

Emergent Symmetries in Ensemble Averages

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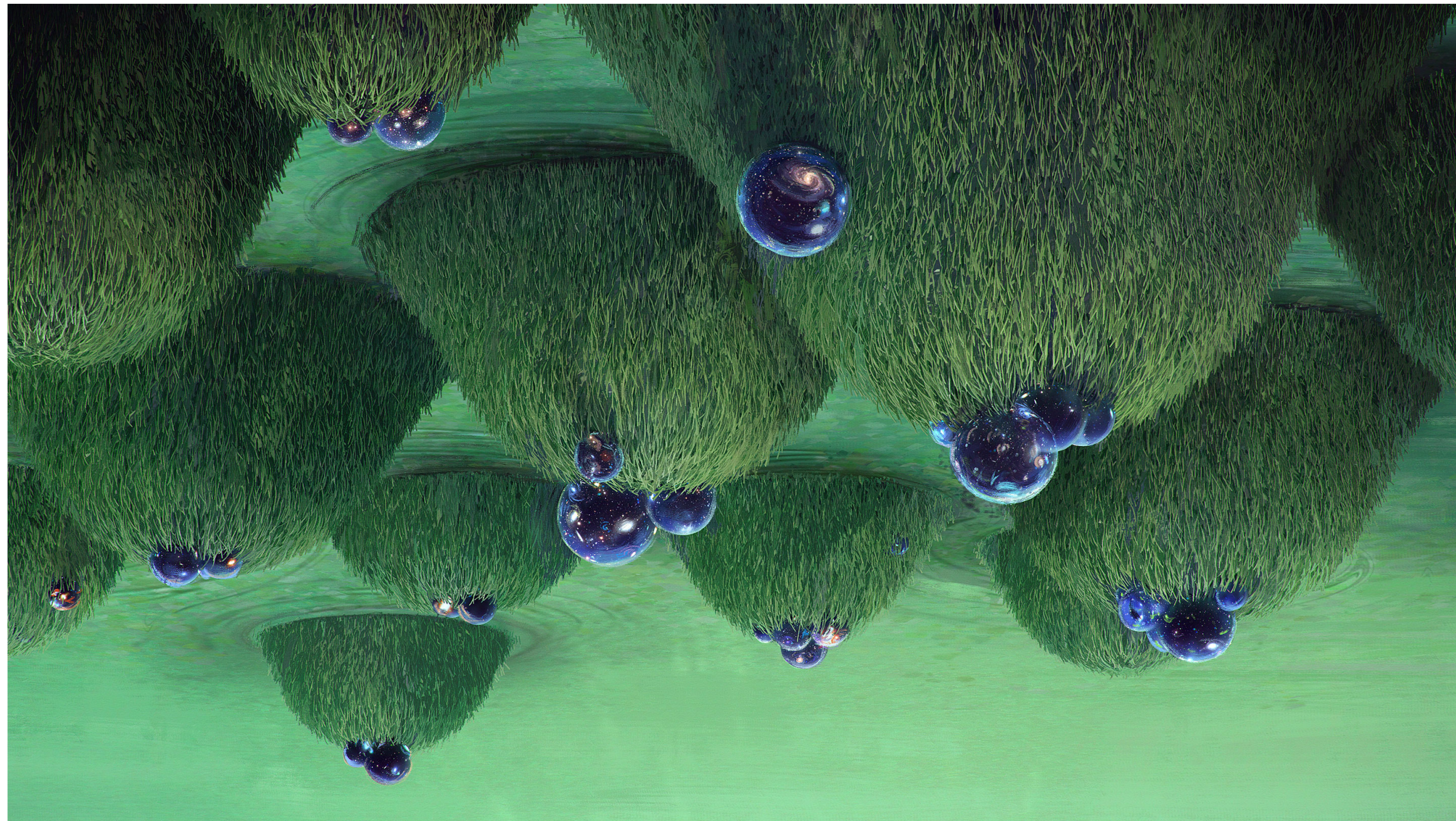
Symmetry : guiding principle of physics



What about in quantum gravity ?

Canonical answer:

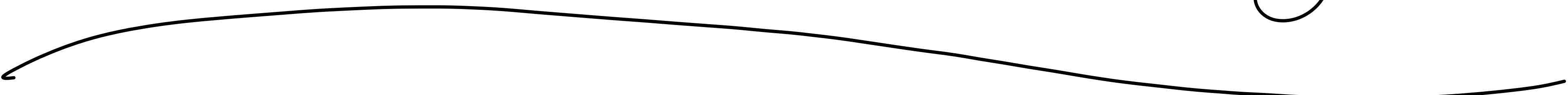
"No exact global sym. in QG"



[Misner-Wheeler ('57)
....]

