Is $\mathcal{N}=2$ Large?

Masahito Yamazaki (Kavli IPMU)

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w/ Ryuichiro Kitano & Norikazu Yamada (KEK)

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Problem

Today:

Pure Yang-Mills Theory

w/ θ-angle w/ $G = SU(N)$

vacuum energy $E(\theta, N) = ?$

expansion around $\theta = 0$

$$E(\theta) - E(0) = \frac{1}{2}\chi\theta^2\left(1 + b_2\theta^2 + b_4\theta^6 + \cdots\right)$$

topological susceptibility

dimensionless

(also motivation from axionic inflation [Nomura-Watari-Y, Nomura-Y ('17)])

Instanton (DIGA) ['t Hooft]

$$E(\theta) \sim 1 - \cos\theta \rightsquigarrow b_2 = -\frac{1}{12}, \quad b_4 = \frac{1}{360}, \quad \cdots$$

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Large N ['t Hooft, Witten, ...]

$$\mathcal{L} \sim \frac{1}{N^{-1}}\left(\frac{1}{g^2 N} \operatorname{Tr} F \wedge * F + \frac{\theta}{N} \operatorname{Tr} F \wedge F\right)$$

$$\downarrow$$

$$E(\theta) = N^2 f\left(\frac{\theta}{N}\right) = \frac{1}{2}\chi\,\theta^2\left(1 + b_2\theta^2 + \cdots\right)$$

$$\downarrow$$

$$\chi = \chi^{(0)} + O\left(\frac{1}{N^2}\right)$$

$$b_{2n} = \frac{b_{2n}^{(0)}}{N^{2n}} + O\left(\frac{1}{N^{2n+2}}\right)$$

NOT 2π-periodic

Instanton

$$Z \sim \int_{\rho\sim 0}^{\rho\sim\infty} \frac{d\rho}{\rho}\,\frac{1}{\rho^4}\, e^{-\frac{8\pi}{g^2(\cancel{\mu})}+i\theta}\,(\mu\rho)^{11N/3}$$

1-loop running

Instanton

$$Z \sim \int_{\rho\sim 0}^{\rho\sim\infty} \frac{d\rho}{\rho}\frac{1}{\rho^4}\, e^{-\frac{8\pi}{g^2(\mu)}+i\theta}\,(\mu\rho)^{\underbrace{11N/3}}$$

1-loop running

$$N \gtrsim N_*^{1\text{-loop}} = \frac{12}{11}$$: IR divergence (IR problem)

Instanton

$$Z \sim \int_{\rho\sim 0}^{\rho\sim\infty} \frac{d\rho}{\rho}\frac{1}{\rho^4}\, e^{-\frac{8\pi}{g^2(\mu)}+i\theta}\, (\mu\rho)^{\underbrace{11N/3}}$$

