Tensor Network and Renormalization

Masahito Yamazaki

KAVLI IPMU

THE UNIVERSITY OF TOKYO

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Currently at Kavli IPMU, University of Tokyo

hep-th (many) hep-ph (10) hep-lat (3)

cond-mat (3) quant-ph (1) astro-ph (1)

math-ph (2) math (8) nlin.SI (1)

Renormalization

UV

IR

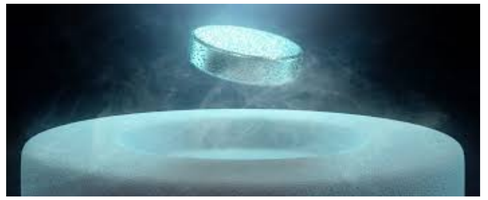
microscopic

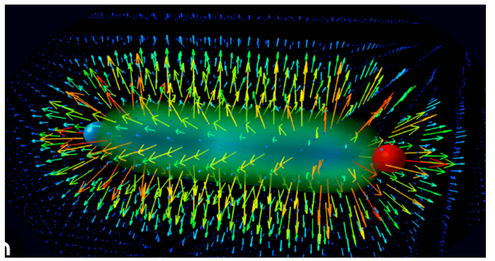
?

macroscopic

$\mathcal{L}$ / H

"emergent" phenomena

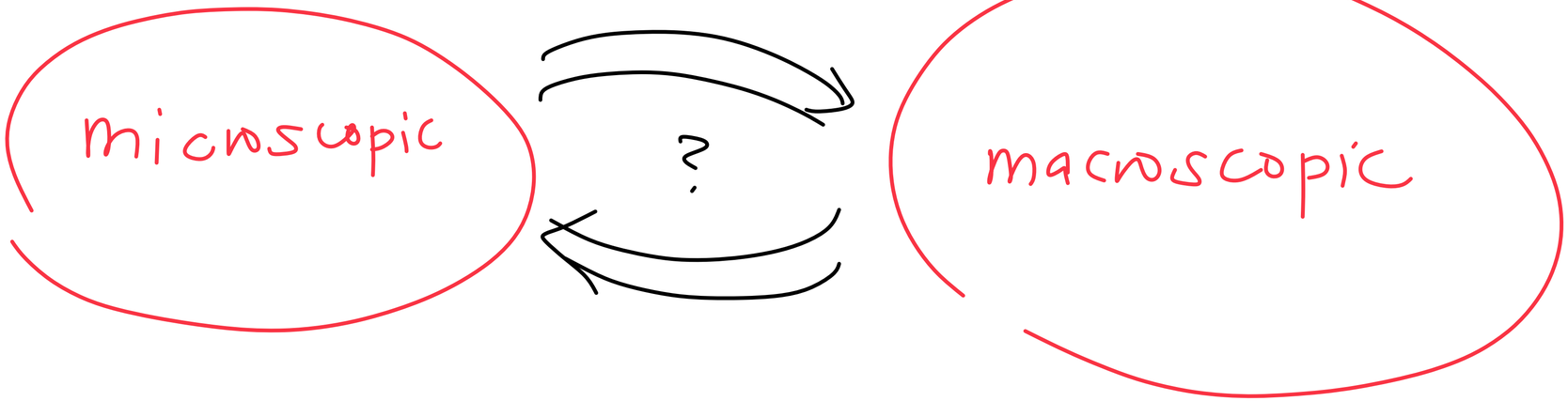


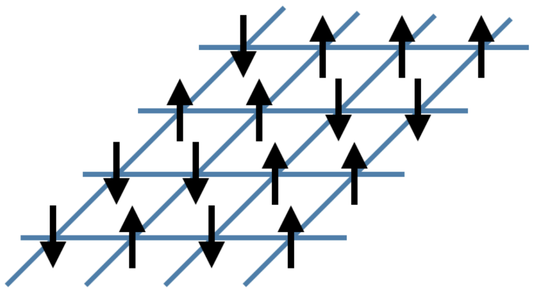


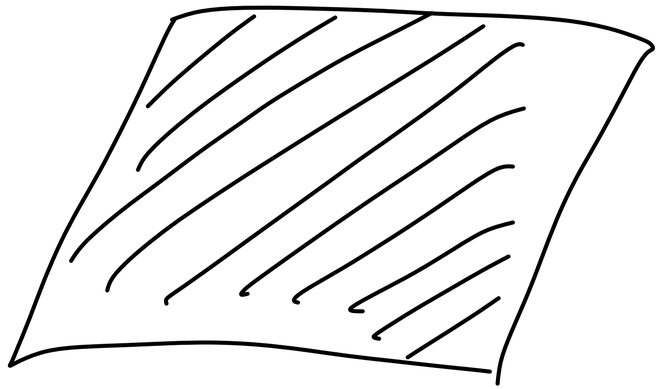
Renormalization

UV

IR







?

Lattice

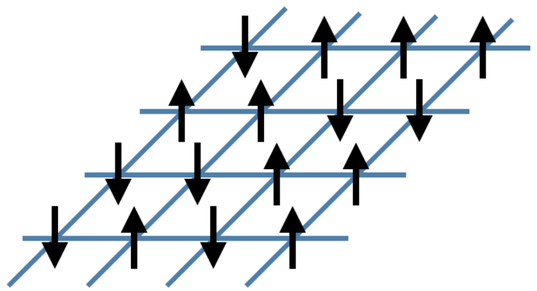
QFT/CFT

?

Renormalization

- often checks with symmetries / critical exponents + universality

- can we discuss the RG flow more directly ?

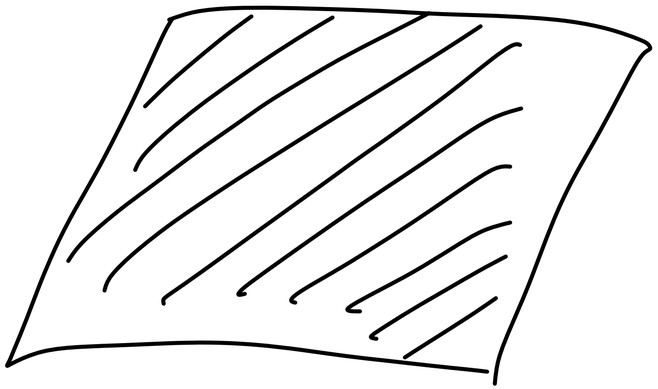


Lattice

thermodynamic/ scaling limit

discretization

?



QFT / CFT

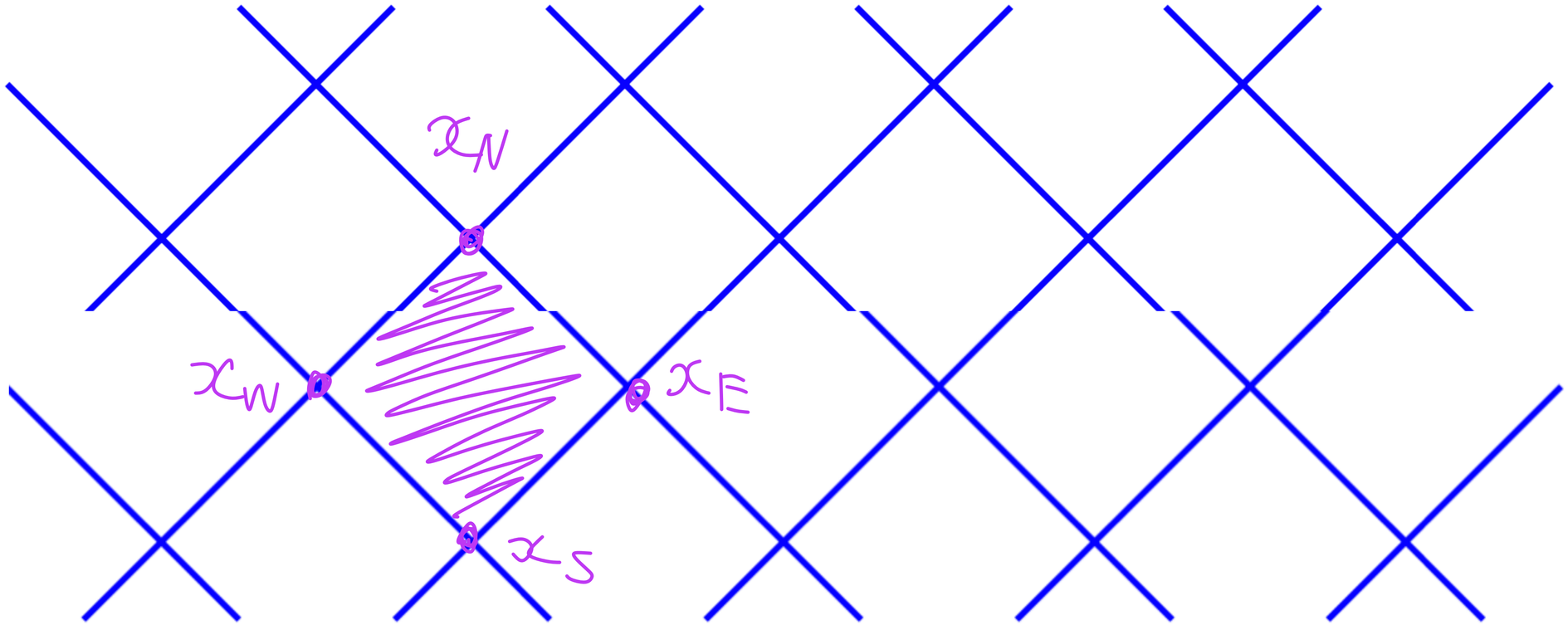
"Digression" on Integrable Models

Discretization / Regularization of QFT:

One of early-day motivations of integrable models

[Many papers by Destri-de Vega
also Faddeev, Reshetikhin, Volkov, ...]

time



$$x_N = f(x_W, x_E, x_S)$$

This is revisited in the context of

quantum simulation of integrable models

[Vovicat - Zadnik - Prosen ('17)]

⇒ Integrable conserved charges on

IBM / Ion Q quantum computers

[Moriyoshi - Okuda - Pedersen - Suzuki - MY - Yoshida ('22)]

- Original discussion of light-cone discretization:

  Case-by-case analysis

- More systematic analysis in 4d Chern-Simons theory

  [Ashwinkumar - Sakamoto - MY (23)]

  → Sakamoto's talk on Friday

Beyond Integrability ?