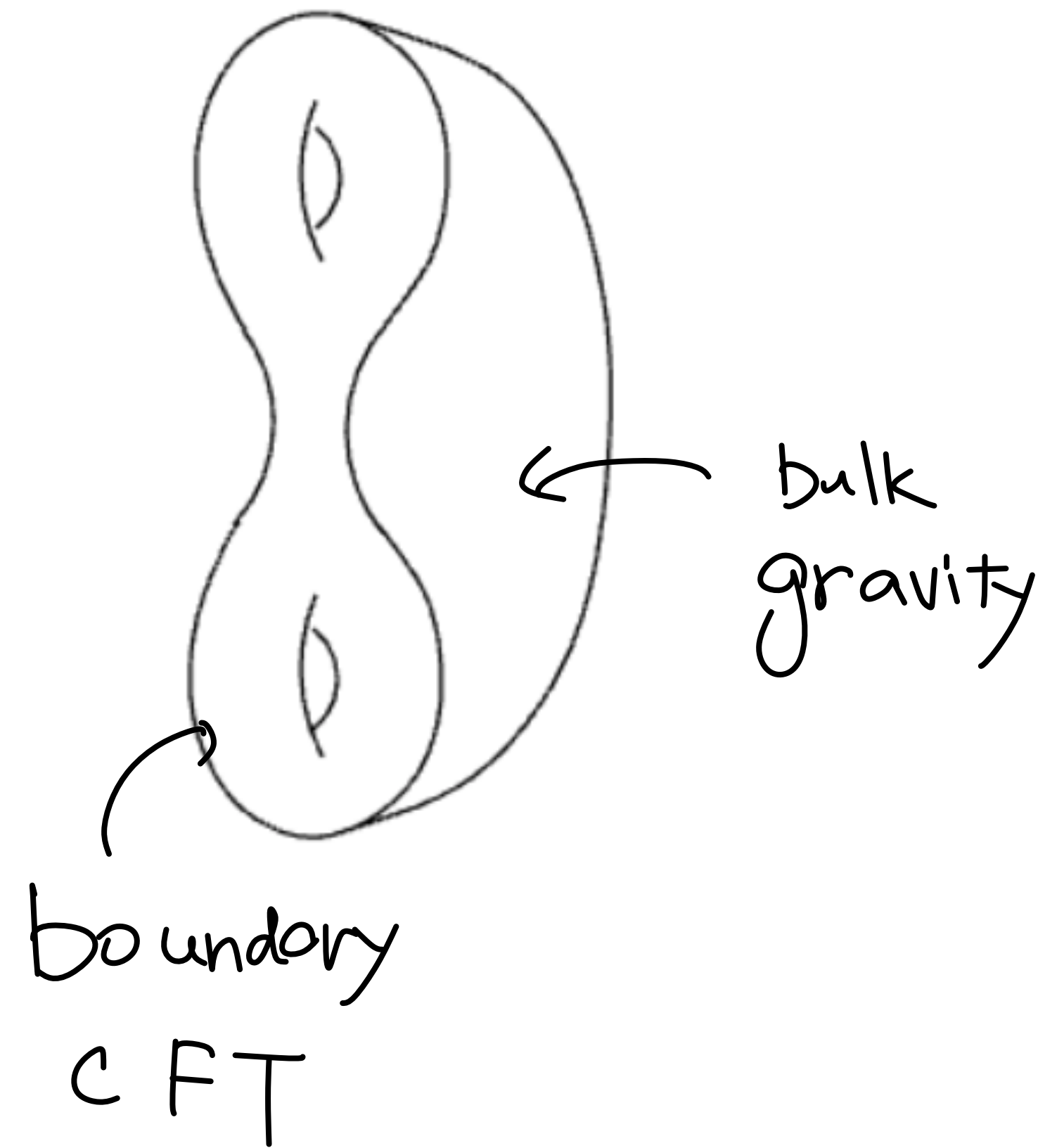
Part of the Swampland program

Ensemble Average



Factorization Puzzle

[Maldacena-Maoz '04]



$$Z_{\text{CFT}} = Z_{\text{AdS}}$$

Factorization Puzzle

[Maldacena-Maoz '04]



CFT₁

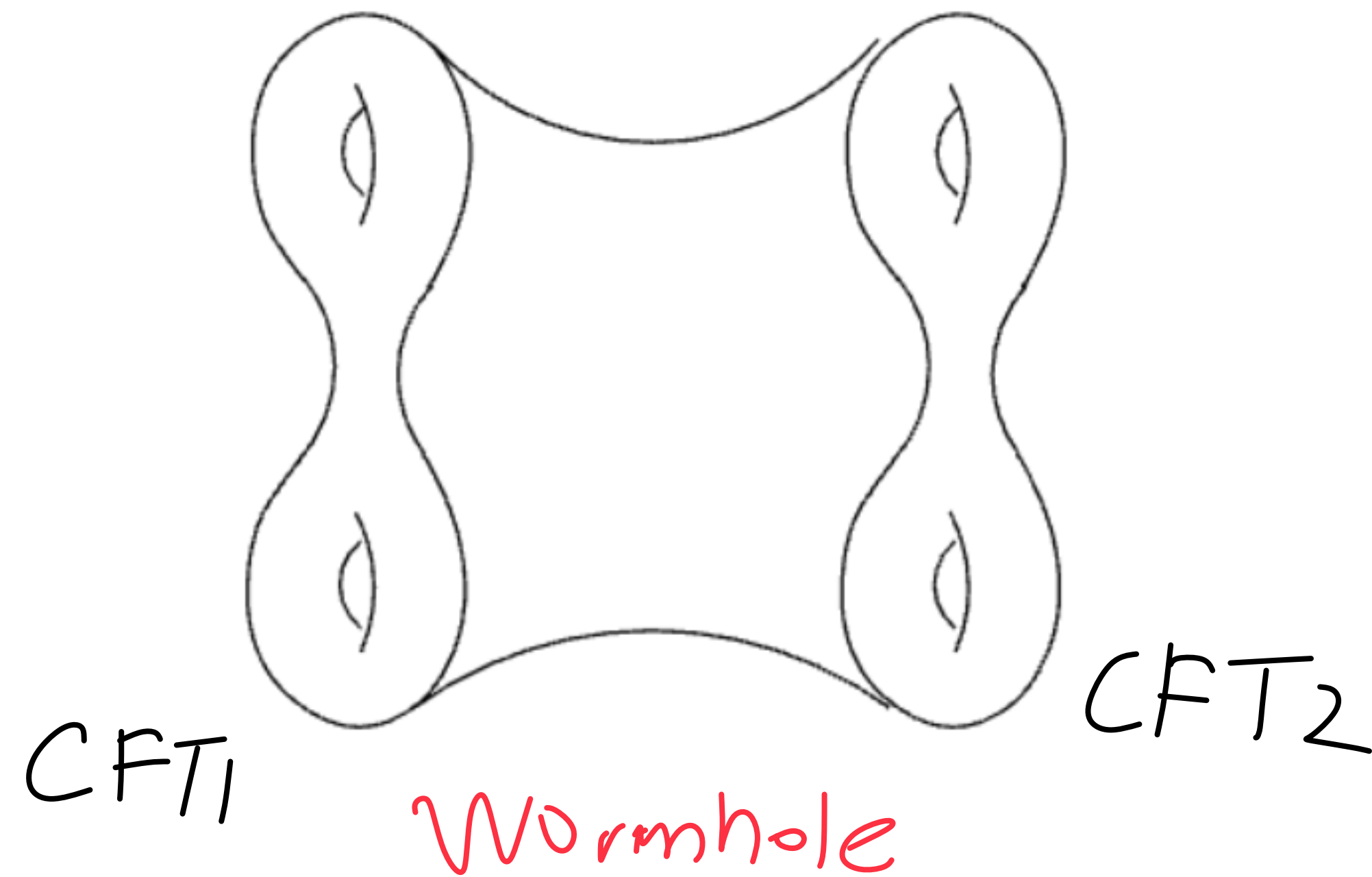


CFT₂

$$\mathcal{Z}_{\text{tot}} = \mathcal{Z}_{\text{CFT}_1} * \mathcal{Z}_{\text{CFT}_2} = \mathcal{Z}_{\text{AdS}_1} * \mathcal{Z}_{\text{AdS}_2}$$

Factorization Puzzle

[Maldacena-Maoz '04]

