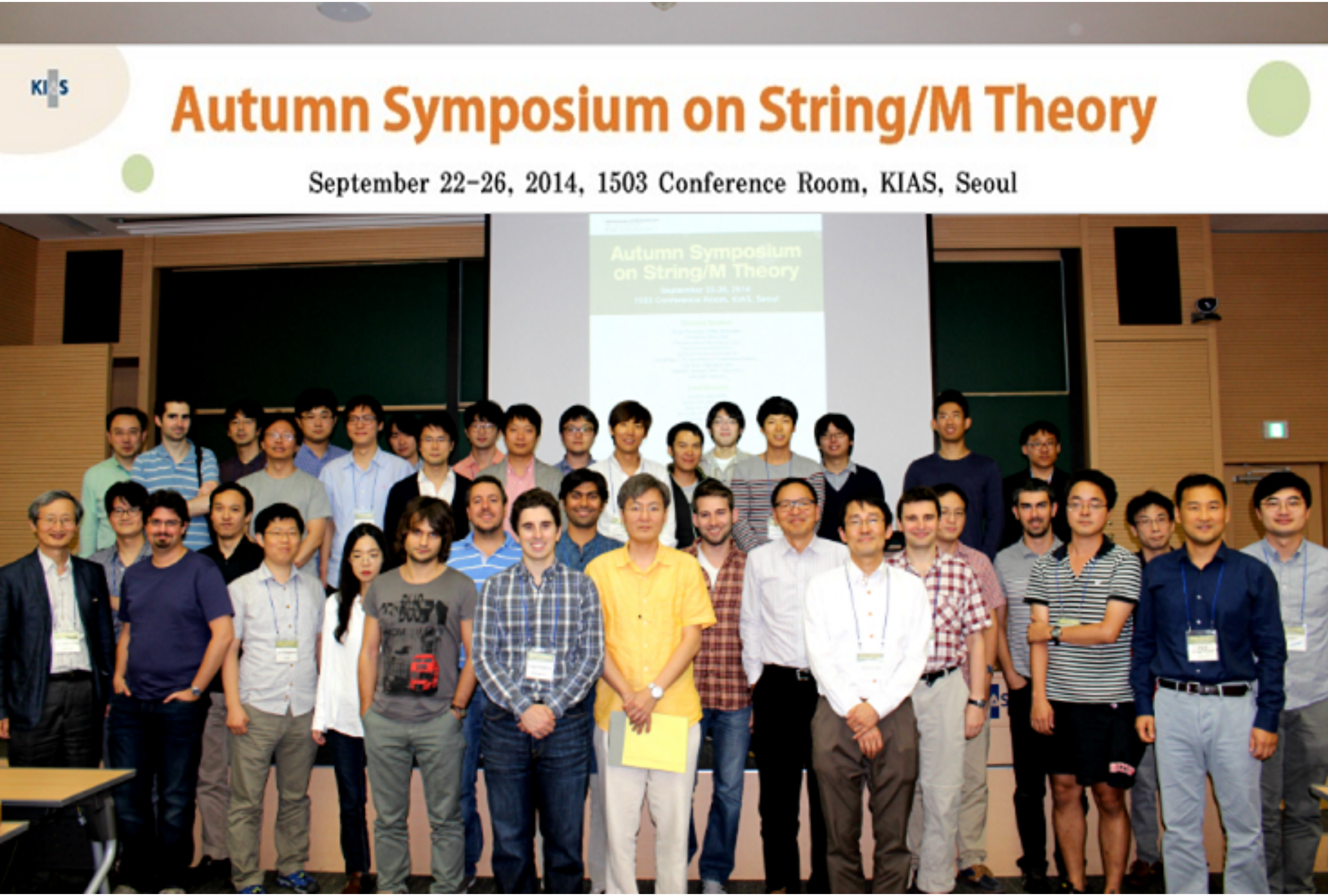
A New Class of Integrable Field Theories

Masahito Yamazaki
(Kavli IPMU, Tokyo)

Sep/23/2019
KIAS-YITP 2019

2007
2007
2008
2013
2014
2014
2017
2017
2017
2019


Exact Results in String / M Theory
Pre Strings 2013 Workshop
Date : June 17 - 21, 2013
Place : KIAS 1st fl, Meeting Room
KIAS 고등과학원
TOTEM


Autumn Symposium on String/M Theory
September 22-26, 2014, 1503 Conference Room, KIAS, Seoul
Autumn Symposium on String/M Theory

refs

Costello - MY

Part III 1908

refs

Costello - Witten - MY

Part I 1709

Part II 1802

integrable lattice models

RTT

Costello - MY

classical

Part III 1908

integrable field theories

Part IV to appear

quantum

MY 1904

T-dual

Setup

4d CS

$$S = \frac{1}{2\pi\hbar} \int_{\mathbb{C} \times C} \omega \wedge CS(A)$$

4d CS

$$S = \frac{1}{2\pi\hbar} \int_{\mathbb{C}_{w,\bar{w}} \times C_{z,\bar{z}}} \omega \wedge CS(A)$$

dimensionful expansion parameter

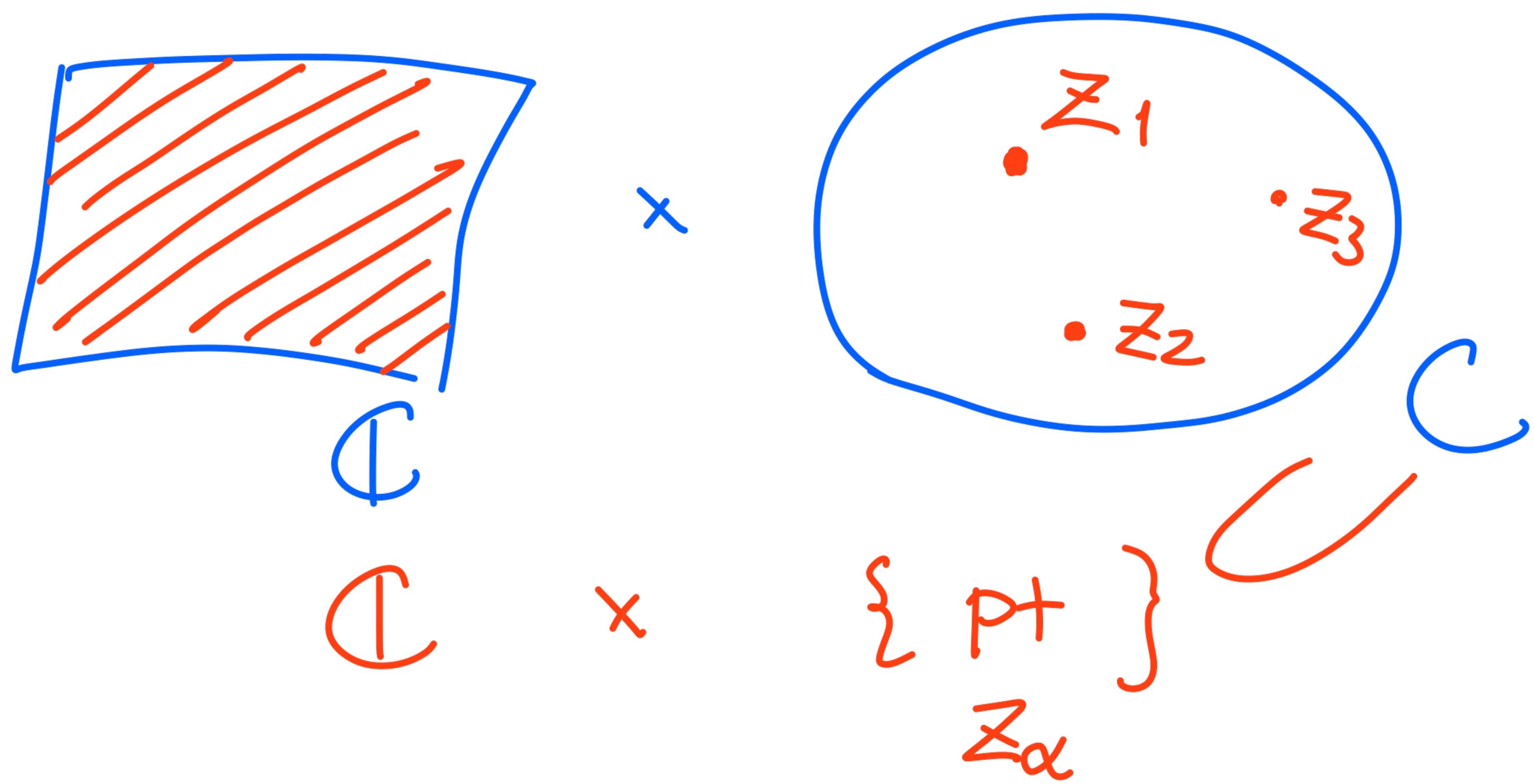
spectral curve (holomorphic z)