Keep track RG flows ?

IR CFT ?

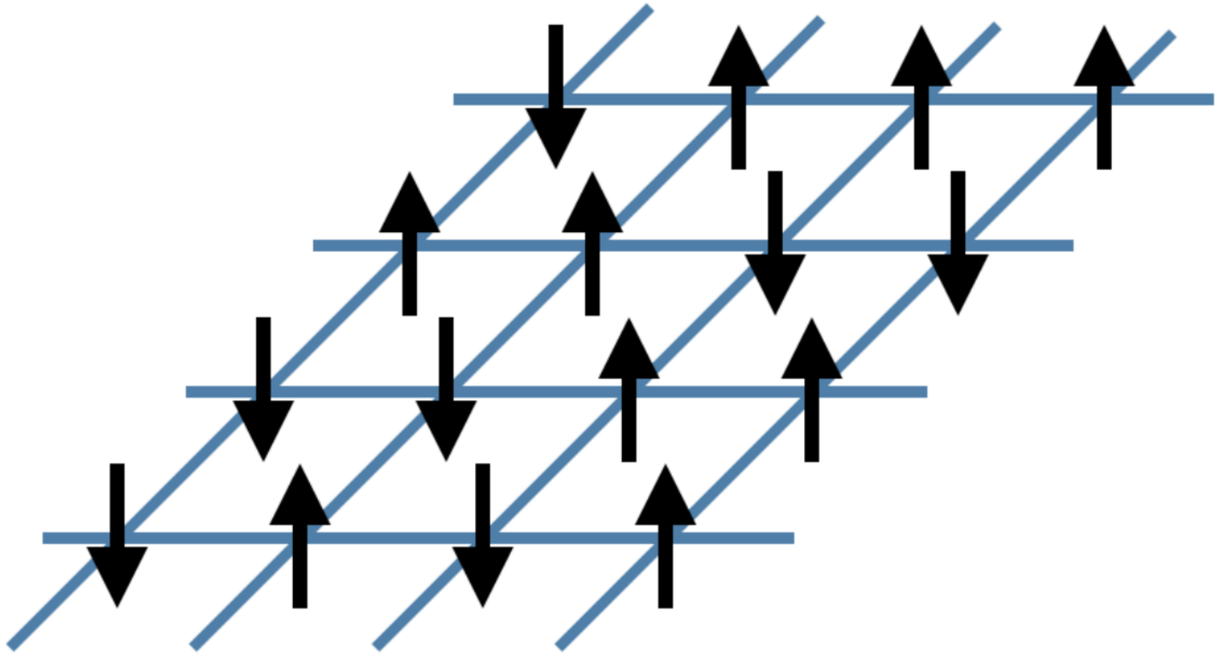
Tensor Network

eg

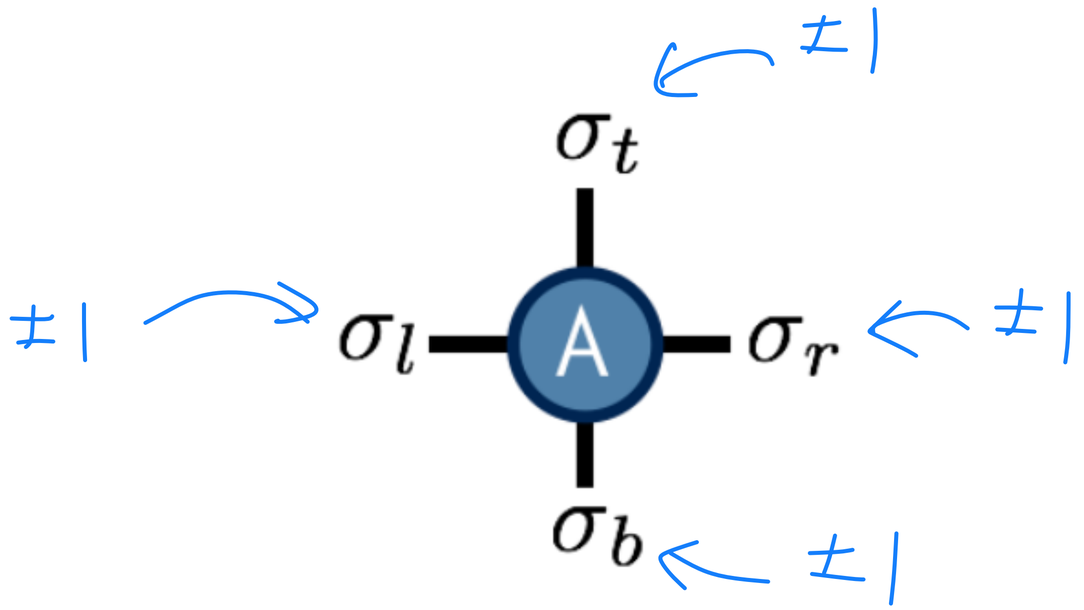
Ising

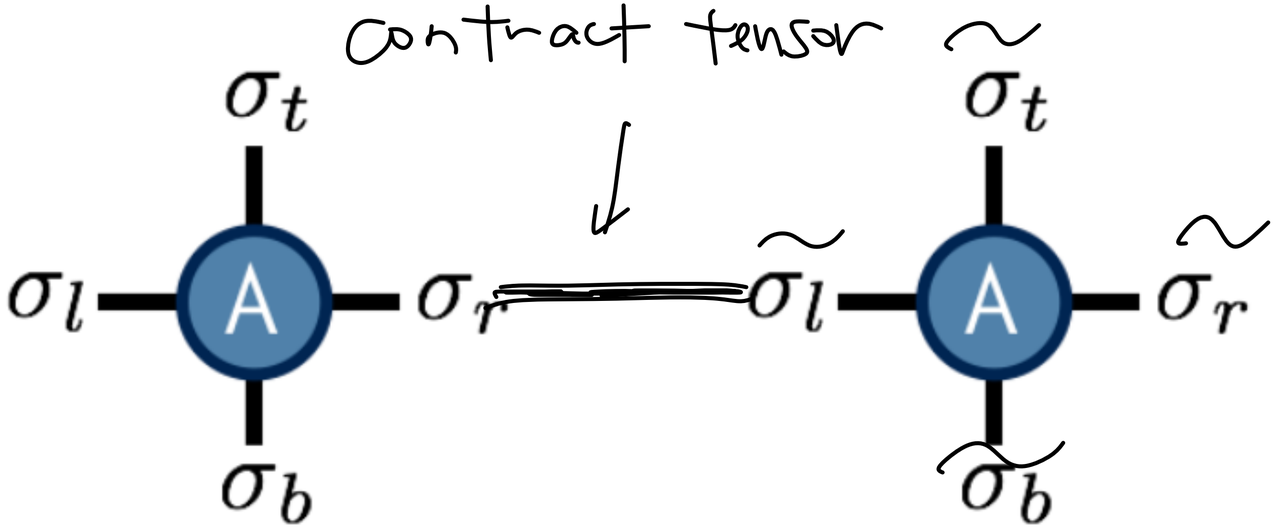
$$Z = \sum_{\{\sigma_i\}} e^{-\left(\sum_{\langle ij \rangle} \sigma_i \sigma_j\right)/T}$$

$\sigma_i = \pm 1$



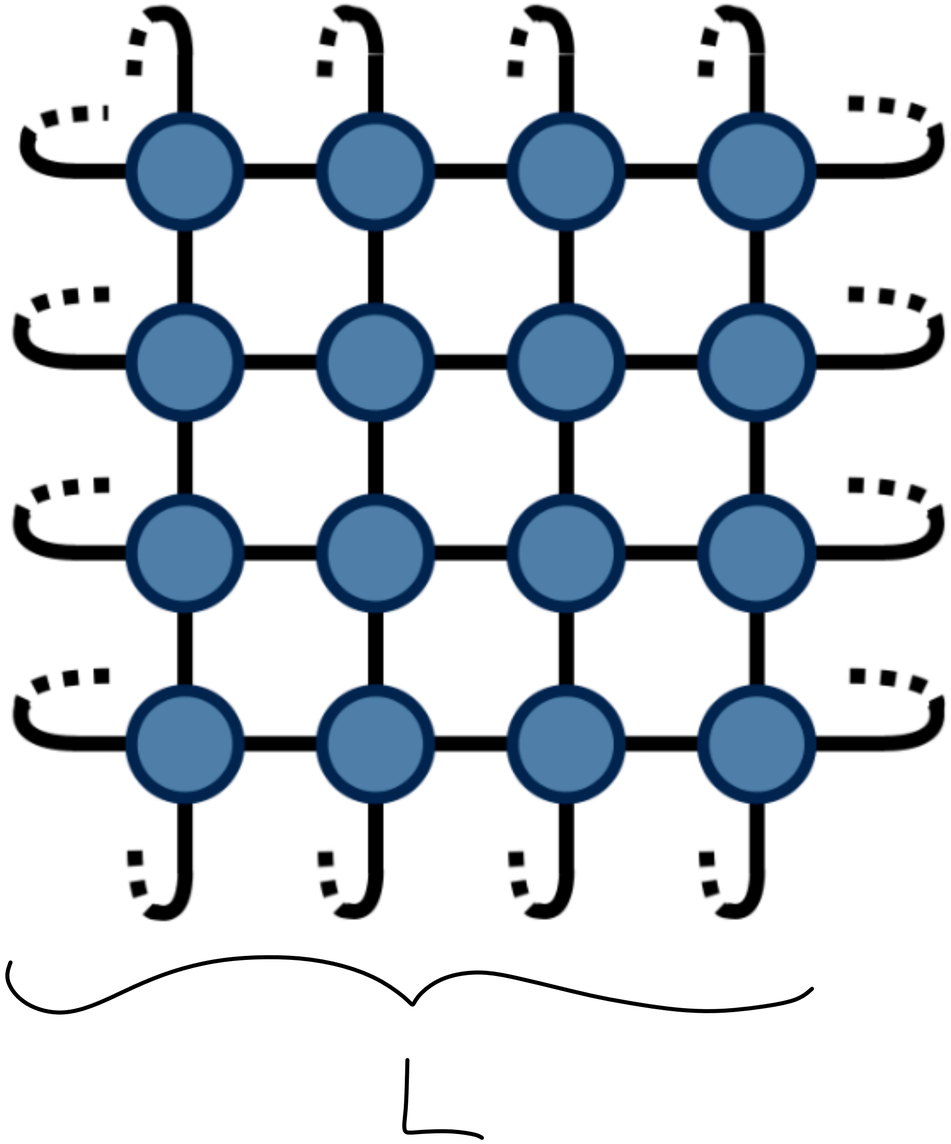
$$A^{\sigma_t\sigma_r\sigma_b\sigma_l} = e^{-(\sigma_t\sigma_r+\sigma_r\sigma_b+\sigma_b\sigma_l+\sigma_l\sigma_t)/T}$$





