Lec 2

2021/Jul/5

Masahito Yamazaki

Knot (Reidemeister move)

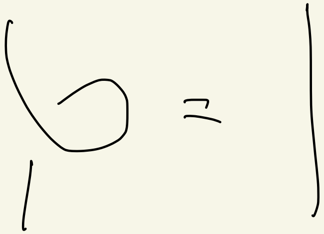
"anomoly"

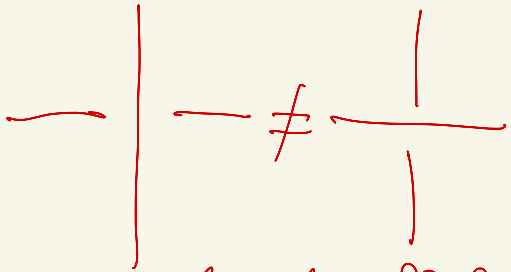
(over/under crossings do not matter

IM

~~RIの対応物はない~~

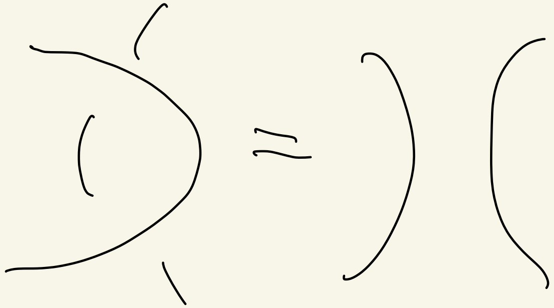
RI



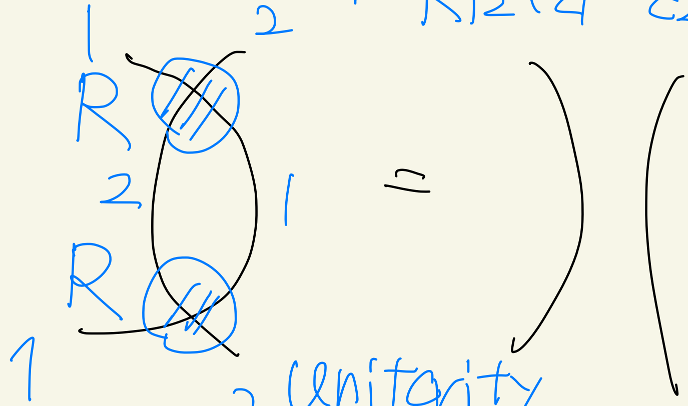


over/under crossing

RII

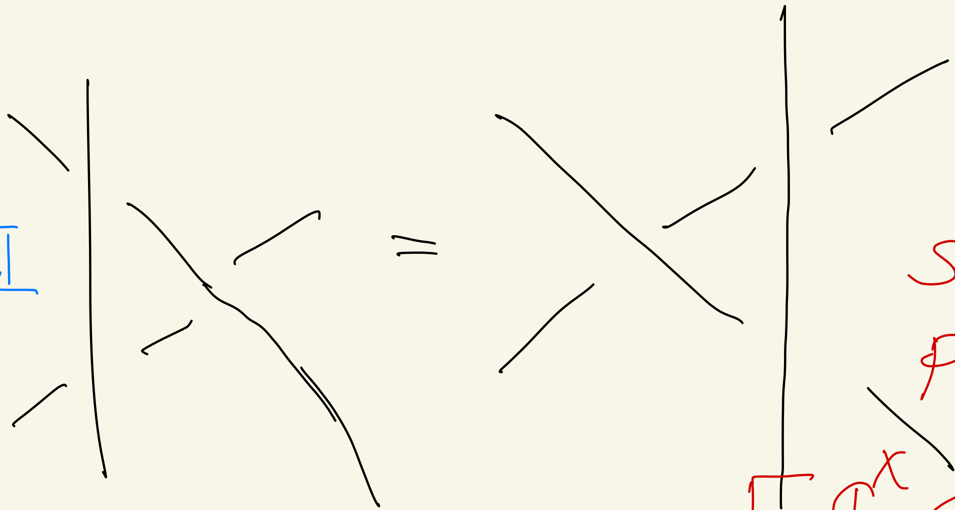


$R_{21}(z_2 - z_1) R_{12}(z_1 - z_2) = I$



