M5-branes / 3d SUSY QFT

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Topological Phases of Matter

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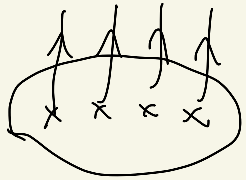
Lec II

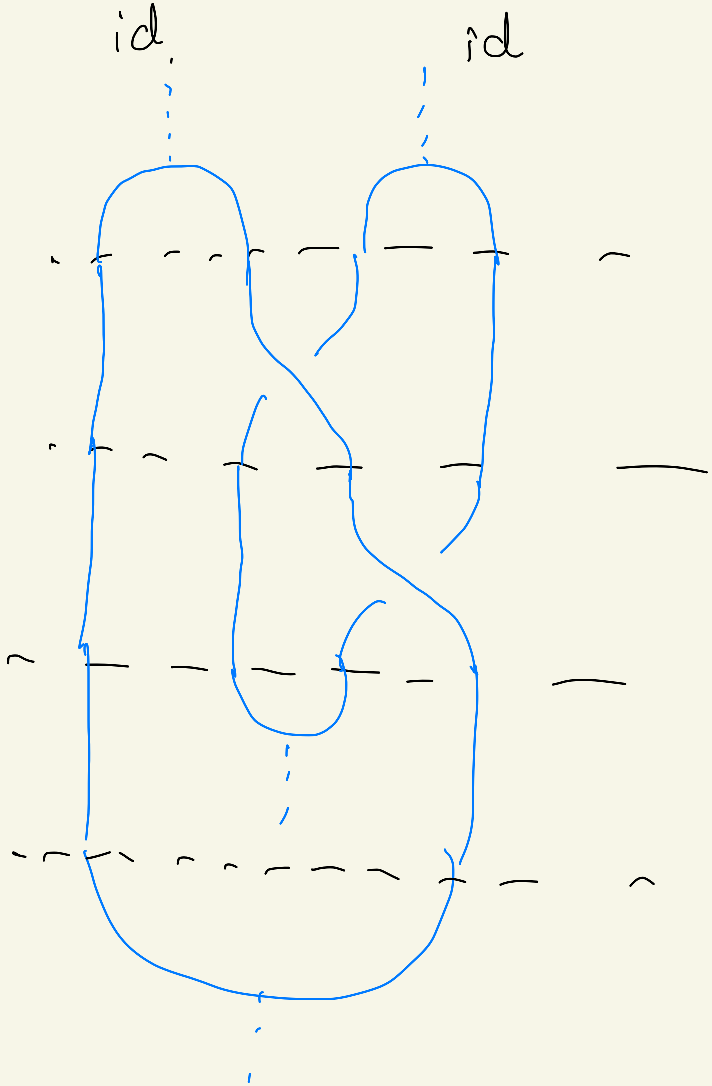
Anyon data = modular tensor category
= Moore-Seiberg data

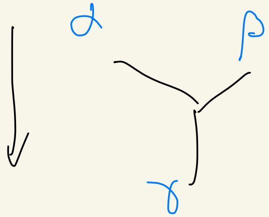
"genus 0"