holomorphic 1-form on C (locally $\omega = dz$)

CS 3-form $\mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A^3\right)$

$$A = A_w\, dw + A_{\bar{w}}\, d\bar{w} + A_{\bar{z}}\, d\bar{z}$$

surface defects



Integrability

* 4d bulk e.o.m. (away from surface defects)

$$F_{w\bar{w}} = F_{w\bar{z}} = F_{\bar{w}\bar{z}} = 0$$

* 4d bulk e.o.m. (away from surface defects)

$$F_{w\bar{w}} = F_{w\bar{z}} = F_{\bar{w}\bar{z}} = 0$$

* choose the gauge $A_{\bar{z}} = 0$

$$F_{w\bar{w}} = 0 \,, \quad \partial_{\bar{z}} A_w = \partial_{\bar{z}} A_{\bar{w}} = 0$$

* 4d bulk e.o.m. (away from surface defects)

$$F_{w\bar{w}} = F_{w\bar{z}} = F_{\bar{w}\bar{z}} = 0$$

* choose the gauge $A_{\bar{z}} = 0$

$$F_{w\bar{w}} = 0 , \quad \partial_{\bar{z}} A_w = \partial_{\bar{z}} A_{\bar{w}} = 0$$

* one-form on $\mathbb{C}_{w,\bar{w}}$

$$\mathcal{L}(z) := A_w(z, w, \bar{w})\, dw + A_{\bar{w}}(z, w, \bar{w})\, d\bar{w}$$

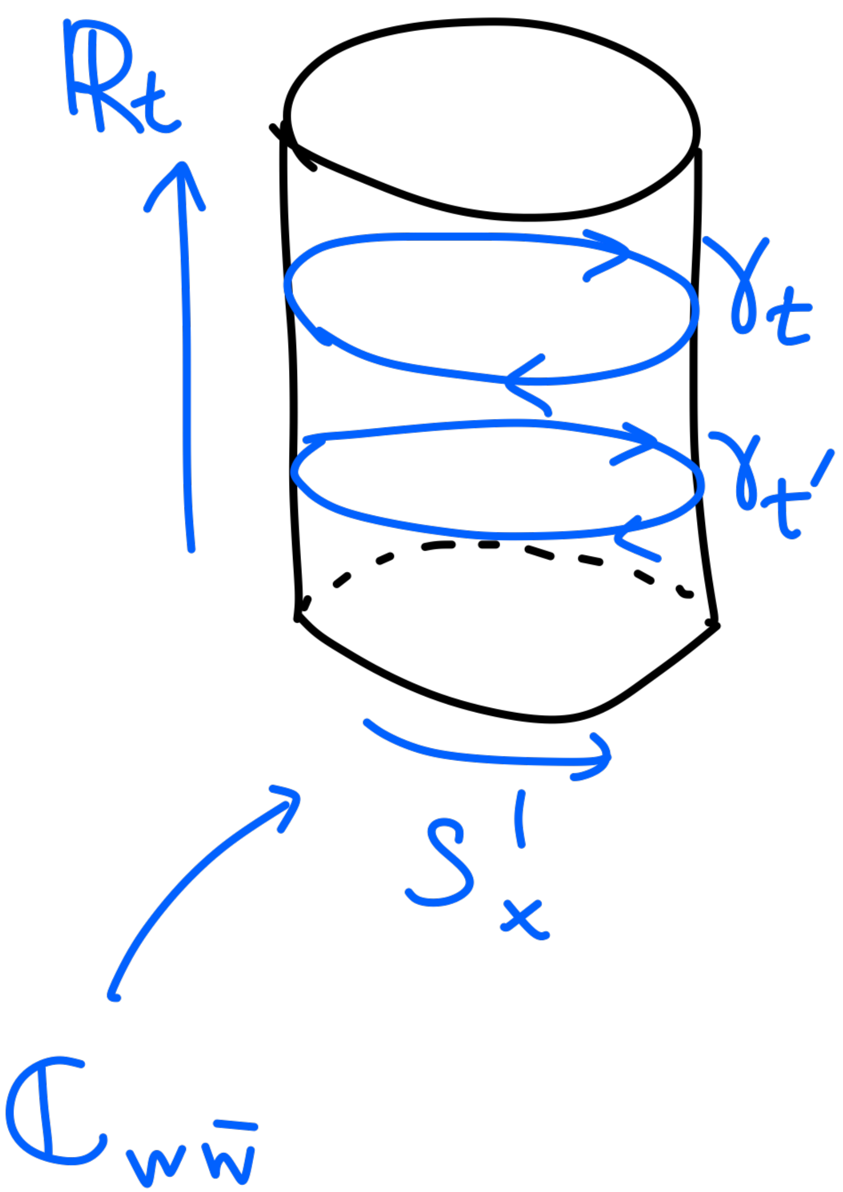
is flat

$$d\mathcal{L}(z) + \mathcal{L}(z) \wedge \mathcal{L}(z) = 0$$

* existence of hol. flat connection $\mathcal{L}(z)$:
  classical definition of integrability

