Brane Tilings, Algae and Quiver Gauge Theories
− with Application to Homological Mirror Symmetry−

Masahito Yamazaki
Univ. of Tokyo (Hongo)

In collaboration with Kazushi Ueda (Osaka)

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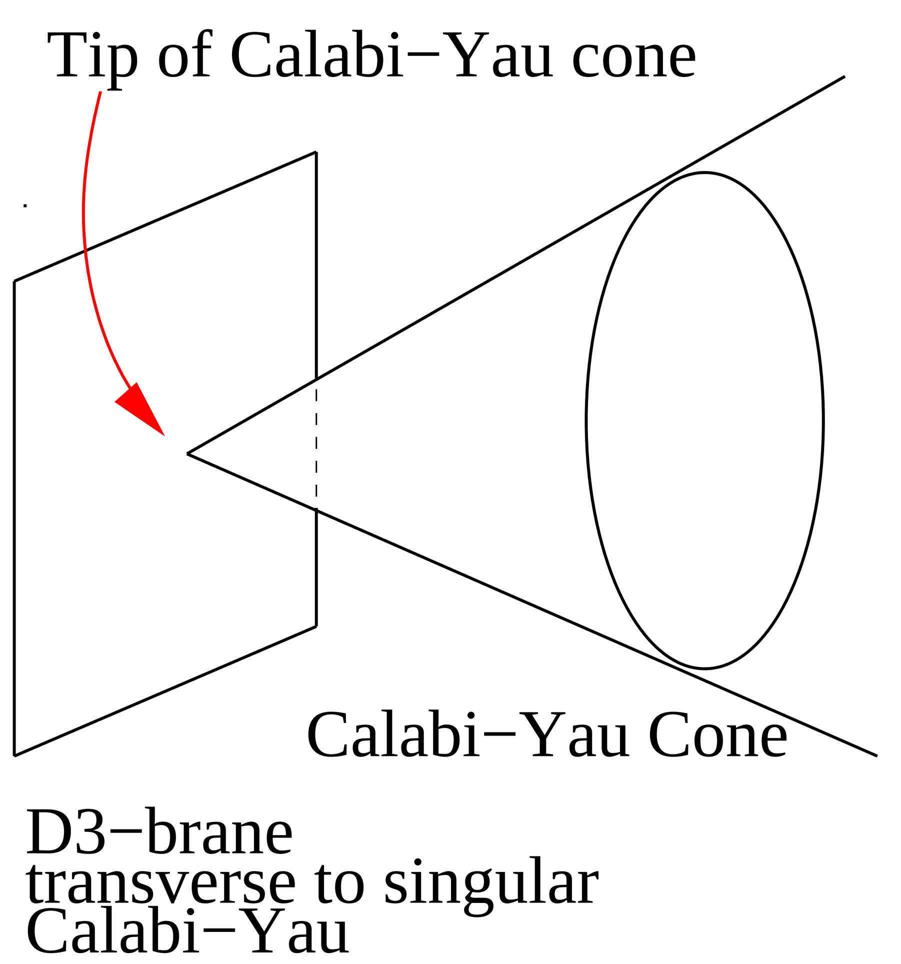
math.AG/0606548

1. Introduction

Geometry (Toric Diagram)

D3−brane probing singular Toric CY



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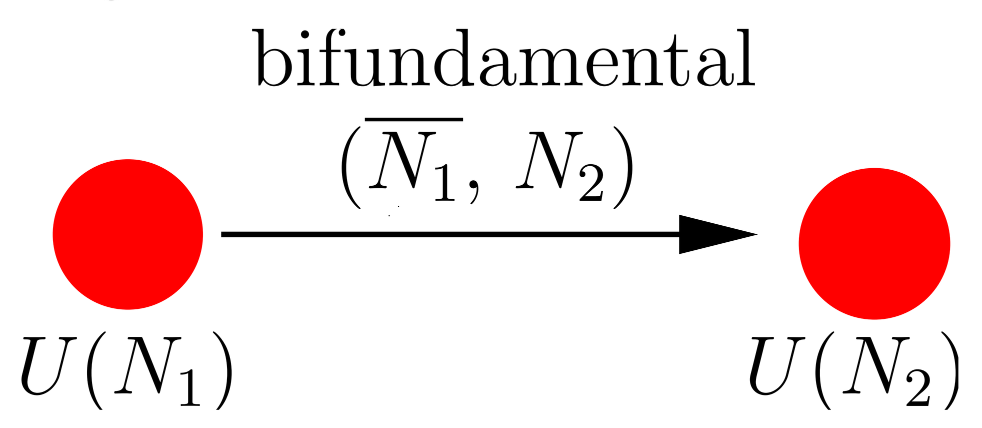
Any Algorithm?
cf. AdS/CFT
Z−min=a−max

