Computation in dimensional regularization

Here, we use dimensional regularization in the computation of one-loop amplitudes in QED.

Let us start from general comments.

Gamma matrices

$$\{\gamma^\mu, \gamma^\nu\} = -2\delta^{\mu\nu}$$

$$\gamma^\mu \gamma_\mu = \frac{1}{2}\delta_{\mu\nu}\{\gamma^\mu, \gamma^\nu\} = -\delta_{\mu\nu}\delta^{\mu\nu} = -d$$

$$\mathrm{tr}(\text{odd number of } \gamma^\mu\text{'s}) = 0$$

$$\mathrm{tr}\,1 = ?$$

in $d=4$, $\mathrm{tr}\,1 = 4$

in a general even d, $\mathrm{tr}\,1 = 2^{d/2}$?

Which one should we take?

We could take any function $f(d)$ s.t. $f(4) = 4$.

The result depends on the choice, but, as we will see, the dependence disappears after renormalization.

So, for simplicity we take $\mathrm{tr}\,1 = 4$.

(We shall comment what will happen for another choice.)

Momentum integrals

We shall encounter memomentum integrals of the form

$$I_n(f) = \mu_{DR}^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{f(k)}{(k^2+m^2)((k-p)^2+\mu^2)^n}$$

$$J_n(f) = \mu_{DR}^{4-d} \int \frac{d^d k}{(2\pi)^d} \frac{f(k)}{((k+q)^2+m^2)(k^2+m^2)(k-p)^{2n}}$$

for a polynomial $f(k)$ of momenta k^μ's. We use

$$\frac{1}{AB^n} = \int_0^1 \frac{n x^{n-1} dx}{((1-x)A + xB)^{n+1}}$$

$$\frac{1}{ABC^n} = \int_{\triangle} \frac{(n+1)n\, z^{n-1}\, dy\, dz}{((1-y-z)A + yB + zC)^{n+2}}$$

where $\triangle := \{ (y,z) \in \mathbb{R}^2 \mid y \geq 0,\ z \geq 0,\ y+z \leq 1 \}$

Computation goes as follows

$$I_n(f) = \mu_{DR}^{4-d} \int \frac{d^d k}{(2\pi)^d} \int_0^1 \frac{n x^{n-1} dx\, f(k)}{(\underbrace{(1-x)(k^2+m^2) + x((k-p)^2+\mu^2)})^{n+1}}$$

$$k^2 - 2xpk + xp^2 + (1-x)m^2 + x\mu^2$$

$$= (\underbrace{k - xp}_{\ell})^2 + \underbrace{x(1-x)p^2 + (1-x)m^2 + x\mu^2}_{\Delta}$$

$$= \int_0^1 n x^{n-1} dx \, \mu_{DR}^{4-d} \int \frac{d^d \ell}{(2\pi)^d} \frac{f(\ell + xp)}{(\ell^2 + \Delta)^{n+1}}$$

- We expand $f(\ell + xp)$ in ℓ^μ's, drop odd power terms and replace even power terms by a function of ℓ^2

e.g. $\ell^\mu \ell^\nu \to \frac{1}{d} \delta^{\mu\nu} \ell^2$

$$f(\ell + xp) \to \tilde{f}(\ell^2, xp)$$

- Use $\displaystyle \int \frac{d^d \ell}{(2\pi)^d} F(\ell^2) = \frac{Vol(S^{d-1})}{(2\pi)^d} \int_0^\infty \ell^{d-1} d\ell \, F(\ell^2)$

$$= \frac{Vol(S^{d-1})}{2(2\pi)^d} \int_0^\infty \ell^{d-2} d\ell^2 \, F(\ell^2)$$

$$= \frac{1}{(4\pi)^{d/2} \Gamma(d/2)} \int_0^\infty t^{\frac{d}{2}-1} dt \, F(t)$$

$$= \frac{\mu_{DR}^{4-d}}{(4\pi)^{d/2} \Gamma(d/2)} \int_0^1 n x^{n-1} dx \int_0^\infty \frac{t^{\frac{d}{2}-1} dt \, \tilde{f}(t, xp)}{(t + \Delta)^{n+1}}$$

We may use $\displaystyle \int_0^\infty \frac{t^{p-1} dt}{(t+\Delta)^{p+q}} = \frac{B(p,q)}{\Delta^q} = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)\Delta^q}$