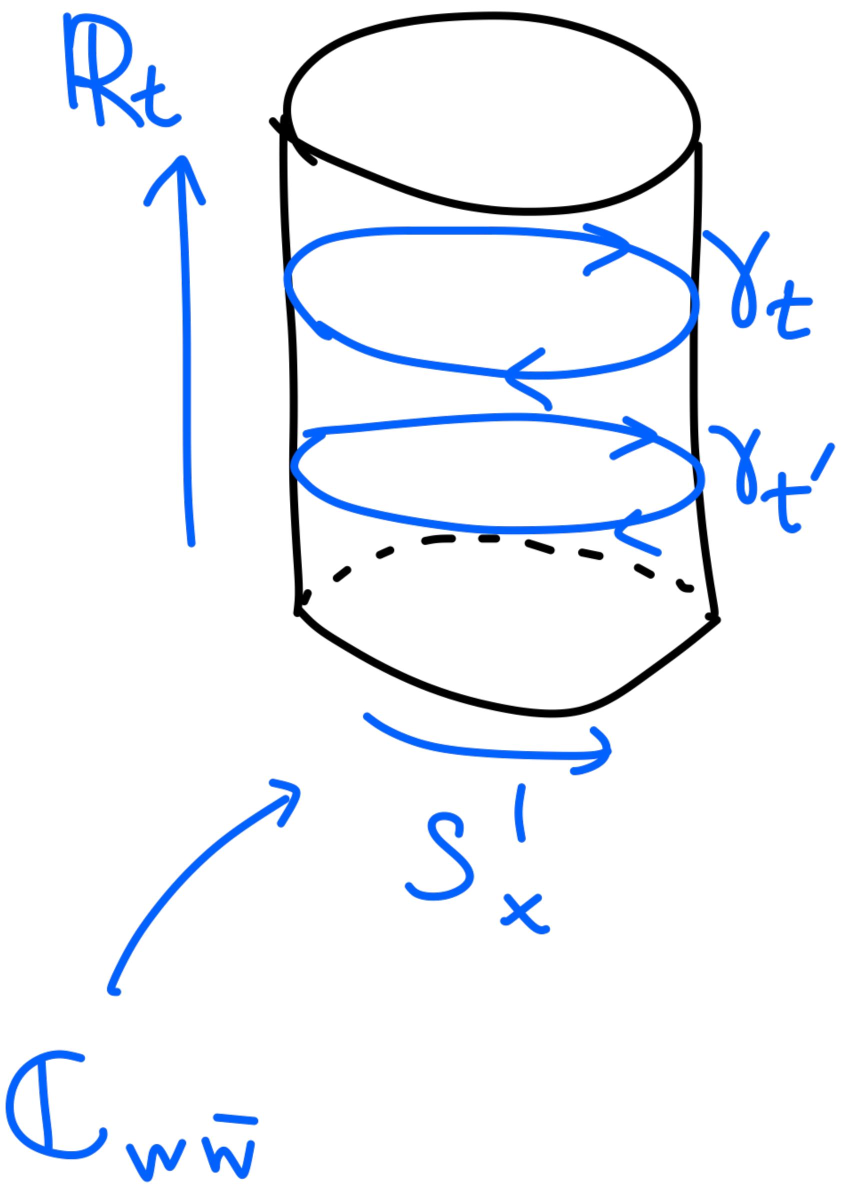
* existence of hol. flat connection $\mathcal{L}(z)$: classical definition of integrability

* Indeed, $\mathbb{C} \simeq \mathbb{R}^2 \to \mathbb{R}_t \times S^1_x$



* existence of hol. flat connection $\mathcal{L}(z)$: classical definition of integrability

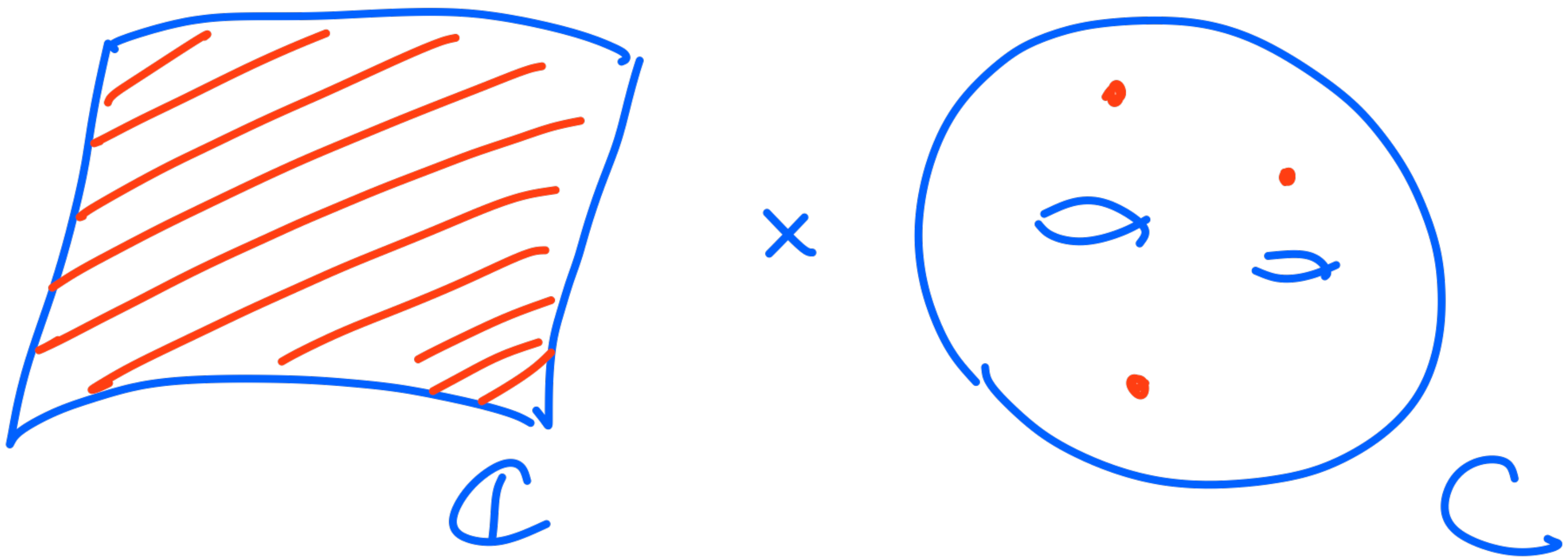
* Indeed, $\mathbb{C} \simeq \mathbb{R}^2 \to \mathbb{R}_t \times S^1_x$



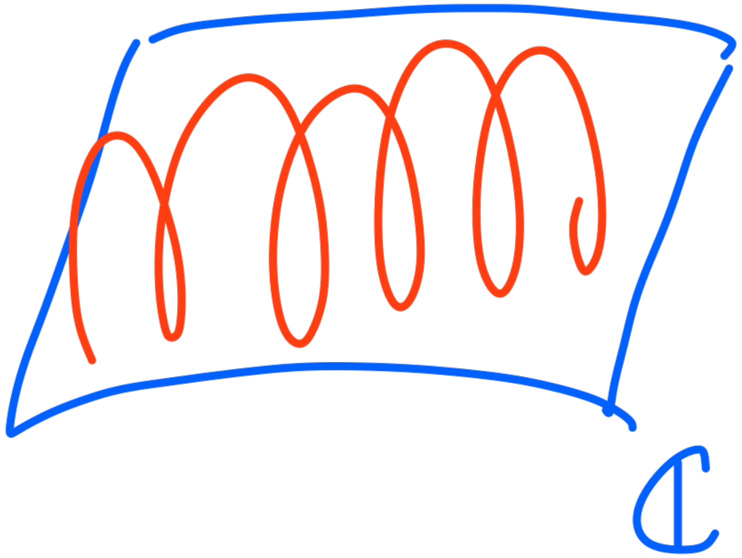
$$\partial_t \underbrace{\left[\overbrace{\mathrm{Tr}\, \exp \int_{\gamma_t} \mathcal{L}(z)\, dz}^{\text{Wilson line } W_\gamma} \right]}_{\parallel \atop \exp\left(\sum Q_n z^n\right)} = 0$$

$$\left(\partial_t Q_n = 0 \quad \forall n \right)$$

We still have coupled 4d/2d system



integrate out along C



effective 2d theory

[Lagrangian, Lax]

Recap

"4d CS" on $\mathbb{C} \times C$ + 2d surface defect at $\mathbb{C} \times \{pt\}$

$\Big\downarrow$ integrate out along C

2d (effective) integrable FT on $\mathbb{C}$

※ integrability <u>automatic</u>

Which models?