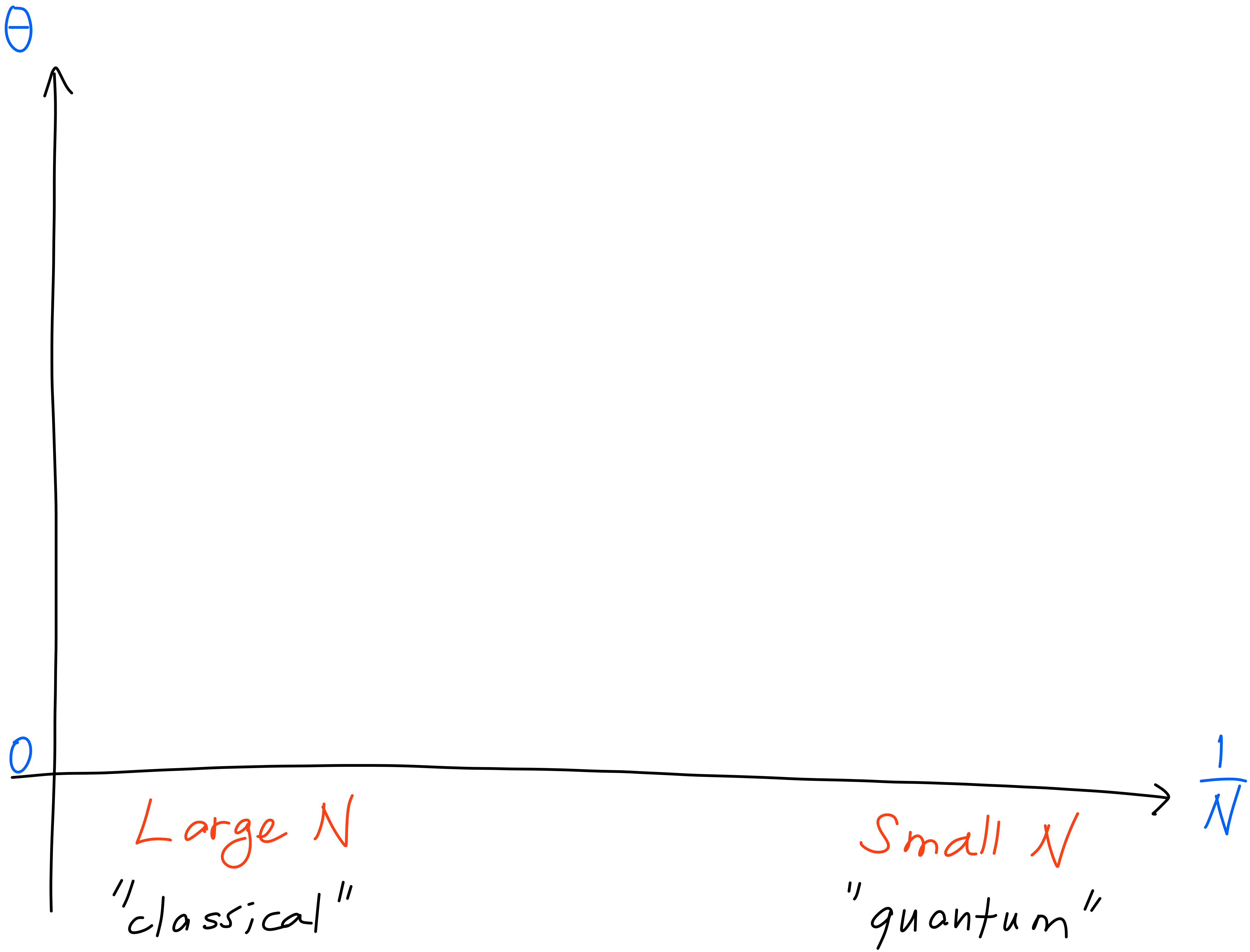
1-loop running

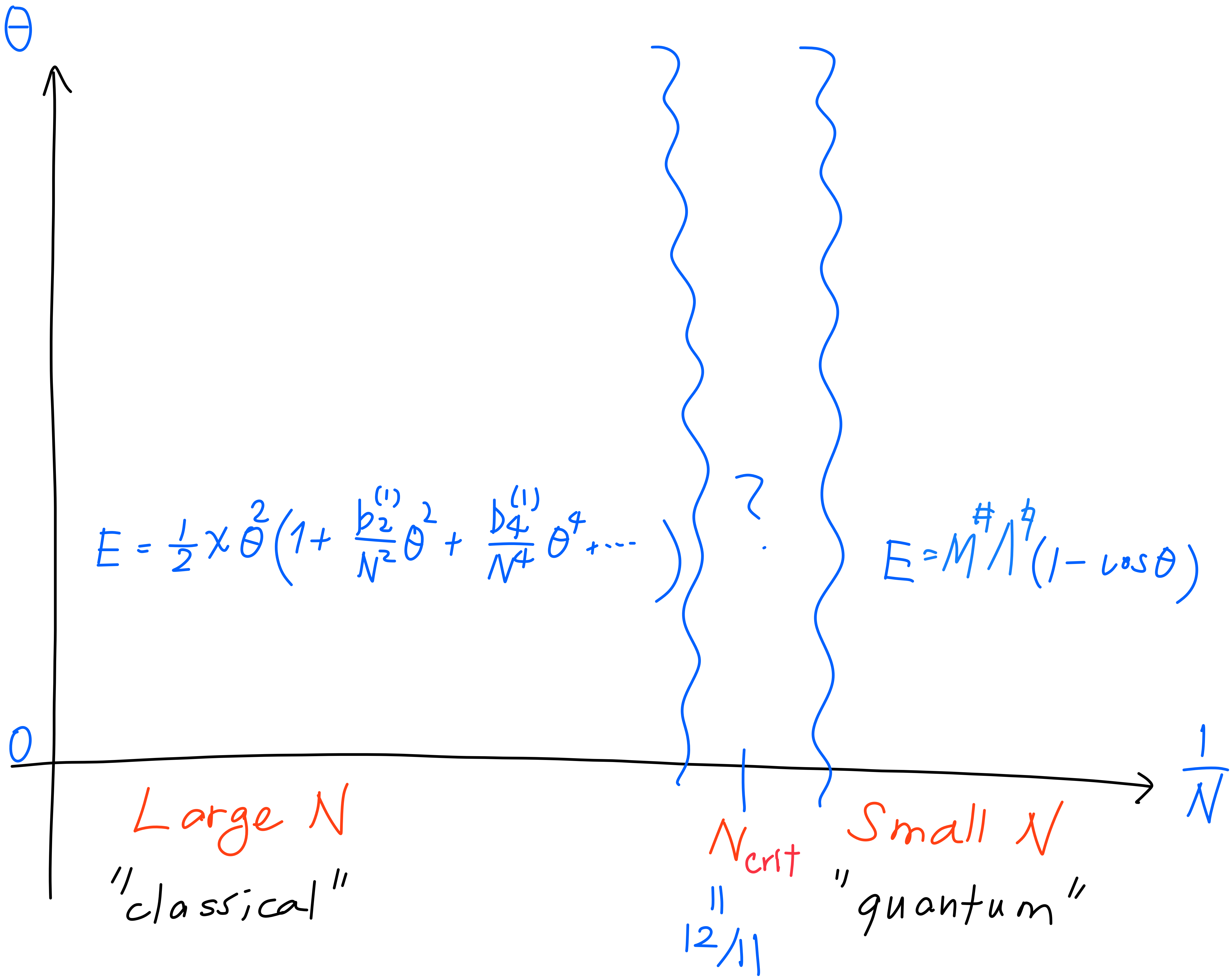
$$\begin{cases} N \gtrsim N_*^{\text{1-loop}} = \dfrac{12}{11} : \text{IR divergence (IR problem)} \\ N \lesssim N_*^{\text{1-loop}} = \dfrac{12}{11} : \text{UV divergence } (\rho \gtrsim M^{-1}) \end{cases}$$