In this way we find

$$I_n(f) = \frac{1}{(4\pi)^2}\int_0^1 \frac{x^{n-1}\,dx}{(n-1)!}\left(\frac{4\pi\mu_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \frac{\Gamma(n+1-\frac{d}{2})}{\Delta^{n-1}}\,\hat{f}$$

where $\Delta = x(1-x)p^2 + (1-x)m^2 + x\mu^2$ and

$$\hat{1} = 1, \qquad \widehat{k^\mu} = x p^\mu$$

$$\widehat{k^\mu k^\nu} = \delta^{\mu\nu}\frac{\Delta}{2n-d} + x^2 p^\mu p^\nu$$

$$\widehat{k^\mu (k-p)^\nu} = \delta^{\mu\nu}\frac{\Delta}{2n-d} - x(1-x)p^\mu p^\nu$$

$$\widehat{(k-p)^\mu (k-p)^\nu} = \delta^{\mu\nu}\frac{\Delta}{2n-d} + (1-x)^2 p^\mu p^\nu$$

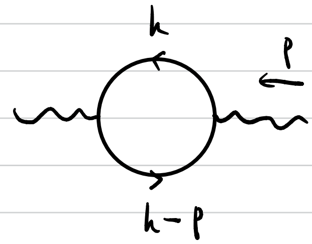
$$J_n(g) = \frac{1}{(4\pi)^2}\int_{\triangle} \frac{z^{n-1}\,dy\,dz}{(n-1)!}\left(\frac{4\pi\mu_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \frac{\Gamma(n+2-\frac{d}{2})}{\Delta^{n}}\,\hat{g}$$

where $\Delta = y(1-y)q^2 + z(1-z)p^2 + 2yzqp + (1-z)m^2$ and

$$\hat{1} = 1, \qquad \widehat{k^\mu} = -yq^\mu + zp^\mu$$

$$\widehat{k^\mu k^\nu} = \delta^{\mu\nu}\frac{\Delta}{2n+2-d} + (-yq^\mu + zp^\mu)(-yq^\nu + zp^\nu)$$

$$\Pi_2^{\mu\nu}(p) = -e^2 \mu_{DR}^{4-d} \int \frac{d^dk}{(2\pi)^d} \, \mathrm{tr}\left(\gamma^\mu \frac{1}{-\not{k}+m} \gamma^\nu \frac{1}{-(\not{k}-\not{p})+m} \right)$$



$$= -e^2 \mu_{DR}^{4-d} \int \frac{d^dk}{(2\pi)^d} \frac{\mathrm{tr}\left(\gamma^\mu(\not{k}+m)\gamma^\nu(\not{k}-\not{p}+m)\right)}{(k^2+m^2)((k-p)^2+m^2)}$$

numerator $= \mathrm{tr}(\gamma^\mu\gamma^\nu)m^2 + \mathrm{tr}(\gamma^\mu \not{k} \gamma^\nu (\not{k}-\not{p}))$

- $\mathrm{tr}(\gamma^\mu\gamma^\nu) = -\delta^{\mu\nu}\,\mathrm{tr}(1)$
- $\mathrm{tr}(\gamma^\mu\gamma^\rho\gamma^\nu\gamma^\lambda) = (\delta^{\mu\rho}\delta^{\nu\lambda} - \delta^{\mu\nu}\delta^{\rho\lambda} + \delta^{\mu\lambda}\delta^{\rho\nu})\,\mathrm{tr}(1)$

Let us use $\mathrm{tr}(1)=4$. then

$$= 4\left(-\delta^{\mu\nu}(m^2+k\cdot(k-p)) + k^\mu(k-p)^\nu + k^\nu(k-p)^\mu\right)$$

$$= -4e^2 \mu_{DR}^{4-d} \int \frac{d^dk}{(2\pi)^d} \frac{-\delta^{\mu\nu}(m^2+k\cdot(k-p)) + k^\mu(k-p)^\nu + k^\nu(k-p)^\mu}{(k^2+m^2)((k-p)^2+m^2)}$$

Rmk If we replace $\mathrm{tr}(1)=4 \to \mathrm{tr}(1)=2^{d/2} = 4\cdot 2^{\frac{d}{2}-2}$,

the effect is the same as $\mu_{DR}^2 \to \frac{1}{2}\mu_{DR}^2$.

