad B^\dagger = B, \quad \bar{c}^\dagger = \bar{c}, \quad c^\dagger = c$$

in the operator version, the BRST transformation is compatible with hermiticity of the BRST charge:

$$Q_B^\dagger = Q_B.$$