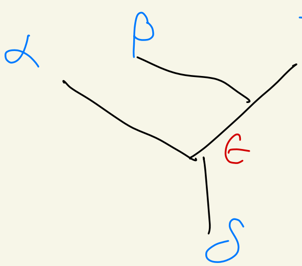
S^2

Knot/braid

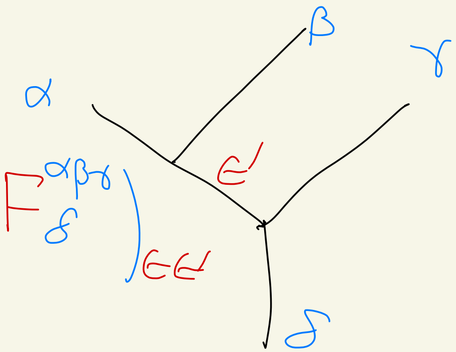


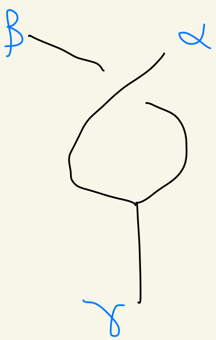


$$\alpha \otimes \beta = \bigoplus_{\gamma} N_{\alpha\beta}^{\gamma}\, \gamma$$

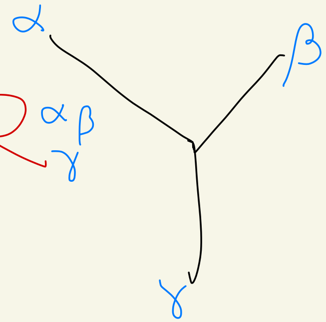


$$= \left(F^{\alpha\beta\gamma}_{\delta}\right)_{\epsilon\epsilon'}$$



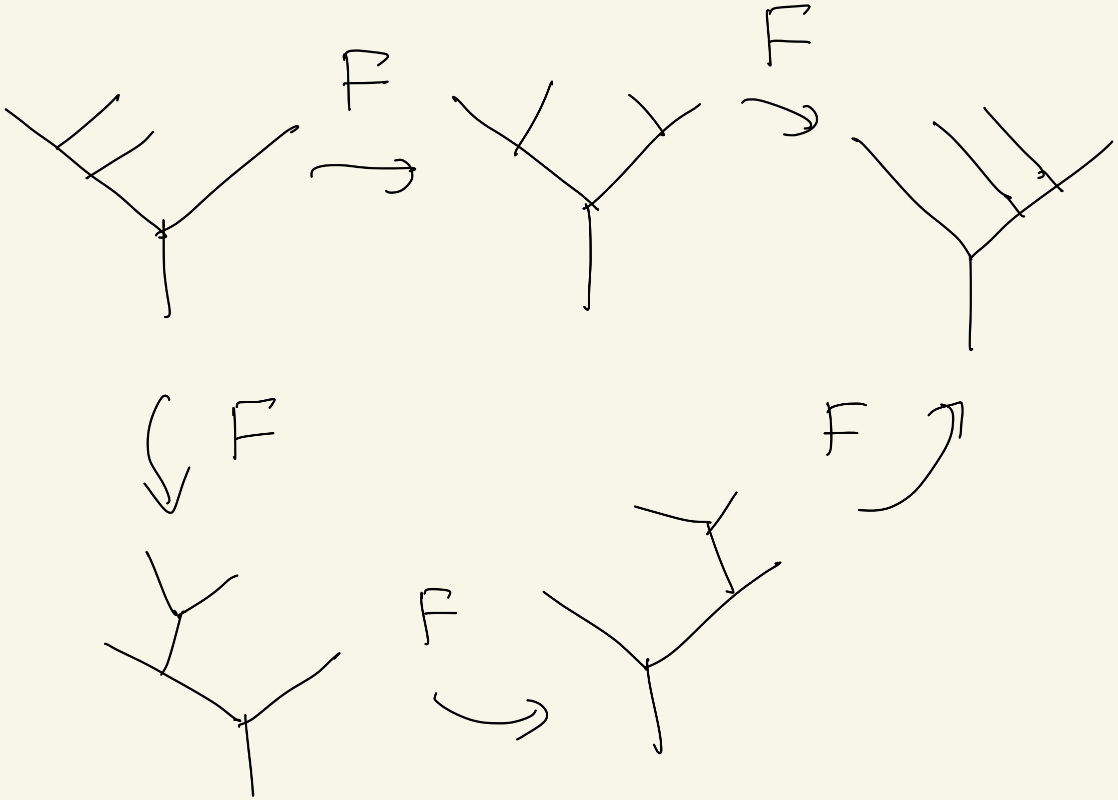