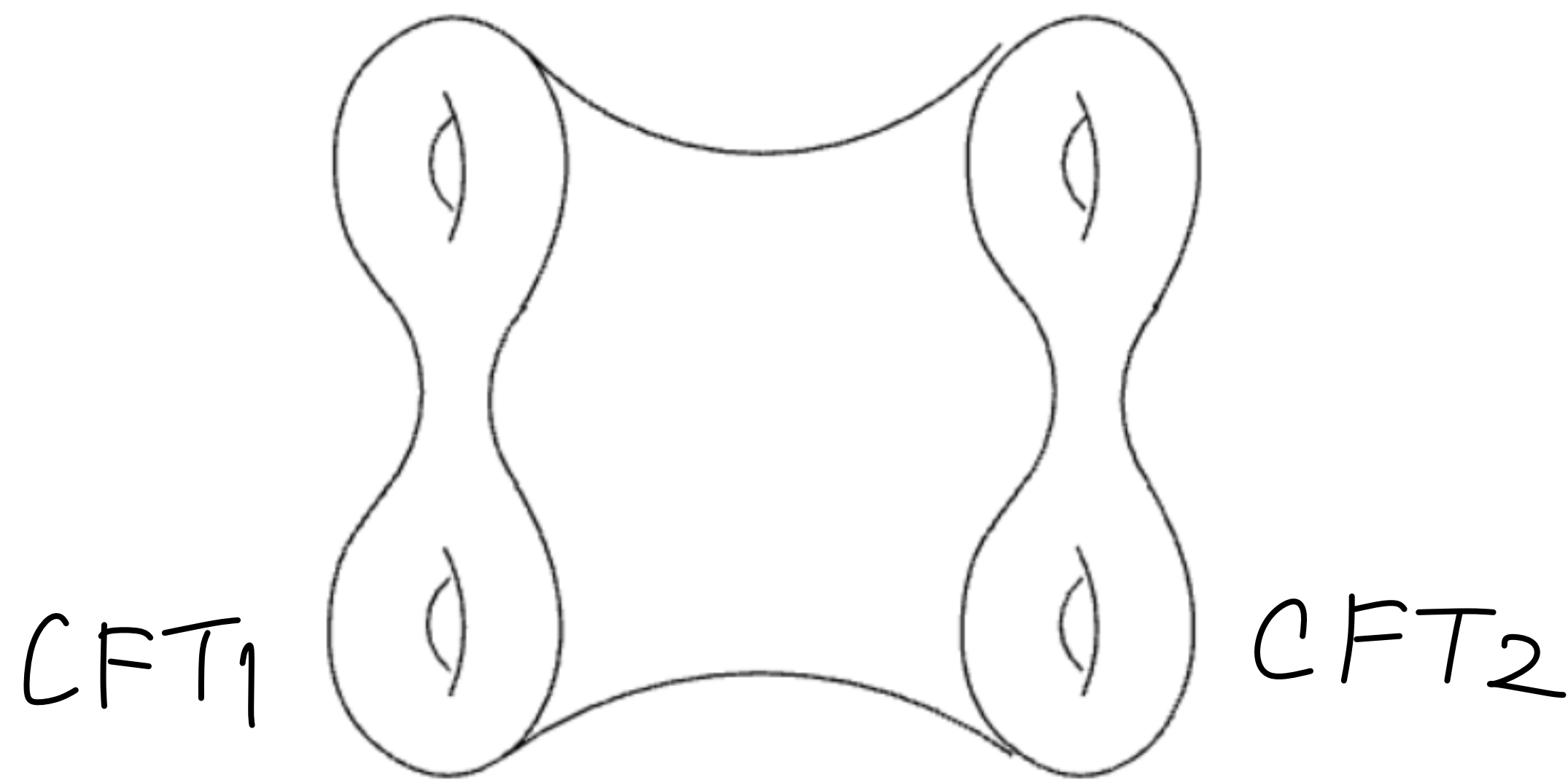


$$Z_{CFT_1} \cdot Z_{CFT_2} \neq Z_{AdS}$$

does not
factorize

Factorization Puzzle

[Maldacena-Maoz '04]



$\langle - \rangle$: ensemble average

ensemble?

$$\langle Z_1 \rangle \langle Z_2 \rangle \neq \langle Z_1 \cdot Z_2 \rangle$$

While ensembles inside a theory is common

e.g. in stat-mech,

here we consider ensembles of THEORIES

$$\langle \hat{O} \rangle = \int_{\mathcal{M}} [dm] \mathcal{O}(m)$$

moduli space of theories

measure

parameters of theories

Precision Case Study

CFT_2 / AdS_3 for Narain-type theories

[Ashwinkumar - Dodelson - Kidambi - Leedom - MY ('21)
Ashwinkumar - Kidambi - Leedom - MY ('23)]



See also

[Afkhami-Jeddi, Cohen, Hartman, Tajdini ('20)
Maloney - Witten ('20)]

Data

of signature (p, g)

• Q : even quadratic form

lattice Λ

• H : Hamiltonian

moduli dependent

$$\begin{cases} Q = P_L^2 - P_R^2 \\ H = P_L^2 + P_R^2 \end{cases}$$

• $\mathcal{M}_Q$: CFT moduli space

$$\left(\begin{array}{c} \text{T-duality} \\ \text{group} \end{array} \right) \xrightarrow{\quad} \mathcal{M}_Q = \underbrace{O(p, g; \mathbb{Z})}_{\text{signature } (p, g)} \bigg/ O(p) \times O(g)$$

e.g. S^1 -compactification of free boson ($p=g=1$)

$$p_L = \frac{n}{2R} + \omega R, \quad p_R = \frac{n}{2R} - \omega R \quad (n, \omega \in \mathbb{Z})$$

$$Q = p_L^2 - p_R^2 = 2n\omega \in \mathbb{Z}$$

$$H = p_L^2 + p_R^2 = \frac{n^2}{2R^2} + 2\omega^2 R^2$$

$C=1$ moduli space

$$\mathcal{M}_Q = \cdots \cdots \cdots \overset{R=1}{\bullet} \longrightarrow$$

Theta functions

$$\mathcal{V}_{Q,\alpha}(\tau, \bar{\tau}; m) = \sum_{\ell \in \Lambda} e^{\pi i \tau_1 Q(\ell + \alpha) - \pi \tau_2 H(\ell + \alpha)}$$

$\overbrace{m}^{\text{— dependence}}$

$$\left(\begin{array}{l} m: \text{CFT moduli} \\ \tau: \text{spacetime torus moduli } \tau = \tau_1 + i\tau_2 \\ \alpha \in \mathcal{Q} := \Lambda^* / \Lambda \end{array} \right)$$