Which models?

Which
surface defects?

Order defects

: coupling to 2d FT w/ global sym. G

disorder defects

: singularities for A

(※ they can coexist)

Order defects : examples

* Free Fermion

  $\rightsquigarrow$ Gross-Neveu, massive Thirring etc.

* Free Boson

  $\rightsquigarrow$ New ?

* Curved β-γ system

  $\rightsquigarrow$ σ-model G/T etc.

+ trigonometric
  elliptic
  deformations
+ mult-defect
  generalization
  ⋮

Today: disorder defects

[singularities of $A_{\bar{z}}, w, \bar{w}$]

Today: disorder defects
[singularities of $A_{\bar{z},w,\bar{w}}$]

This is NOT possible at a generic point

$$L \sim \underset{\substack{\wr \\ dz}}{\omega} \wedge \underbrace{\mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)}_{\to\, \infty} \to \infty$$

since this does not give proper b.c.
(no well-defined propagator)

Today: disorder defects

[singularities of $A_{\bar{z}, w, \bar{w}}$]

.... but possible at zeros of ω

$$L \sim \underbrace{\omega}_{\to 0} \wedge \underbrace{\mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)}_{\to \infty} \sim (\text{finite})$$

Today: disorder defects

[singularities of $A_{\bar{z}, w, \bar{w}}$]

.... but possible at zeros of ω

$$L \sim \underbrace{\omega}_{\to 0} \wedge \underbrace{\mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)}_{\to \infty} \sim (\text{finite})$$

If $\omega \sim z\,dz$ (1st order zero)

then $A_w = \infty$ or $A_{\bar{w}} = \infty$

We also have poles of ω

$$L \sim \underbrace{\omega}_{\to\infty} \wedge \underbrace{\mathrm{Tr}\left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A\right)}_{\to 0} \sim (\text{finite})$$

* If $\omega \sim dz/z$ (1st order pole)

  then $A_w = A_{\bar{z}} = 0$ or $A_{\bar{w}} = A_{\bar{z}} = 0$

* If $\omega \sim dz/z^2$ (2nd order pole)

  then $A_w = A_{\bar{w}} = A_{\bar{z}} = 0$

Consider $(\mathbb{C}, \omega)$

ω: - 1st order zeros

$$A_w = \infty \quad \text{or} \quad A_{\bar{w}} = \infty$$

- 1st order poles

$$A_w = A_{\bar{z}} = 0 \quad \text{or} \quad A_{\bar{w}} = A_{\bar{z}} = 0$$

- 2nd order poles

$$A_w = A_{\bar{w}} = A_{\bar{z}} = 0$$

Once we fix $(C, w, b.c.)$

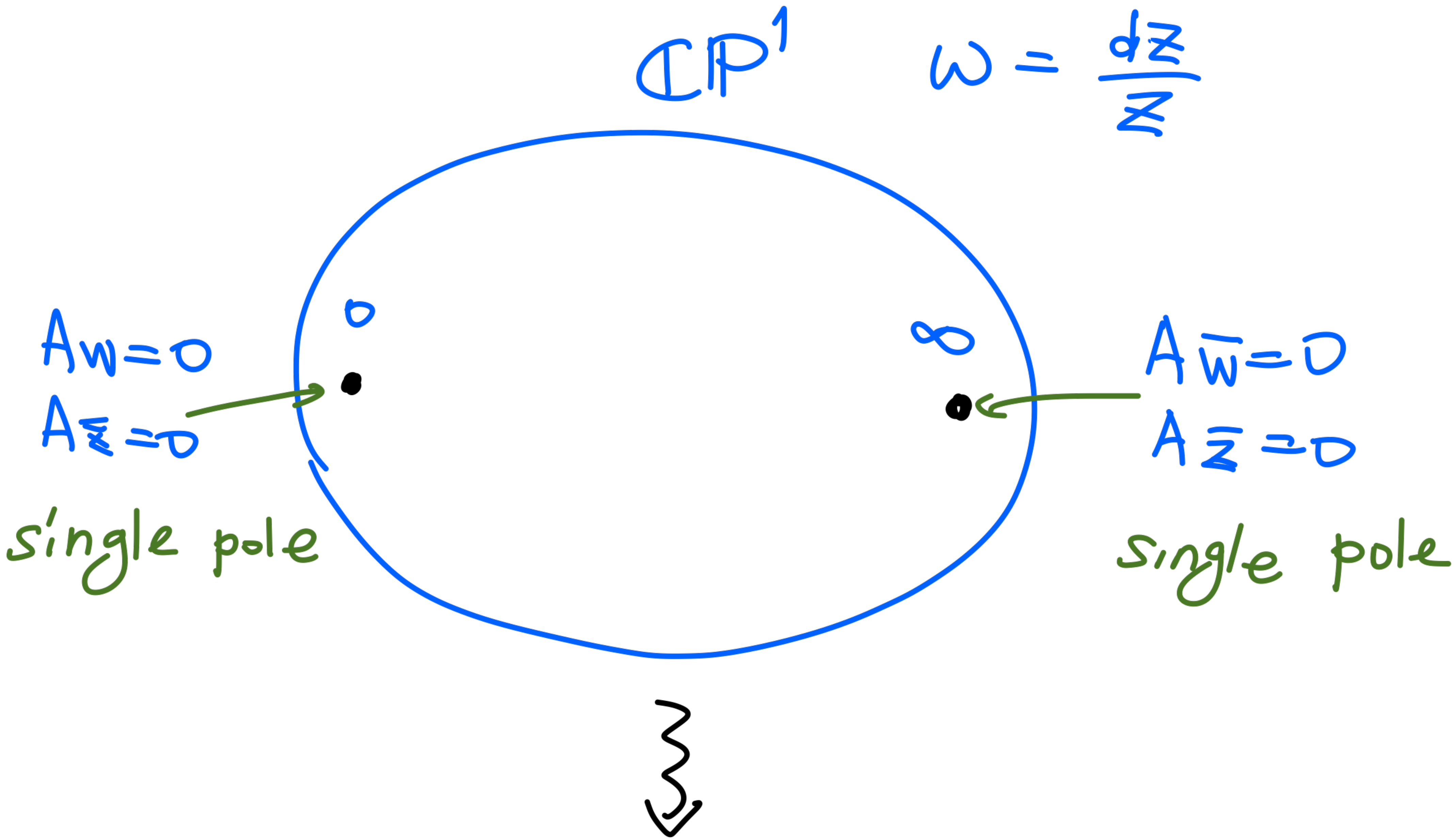
we can solve e.o.m.