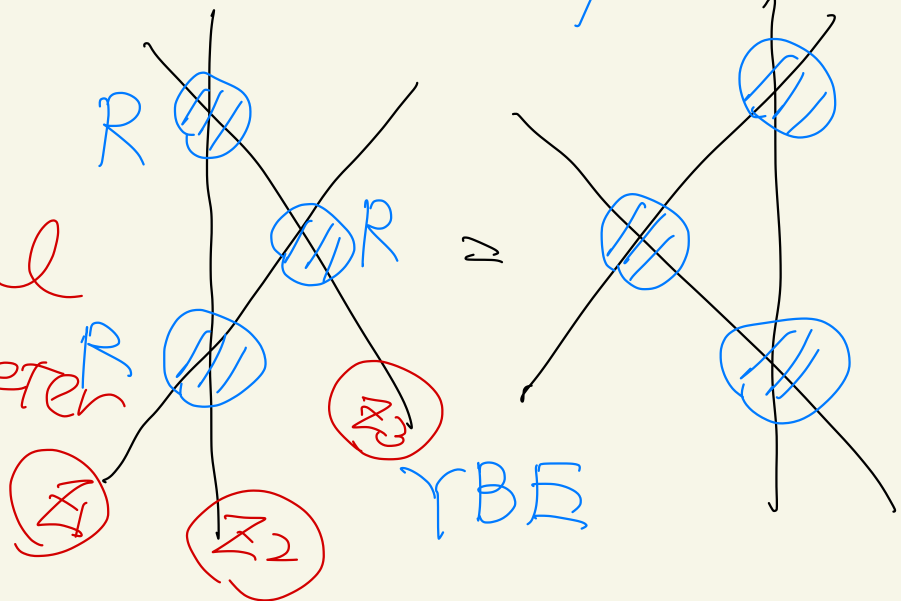
Unitarity

RIII

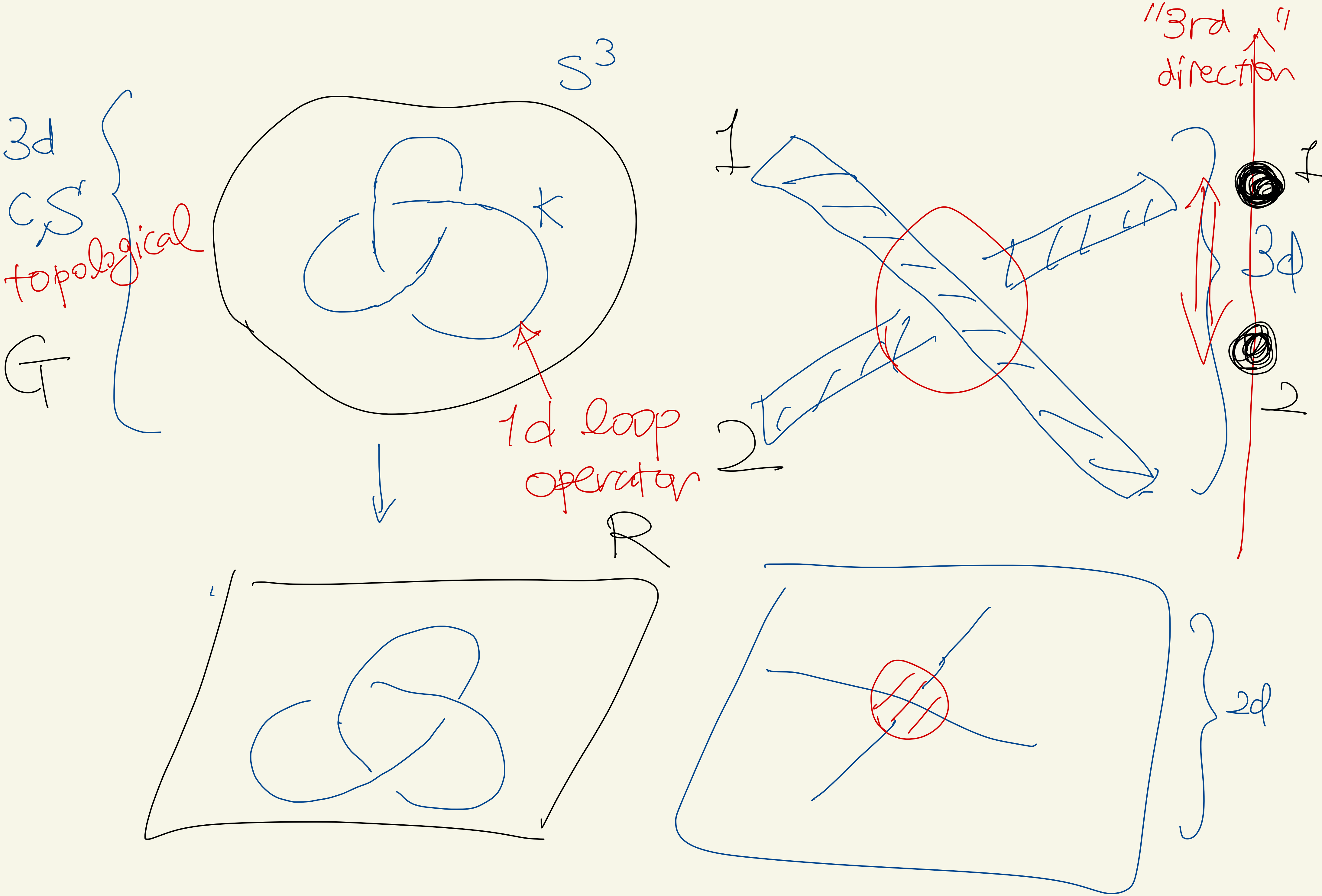


spectral parameter

$E \subset \mathbb{C}^\times \subset \mathbb{C}$



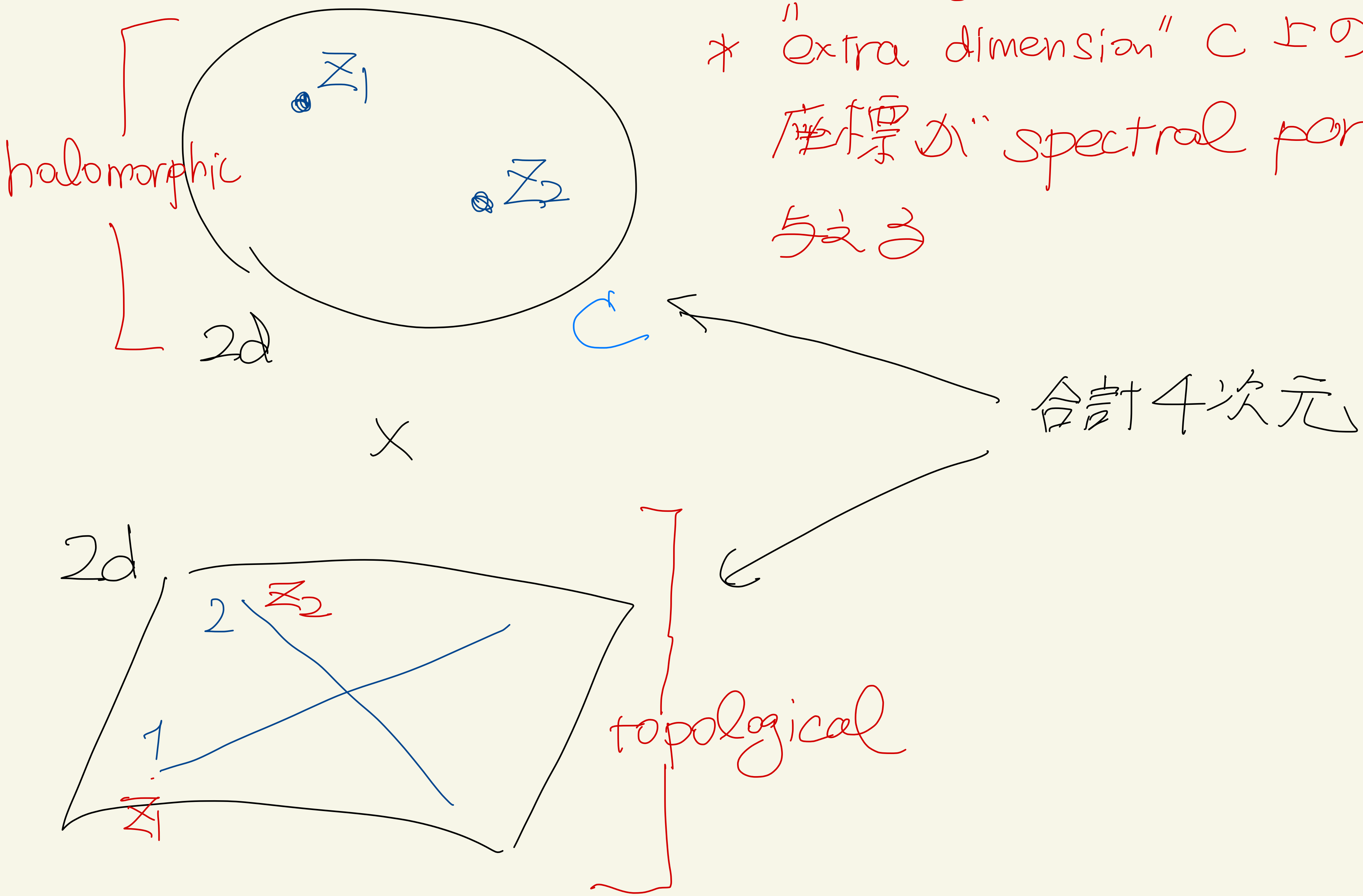
YBE



3d
CS
topological