Since the physics does not depend on the choice of μ_{DR},

the convention of $\mathrm{tr}(1)$ is irrelevant.

Thus, we find

$$\Pi_2^{\mu\nu}(p) = -4e^2 I_1 \Big(-\delta^{\mu\nu} \big(m^2 + \overbrace{k\cdot(k-p)}^{\delta_{\rho\lambda} k^\rho (k^\lambda - p^\lambda)} \big) + k^\mu (k-p)^\nu + k^\nu (k-p)^\mu \Big)$$

$$\left(\text{with } \mu = m \ ; \ \Delta = x(1-x)p^2 + m^2 \right)$$

$$= -\frac{4e^2}{(4\pi)^2} \int_0^1 dx \left(\frac{4\pi \mu_{DR}^2}{\Delta} \right)^{2-\frac{d}{2}} \Gamma\!\left(2-\frac{d}{2}\right) \Big[-\delta^{\mu\nu} \cancel{m^2}$$

$$\underbrace{-\delta^{\mu\nu} \delta_{\rho\lambda} \left(\delta^{\rho\lambda} \frac{\Delta}{2-d} - x(1-x) p^\rho p^\lambda \right)}_{-\delta^{\mu\nu} d \frac{\Delta}{2-d} + \delta^{\mu\nu} x(1-x)p^2} + 2\left(\delta^{\mu\nu} \frac{\Delta}{2-d} - x(1-x) p^\mu p^\nu \right) \Big]$$

$$\delta^{\mu\nu}\Delta$$

$$= \delta^{\mu\nu} \left(x(1-x)p^2 + \cancel{m^2} \right)$$

$$= -\frac{4e^2}{(4\pi)^2} \int_0^1 dx \left(\frac{4\pi \mu_{DR}^2}{\Delta} \right)^{2-\frac{d}{2}} \Gamma\!\left(2-\frac{d}{2}\right) \left[2\delta^{\mu\nu} x(1-x)p^2 - 2x(1-x) p^\mu p^\nu \right]$$

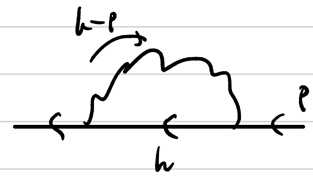
$$= -\frac{8e^2}{(4\pi)^2} \int_0^1 dx\, x(1-x) \underbrace{\left(\frac{4\pi \mu_{DR}^2}{\Delta} \right)^{2-\frac{d}{2}} \Gamma\!\left(2-\frac{d}{2}\right)} \left(\delta^{\mu\nu} p^2 - p^\mu p^\nu \right)$$

$$\left(\frac{4\pi \mu_{DR}^2}{\Delta} \right)^{\frac{\epsilon}{2}} \Gamma\!\left(\frac{\epsilon}{2}\right) = \left(\frac{2}{\epsilon} - \gamma + \log\left(\frac{4\pi \mu_{DR}^2}{\Delta} \right) + O(\epsilon) \right)$$

$$= \left(\delta^{\mu\nu} p^2 - p^\mu p^\nu \right) \Pi_2(p) \quad \text{where}$$

$$\Pi_2(p) = -\frac{8e^2}{(4\pi)^2} \int_0^1 dx\, x(1-x) \left(\frac{2}{\epsilon} - \gamma + \log\left(\frac{4\pi \mu_{DR}^2}{m^2 + x(1-x)p^2} \right) + O(\epsilon) \right).$$

$$\Sigma_2(p) = -e^2 \mu_{DR}^{4-d}\int \frac{d^dk}{(2\pi)^d}\,\gamma^\mu \underbrace{\frac{1}{-\not k+m}}_{\frac{\not k+m}{k^2+m^2}}\, \gamma^\nu \left(\frac{\delta_{\mu\nu}}{(k-p)^2} + (\xi-1)\frac{(k-p)_\mu(k-p)_\nu}{(k-p)^4}\right)$$