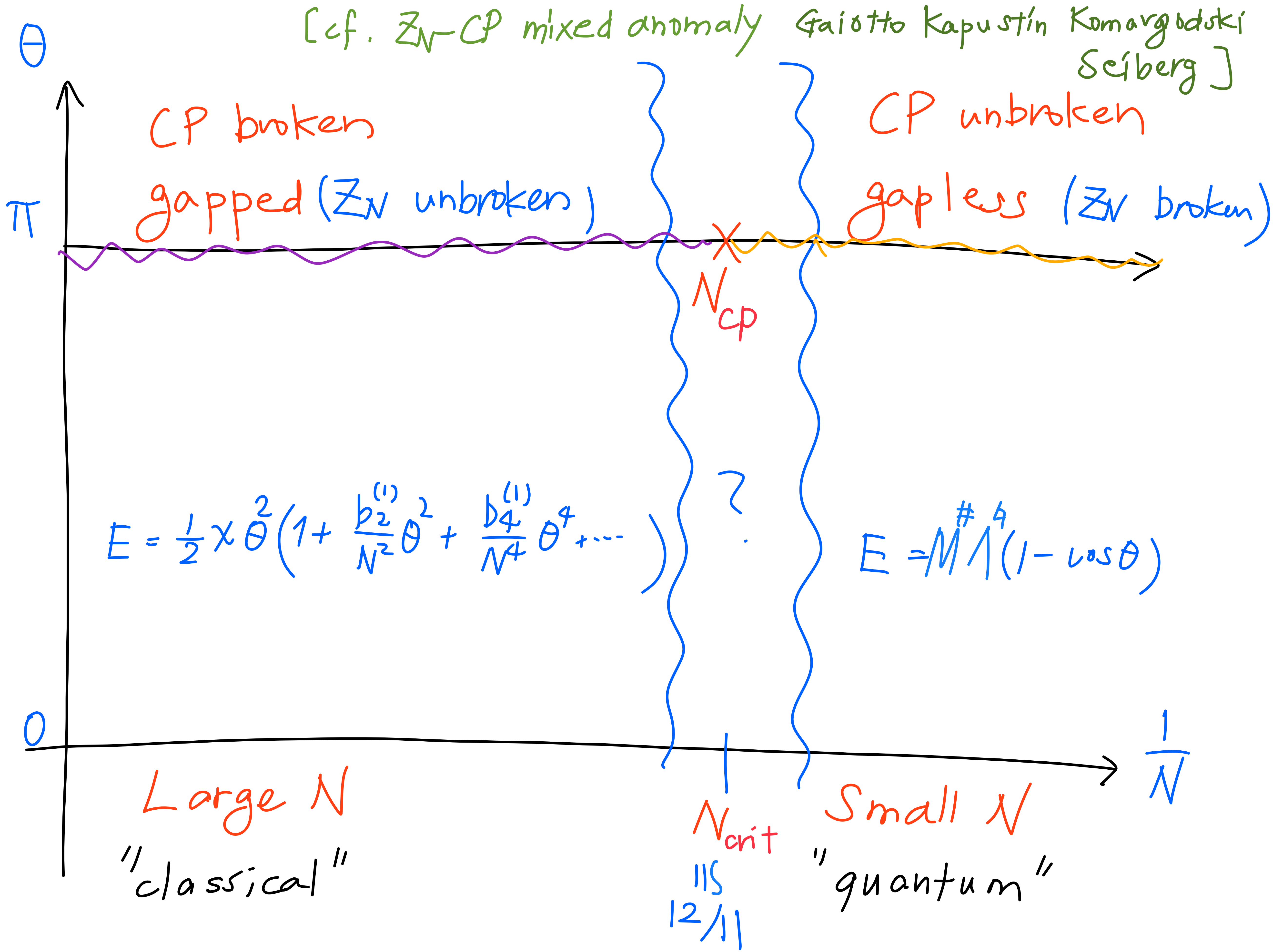
cutoff scale

$$\longrightarrow Z \sim M^{4-\frac{11}{3}N}\Lambda^{\frac{11N}{3}}(1-\cos\theta)$$ dominates over other contributions

[This happens for $SU(2)_{EW}$ in SM $\rightsquigarrow \Lambda_{c.c}$?
[Nomura-Watari-Yanagida ('00), ⋯, Ibe-Yanagida-Y ('18)]]

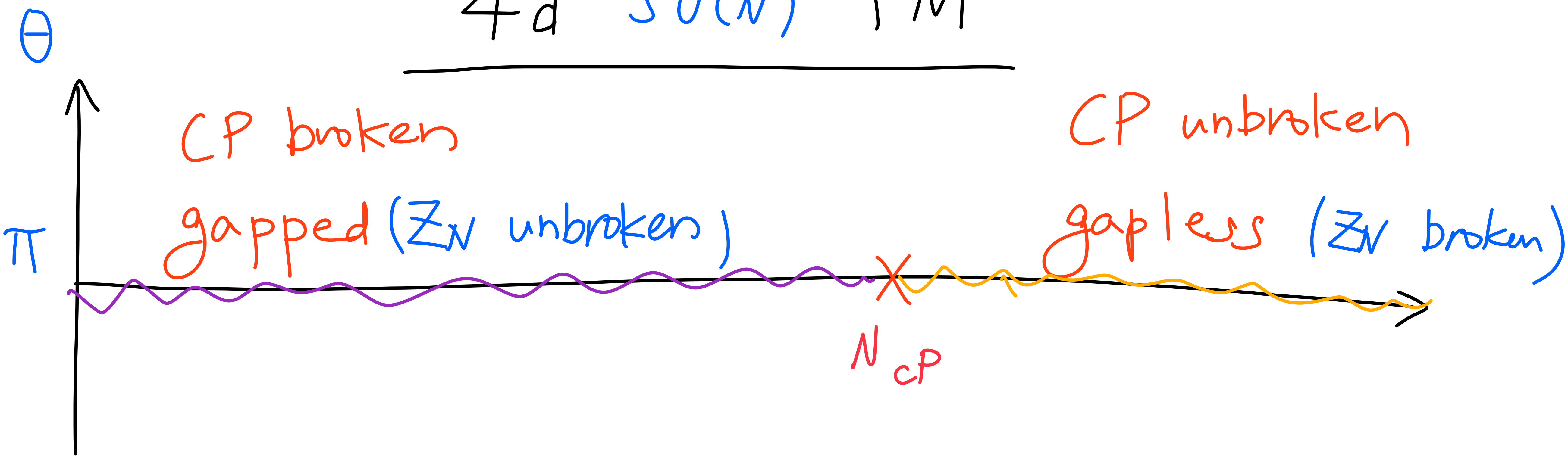

Θ
0
1/N
Large N
"classical"
Small N
"quantum"


Θ
0
$\frac{1}{N}$
$E = \frac{1}{2}\chi\theta^2\left(1 + \frac{b_2^{(1)}}{N^2}\theta^2 + \frac{b_4^{(1)}}{N^4}\theta^4 + \cdots\right)$
?
$E = M^{\#}\Lambda^4(1 - \cos\theta)$
Large N
"classical"
N_{crit}
=
12/11
Small N
"quantum"