G
S^3
K
1d loop
operator
R
1
2
"3rd direction"
3d
1
2
2d

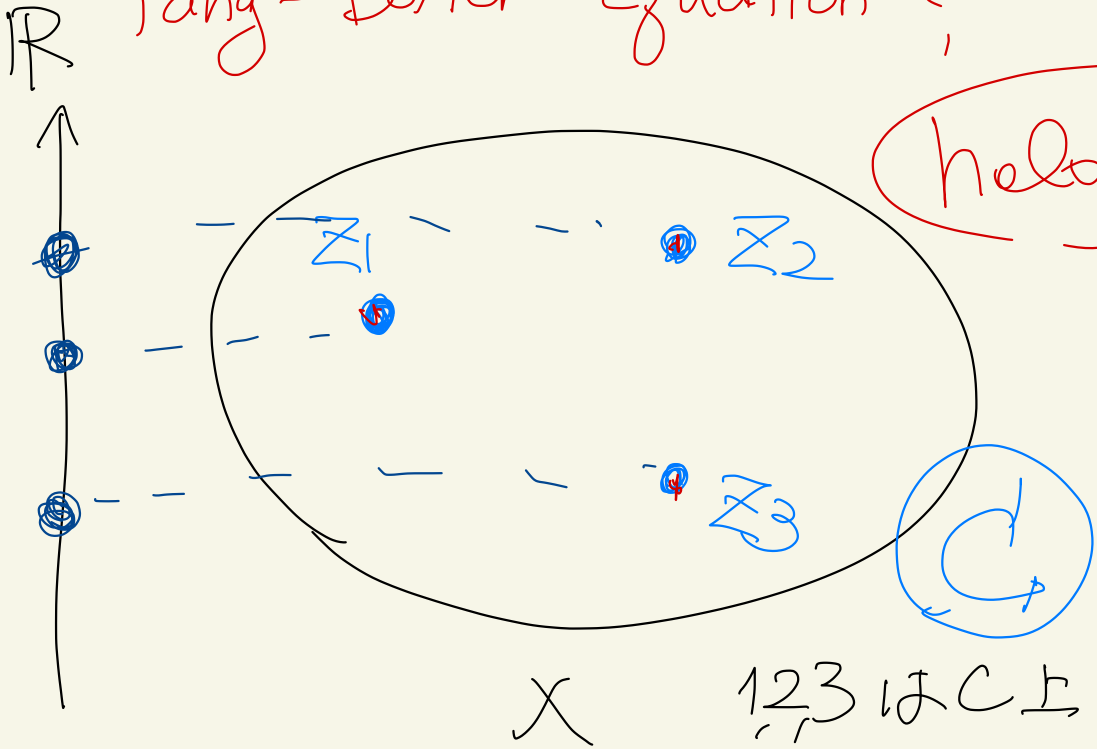
* C が2次元以上あれば over / under crossing はない

* "extra dimension" C 上の座標が spectral param を与える



Yang-Baxter equation ?