$$= R^{\alpha\beta}_{\gamma}$$

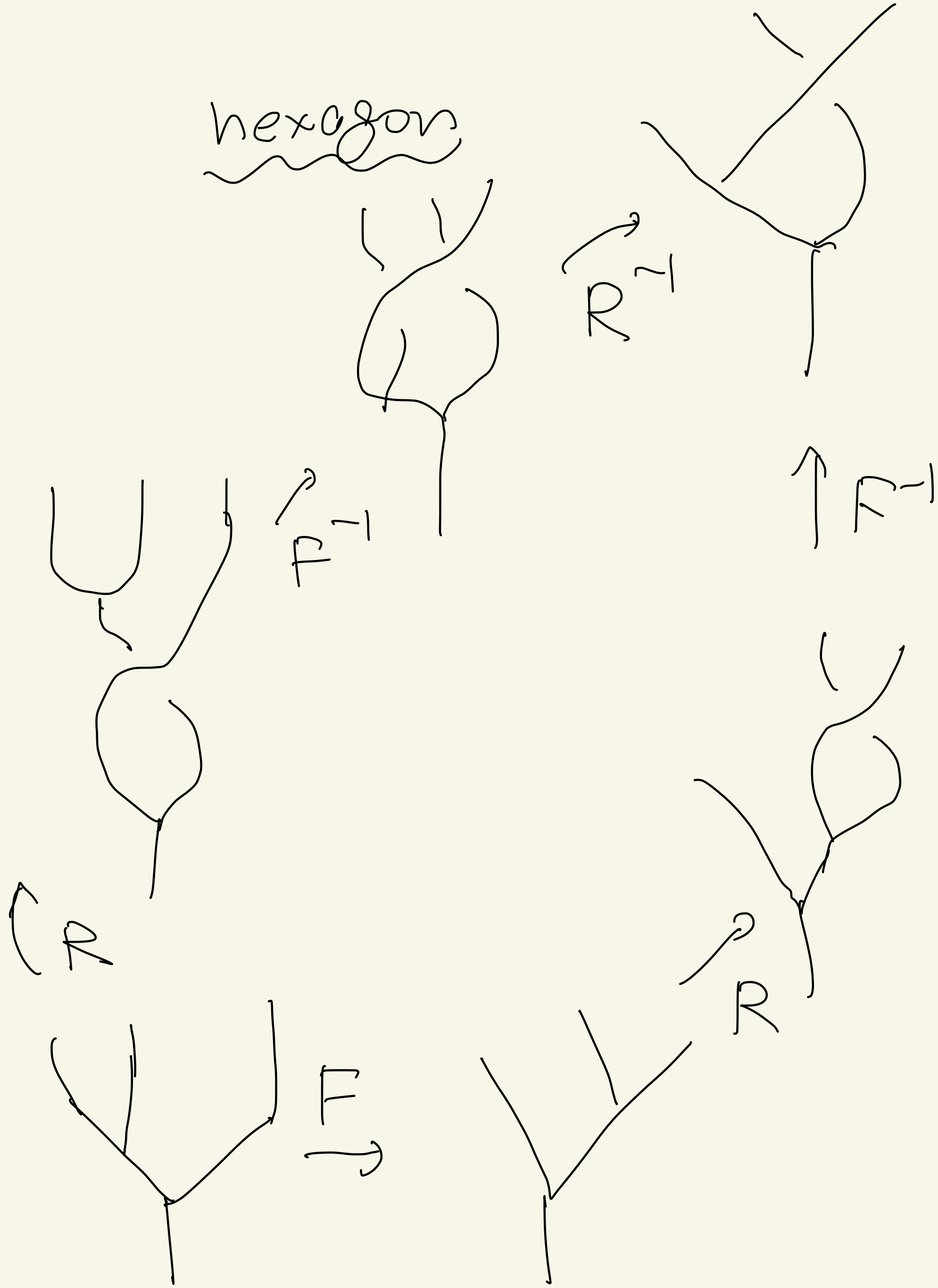


consistency conditions

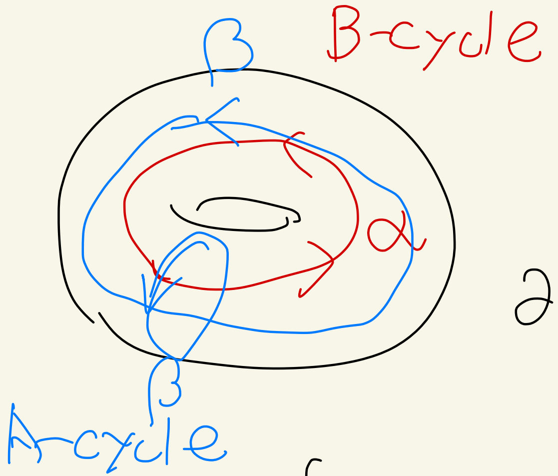
pentagon



hexagon



genus 1



$D^2 \times S^1$: solid torus

$\partial(D^2 \times S^1) = T^2$

$$\mathcal{H}(T^2) := \mathrm{span}\left\{ |\alpha\rangle = \mathcal{O}_{\alpha}^{B\text{-cycle}} |0\rangle \right\}$$