Gauge Theory (Quiver)

N=1 Quiver Gauge Theory

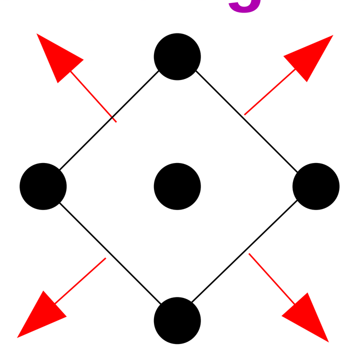
Quiver
node: gauge group
edge: bidundamentals



2. Brane Tilings and Quivers [Hanany−Vegh]

Example: Hirzebruch 0

Toric diagram



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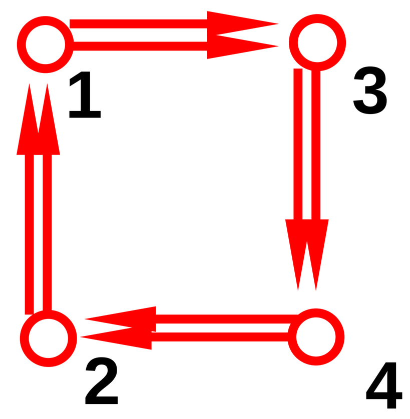
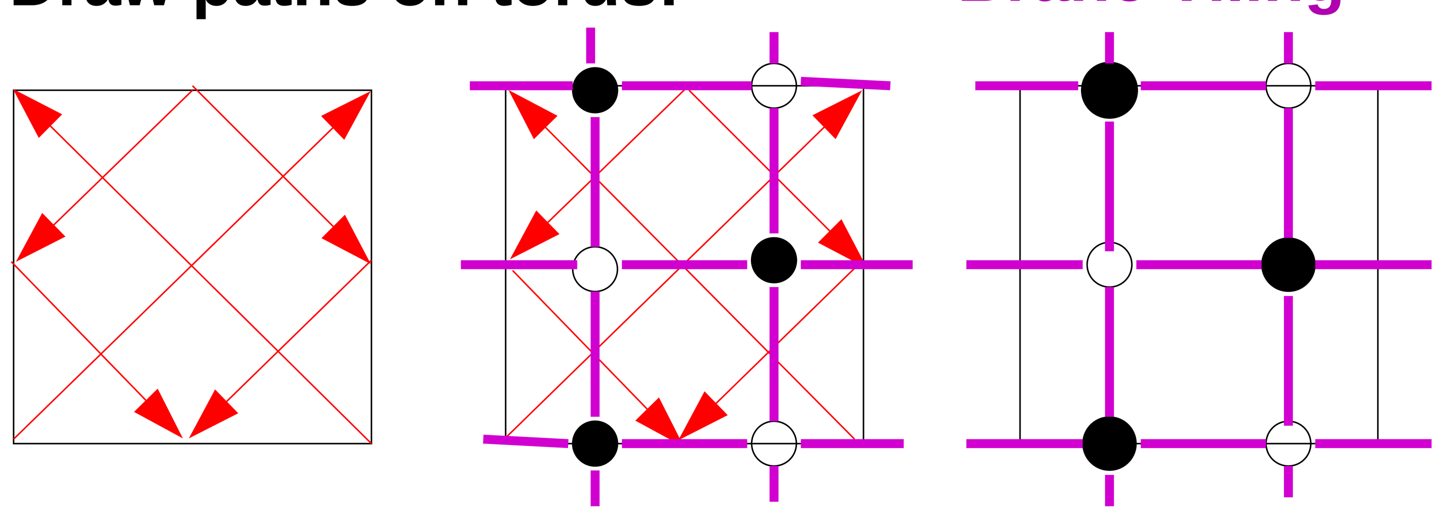
Quiver, Superpotential