holomorphic

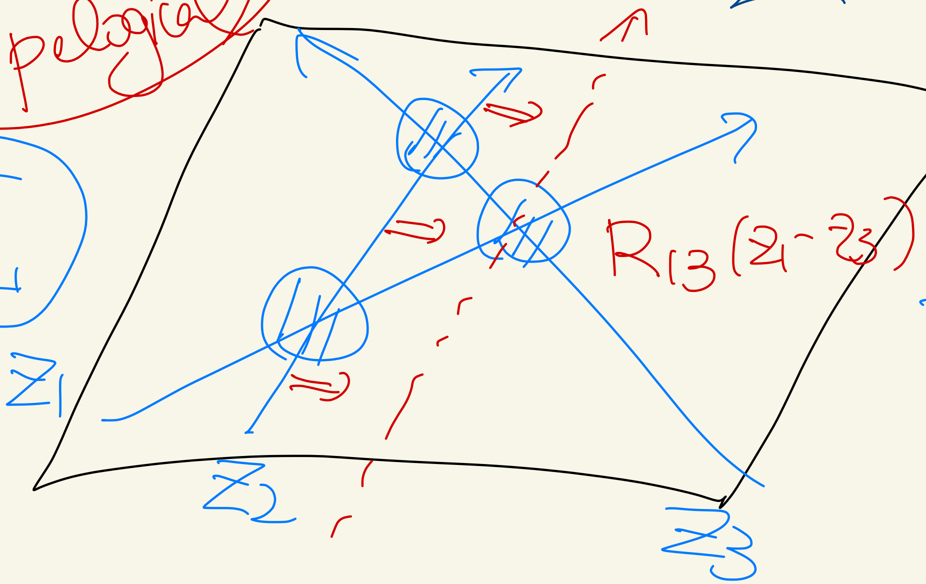


1,2,3は$\mathbb{C}$上別の点にいる → 重ならない

□方向のtopological inv. ⇒ YBE

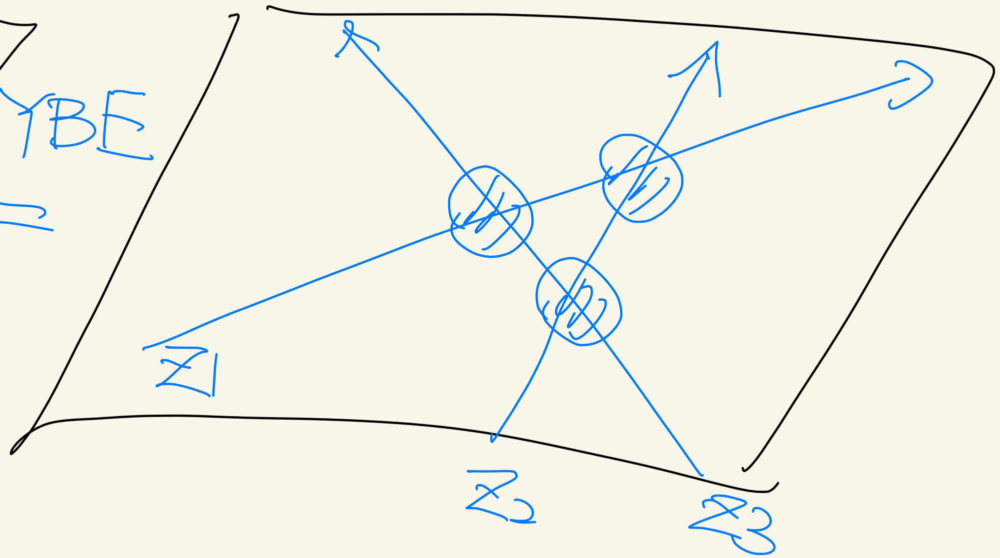
topological

Σ



YBE

=



4d theory ?

- $\Sigma \times \mathbb{C}$ 上定義

  Σ: top　　$\mathbb{C}$: hol.

- 1次元の defect を持つ

- 3d CS theory に"近い"

  結び目

$\rightsquigarrow$ 4d Chern-Simons theory

4d CS

$$S = \frac{1}{2\pi\hbar}\int_{\underbrace{\Sigma \times \mathbb{C}^{1}}_{2}} \overbrace{\omega}^{\text{1-form}}\wedge \underbrace{\mathrm{Tr}\left(A\wedge dA + \tfrac{2}{3}A^{3}\right)}_{\text{3-form}}$$

Killing form

$\omega = dz$, 2

- A : gauge field (principal G-bun$\oslash$ connection)

1-form, gauge group

$$A \to g^{-1}Ag + g^{-1}dg \quad (g \in G)$$

$$F = dA + A\wedge A \longrightarrow g^{-1}Fg$$

field strength

- ω : hol. 1-form on $\mathbb{C}^{1}$

without zero

$(\omega = dz,\ \ z\in\mathbb{C},\ \mathbb{C}^{\times} = \mathbb{C}\setminus\{0\},\ E)$

dz/z

no pole

rational, trig, ell.

∞ 2nd order pole