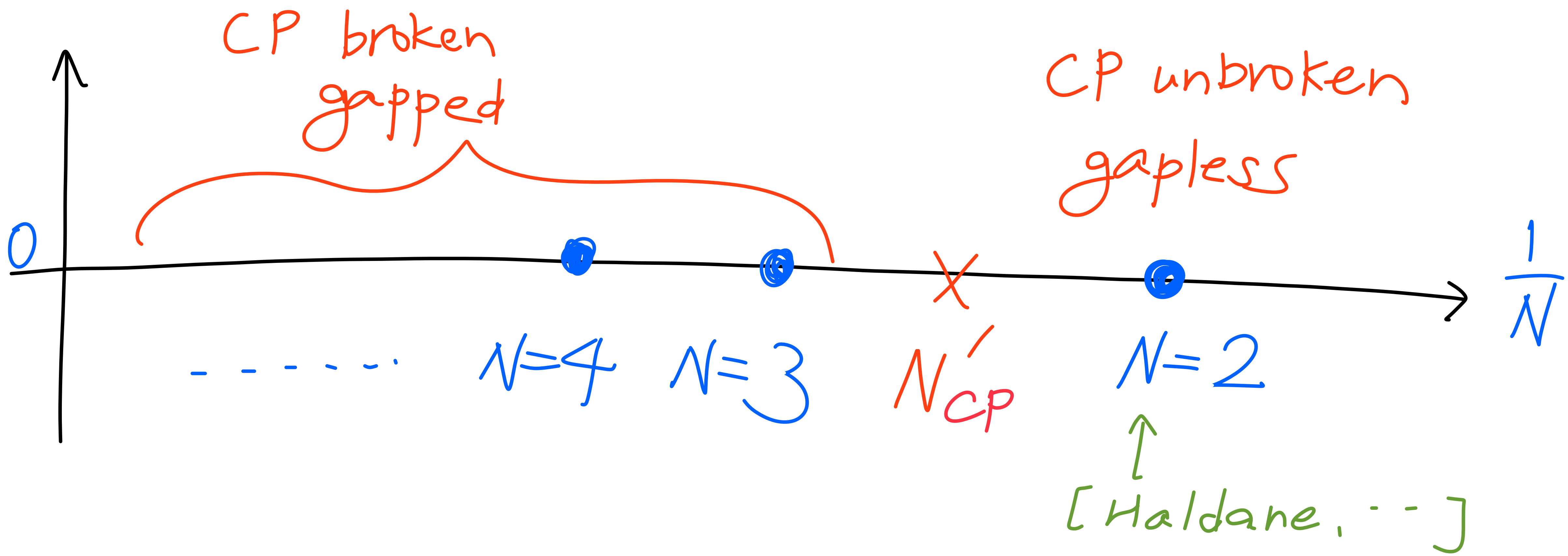


[cf. $\mathbb{Z}_N$-CP mixed anomaly Gaiotto Kapustin Komargodski Seiberg]
θ
CP broken
gapped ($\mathbb{Z}_N$ unbroken)
CP unbroken
gapless ($\mathbb{Z}_N$ broken)
π
N_{cp}
?
$E = \frac{1}{2}\chi\theta^2\left(1 + \frac{b_2^{(1)}}{N^2}\theta^2 + \frac{b_4^{(1)}}{N^4}\theta^4 + \cdots\right)$
$E = M^{\#}\Lambda^4(1-\cos\theta)$
0
$\frac{1}{N}$
Large N
"classical"
N_{crit}
$\simeq$
12/11
Small N
"quantum"

4d $SU(N)$ YM



2d CP^{N-1} model



4d SU(2) YM + θ-angle

$$E(\theta) - E(0) = \frac{1}{2}\chi\theta^2\left(1 + b_2\theta^2 + b_4\theta^4 + \cdots\cdots\right)$$

compute

Is $N=2$ large (large N) or small (inst.)?

Is CP preserved/broken @ $\theta=\pi$?

gapped/gapless

Lattice

* conceptually "simple"

generate gauge conf. at $\theta = 0$ ← no sign problem

↓

measure top. charge Q

↓

$$\chi = \frac{\langle Q^2 \rangle_{\theta=0}}{V} \,,$$

$$b_2 = -\frac{\langle Q^4 \rangle_{\theta=0} - 3 \langle Q^2 \rangle^2_{\theta=0}}{12 \langle Q^2 \rangle_{\theta=0}} \,,$$

$$b_4 = \frac{\langle Q^6 \rangle_{\theta=0} - 15 \langle Q^2 \rangle_{\theta=0} \langle Q^4 \rangle_{\theta=0} + 30 \langle Q^2 \rangle^3_{\theta=0}}{360 \langle Q^2 \rangle_{\theta=0}} \,,$$

* in practice several subtleties / difficulties

Need statistics $\leftarrow$ deviation from Gaussian

$$\left(e^{-\frac{1}{2}\chi\theta^2} \sim Z(\theta) = \sum_Q Z_Q \, e^{iQ\theta} \;\; \downarrow \;\; Z_Q \sim e^{-Q^2/2\chi} \right)$$

1,75
1,85
1,975

$16^3 \times 32$
$24^3 \times 48$

$(aT_c)^{-2}$

many

β	N_S	N_{T_c}	$(a\,T_c)^2$	$L\,\sigma_{\mathrm{str}}^{1/2}$	statistics
1.750	16	4.65	0.0462	4.9	80,100
1.850	16	6.50	0.0237	3.5	71,040
1.975	16	9.50	0.0111	2.4	30,490
1.975	24	9.50	0.0111	3.6	131,830

(symanzik action, HMC, Bridge++)

Short-Distance Fluctuations

We have fluctuations of size $\sim O(a)$

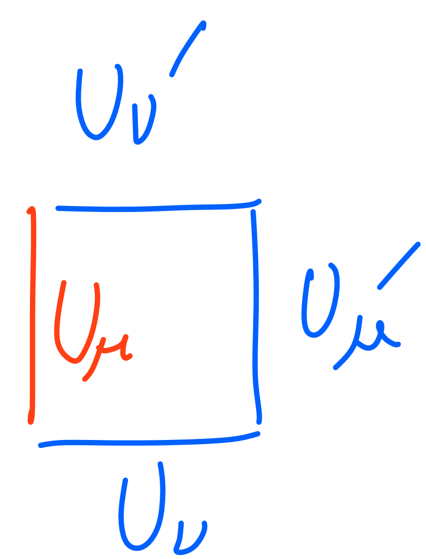
$\rightarrow$ removed by "smearing"