$$\sum_{\sigma_r = \tilde{\sigma}_l} A^{\sigma_t \sigma_r \sigma_b \sigma_l} \tilde{A}_{\tilde{\sigma}_t \tilde{\sigma}_r \tilde{\sigma}_b \tilde{\sigma}_l}$$

$$Z = \quad = \mathrm{Tr}\left(\begin{matrix}\text{transfer}\\ \text{matrix}\end{matrix}\right)^{L}$$



Many ingredients for integrable models:

reformulated in terms of tensor networks

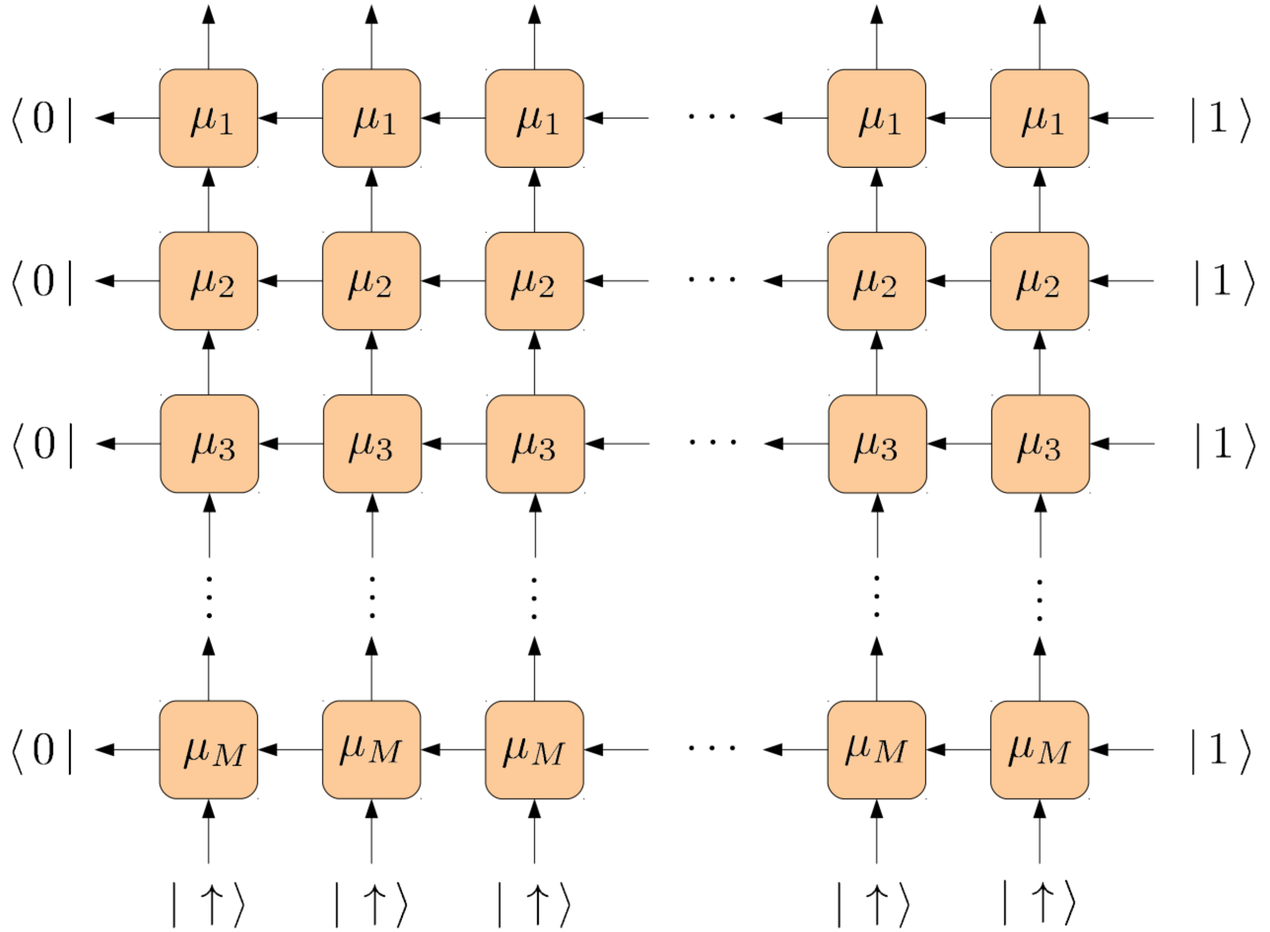
e.g.

Bethe state is

a MPS

[Katsura-Maruyama('09)

Murg-Korepin-Verstraete ('12)]



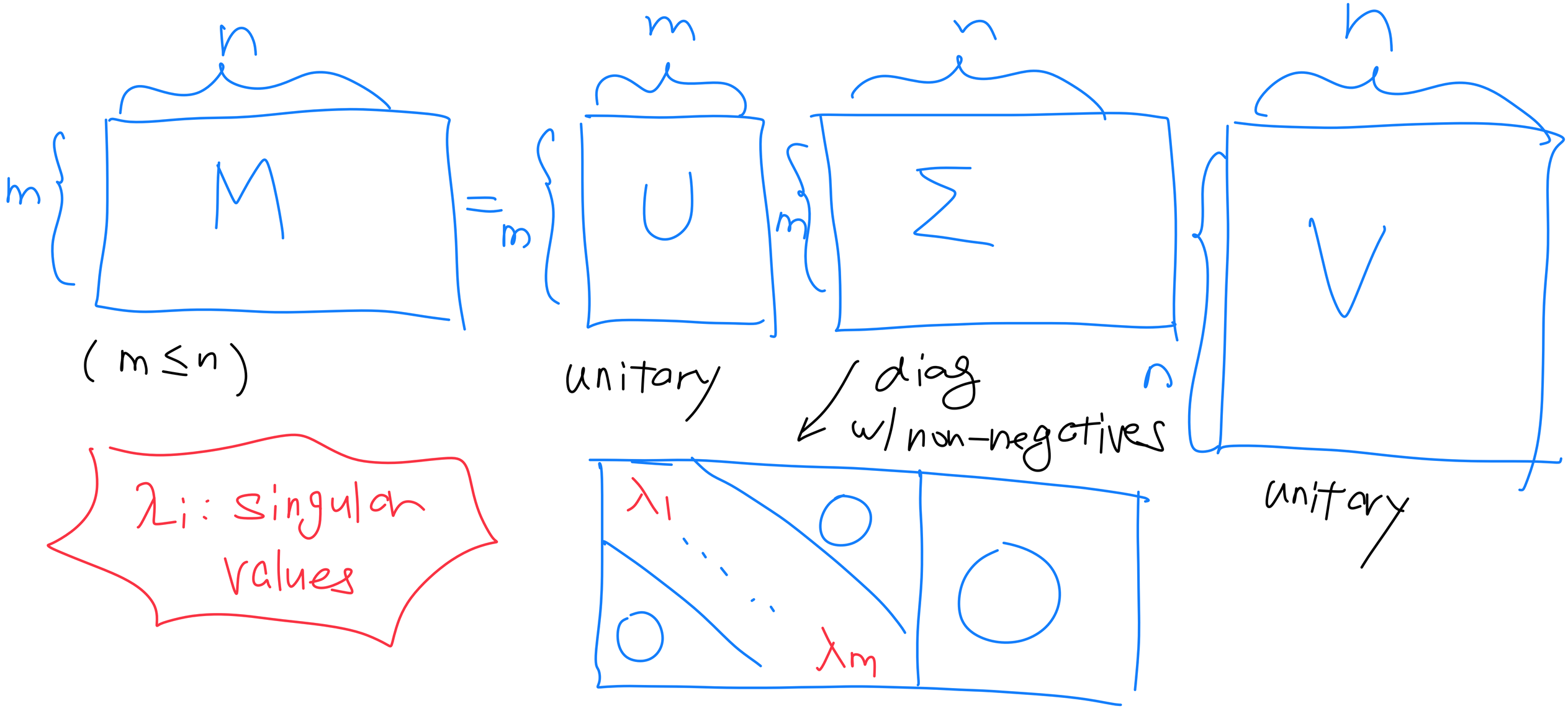
RG flow in Tensor Networks ?

- modern incarnation of Kadanoff-type RG
- comes in many variants eg.
  - TRG [Levin-Nave ('07)]
  - TNR [Evenbly-Vidal ('15), cf. Gu-Wen ('09)]
  - Loop-TNR [Yang-Gu-Wen ('17)]
  - Gilt-TNR [Hauru-Delcamp-Mizera ('17)]

  etc.

