Back to the case of gauge theory :

$$M \rightsquigarrow \mathcal{M} = \{ (A_\mu(x), \varphi(x), \psi(x), \cdots) \text{ field config.} \}$$

$$G \rightsquigarrow \mathcal{G} = \{ g(x) \mid G\text{-valued function} \}$$

$$\mathfrak{g} \rightsquigarrow \mathrm{Lie}(\mathcal{G}) = \{ \epsilon(x) \mid \mathfrak{g}\text{-valued function} \}$$

As gauge fixing function, we can take

$$\chi[A](x) = \partial^\mu A_\mu(x) \qquad \text{Lorentz gauge}$$

$$\delta_\epsilon \chi[A](x) = \partial^\mu D_\mu \epsilon(x)$$

gauge fixed Lagrangian

$$\tilde{\mathcal{L}}_E = \mathcal{L}_E + \frac{\xi}{2} B^2 - i B \cdot \partial^\mu A_\mu + \bar{c} \cdot \partial^\mu D_\mu c$$

Inverse Wick rotation to real time

(with $B \to -iB$, $\bar{c} \to -i\bar{c}$ & $\xi \to e^2 \xi$ for convenience)

$$\tilde{\mathcal{L}} = \mathcal{L} + \frac{e^2 \xi}{2} B^2 - B \cdot \partial^\mu A_\mu - i \bar{c}\, \partial^\mu D_\mu c$$

Correction

In the original note, the inverse Wick rotation was

$$B \to iB, \quad \bar{C} \to i\bar{C}, \quad \ldots$$

But it must be corrected to

$$B \to -iB, \quad \bar{C} \to -i\bar{C}, \quad \ldots$$

Note that inverse Wick rotation does

$$\eta^{\mu\nu} \to -\delta^{\mu\nu}. \qquad (*)$$

As a consequence, for example, inv. W. rot does

$$-iB \cdot \partial^\mu A_\mu \to -i(-iB)\cdot(-\partial^\mu A_\mu) = B\cdot\partial^\mu A_\mu$$

$$\bar{C}\cdot\partial^\mu D_\mu C \to -i\bar{C}\cdot(-\partial^\mu D_\mu C) = i\bar{C}\cdot\partial^\mu D_\mu C$$

and the real time Lagrangian (obtained by $\tilde{\mathcal{L}}_E \to -\tilde{\mathcal{L}}$) is $\tilde{\mathcal{L}}$ as in the note (before & after the correction).

The problem in the old note was that the minus sign in $(*)$ was forgotten.

End of correction.