0, ∞ 1st order poles

- $\hbar$: Planck const.

$\to$ perturbation

* Riemann 面 C 上 hol. 1-form without zero ω が…

存在 $\xrightarrow[\substack{\Uparrow \\ \text{RH}}]{}$ $C = \mathbb{C}, \mathbb{C}^\times, E$

$g=0$, $g=0$, $g=1$

R-matrixの分類 Belavin-Drinfeld

$$R_\hbar(z) = I + \hbar \underbrace{r(z)}_{\substack{\text{classical} \\ \text{r-matrix}}} + \hbar^2 r'(z) + \cdots$$

quasi-classical R-matrix

$\longrightarrow$ rational, trig, ell.

* $A = A_x dx + A_y dy + \cancel{A_z dz} + A_{\bar z} d\bar z$

$x, y \in \Sigma$

$z, \bar z \in \mathbb{C}$

— equation of motion

$$\frac{\delta S[A]}{\delta A} = 0 \rightsquigarrow F_{xy} = \underbrace{F_{x\bar{z}}}_{\parallel} = \underbrace{F_{y\bar{z}}}_{} = 0$$

$$\partial_x A_{\bar{z}} - \partial_{\bar{z}} A_x + [A_x, A_{\bar{z}}]$$

$\downarrow$ choose gauge s.t. $A_{\bar{z}} = 0$

$$F_{xy} = \partial_{\bar{z}} A_x = \partial_{\bar{z}} A_y = 0$$

Σ方向に flat

C方向に holomorphic

$\underbrace{\Sigma}_{x,y} \times C$

A_x, A_y