TEFR

Singular Value Decomposition (SVD)



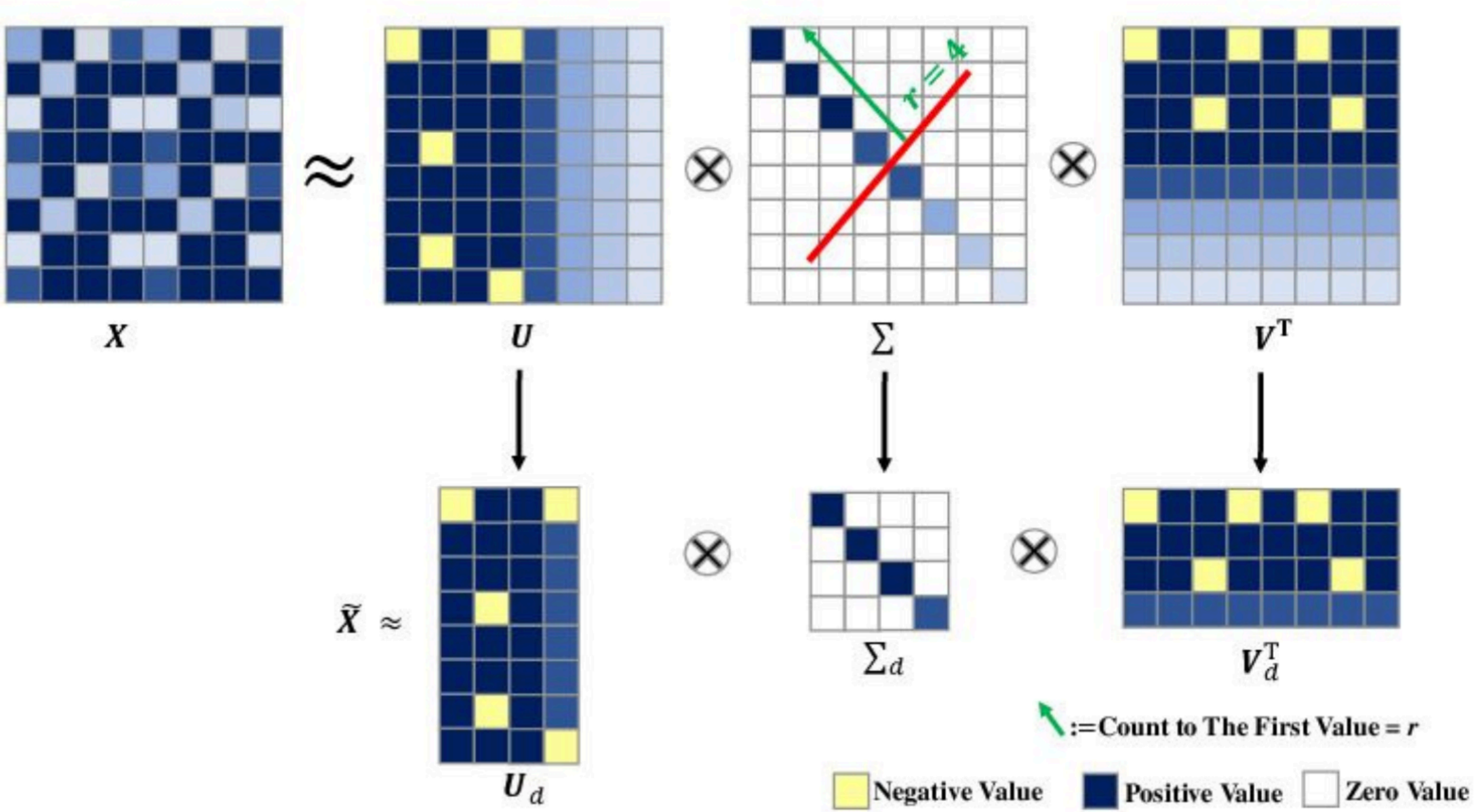


Figure borrowed from https://ai-artificial-intelligence.webyes.com.br/singular-value-decomposition-svd/

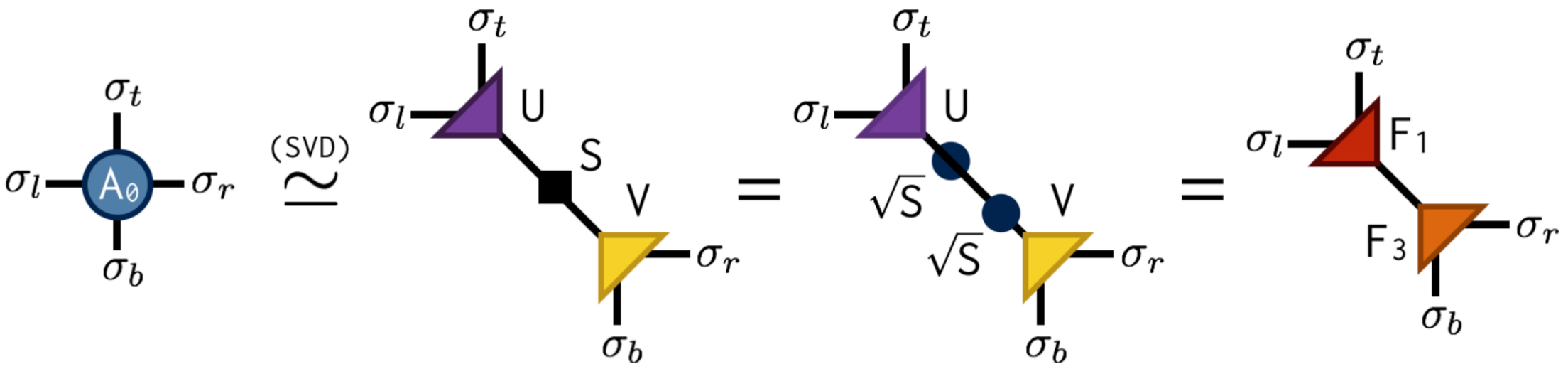
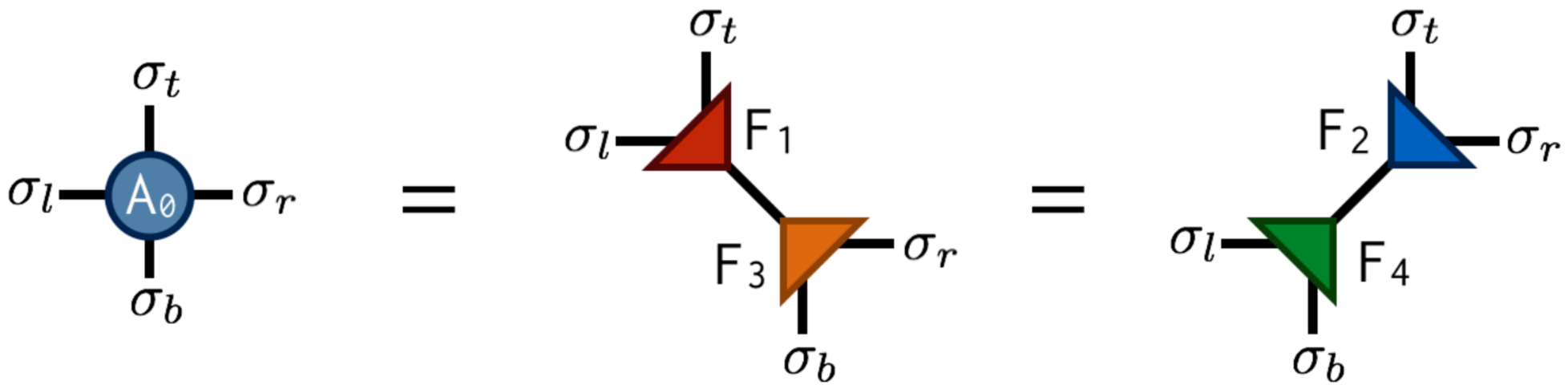
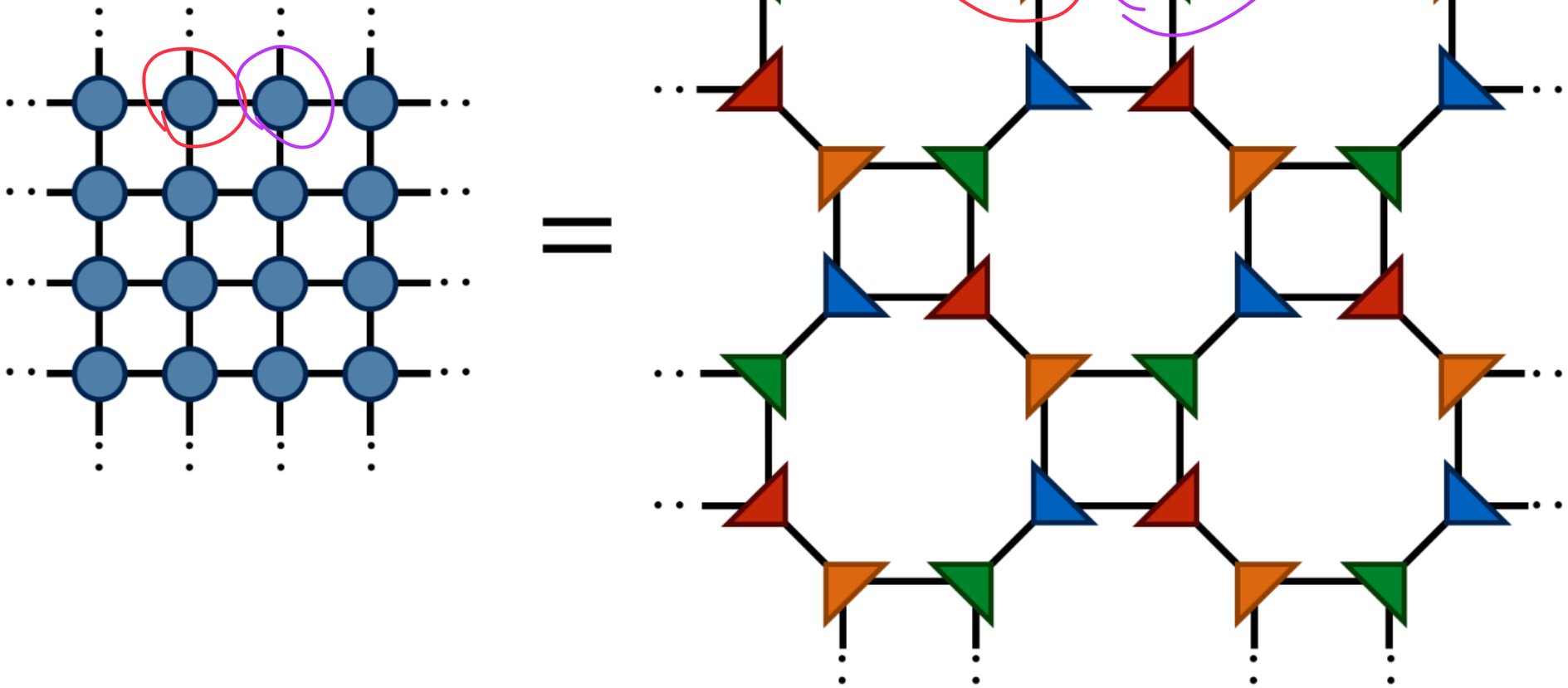
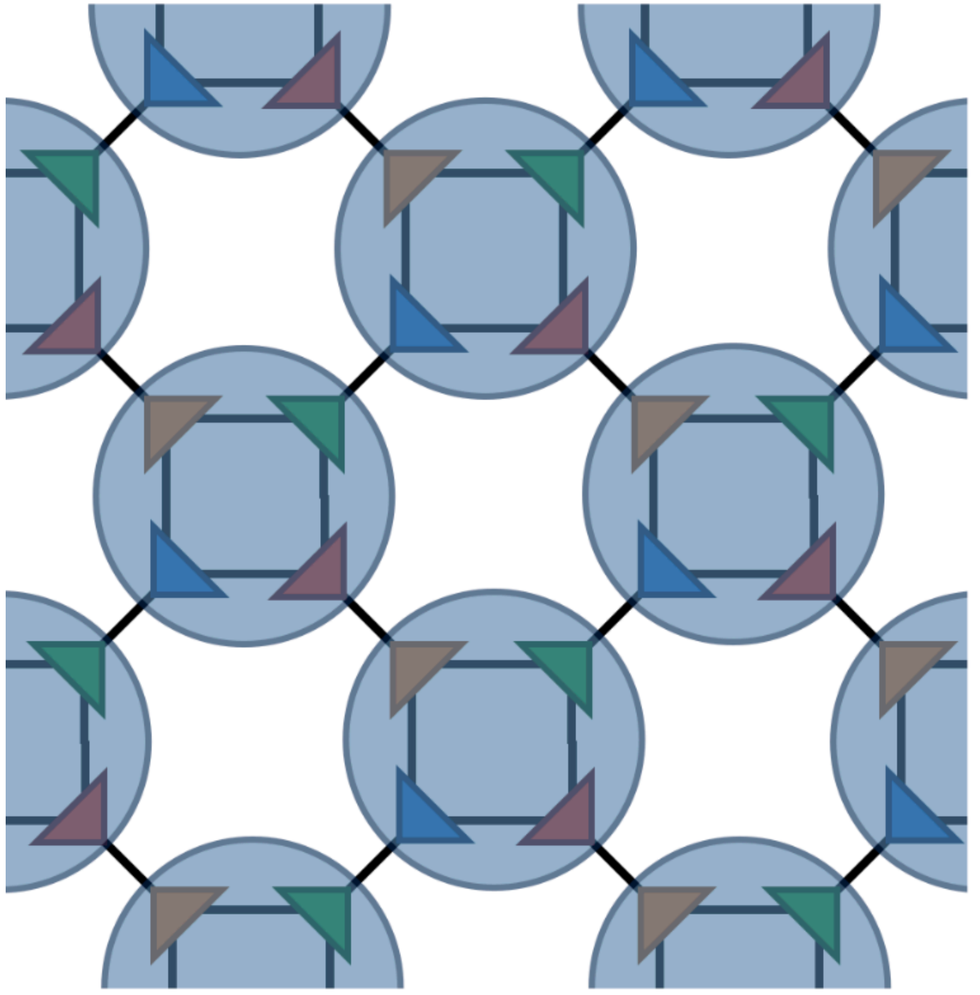