S, T: generators of $SL(2, \mathbb{Z})$

$$S = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \qquad T = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$$

$$S_{\alpha\beta} = \langle \alpha | S | \beta \rangle$$

$$T_{\alpha\beta} = \langle \alpha | T | \beta \rangle = e^{2\pi i \underbrace{h_\alpha}_{\text{topological spin}}} \delta_{\alpha\beta}$$

$$\mathcal{O}_{\beta}^{B\text{-cycle}} |\alpha\rangle = \sum_{\gamma} (N_\beta)_{\alpha}{}^{\gamma} |\gamma\rangle$$

Verlinde formula

$$\mathcal{O}_{\beta}^{A\text{-cycle}} |\alpha\rangle = (S^{-1} N_\beta S) |\alpha\rangle = \frac{S_{\beta\alpha}}{S_{0\alpha}} |\alpha\rangle$$

3d $\mathcal{N}=2$ SUSY QFT $\rightsquigarrow$ 3d TQFT ?

F, R, S, T

α : anyon

$Z^{\mathcal{M}}$ $\langle$ Wilson line $\rangle$

$Z^{\mathcal{M}}$ closed 3-manifold

O_β

$S^1 \to \mathcal{M}_{g,p}$

deg = p

$\downarrow$

Σ_g

e.g. $g=0,\ p=0$

$S^1\times S^2$

$g=0,\ p=+1$

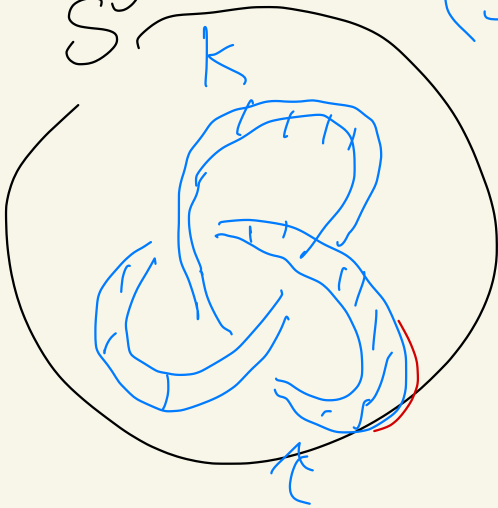
$S^1 \to S^3$

$\downarrow$

S^2

S^3

K



$\left(S^3 \setminus (D^2\times S^1)\right)$

$\cup\ (D^2\times S^1)$

$\uparrow$

$SL(2,\mathbb{Z})$ action

solid torus

$D^2\times S^1$

Dehn surgery

$$\boxed{Z^{\mathcal{M}_{g,p}} = \sum_\alpha (S_{0\alpha})^{2-2g}\,(T_{\alpha\alpha})^p}$$

$$\boxed{\langle O_\beta\rangle = \sum_\alpha (S_{0\alpha})^{2-2g}\,(T_{\alpha\alpha})^p\,\frac{S_{\beta\alpha}}{S_{0\alpha}}}$$

$S^3 \longrightarrow M$

K Dehn surgery

3d $\mathcal{N}=2$ theory on $\mathcal{M}_{g,p} \Longrightarrow S, T$

$$\left(Z^{\mathcal{M}_{g,p}} = \sum_{\alpha\text{: Bethe vacua}} (\mathcal{H}_\alpha)^{g-1} (\mathcal{F}_\alpha)^p \right.$$

fibering op

$$\left. \langle \mathcal{O}_\beta \rangle = \sum_{\alpha\text{ : vacua}} (\mathcal{H}_\alpha)^{g-1} (\mathcal{F}_\alpha)^p \mathcal{O}_\beta(z_\alpha) \right)$$

handle-gluing operator

3d TQFT

$$\boxed{Z^{\mathcal{M}_{g,p}} = \sum_{\alpha\text{: anyon}} (S_{0\alpha})^{2-2g} (T_{\alpha\alpha})^p}$$

$$\boxed{\langle \mathcal{O}_\beta \rangle = \sum_\alpha (S_{0\alpha})^{2-2g} (T_{\alpha\alpha})^p \frac{S_{\beta\alpha}}{S_{0\alpha}}}$$