Ensemble Average

Haar measure / Zamolodchikov metric

$$\frac{1}{\text{Vol}(\mu_{Q,\alpha})} \int \overbrace{[dm]}^{\text{Haar measure / Zamolodchikov metric}} \vartheta_{Q,\alpha}(\tau, \bar{\tau}; m) = ??$$

$\uparrow$
T-duality should
preserve α

Siegel-Weil formula $\left[\begin{array}{l} \text{Siegel ('51)} \\ \text{Weil ('64)} \end{array} \right]$

$$\underbrace{\frac{1}{\text{Vol}(\mu_{Q,\alpha})} \int_{\mu_{Q,\alpha}} [dm] \, \theta_{Q,\alpha}(\tau, \bar{\tau}; m)}_{\substack{\text{ensemble average} \\ \text{over CFT moduli space}}} = \underbrace{E_{Q,\alpha}(\tau, \bar{\tau})}_{\substack{\text{non-hol.} \\ \text{Eisenstein series}}}$$

Poincaré sum

$$\begin{pmatrix} * & * \\ c & d \end{pmatrix} : SL(2, \mathbb{Z}) / \Gamma_\infty$$

$\downarrow$
 $\mathbb{Z}$

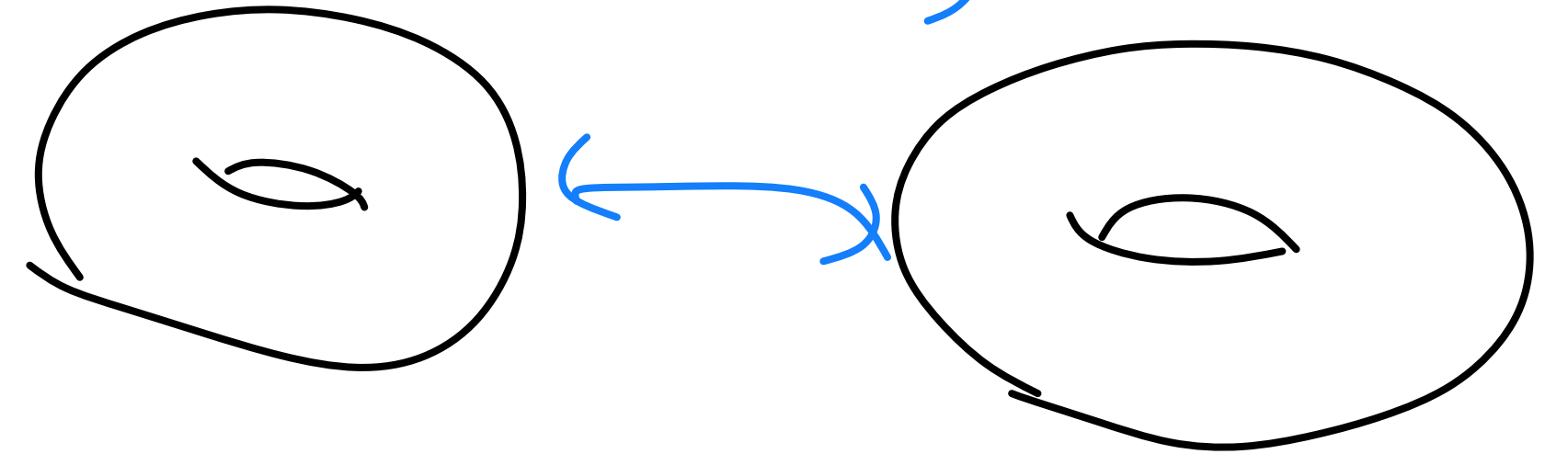
$$E_{Q, \alpha}(\tau) := \int_{\alpha \in \Lambda} + \sum_{\substack{(c, d)=1 \\ c > 0}} \frac{\gamma_{Q, \alpha}(c, d)}{(c\tau + d)^{\frac{p}{2}} (c\bar{\tau} + d)^{\frac{q}{2}}}$$

$$\gamma_{Q, \alpha}(c, d) = \frac{e^{\frac{\pi i (p-q)}{4}}}{\sqrt{|\det Q|}} c^{-\frac{p+q}{2}} \sum_{\ell \in \Lambda/c\Lambda} \exp \left[-\pi i \frac{d}{c} Q(\ell + \alpha) \right]$$

lens space partition function

Sum over
 $PSL(2, \mathbb{Z})$ BH

$$\langle \psi_0 | \begin{pmatrix} * & * \\ c & d \end{pmatrix} | \psi_\alpha \rangle$$



$$E_{Q, \alpha}(\tau) := \delta_{\alpha \in \Lambda} + \sum_{\substack{(c, d)=1 \\ c > 0}} \frac{\gamma_{Q, \alpha}(c, d)}{(c\bar{\tau} + d)^{\frac{p}{2}} (c\tau + d)^{\frac{q}{2}}}$$

$$\gamma_{Q, \alpha}(c, d) = \frac{e^{\frac{\pi i (p-q)}{4}}}{\sqrt{|\det Q|}} c^{-\frac{p+q}{2}} \sum_{\ell \in N_{c\Lambda}} \exp \left[-\pi i \frac{d}{c} Q(\ell + \alpha) \right]$$

Holographic dual after averaging:

Abelian CS

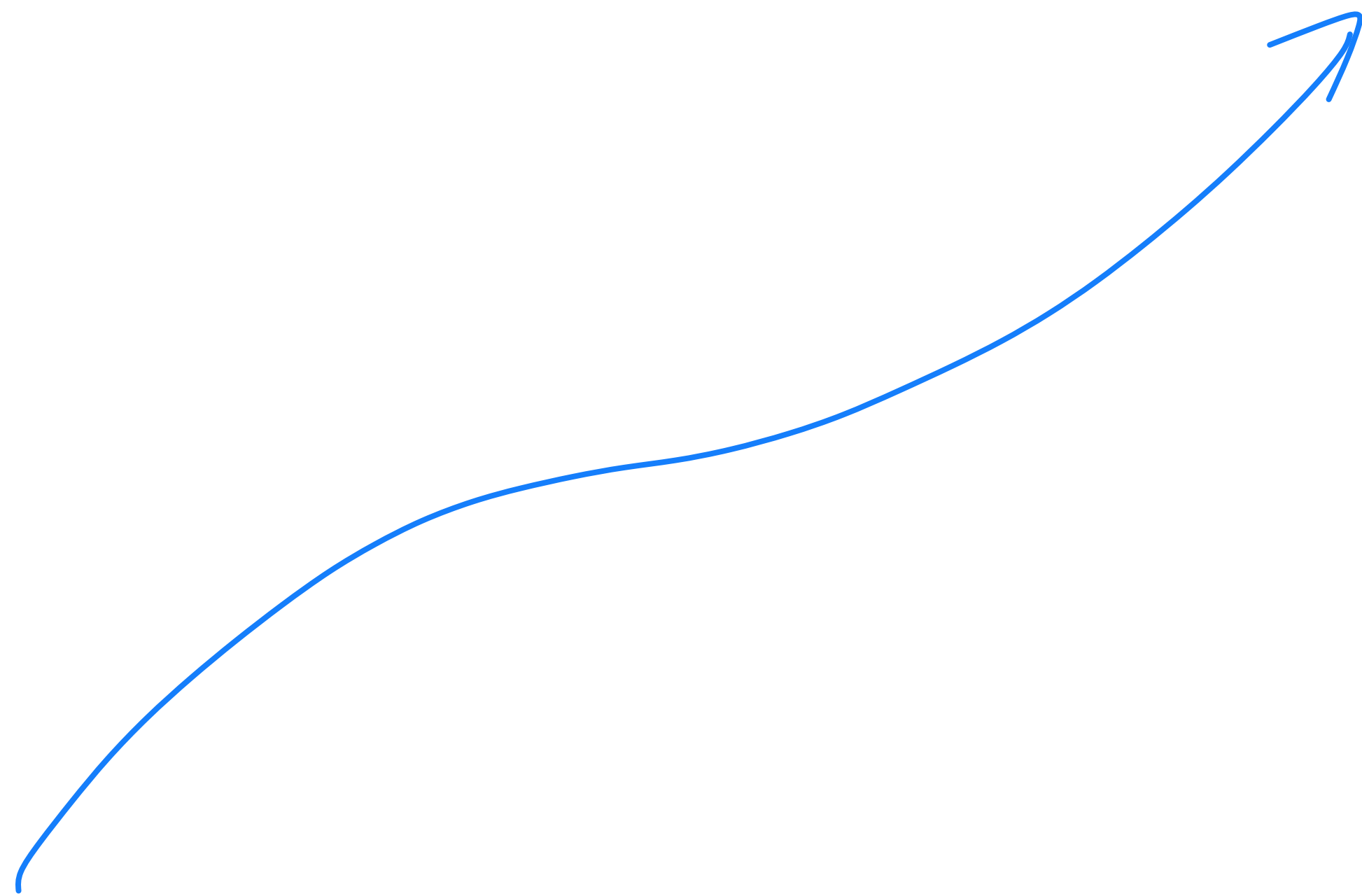
$$S_{CS} = \sum_{I,J=1}^{p+8} \frac{1}{4\pi} Q_{IJ} \int A_I \wedge dA_J$$

↑
"K-matrix"

in cond-mat

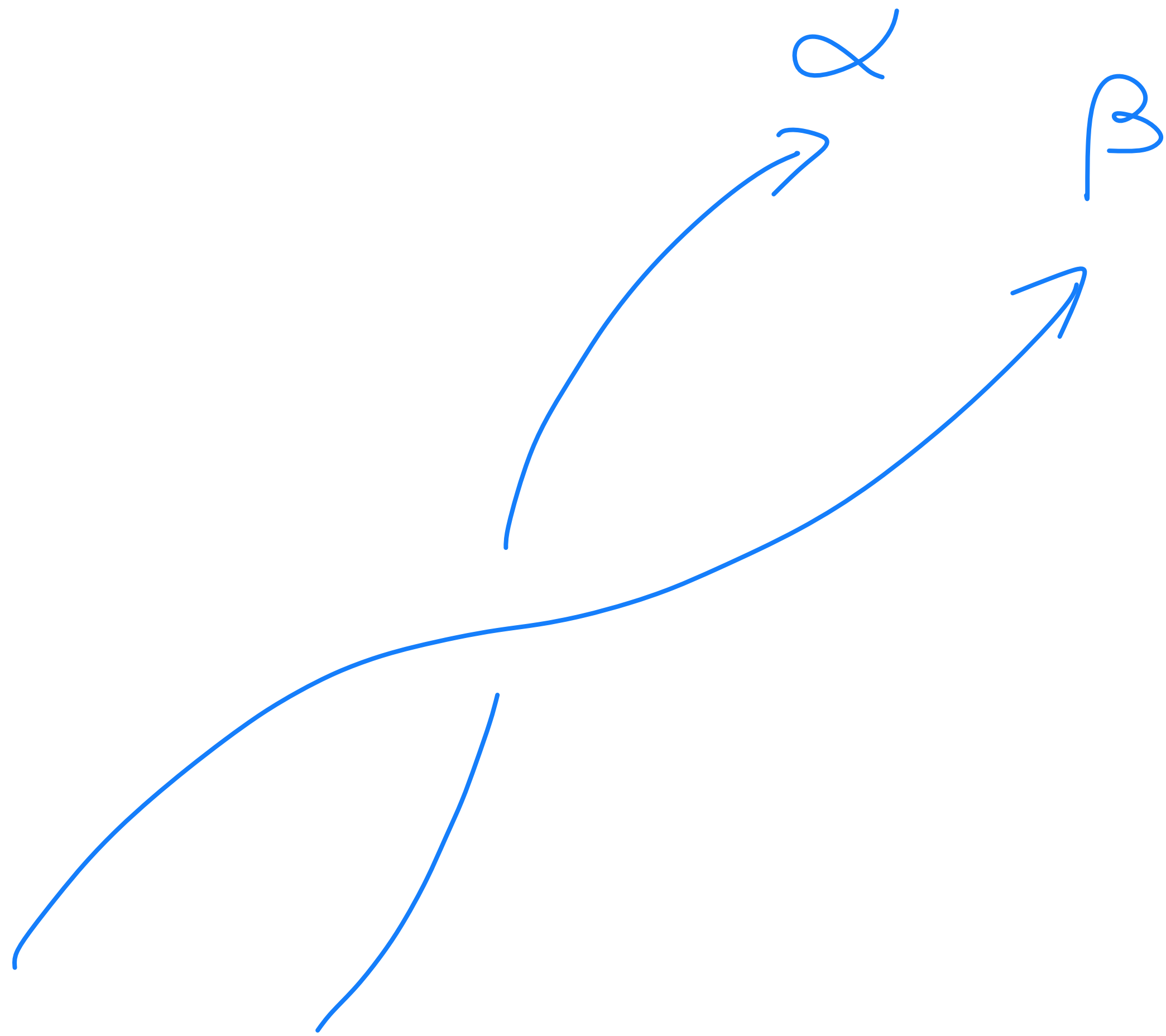
Abelian anyons described by $\mathbb{Q}$