Figure borrowed from https://tensornetwork.org/trg/


σ_t
σ_l
A_0
σ_r
σ_b
=
σ_t
σ_l
F_1
F_3
σ_r
σ_b
=
σ_t
F_2
σ_r
σ_l
F_4
σ_b





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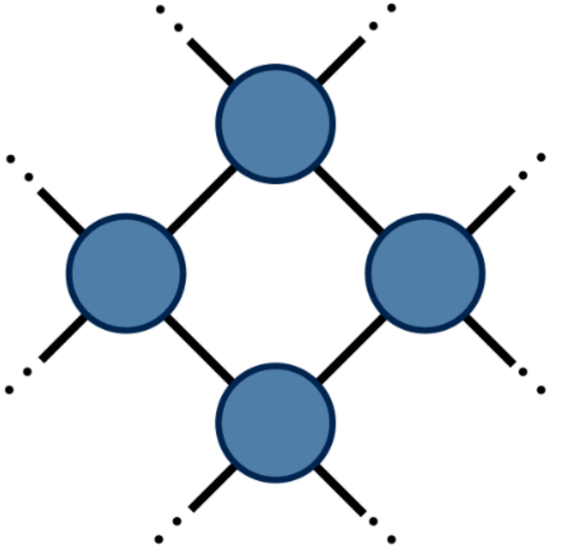

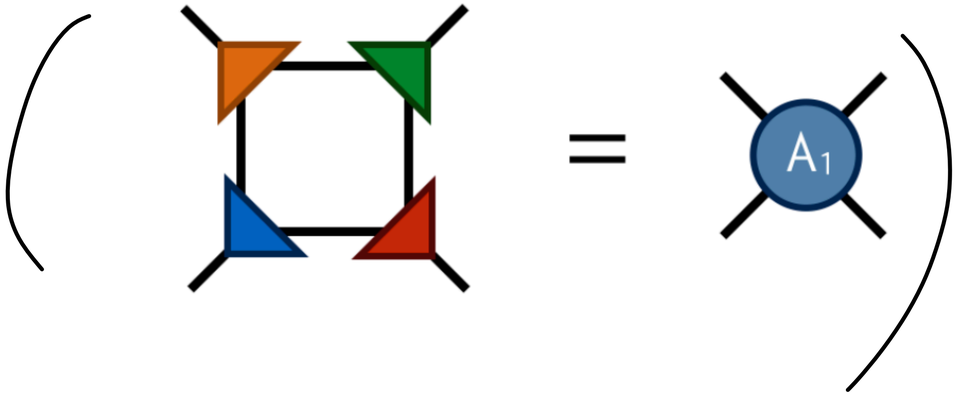
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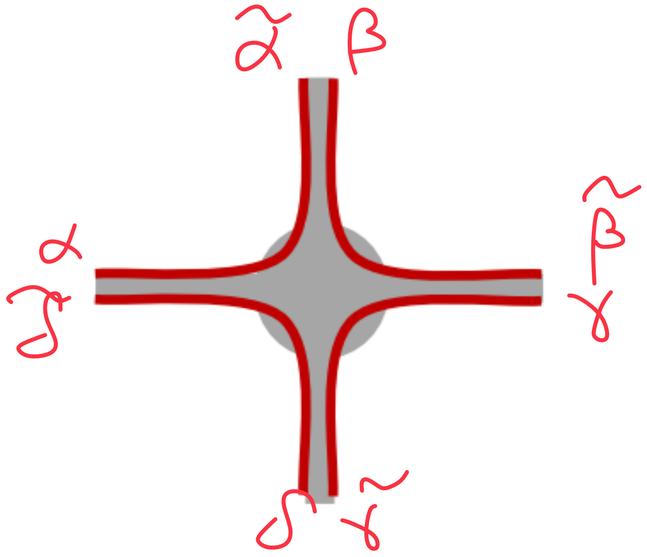
A1

