# Canonical quantization of Maxwell theory

From Lecture 5: Maxwell theory in $d$-dimensions, with variable $A_\mu$ and action $S[A] = \int d^dx \left(-\frac{1}{4e^2} F^{\mu\nu} F_{\mu\nu}\right)$ is equivalent to the theory of variable $(\mathbb{A}, \mathbb{E}; A_0)$ with action

$$S[\mathbb{A}, \mathbb{E}; A_0] = \int d^dx \left( \sum_i E_i F_{0i} - \frac{e^2}{2} \sum_i E_i^2 - \frac{1}{2e^2} \sum_{i<j} F_{ij}^2 \right)$$

$$= \int d^dx \left( \sum_i E_i \dot{A}_i - \frac{e^2}{2} \sum_i E_i^2 - \frac{1}{2e^2} \sum_{i<j} F_{ij}^2 + A_0 \sum_i \partial_i E_i \right).$$

Note that EOM for $E_i$ is $\quad E_i = \frac{1}{e^2} F_{0i}$.

Looking at the second expression for $S[\mathbb{A}, \mathbb{E}; A_0]$, we can view this as a Hamiltonian system with conjugate variables $(\mathbb{A}, \mathbb{E})$ with 1st class constraint $\nabla \cdot \mathbb{E} = 0$.

We may take Coulomb gauge $\nabla \cdot \mathbb{A} = 0$ for a slice.

Since $\{\nabla \cdot \mathbb{A}(x), \nabla \cdot \mathbb{A}(y)\} = 0$, we may quantize the system with conjugate variables $(\mathbb{A}, \mathbb{E})$ obeying $\nabla \cdot \mathbb{E} = \nabla \cdot \mathbb{A} = 0$.

<u>Exercise</u> ① Carry out this quantization and find the space of states.

② Compute $\langle 0 | T E_i(x) E_j(y) | 0 \rangle$, $\langle 0 | T E_i(x) F_{jk}(y) | 0 \rangle$, $\langle 0 | T F_{ij}(x) F_{kl}(y) | 0 \rangle$ and compare the result with $\langle F_{\mu\nu}(x) F_{\rho\lambda}(y) \rangle = \langle 0 | T F_{\mu\nu}(x) F_{\rho\lambda}(y) | 0 \rangle$ obtained in Lecture 6.