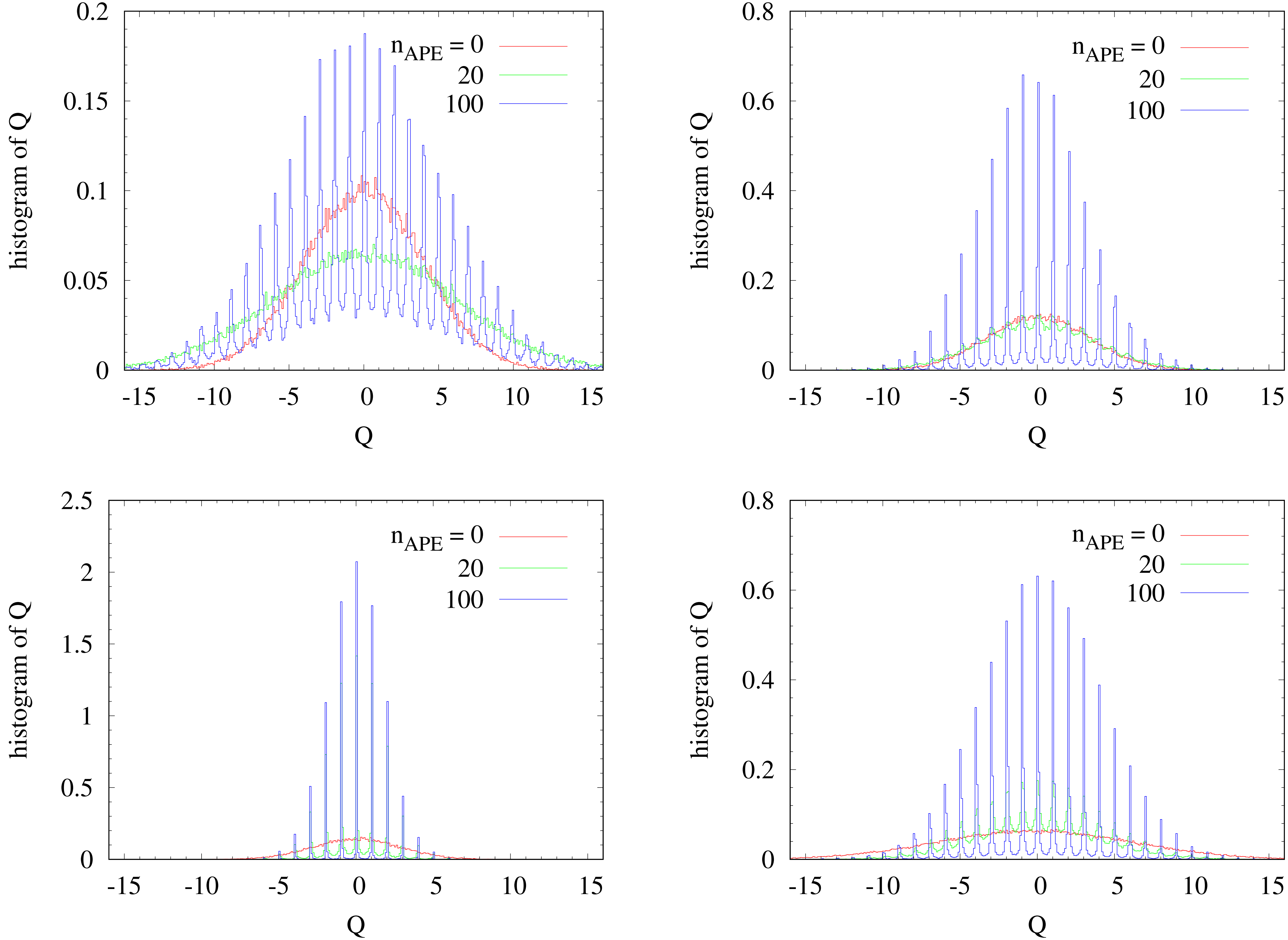
[many different methods, see Alexandrou+ ('17) for comparison]

We use APE smearing (& gradient flow)
[Albanese+ ('87)]

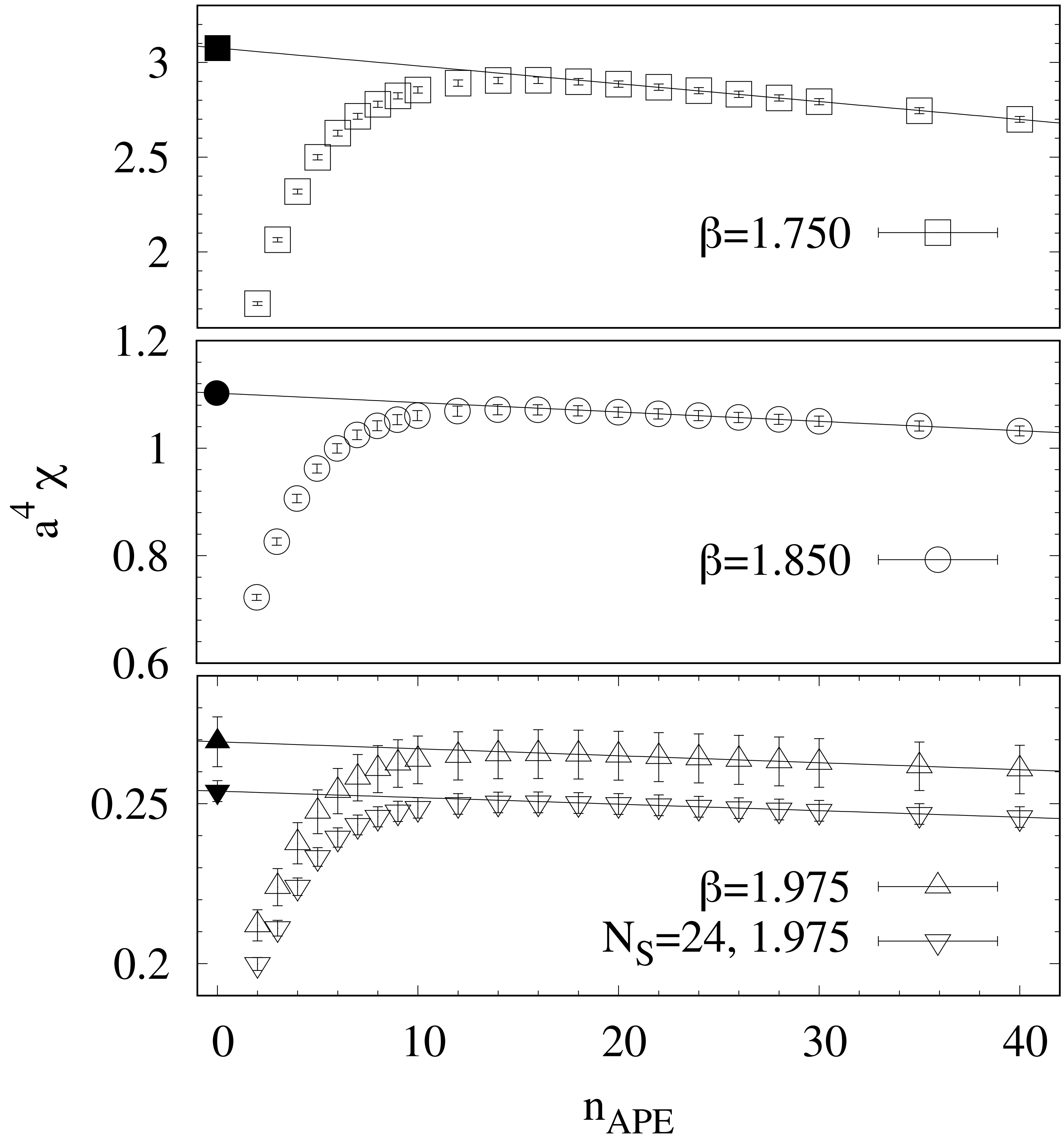
$$U_\mu^{(\mathrm{new})} = \mathrm{Proj}\left[(1-\rho)\mathrm{U}_\mu^{(\mathrm{old})}(\mathrm{x}) + \rho\,\mathrm{X}_\mu(\mathrm{x})\right] ,$$