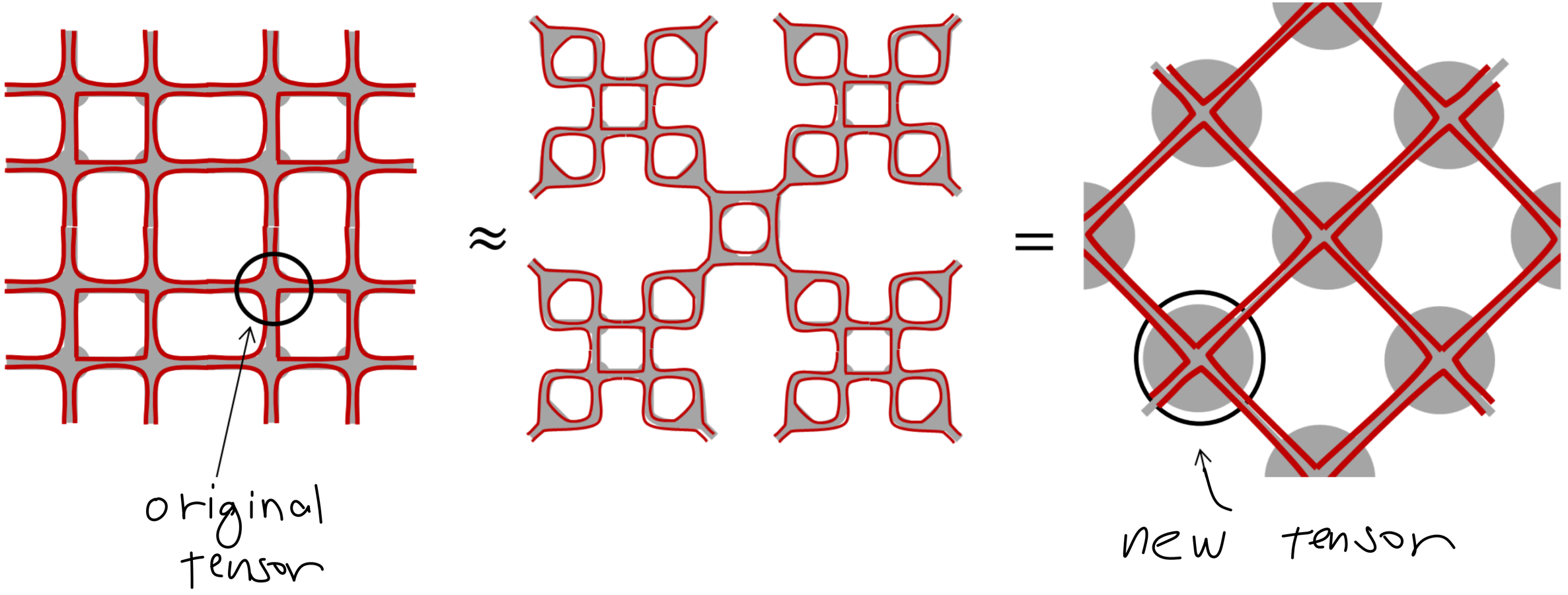
(Caveat: TRG does not remove some high-scale entanglement [Levin])

- Corner-Double Line (CDL) tensor

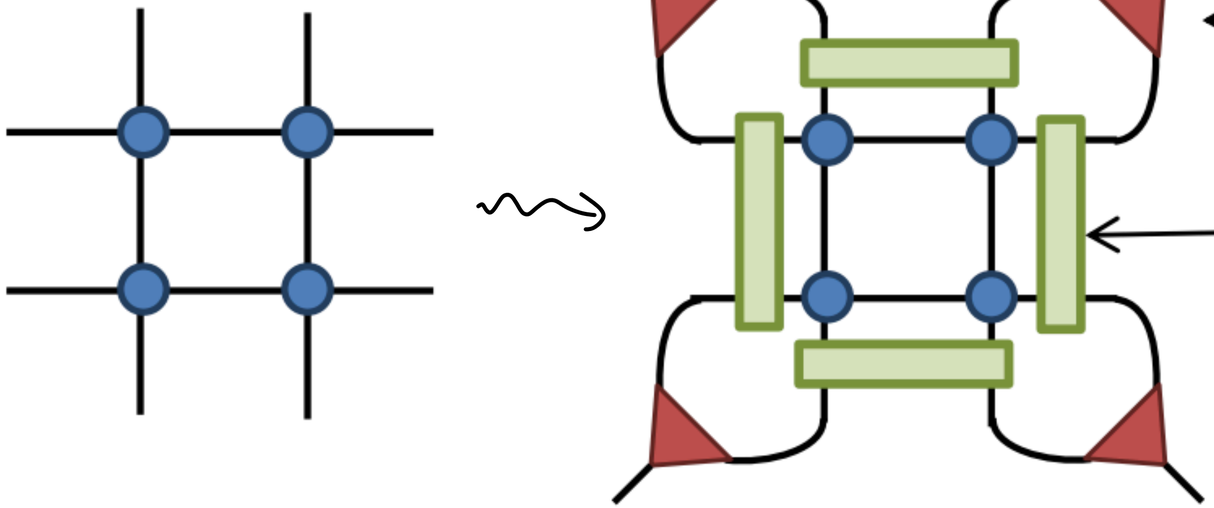


$$M^{\tilde{\alpha}\beta,\tilde{\beta}\gamma,\tilde{\gamma}\delta,\tilde{\delta}\alpha} = A^{\alpha\tilde{\alpha}} A^{\beta\tilde{\beta}} A^{\gamma\tilde{\gamma}} A^{\delta\tilde{\delta}}$$

Figures borrowed from Prof. Vidal's slides


original
tensor
≈
=
new tensor

Resolution : TNR [Evenbly - Vidal] (cf. MERA [Vidal])



$v : V \to V \otimes V$

isometry

disentangler!

$u : V \otimes V \to V \otimes V$

unitary

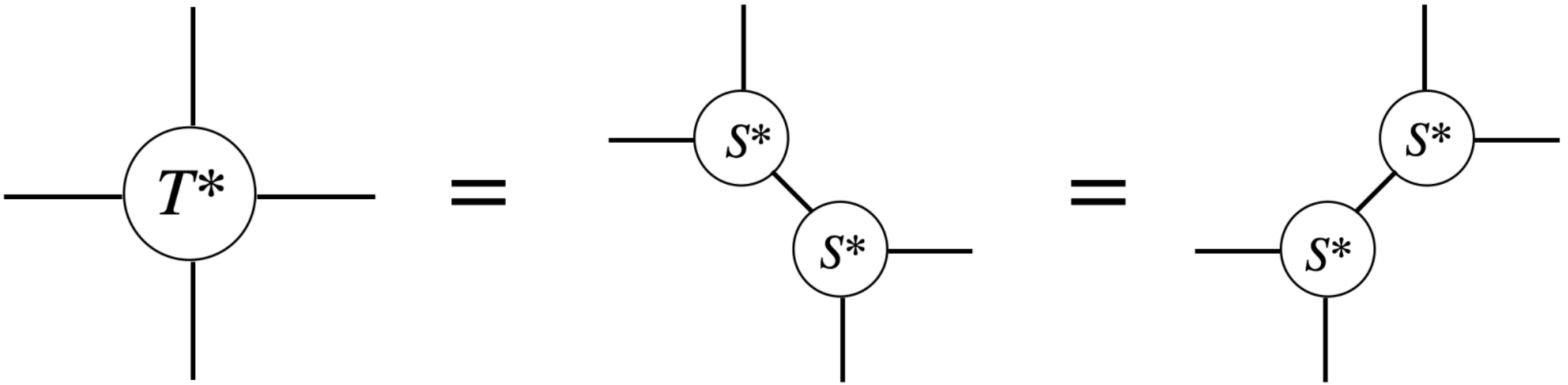
optimize over
isometry & disentangler

Figures borrowed from Prof. Vidal's slides

(1+1) dim. CFT data from TN RG

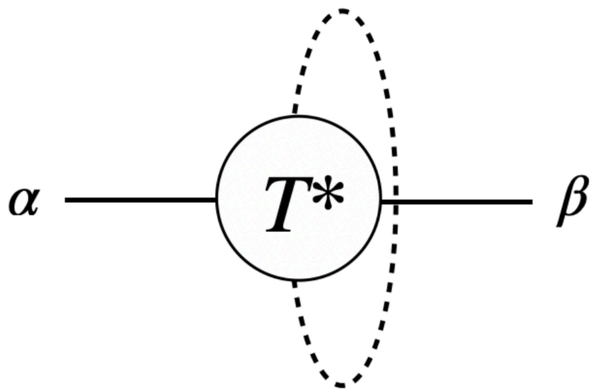
[Ueda-MY, 2307.02523 [hep-th]]

Endpoint of RG = Fixed-Point Tenson



Q: CFT data?

Before our work ...



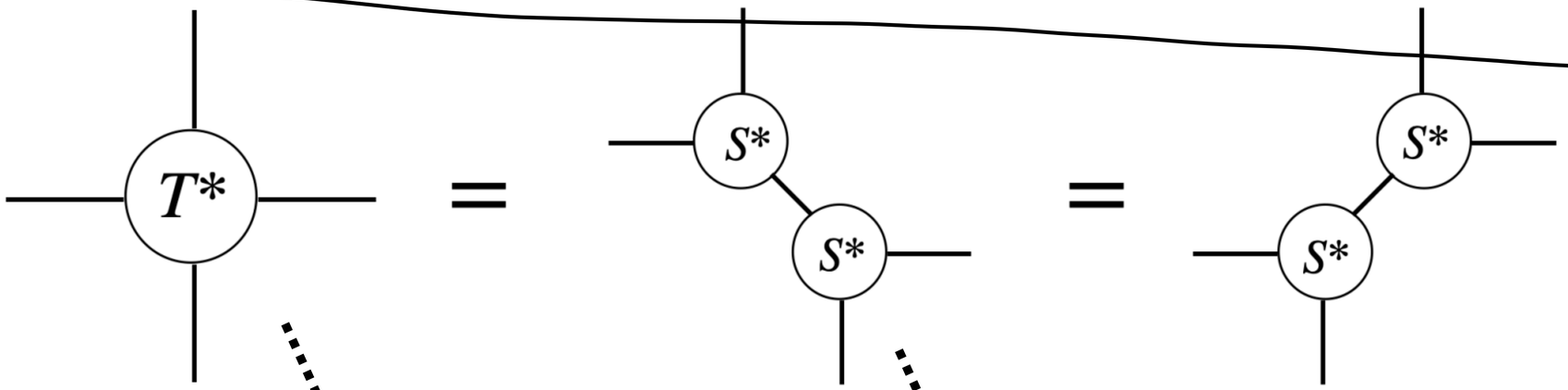
$$= \delta_{\alpha\beta}\, e^{-2\pi\Delta_\alpha}$$

operator dimension & central charge

[Gu - Wen ('09)]