α



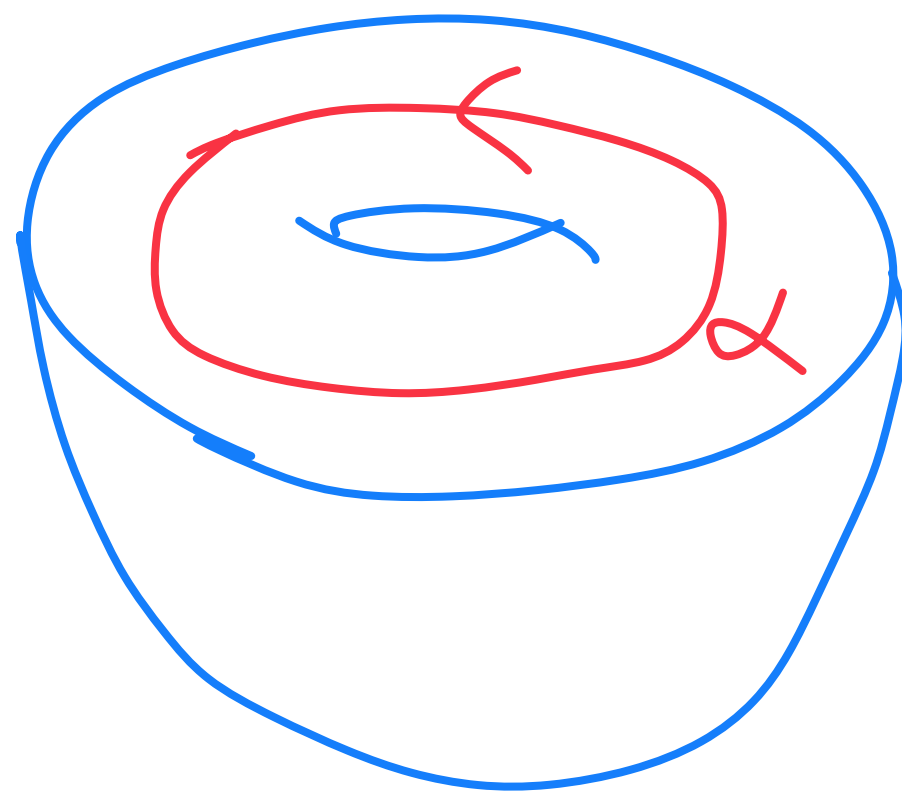
$$\alpha \in \underbrace{\Lambda^*}_{\substack{\text{charge} \\ \text{lattice}}} / \underbrace{\Lambda}_{\substack{\text{gauge equiv.}}}$$

Abelian anyons described by Q



$$|\psi\rangle \rightarrow e^{\pi i Q(\alpha, \beta)} |\psi\rangle$$

$|E_{Q,\alpha}(\tau)\rangle$: wavefunction on T^2



anyon
insertion

Emergent Global Symmetries

[Ashwinkumar, Leedom, MY ('23)]

QG: "No exact global symmetries"

However, ensemble-averaged theories

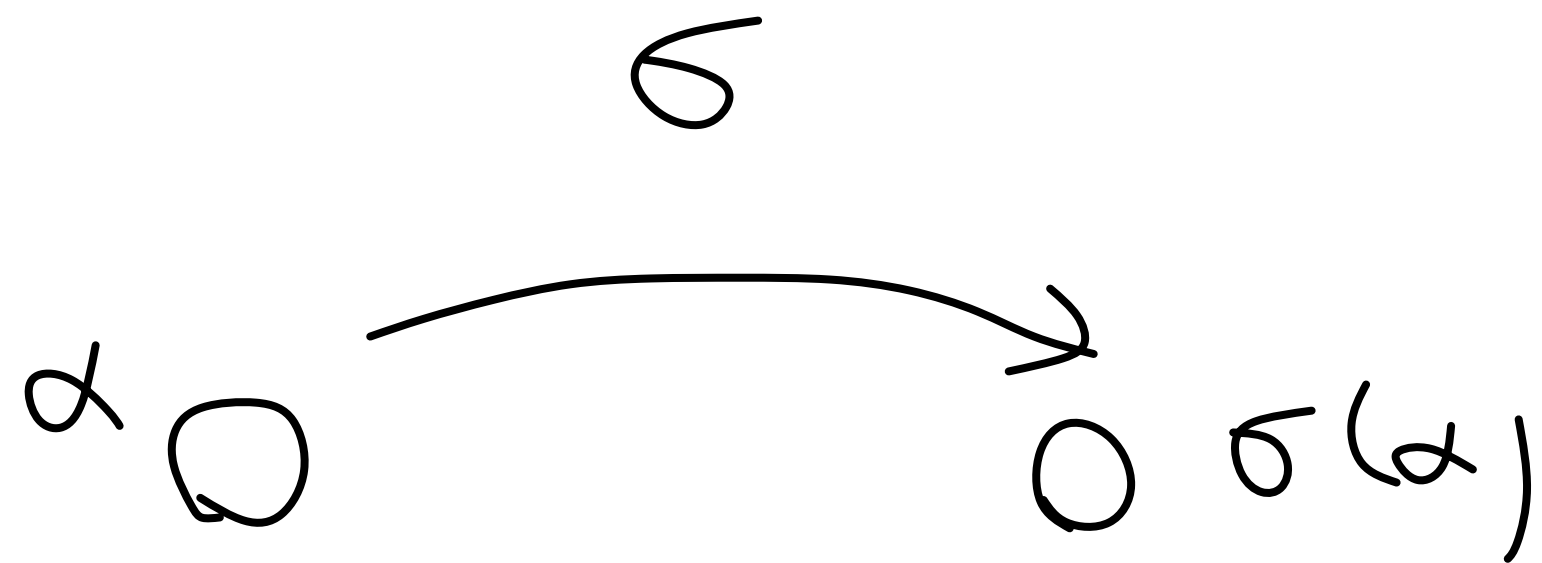
can have emergent global symmetries

Symmetries of anyons

• $\alpha \in \mathcal{D} = \Lambda^* / \Lambda$: anyons w/ spin $\theta(\alpha) = e^{\pi i Q(\alpha)}$

• Symmetry: $\sigma \in \text{Aut}(\mathcal{D})$

$$\alpha \mapsto \sigma \cdot \alpha \quad \text{s.t.} \quad \theta(\alpha) = \theta(\sigma \cdot \alpha)$$



Emergent symmetry after averaging

• $\langle \vartheta_{Q, \sigma, \alpha} \rangle = \langle \vartheta_{Q, \alpha} \rangle$ 😊

NOT a symmetry before averaging

• $\vartheta_{Q, \sigma, \alpha} \neq \vartheta_{Q, \alpha}$ 😞??