$$X_\mu(x) = \sum_{\nu\neq\mu}\Big[U_\nu^{(\mathrm{old})}(x)U_\mu^{(\mathrm{old})}(x+\hat\nu)U_\nu^{(\mathrm{old})\dagger}(x+\hat\mu)$$
$$+U_\nu^{(\mathrm{old})\dagger}(x-\hat\nu)U_\mu^{(\mathrm{old})}(x-\hat\nu)U_\nu^{(\mathrm{old})}(x-\hat\nu+\hat\mu)\Big] ,$$

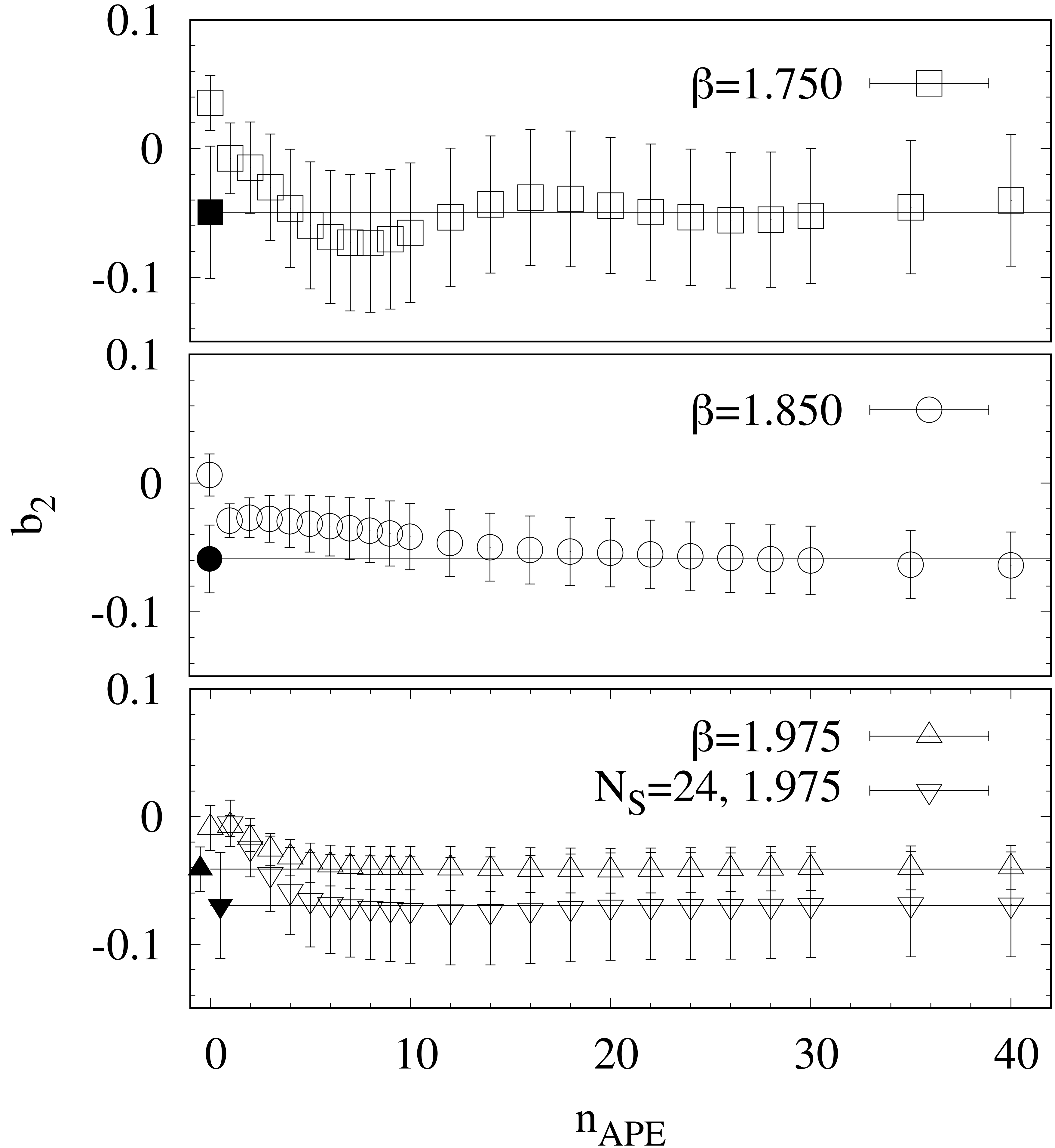




histogram of Q
Q
n_APE = 0
20
100



3
2.5
2
1.2
1
0.8
0.6
0.25
0.2
β=1.750
β=1.850
β=1.975
N_S=24, 1.975
a^4 χ
0
10
20
30
40
n_APE