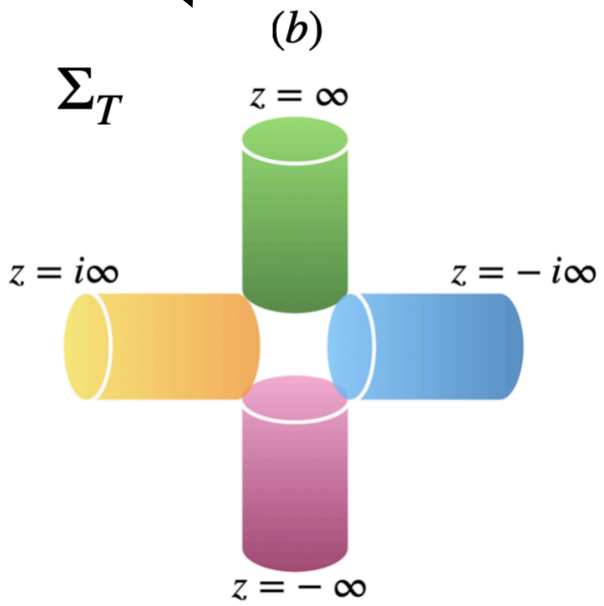
OPE data not clear-cut (3-pt function)

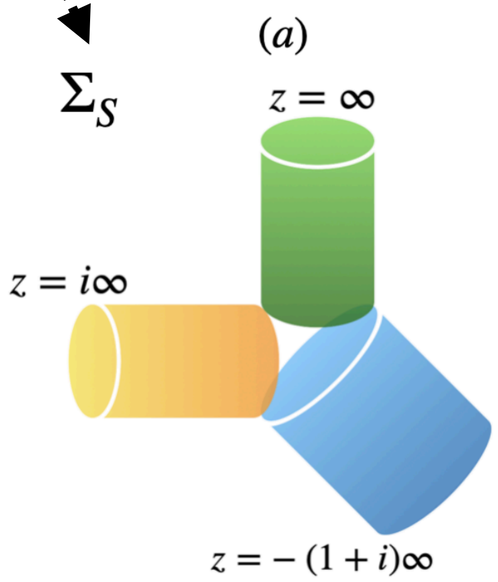
It seems natural to regard "tensor = spacetime"



"fatten"

"fatten"





$$\frac{S^*_{\alpha\beta\gamma}}{S^*_{111}} = \langle \phi_\alpha(\infty)\phi_\beta(i\infty)\phi_\gamma(-(1+i)\infty)\rangle_{\Sigma_S}$$

$z_S = \infty, i\infty, -(1+i)\infty$

$$\frac{T^*_{\alpha\beta\gamma\delta}}{T^*_{1111}} = \langle \phi_\alpha(-\infty)\phi_\beta(i\infty)\phi_\gamma(\infty)\phi_\delta(-i\infty)\rangle_{\Sigma_T}$$

$z_T = \pm\infty, \pm i\infty$

$$z_S = \frac{L}{2\pi}[-\ln(w-i) - i\ln(w+1) + (1+i)\ln w]$$

$$z_T = \frac{L}{2\pi}\left[\ln\left(\frac{w+i}{w-i}\right) + i\ln\left(\frac{w-1}{w+1}\right)\right]$$

$$\frac{S^*_{\alpha\beta\gamma}}{S^*_{111}} = \langle \phi_\alpha(-1)\phi_\beta(i)\phi_\gamma(0)\rangle_{\text{pl}} \prod_{n\in(\alpha,\beta,\gamma)} |J_n|^{\Delta_n}$$

$w_S = i, -1, 0$

$$\frac{T^*_{\alpha\beta\gamma\delta}}{T^*_{1111}} = \langle \phi_\alpha(-1)\phi_\beta(i)\phi_\gamma(1)\phi_\delta(-i)\rangle_{\text{pl}} \prod_{n\in(\alpha,\beta,\gamma,\delta)} |J_n|^{\Delta_n}$$

$w_T = \pm i, \mp 1$

✡ Our proposal in Ueda-MY 2307.02523 [hep-th]

$$\frac{S^*_{\alpha\beta\gamma}}{S^*_{111}} = e^{-\frac{\pi}{4}(\Delta_\alpha+\Delta_\beta+\Delta_\gamma)}\langle\phi_\alpha(-1)\phi_\beta(i)\phi_\gamma(0)\rangle_{\rm pl}$$

$$\frac{T^*_{\alpha\beta\gamma\delta}}{T^*_{1111}} = \left(\frac{e^{\frac{\pi}{2}}}{2}\right)^{-\Delta_{\rm tot}} \langle\phi_\alpha(-1)\phi_\beta(i)\phi_\gamma(1)\phi_\delta(-i)\rangle_{\rm pl}$$

$\Delta_{\rm tot} = \Delta_\alpha+\Delta_\beta+\Delta_\gamma+\Delta_\delta$

i.e.

$$\frac{S^*_{\alpha\beta\gamma}}{S^*_{111}} = \langle\phi_\alpha(-x_S)\phi_\beta(ix_S)\phi_\gamma(0)\rangle_{\rm pl}$$

$$\frac{T^*_{\alpha\beta\gamma\delta}}{T^*_{1111}} = \langle\phi_\alpha(-x_T)\phi_\beta(ix_T)\phi_\gamma(x_T)\phi_\delta(-ix_T)\rangle_{\rm pl}$$

$x_S = e^{\frac{\pi}{4}}$

$x_T = e^{\frac{\pi}{2}}/2$

We can implement TRG / TNR

& Extract CFT data

$\{\Delta_\alpha\}$ $\{C_{\alpha\beta\gamma}\}$

for e.g. Ising / Potts models

TRG code for the Ising model available from https://github.com/dartsushi/TRG_D4_symmetry

TRG_analysis-Ising.ipynb	func_fixed	last year
entanglement_filtering.py	base	last year
opt_3leg.py	base	last year
util.py	base	last year

README

TRG_D4_symmetry

This repository provides numerical details of the paper "Fixed-point tensor is a CFT four-point function" by A.Ueda and M.Yamazaki.

Preprocessed data are available in the directories "/fptensor" and "/Gauge-fixed".

If you would like to compute the data by yourself, "Data_collection.ipynb" gives the instructions.

The main figures in our paper are plotted in "TRG_analysis.ipynb".

No packages published

Languages

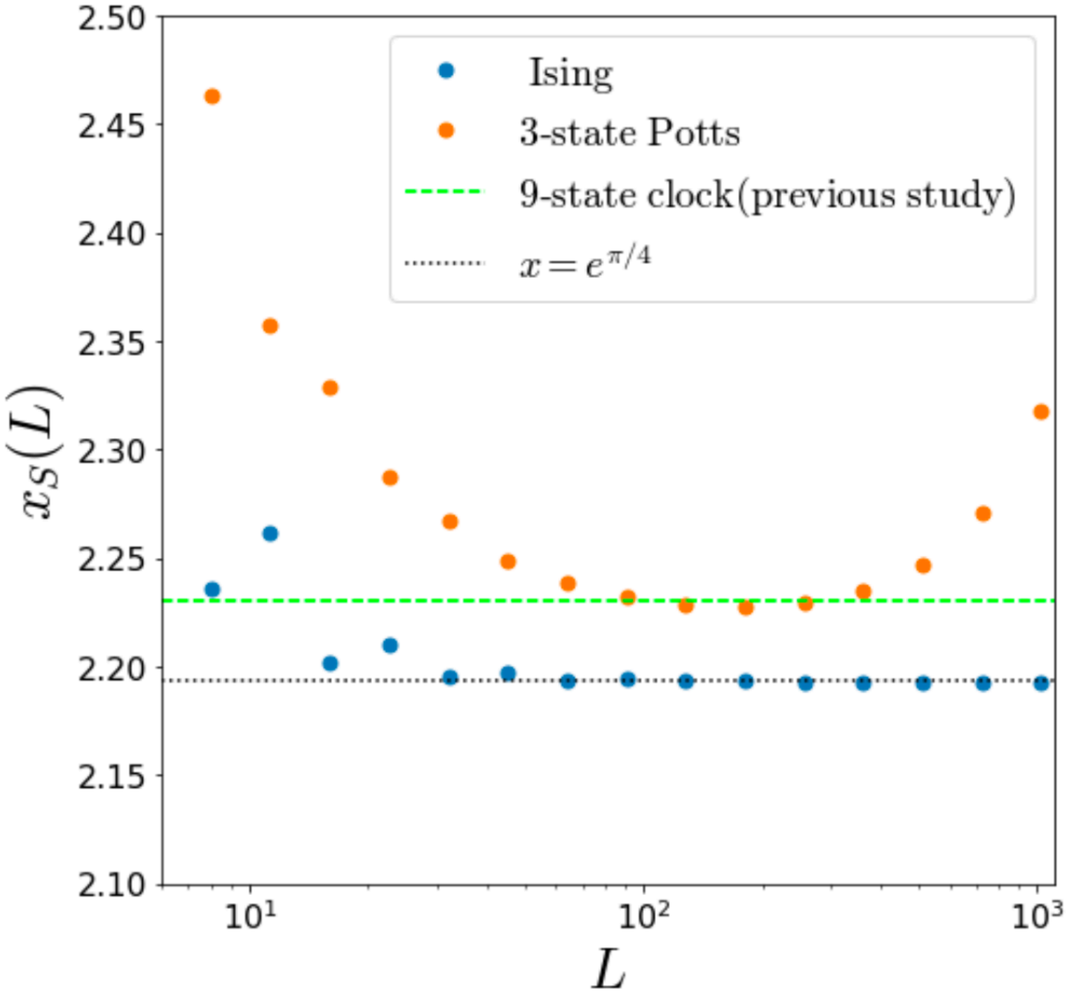
- Jupyter Notebook 95.9%
- Python 4.1%

(2D Ising model)