$$F_{w\bar{w}} = F_{\bar{z}w} = F_{\bar{z}\bar{w}} = 0$$

under the b.c.

and plug in resulting A_w, $A_{\bar{w}}$, $A_{\bar{z}}$

into Lagrangian & Lax



Conformal WZW model

$\mathbb{CP}^1$

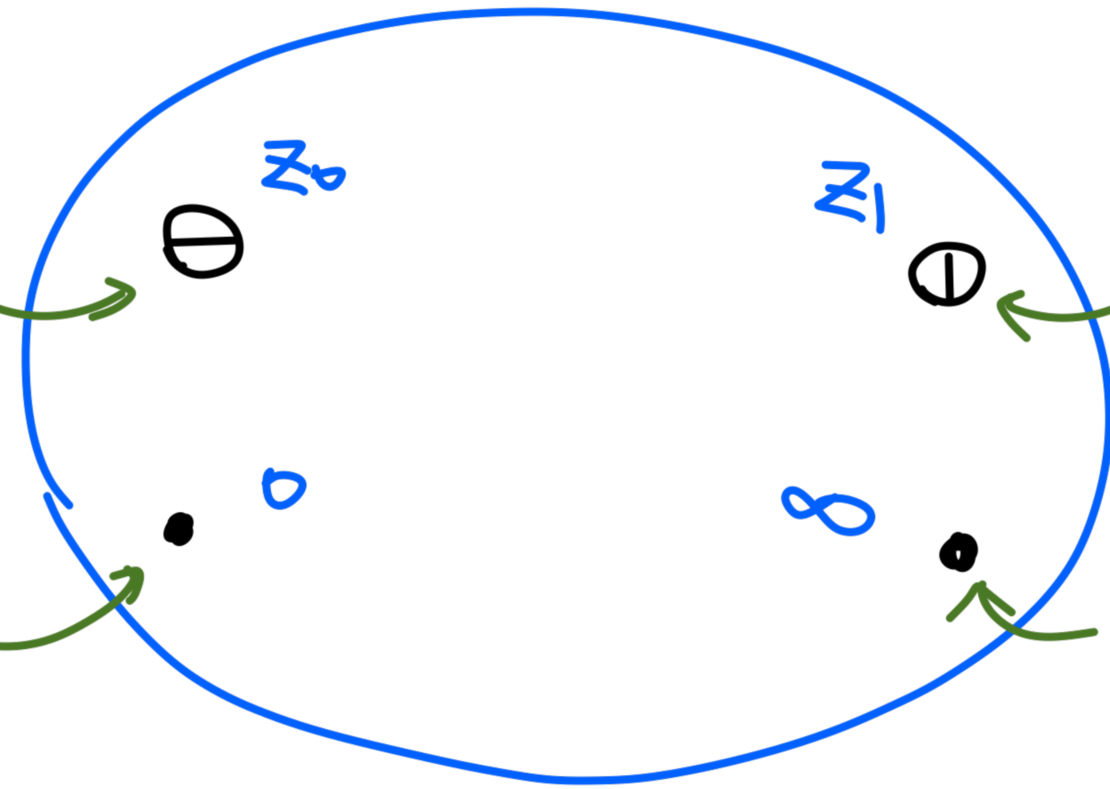
$$\omega = \frac{(z-z_0)(z-z_1)\,dz}{z^2}$$

single zero

$A_{\bar{w}} = \infty$

$A_w = \infty$

single zero



$A_w = A_{\bar{w}} = A_{\bar{z}} = 0$

double pole

$A_w = A_{\bar{w}} = A_{\bar{z}} = 0$

double pole

non-conformal WZW model

(PCM + WZ term)

$\propto (z_1 - z_0)$ $\quad$ $\propto (z_1 + z_0)$

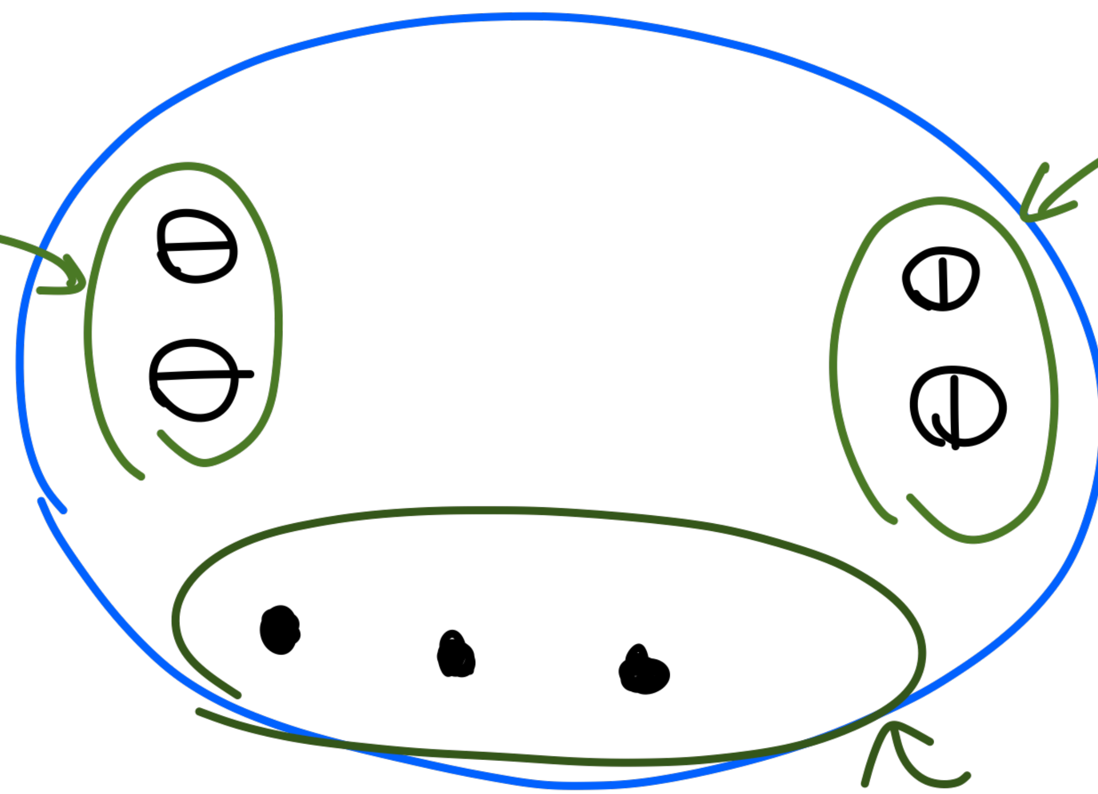
$\mathbb{CP}^1$

(n-1) single zero

$A_{\bar{w}} = \infty$

(n-1) single zeros

$A_w = \infty$



n double poles

$A_w = A_{\bar{w}} = A_{\bar{z}} = \infty$

G^n/G – "coupled" WZW-type model

[also Delduc–Lacroix–Magno–Vicedo '18

Vicedo '19]

Generalized Symmetric Space

G/G_0 ← fixed by σ

$\mathbb{Z}_n$-grading

$\mathfrak{g} = \mathfrak{g}_0 \oplus \mathfrak{g}_1 \oplus \cdots \oplus \mathfrak{g}_{n-1}$