$$W = X_{21}X_{13}X_{34}X_{42} - Y_{34}X_{42}Y_{21}X_{13} + X_{21}Y_{13}X_{34}Y_{42} - Y_{13}Y_{34}Y_{42}Y_{21}$$

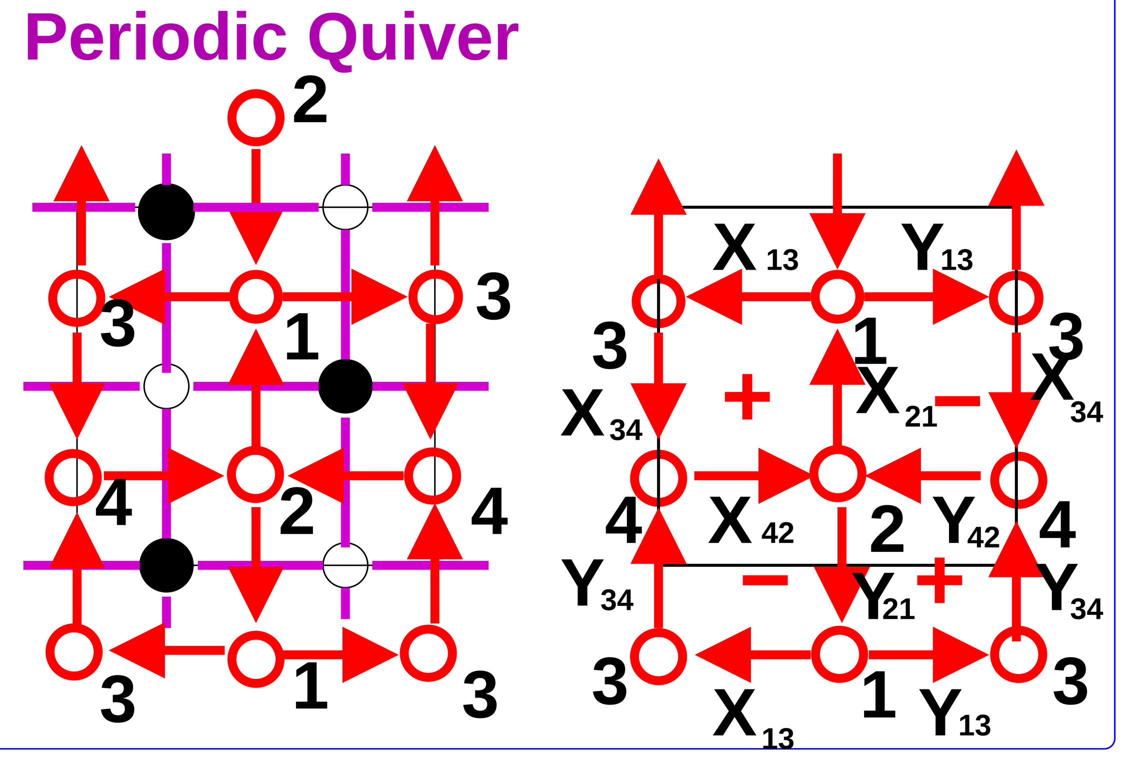
Draw paths on torus: **Brane Tiling**



Place white (black) vertex to each face
if arrows go counterclockwise (clockwise)

Dual

Periodic Quiver



Theorem [Ueda−Y]

The above algorithm is correct when toric diagram is triangle or parallelogram
that is, (Calabi−Yau)=(vacuum moduli of gauge theory): $D^b \operatorname{coh} Y \cong D^b \operatorname{mod} \mathcal{P}/\mathcal{I}$
where Y is crepant resolution of CY, $\mathcal{P}/\mathcal{I}$ **is quiver with relations**

3. Brane Tilings as D5/NS5−System

Take T−dual twice, then we have D5/NS5−system

	0	1	2	3	4	5	6	7	8	9
D5	○	○	○	○		○		○		
NS5	○	○	○	○		Σ				

Here $\Sigma(x,y)$ **is a Newton Polynomial of toric diagram**

where $x = \exp(x_4 + ix_5), y = \exp(x_6 + ix_7)$

The Newton Polynomial of lattice polygon $\Delta \in \mathbb{Z}^2$ is

$$W(x,y) = \sum_{(i,j)\in\Delta} c_{(i,j)} x^i y^j$$