	Tensor RG	Exact
	$T_{\alpha\beta\gamma\delta}\ (L = 2048)$	$\langle \phi_\alpha \phi_\beta \phi_\gamma \phi_\delta \rangle$
1111	1	1
$\sigma\sigma\sigma\sigma$	0.610	0.645
$\sigma\sigma\epsilon\epsilon$	0.0714	0.0716
$\sigma\epsilon\sigma\epsilon$	0.000	0
$\epsilon\epsilon\epsilon\epsilon$	0.0168	0.0168
$\sigma\sigma\epsilon 1$	0.0618	0.0765
$\sigma\epsilon\sigma 1$	0.133	0.140
$\sigma\sigma\sigma 1$	0.000	0
$\epsilon\epsilon\epsilon 1$	0.001	0
$\sigma\sigma 11$	0.708	0.736
$\sigma 1\sigma 1$	0.639	0.675
$\epsilon\epsilon 11$	0.0863	0.0864
$\epsilon 1\epsilon 1$	0.0439	0.0432
$\epsilon\sigma 11$	0.000	0

When we fit x_S

$$\frac{S^*_{\alpha\beta\gamma}}{S^*_{111}} = \langle \phi_\alpha(-x_S)\phi_\beta(ix_S)\phi_\gamma(0)\rangle_{\rm pl}$$

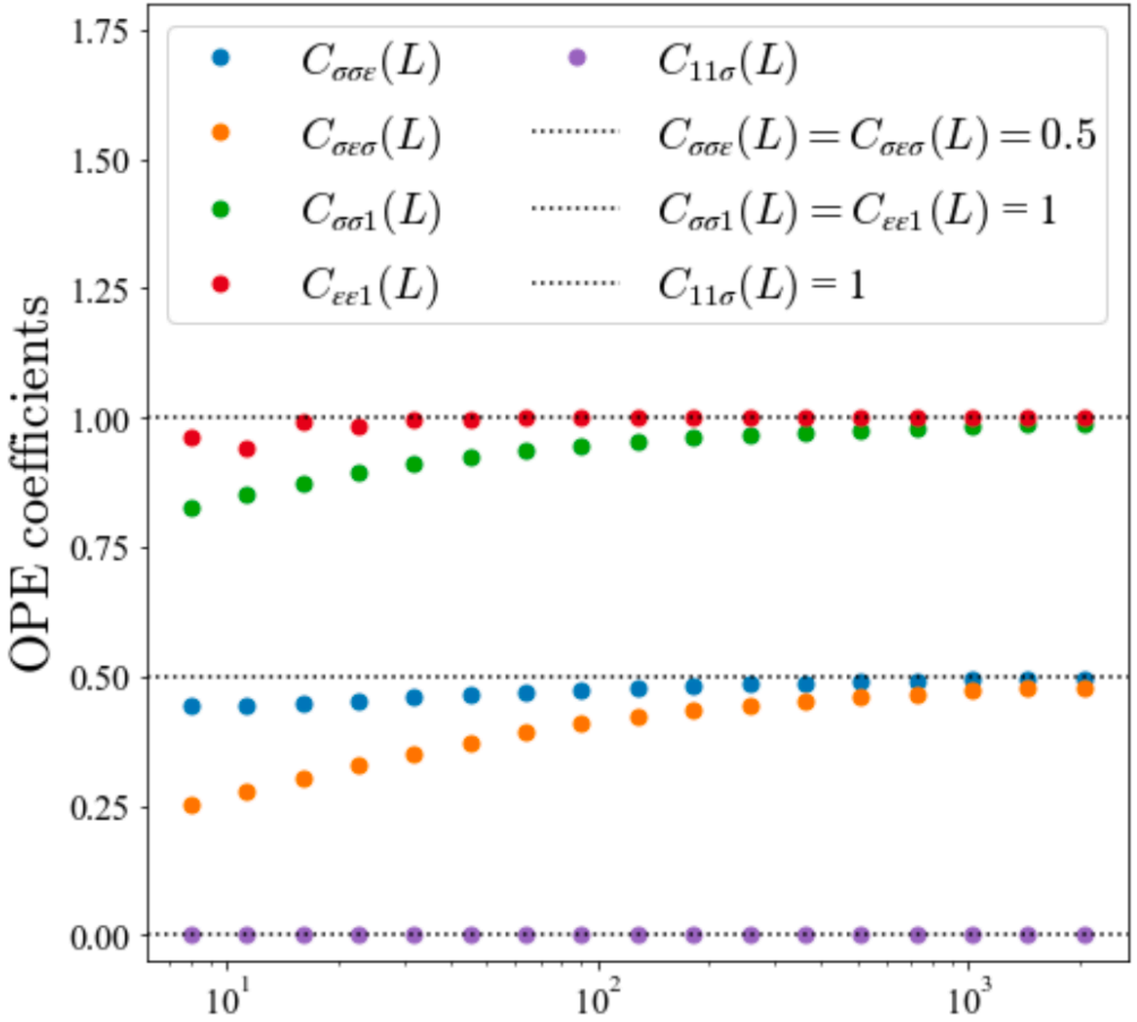


[Li-Pai-Gu ('22)]

← 2.23035

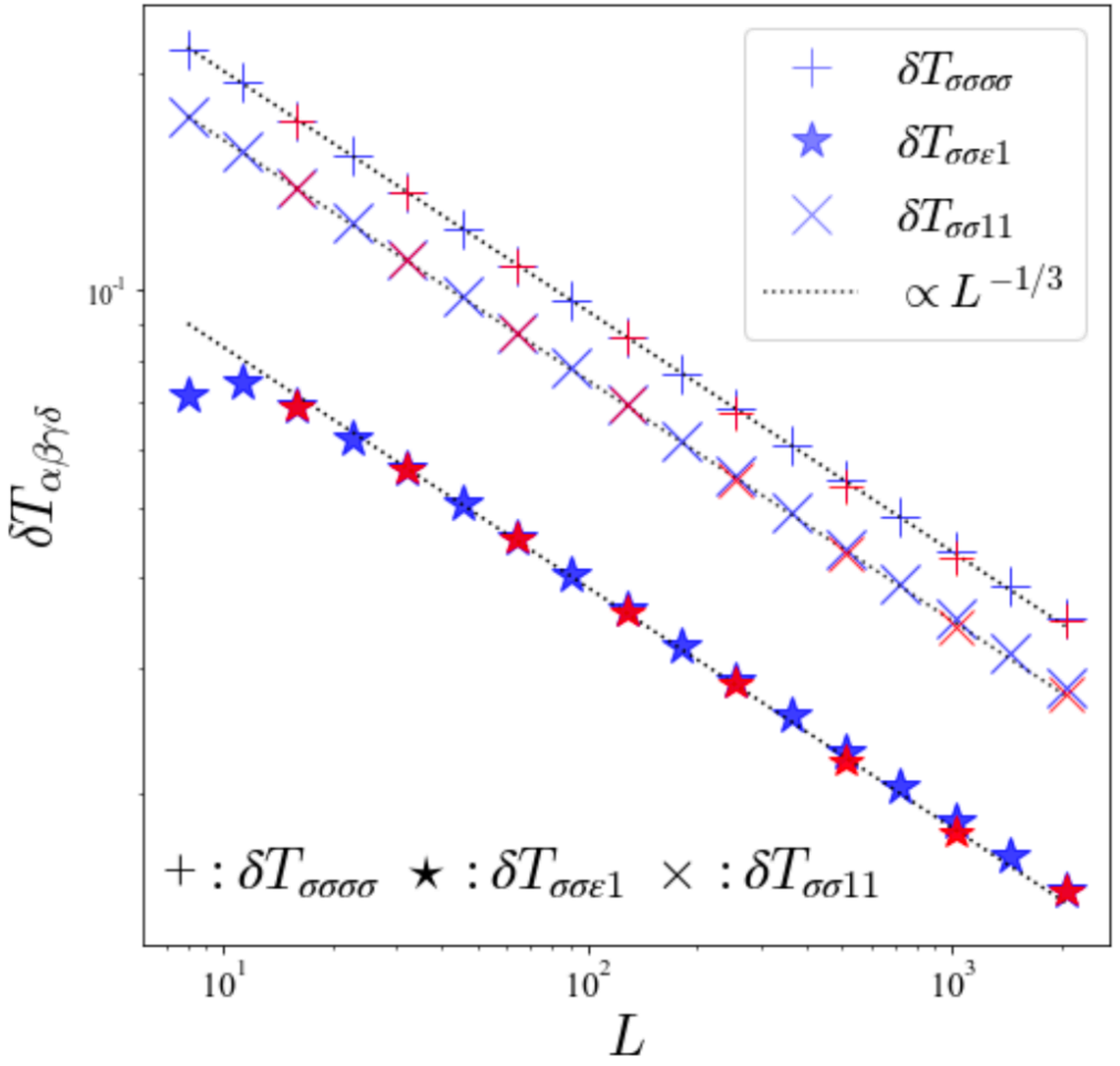
← 2.193257

Our value



K tensor size

finite size effect



Summary

* Tensor RG : powerful to study RG flow

* We can numerically extract full data of 2D CFT
  (full data = defining)

* Can follow RG flow

Outlook

* Generalizations ? (e.g. dim > 2, c large)

* RG flow ? Numerical / Exact

* Interplay with (near) integrability ?

* Holography ?

“Focus Week on Non-equilibrium Dynamics,”
Sep. 30 - Oct.4, 2024, Kavli IPMU, Tokyo, Japan

Organizers: H. Katsura, Y. Miao, M. Oshikawa, M. Yamazaki

KAVLI IPMU
東京大学 国際高等研究所 カブリ数物連携宇宙研究機構
KAVLI INSTITUTE FOR THE PHYSICS AND MATHEMATICS OF THE UNIVERSE

“High-Energy Physics in the Quantum Era,”
Dec. 2-4, 2024, KEK, Japan

Organizers: D. Grabowska, R. Harnak, S. Hashimoto,
M. Honda, R. Kitano, M. Yamazaki

KEK
加速器だから見える世界。
Insight through Accelerators.

“Exact Solvability and Quantum Information,”
Aug. 4-29, 2025, Les Houches, France

Organizers: S. Ouvry, T. Prosen, D. Serban, M. Yamazaki

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PHYSIQUE
DES HOUCHES