σ: n-fold involution

for $n=2$ symmetric space

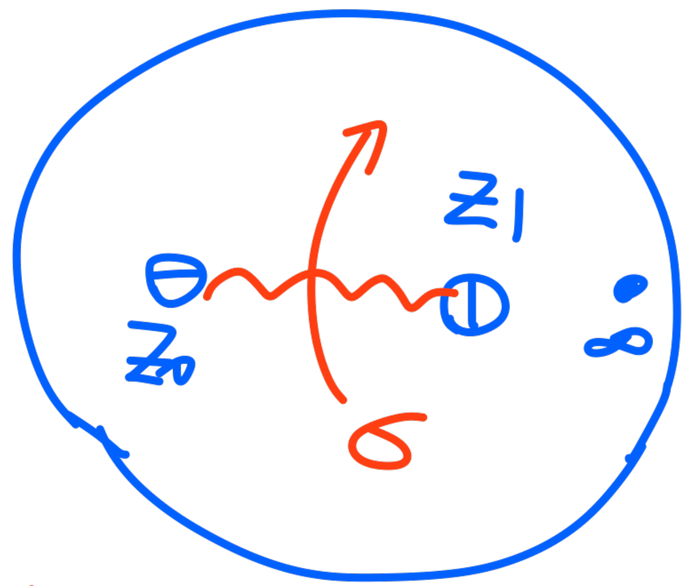
$n=4$ super coset e.g. $\dfrac{PSU(2,2|4)}{SU(2,2) \times SU(4)}$

IS

$AdS_5 \times S^5$

$\mathbb{CP}^1$

$\omega = dz$

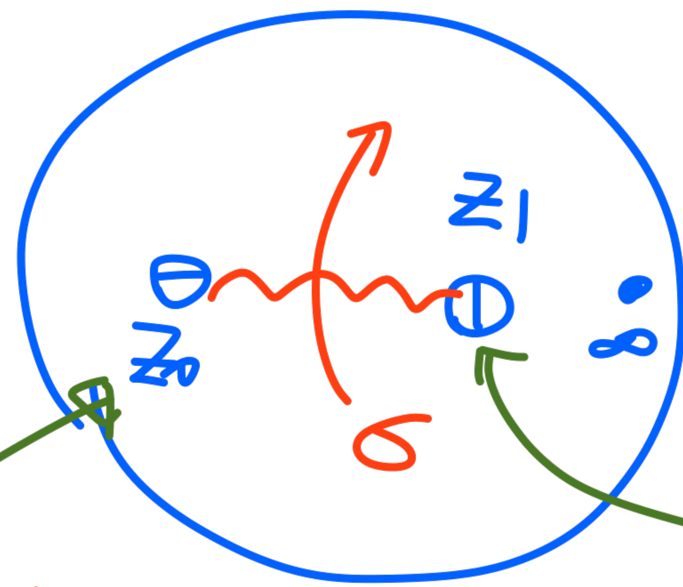


"topological line defect"

action by σ

$\mathbb{CP}^1$

$\omega = dz$



"topological line defect"

$A^{(k)}_{\bar{w}} \sim (z - z_1)^{-\frac{k}{n}}$

action by σ

$A^{(k)}_{w} \sim (z - z_0)^{-\frac{k}{n}}$

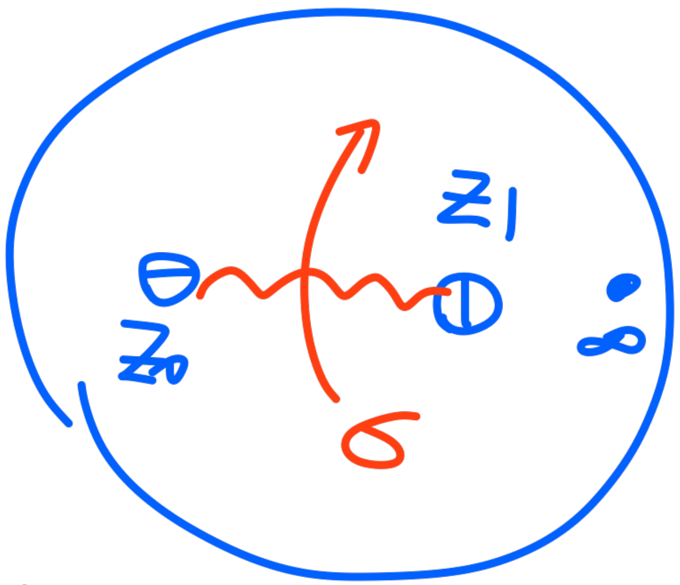
$\infty \to 1, \omega, \ldots \omega^{n-1}$