NOT a symmetry before averaging

• $\partial Q_{\delta, \alpha}(m) \neq \partial Q_{\alpha}(m)$ 😞??

NOT a symmetry before averaging

• $\partial_{Q, \sigma \cdot \alpha}(m) \neq \partial_{Q, \alpha}(m)$ 😞??

However, we have

• $\partial_{Q, \sigma \cdot \alpha}(\sigma \cdot m) = \partial_{Q, \alpha}(m)$ 😐!?

In general,

"ensemble sym."

average over $\mathcal{M} \subset \sigma \in G$ s.t. $[d(\sigma(m))] = [dm]$
of $\Theta_\alpha(m)$ s.t. $\Theta_{\sigma \cdot \alpha}(\sigma \cdot m) = \Theta_\alpha(m)$

$$\leadsto \langle \Theta_\alpha \rangle := \int [dm] \Theta_\alpha(m) = \langle \Theta_{\sigma \cdot \alpha} \rangle$$

$$\langle \theta_\alpha \rangle = \int [dm] \theta_\alpha(m)$$

$$= \int [dm] \theta_{\sigma \cdot \alpha}(\sigma \cdot m)$$

$$= \int [d(\sigma^{-1}m)] \theta_{\sigma \cdot \alpha}(m)$$

$$= \int [dm] \theta_{\sigma \cdot \alpha}(m)$$

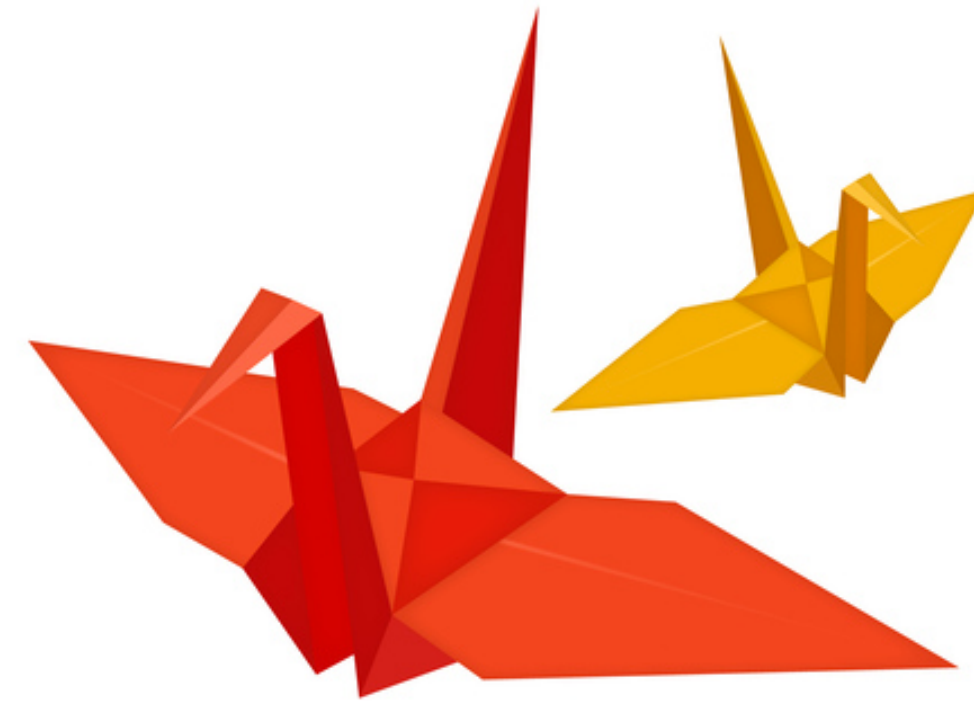
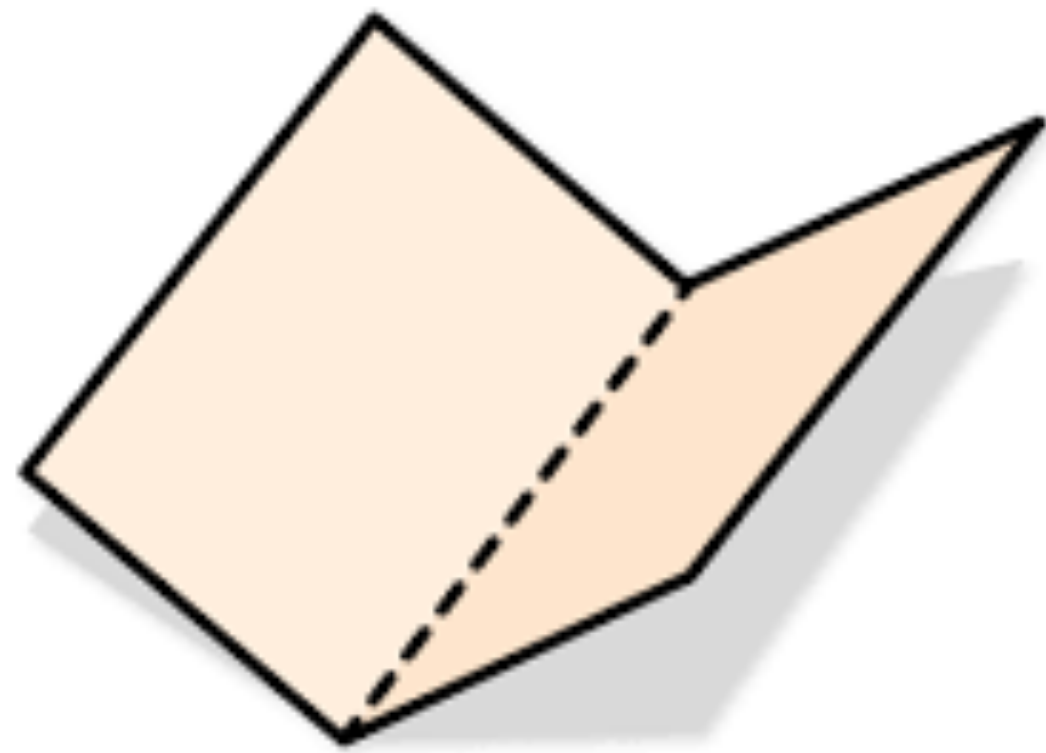
$$= \langle \theta_{\sigma \cdot \alpha} \rangle$$

covariance of θ

change of variable

invariance of measure

"duality origami"



ensemble symmetry



global symmetry

(e.g. T-duality)

ensemble
average

Rem: "Emergent Sym." can be
detected by different observables

$$\langle O \rangle = 0, \quad \langle \underbrace{O^2}_{\uparrow} \rangle \neq 0 //$$

cf. Edwards - Anderson
order parameter
in spin glass

Back to Swamp/land?