$\mathcal{H}_\alpha \longleftrightarrow (S_{0\alpha})^{-2}$

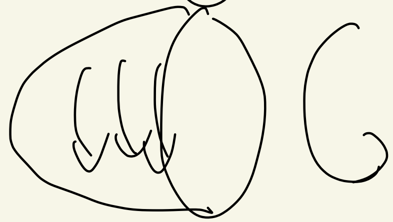
$\mathcal{F}_\alpha \longleftrightarrow T_{\alpha\alpha}$

$\mathcal{O}_\beta(z_\alpha) \Updownarrow \dfrac{S_{\beta\alpha}}{S_{0\alpha}}$

$\mathcal{N}=2$

3d theory on $S^1 \times \mathbb{R}^2_\epsilon$

cigar



$\underbrace{S^1 \times \mathbb{R}^2_\epsilon}_{\text{2d } \mathcal{N}=(2,2)}$

$G_{\mathbb{C}}$

3d CS theory

$$\exp\left(\frac{\partial W}{\partial \sigma}\right) = 1$$

shape angle

gauge potential

scalar

(A, σ)

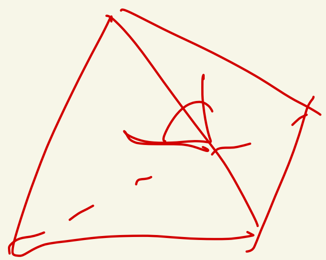
complex

vacua

$W(\vec{\sigma})$

$$\exp\left(\frac{\partial W}{\partial \vec{\sigma}}\right) = 1$$

solution Bethe value



3d $\mathcal{N}=2$ in general $\rightsquigarrow$ 3d TQFT data?

more special?

$S_{\alpha\beta}$, $T_{\alpha\beta}$

in general

$Z_{S^1\times S^2} \neq 1$

?

Full MTC

F, R, S, T

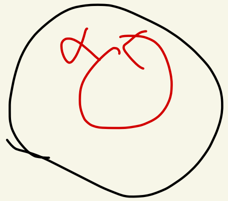
Z^{u}: depends on mass parameter etc.

3d TQFT $\rightsquigarrow$ $\dim \mathcal{H}(S^2) = 1$

T^2

$|\alpha\rangle$

S^2



$\boxed{Z_{S^1\times S^2}}$

$\parallel$

$\mathrm{Tr}_{\mathcal{H}(S^2)}\, 1$

$\parallel$

$\dim \mathcal{H}(S^2) = 1$

3d TQFT $\longleftarrow$ 3d $\mathcal{N}=2$ SUSY QFT $\longleftrightarrow$ 3-manifold M

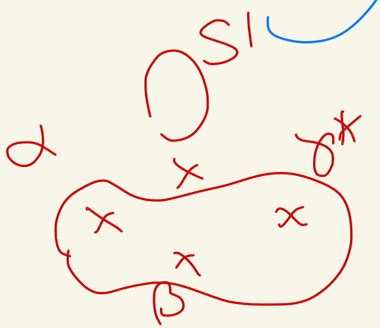
CS

3d TQFT	3d $\mathcal{N}=2$ SUSY QFT	3-manifold M
α : anyon	α : vacua	α : flat connection $A = A_\alpha$
$S_{0\alpha}^{-2}$	$\mathcal{H}_\alpha$	Reidemeister torsion
$T_{\alpha\alpha}$	F_α	CS invariant $CS[A_\alpha]$
Wilson line Θ_β	Wilson line	Wilson line

"chirals"

3 + 3 "AGT"

$\underbrace{3 + 3}_{6}$



$= N_{\alpha\beta}^{\gamma}$??

Strategy (non-generic?)

$\Downarrow$

I : $\underset{\sim\sim\sim}{M}$; special choice $\longrightarrow$ 3d TQFT

(not hyperbolic)

(e.g. Brieskorn sphere)

S, T

II : $\overbrace{\text{3d SUSY}}^{\mathcal{N}=4}$ QFT

non-semisimple

$\rightsquigarrow$ 3d TQFT

topological twist

logarithmic CFT

III : $\overbrace{\text{3d SUSY}}^{\mathcal{N}=4}$ QFT $\xrightarrow[\text{IR}]{}$ SCFT

(limit/subsector

$\longrightarrow$ 3d TQFT