$z_0 \to 0$

$z_1 \to \infty$

$\mathbb{CP}^1$

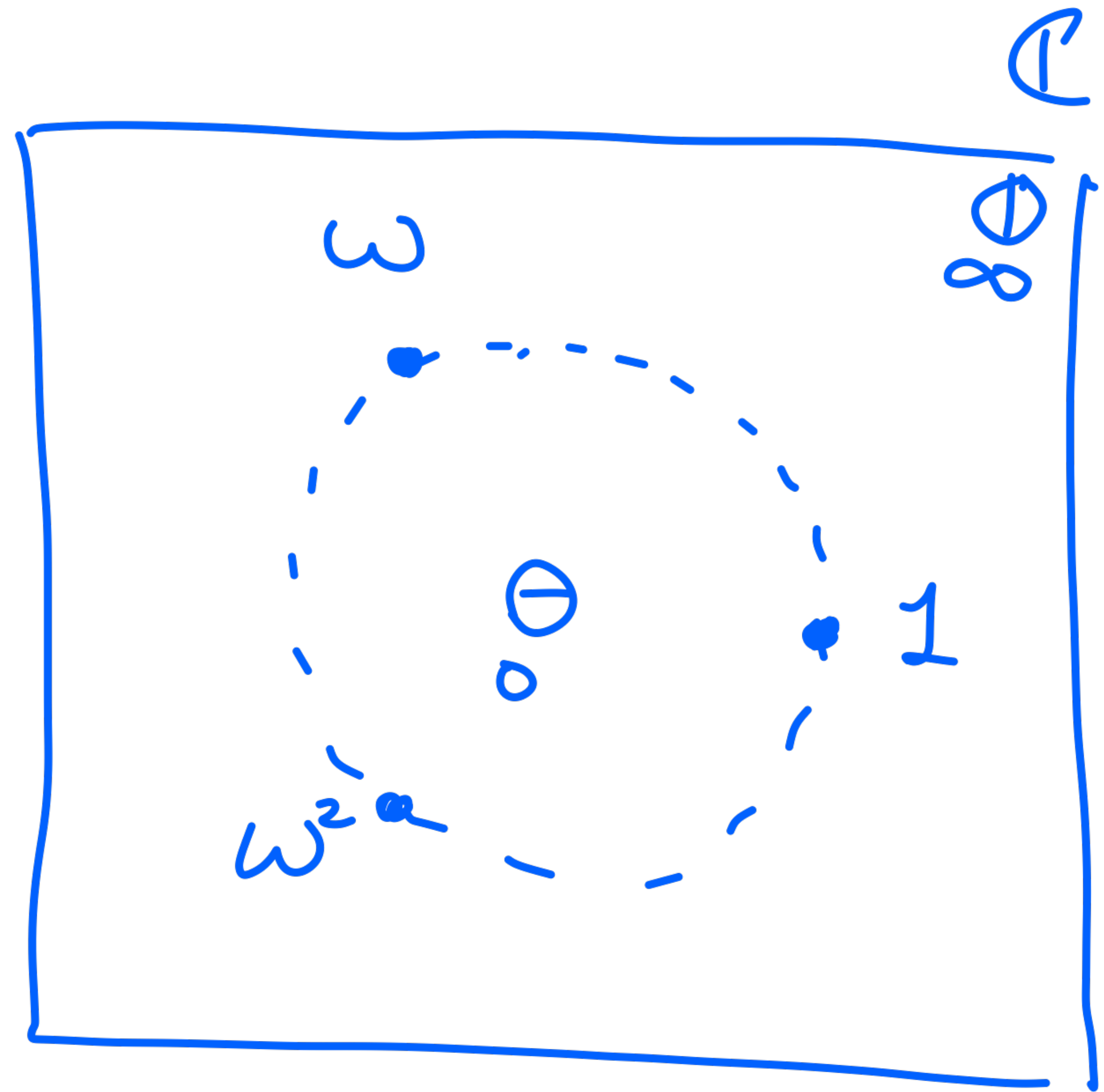
$\omega = dz$



"topological line defect"

action by σ

n - fold cover



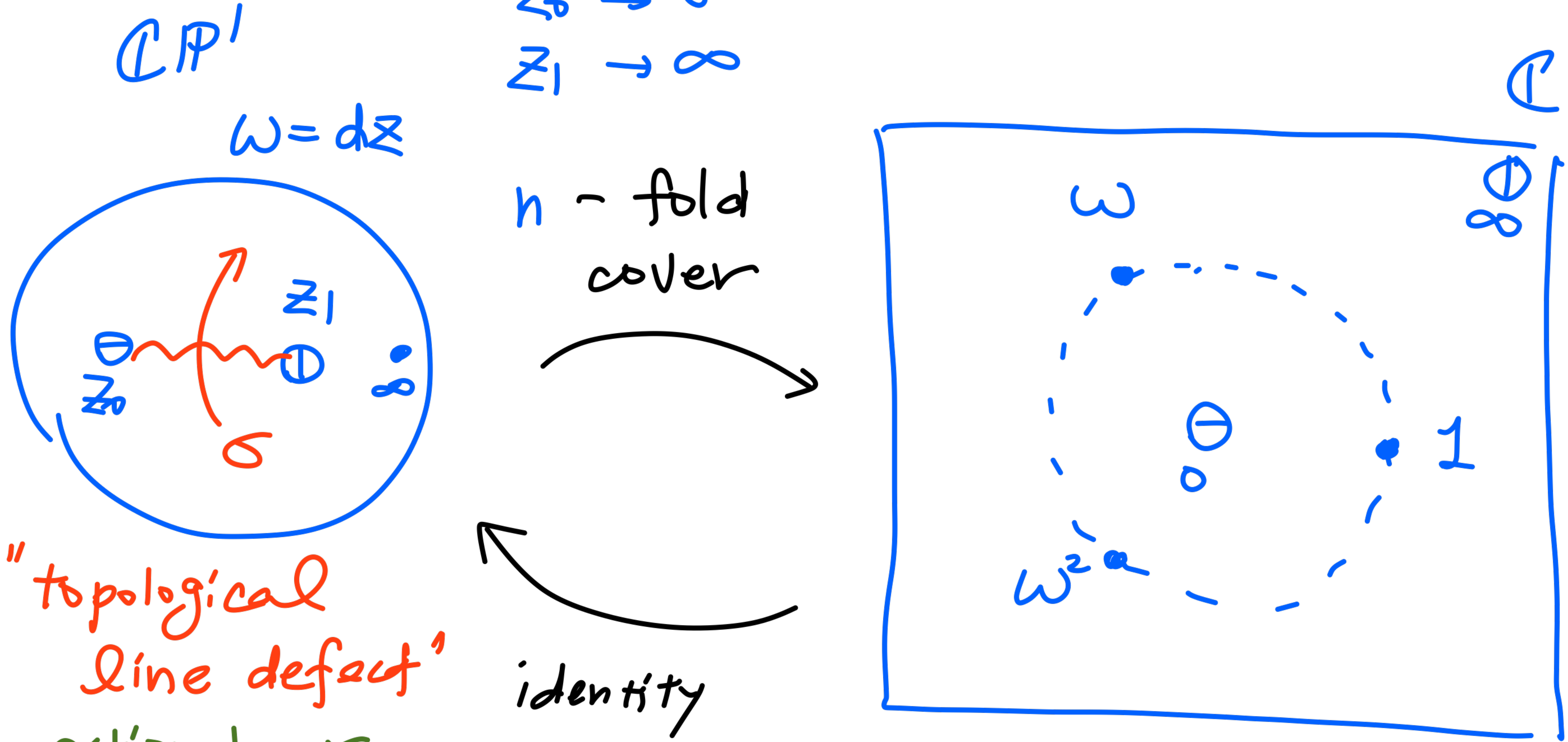
degeneration of

G^n/G - model

$\infty \to 1, \omega, \ldots \omega^{n-1}$

$z_0 \to 0$

$z_1 \to \infty$



"topological line defect"