- $$\gamma^\mu(\not k+m)\gamma_\mu = \underbrace{\{\gamma^\mu,\not k\}}_{-2k^\mu}\gamma_\mu - \not k\,\underbrace{\gamma^\mu\gamma_\mu}_{-d} + m\,\underbrace{\gamma^\mu\gamma_\mu}_{-d}$$

$$= (d-2)\not k - dm$$

- $$(\not k-\not p)(\not k+m)(\not k-\not p) = (\not k-\not p)(\not k-\not p+\not p+m)(\not k-\not p)$$

$$= -(k-p)^2(\not k-\not p) + \underbrace{(\not k-\not p)\not p(\not k-\not p)} - m(k-p)^2$$

$$-2(k-p)\cdot p\,(\not k-\not p) + \not p\,(k-p)^2$$

$$= (k-p)^2(-\not k+2\not p-m) - 2(k-p)\cdot p\,(\not k-\not p)$$

$$= e^2\mu_{DR}^{4-d}\int\frac{d^dk}{(2\pi)^d}\frac{1}{k^2+m^2}\left[\frac{-(d-2)\not k+dm}{(k-p)^2} + (1-\xi)\left(\frac{-\not k+2\not p-m}{(k-p)^2} - \frac{2(k-p)\cdot p\,(\not k-\not p)}{(k-p)^4}\right)\right]$$

$$= e^2 I_1\left(-(d-2)\not k + dm + (1-\xi)(-\not k+2\not p-m)\right)$$

$$+ e^2 I_2\left(-2(1-\xi)(k-p)^\mu p_\mu (k-p)^\nu\gamma_\nu\right)$$

(with $\mu=0$; $\Delta = x(1-x)p^2+(1-x)m^2$)

$$= \frac{e^2}{(4\pi)^2}\int_0^1 dx\left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \Gamma\left(2-\frac{d}{2}\right)\left[-(d-2)\,x\not{p} + d\,m + (1-\xi)(-x\not{p}+2\not{p}-m)\right]$$

$$+ \frac{e^2}{(4\pi)^2}\int_0^1 x\,dx\left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \frac{\Gamma\left(3-\frac{d}{2}\right)}{\Delta}\left[-2(1-\xi)\,p_\mu \gamma_\nu\left(\delta^{\mu\nu}\frac{\Delta}{4-d} + (1-x)^2 p^\mu p^\nu\right)\right]$$

- $\dfrac{\Gamma\left(3-\frac{d}{2}\right)}{\Delta}\dfrac{2\Delta}{4-d} = \left(2-\frac{d}{2}\right)\Gamma\left(2-\frac{d}{2}\right)\dfrac{2}{4-d} = \Gamma\left(2-\frac{d}{2}\right)$

$\therefore$ A part of the second line joins into the first line

as $\quad \cdots\,\Gamma\left(2-\frac{d}{2}\right)\left[\,x\times\left(-(1-\xi)\not{p}\right)\,\right]$

$$= \frac{e^2}{(4\pi)^2}\int_0^1 dx\left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \left\{\Gamma\left(2-\frac{d}{2}\right)\left[-(d-2)\,x\not{p} + d\,m + (1-\xi)(-2x\not{p}+2\not{p}-m)\right]\right.$$

$$\left. + \frac{\Gamma\left(3-\frac{d}{2}\right)}{\Delta}\left[-2(1-\xi)\,p^2\not{p}\,x(1-x)^2\right]\right\}$$

- $\Gamma\left(2-\frac{d}{2}\right)(d-2) = \Gamma\left(2-\frac{d}{2}\right)\left(2-2\left(2-\frac{d}{2}\right)\right)$

$\quad\quad = \Gamma\left(2-\frac{d}{2}\right)\cdot 2 + \Gamma\left(3-\frac{d}{2}\right)\cdot(-2)$

- $\dfrac{x(1-x)^2p^2}{\Delta} = \dfrac{(1-x)xp^2}{m^2+xp^2} = 1-x - \dfrac{(1-x)m^2}{m^2+xp^2}$

$$
= \frac{e^2}{(4\pi)^2}\int_0^1 dx \left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \Bigg\{ \Gamma\left(2-\frac{d}{2}\right)\Big[-2x\not{p} + dm + (1-\xi)\big(\overbrace{-2x\not{p}+2\not{p}}^{2(1-x)\not{p}}-m\big)\Big]
$$

$$
+\Gamma\left(3-\frac{d}{2}\right)\left[2x\not{p} - 2(1-\xi)(1-x)\not{p} + 2(1-\xi)\frac{(1-x)m^2}{m^2+xp^2}\right]\Bigg\}
$$