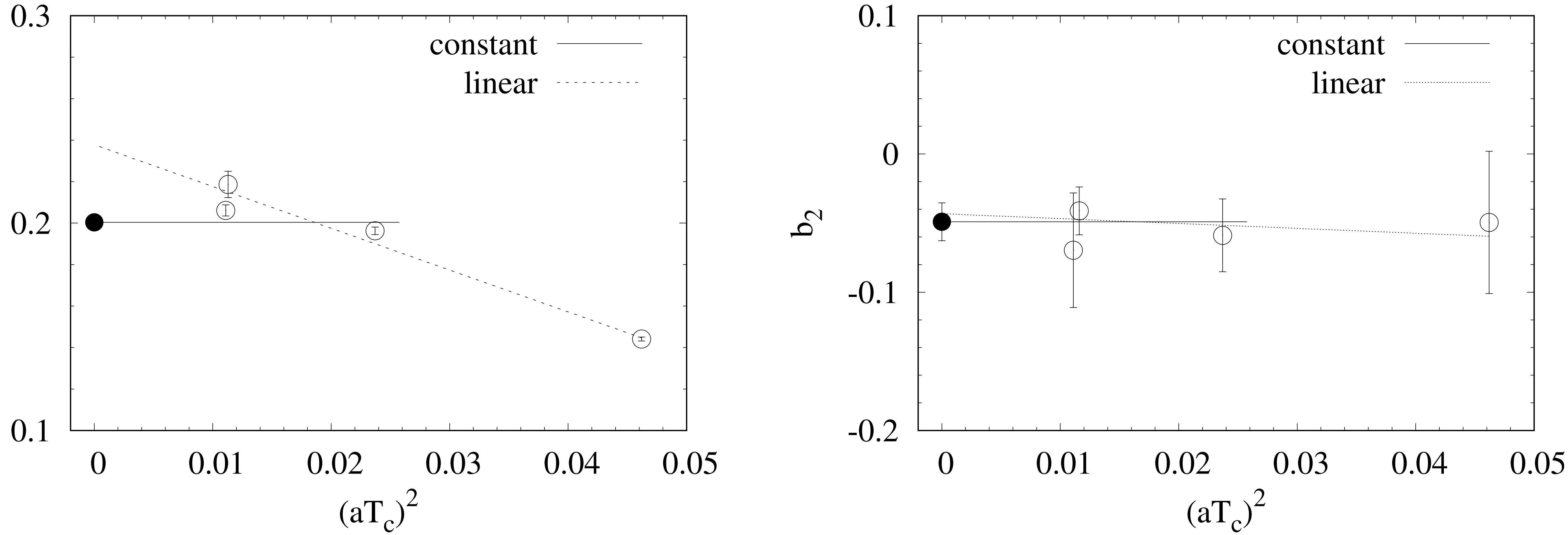


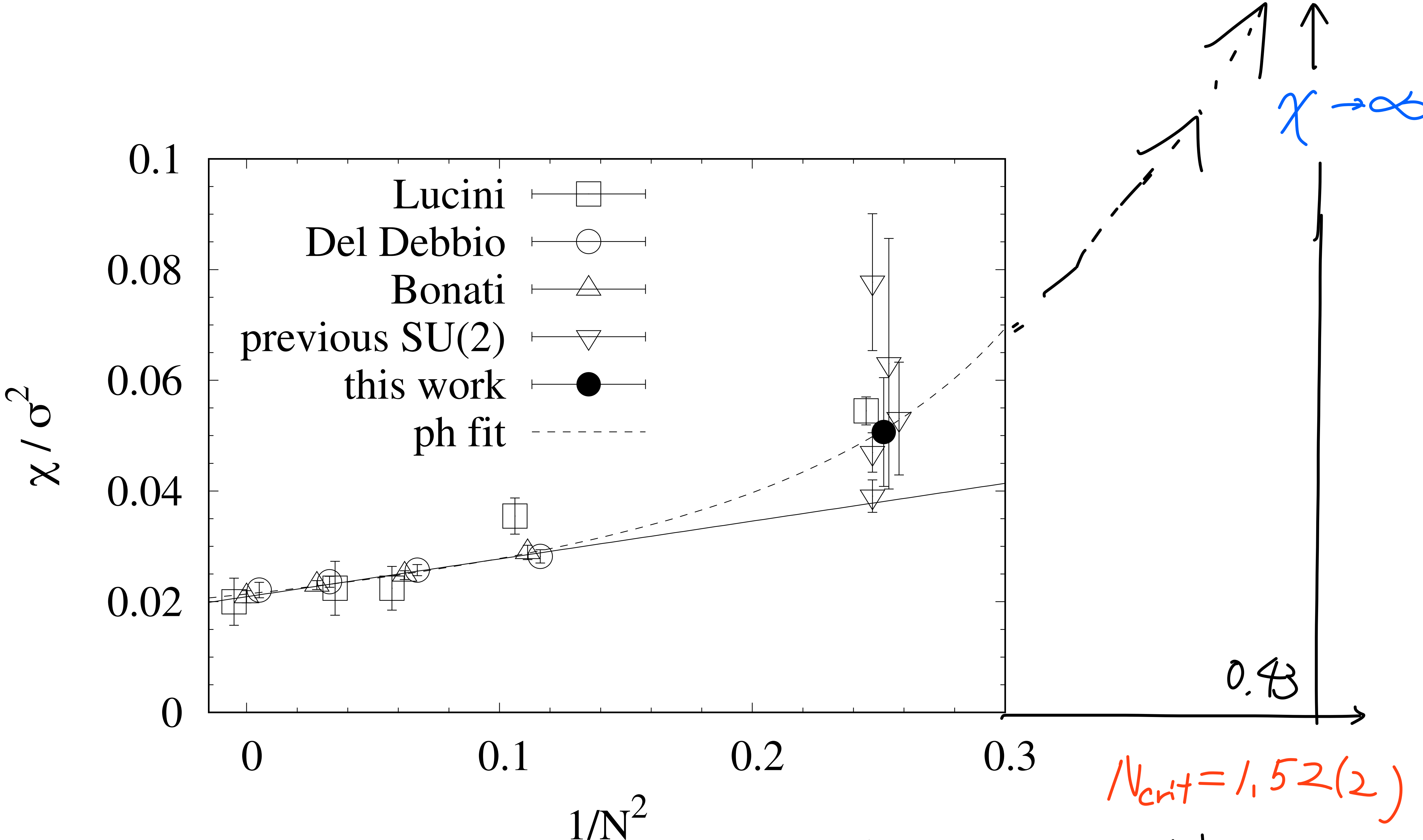
0.1
0
-0.1
β=1.750
β=1.850
β=1.975
N_S=24, 1.975
b_2
0
10
20
30
40
n_APE