action by σ

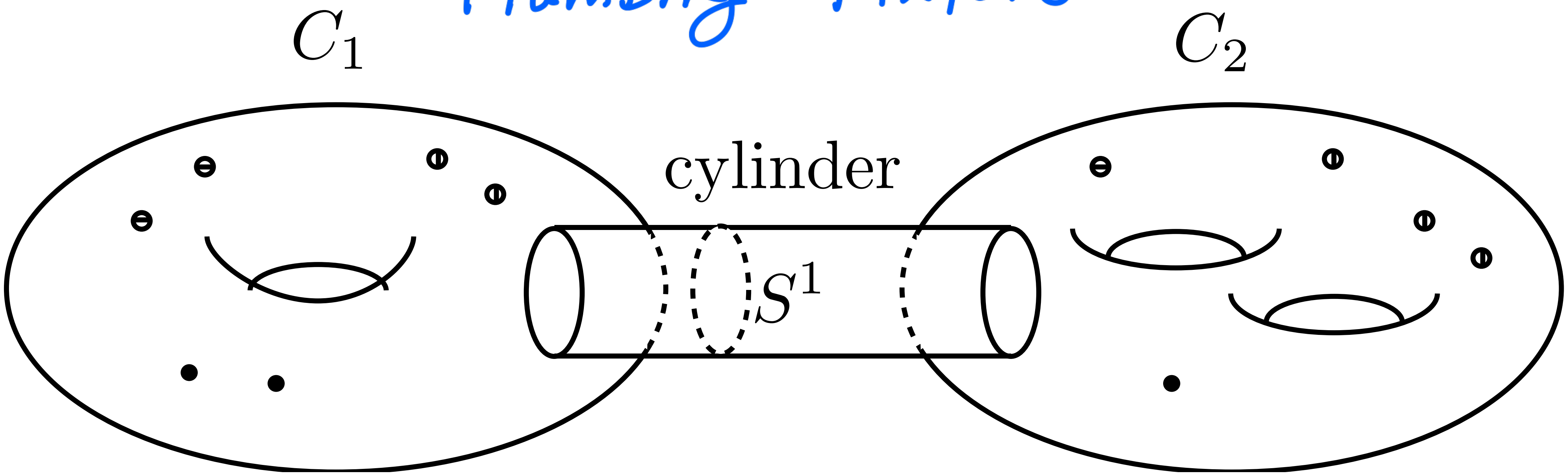
n copies

degeneration of G^n/G - model

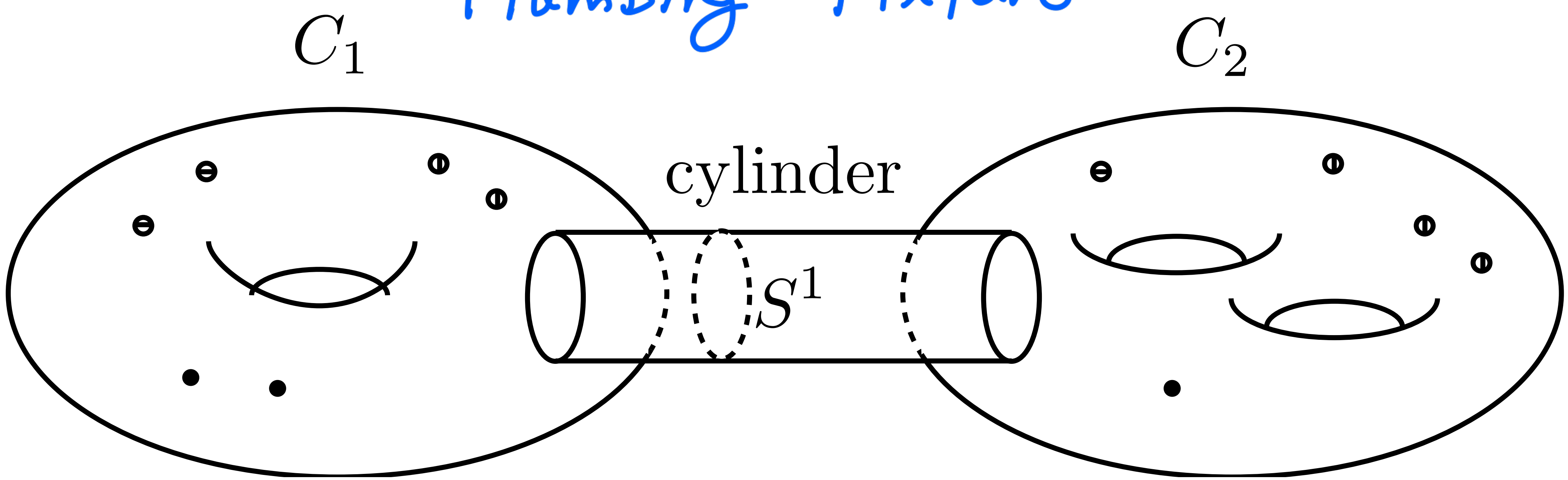
$$G/G_0 = \left(G^n/G\right)_{\mathbb{Z}_n} !$$

Cutting / Gluing

Plumbing Fixture



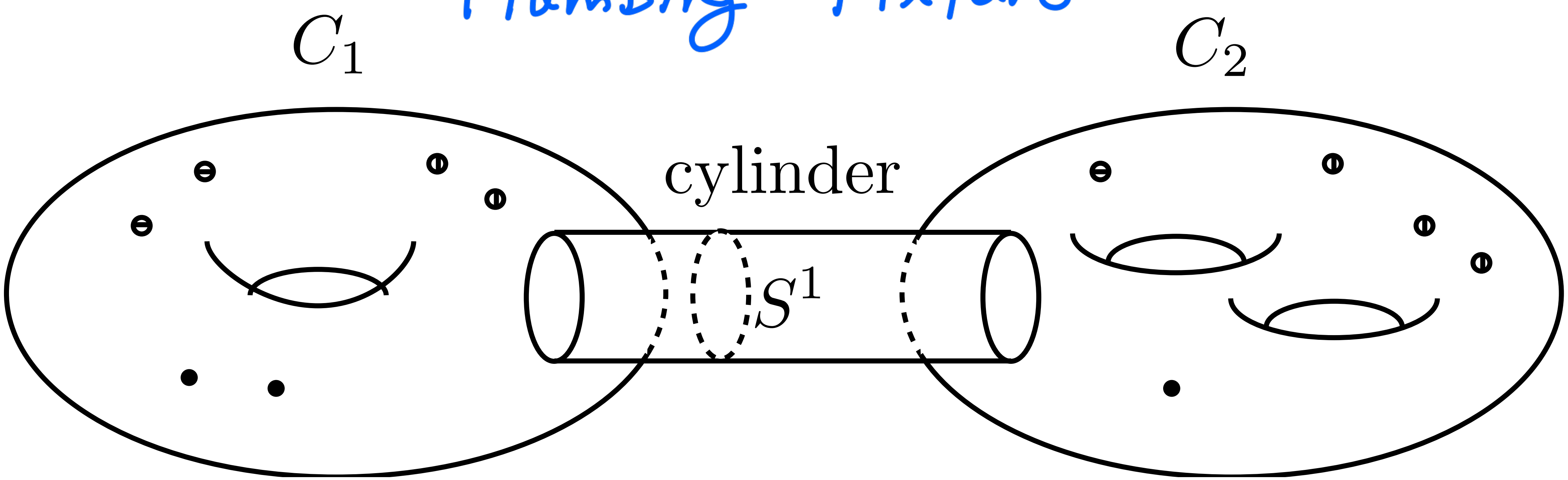
Plumbing Fixture



$$\omega_1 \sim c_1 \frac{dz_1}{z_1} \quad \begin{pmatrix} z_1 z_2 = e^{+i\theta+t} \\ c_1 + c_2 = 0 \end{pmatrix} \quad \omega_2 \sim c_2 \frac{dz_2}{z_2}$$

glue two single poles of ω

Plumbing Fixture



$$\omega_1 \sim c_1 \frac{dz_1}{z_1} \quad \begin{pmatrix} z_1 z_2 = e^{+i\theta+t} \\ c_1 + c_2 = 0 \end{pmatrix} \quad \omega_2 \sim c_2 \frac{dz_2}{z_2}$$

glue two single poles of ω

$$\text{“} \; T_{1 \cup 2} \simeq (T_1 \times T_2)/G \; \text{”}$$

Summary

Result:

C : Riemann surface

ω : has zeros: disorder defects

and

poles

discrete choice of boundary conditions for A_w, $A_{\bar{w}}$, $A_{\bar{z}}$

$\{D\}$: order defects

$\Downarrow$

2d (classically) integrable field theories

* contains

- Gross-Neveu / massive Thirring
- Prinipal Chiral Model (WZW) & Coupled Generalization (G^n/G)
- Generalized ($\mathbb{Z}_n$) symmetric space cosets
- elliptic/trigonometric deformations of many models

$\vdots$

* a new class of
  integrable field theories

* generically has
  higher-genus spectral curves
  and seem to be NEW!

* Cutting / Gluing procedure
  for integrable field theories

* many many questions remain,
  e.g. quantum integrability
  (Part IV)