$$
= A_2(p^2)\not{p} + B_2(p^2)m \qquad \text{where}
$$

$$
A_2(p^2) = \frac{e^2}{(4\pi)^2}\int_0^1 dx \left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \Bigg\{\left(\Gamma\left(2-\frac{d}{2}\right)-\Gamma\left(3-\frac{d}{2}\right)\right)\big(-2x+2(1-\xi)(1-x)\big)
$$

$$
+\Gamma\left(3-\frac{d}{2}\right)2(1-\xi)\frac{(1-x)m^2}{m^2+xp^2}\Bigg\}
$$

$$
= \frac{e^2}{(4\pi)^2}\int_0^1 dx \Bigg[\left(\frac{2}{\epsilon}-\gamma-1+\log\left(\frac{4\pi M_{DR}^2}{\Delta}\right)\right)\big(-2x+2(1-\xi)(1-x)\big)
$$

$$
+ 2(1-\xi)\frac{(1-x)m^2}{m^2+xp^2} + O(\epsilon)\Bigg]
$$

$$
B_2(p^2) = \frac{e^2}{(4\pi)^2}\int_0^1 dx \left(\frac{4\pi M_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \Gamma\left(2-\frac{d}{2}\right)(d+\xi-1)
$$

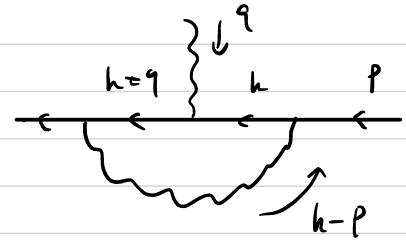
$$
= \frac{e^2}{(4\pi)^2}\int_0^1 dx \left[\left(\frac{2}{\epsilon}-\gamma+\log\left(\frac{4\pi M_{DR}^2}{\Delta}\right)\right)(3+\xi)-2+O(\epsilon)\right]
$$

For $\Gamma_2^\mu(p,q)$, I am confident to make mistakes for $\xi \neq 1$.

Thus, I do the computation only for $\xi = 1$.

$$\Gamma_2^\mu(p,q) = e^2 \mu_{DR}^{4-d} \int \frac{d^dk}{(2\pi)^d}\, \gamma^\rho \frac{1}{-(\not{k}+\not{q})+m}\, \gamma^\mu \frac{1}{-\not{k}+m}\, \gamma^\lambda \frac{\delta_{\rho\lambda}}{(k-p)^2}$$

$$= e^2 \mu_{DR}^{4-d} \int \frac{d^dk}{(2\pi)^d}\, \frac{\gamma^\rho(\not{k}+\not{q}+m)\gamma^\mu(\not{k}+m)\gamma_\rho}{((k+q)^2+m^2)(k^2+m^2)(k-p)^2}$$



$$\gamma^\rho\gamma^\mu\gamma_\rho = \underbrace{\{\gamma^\rho,\gamma^\mu\}}_{-2\delta^{\rho\mu}}\gamma_\rho - \gamma^\mu\underbrace{\gamma^\rho\gamma_\rho}_{-d} = 2\gamma^\mu + (d-4)\gamma^\mu$$

$$\gamma^\rho\gamma^\nu\gamma^\mu\gamma_\rho = \underbrace{\{\gamma^\rho,\gamma^\nu\}}\gamma^\mu\gamma_\rho - \gamma^\nu\underbrace{\{\gamma^\rho,\gamma^\mu\}}\gamma_\rho + \gamma^\nu\gamma^\mu\underbrace{\gamma^\rho\gamma_\rho}_{-d}$$

$$= -2\delta^{\rho\nu}\gamma^\mu\gamma_\rho + 2\delta^{\rho\mu}\gamma^\nu\gamma_\rho - d\,\gamma^\nu\gamma^\mu$$

$$= -2\gamma^\mu\gamma^\nu + (2-d)\gamma^\nu\gamma^\mu = 4\delta^{\mu\nu} - (d-4)\gamma^\nu\gamma^\mu$$

$$\gamma^\rho\gamma^\nu\gamma^\mu\gamma^\sigma\gamma_\rho = \gamma^\rho\gamma^\nu\gamma^\mu\underbrace{\{\gamma_\rho,\gamma^\sigma\}}_{-2\delta^\sigma_\rho} - \underbrace{\gamma^\rho\gamma^\nu\gamma^\mu\gamma_\rho}_{4\delta^{\mu\nu}-(d-4)\gamma^\nu\gamma^\mu}\gamma^\sigma$$