$$\frac{\chi}{T_c^4} = 0.200(39) \ , \qquad \frac{\chi^{1/4}}{T_c} = 0.674(31) \ , \qquad b_2 = -0.049(20) \ ,$$

seems to be the first determination of b_2

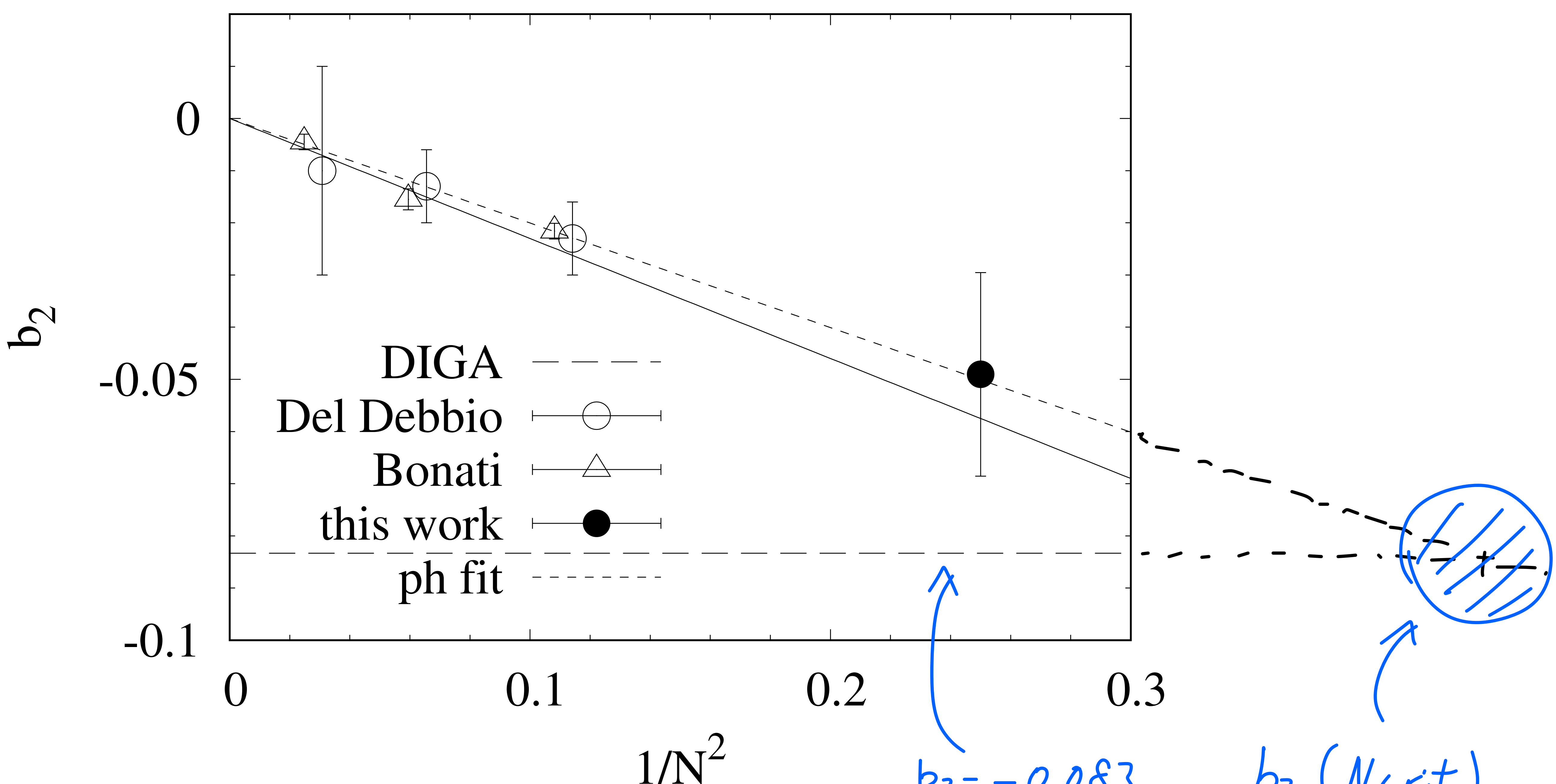
[cf. Bonanno, Bonati, D'Elía ('18) $b_4 = 6(2) \cdot 10^{-4}$]





when fitted with

[cf. Lüscher ('82) for 2d CP^{N-1} model] $\left(\begin{array}{l}\chi\to\infty \\ \text{at} \\ N = N_{crit}\end{array}\right) \rightsquigarrow \dfrac{\chi}{\sigma} = \left(\dfrac{\chi}{\sigma}\right)_{N=\infty} \dfrac{N^2}{N^2 - N_{crit}^2}$



$b_2 = -0.083$

(instanton)

$b_2(N_{crit})$

$\parallel$

$-0.087(5)$

$N_{crit} \sim 1.5$

Summary

* 4d SU(2) YM : still "large N"

spontaneous CP breaking, mass gap @ $\theta = \pi$

$$\frac{\chi^{1/4}}{T_c} = 0.674(31), \quad b_2 = -0.049(20)$$

[Quantitatively different from 2d CP^{N-1}- model]