$$= -2\gamma^\sigma\underbrace{\gamma^\nu\gamma^\mu} - 4\delta^{\mu\nu}\gamma^\sigma + (d-4)\gamma^\nu\gamma^\mu\gamma^\sigma$$

$$= 2\gamma^\sigma\gamma^\mu\gamma^\nu + (d-4)\gamma^\nu\gamma^\mu\gamma^\sigma$$

$$\therefore \text{ numerator} = 2\Big[\not{k}\gamma^\mu(\not{k}+\not{q}) + 2m(2k+q)^\mu + m^2\gamma^\mu + \frac{d-4}{2}(\not{k}+\not{q}-m)\gamma^\mu(\not{k}-m) \Big]$$

$$\Gamma_2^\mu(p,q) = 2e^2 J_1 \Big(\not{k}\gamma^\mu(\not{k}+\not{q}) + 2m(2k+q)^\mu + m^2\gamma^\mu + \frac{d-4}{2}(\not{k}+\not{q}-m)\gamma^\mu(\not{k}-m) \Big)$$

$$= \frac{2e^2}{(4\pi)^2}\int_{\triangle} dy\, dz \left(\frac{4\pi\mu_{DR}^2}{\Delta}\right)^{2-\frac{d}{2}} \frac{\Gamma(3-\frac{d}{2})}{\Delta}\Big[$$

$$\gamma_\rho\gamma^\mu\gamma_\lambda\left(\frac{\delta^{\rho\lambda}\Delta}{4-d} + (-yq+zp)^\rho((1-y)q+zp)^\lambda + \frac{d-4}{2}\left(\frac{\delta^{\rho\lambda}\Delta}{4-d}\right)\right)$$

$$+2m\left(2(-yq+zp)+q\right)^\mu + m^2\gamma^\mu \Big] + O(d-4)$$

$$\left(\text{where } \Delta = y(1-y)q^2 + z(1-z)p^2 + 2yz\,qp + (1-z)m^2\right)$$

- $$\gamma_\rho\gamma^\mu\gamma_\lambda\left(\frac{\delta^{\rho\lambda}\Delta}{4-d} + \frac{d-4}{2}\frac{\delta^{\rho\lambda}\Delta}{4-d}\right) = \underbrace{\gamma_\rho\gamma^\mu\gamma^\rho}_{2\gamma^\mu+(d-4)\gamma^\mu}\left(\frac{1}{4-d}-\frac{1}{2}\right)\Delta$$

$$= \left(\frac{2}{4-d} - 2\right)\gamma^\mu\Delta \quad + O(d-4)$$

- Use $\dfrac{\Gamma(3-\frac{d}{2})}{\Delta}\dfrac{2\Delta}{4-d} = \Gamma(2-\frac{d}{2})$ again.

$$= \frac{2e^2}{(4\pi)^2} \int_{\triangle} dy\, dz \left(\frac{4\pi M_{DR}^2}{\Delta} \right)^{2-\frac{d}{2}} \Bigg\{ \Gamma\left(2-\tfrac{d}{2}\right) \gamma^\mu - 2\, \Gamma\left(3-\tfrac{d}{2}\right) \gamma^\mu$$

$$+ \frac{\Gamma\left(3-\frac{d}{2}\right)}{\Delta} \Bigg[\underbrace{\boxed{\begin{aligned} &(-y \not{q} + z \not{p}) \gamma^\mu ((1-y) \not{q} + z \not{p}) \\ &\quad + 2m \left(2(-y q + z p) + q\right)^\mu + m^2 \gamma^\mu \end{aligned}}}_{=:\, X^\mu} \Bigg] \Bigg\} + O(d-4)$$

$$= \frac{2e^2}{(4\pi)^2} \int_{\triangle} dy\, dz \left[\left(\frac{2}{\epsilon} - \gamma - 2 + \log\left(\frac{4\pi M_{DR}^2}{\Delta} \right) \right) \gamma^\mu + \frac{X^\mu}{\Delta} \right]$$

$$+ O(\epsilon).$$

After renormalization, either "on shell" or "another (μ)", all the results from dimensional regularization match with those from the Pauli-Villars regularization.