[Ashwinkumar - Leedom - MY]

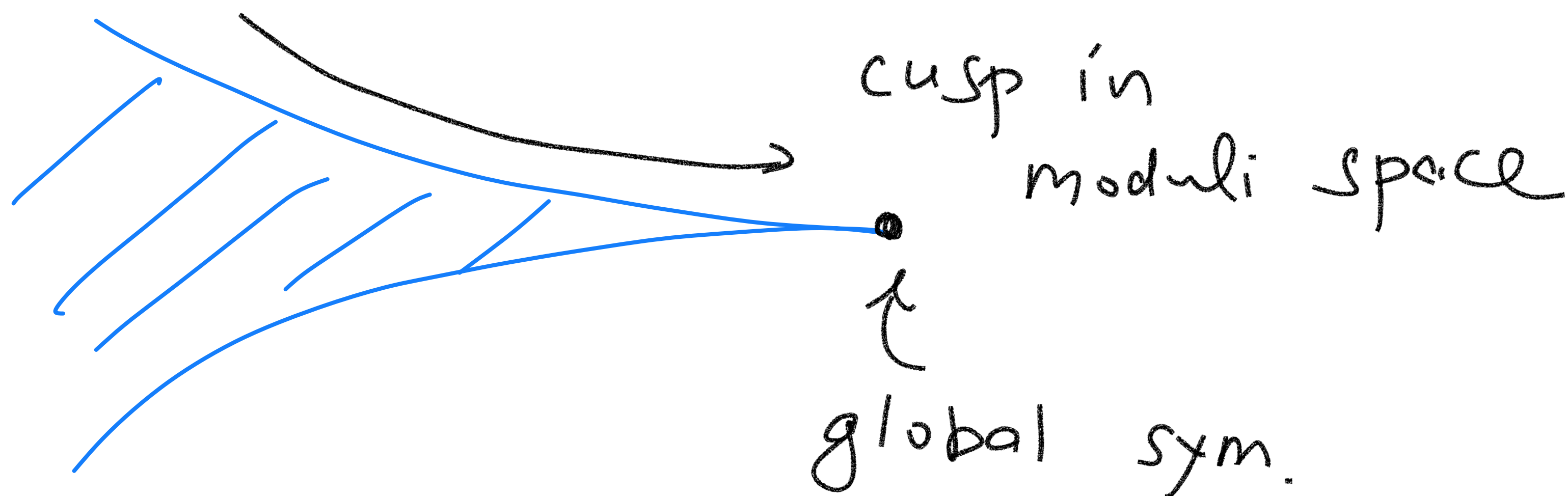
no global sym.



global sym.

@ ∞ -distance

(cf. distance
conjecture)



In our case, "cusp" for the
modular form:

$$\tau \rightarrow i\infty$$

$$\tau \rightarrow \mathbb{Q}$$

$PSL(2, \mathbb{Z})$

$\vartheta_{Q,\alpha}(\tau)$ ∞ -distance $\rightarrow$ asymptotic at cusps as $\tau \rightarrow -d/c$

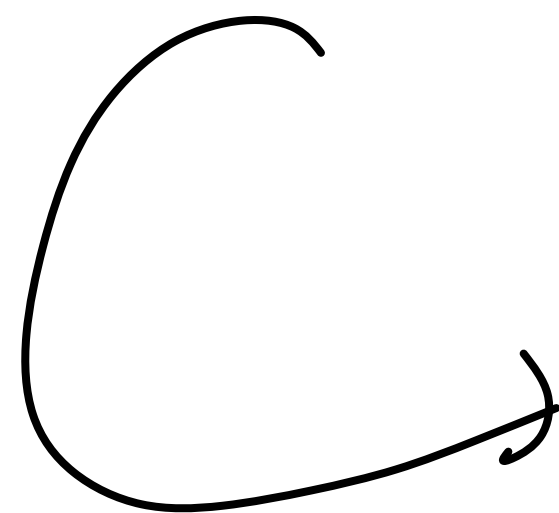
$\downarrow$

lens inv.

$\gamma_{Q,\alpha}(c,d)$

$\vartheta_{Q,\alpha}(\tau)$
 ∞ -distance
asymptotic at cusps as $\tau \rightarrow -d/c$

ensemble average

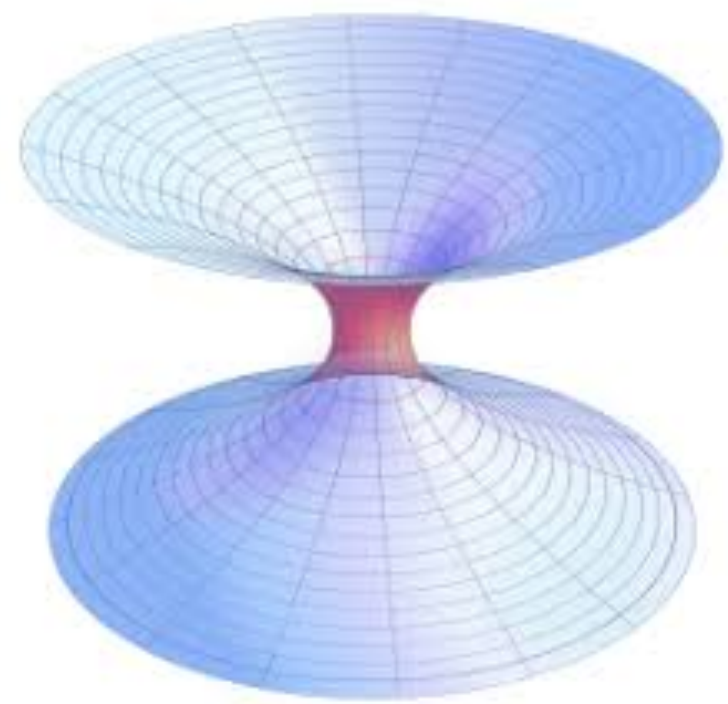


lens inv.
 $\gamma_{Q,\alpha}(c,d)$

$$\langle \vartheta_{Q,\alpha} \rangle(\tau) = \delta_{\alpha \in \Lambda^+} + \sum_{\substack{(k,d)=1 \\ c > 0}} \frac{\gamma_{Q,\alpha}(c,d)}{(c\tau+d)^{\frac{p}{2}} (c\bar{\tau}+d)^{\frac{q}{2}}}$$

building block

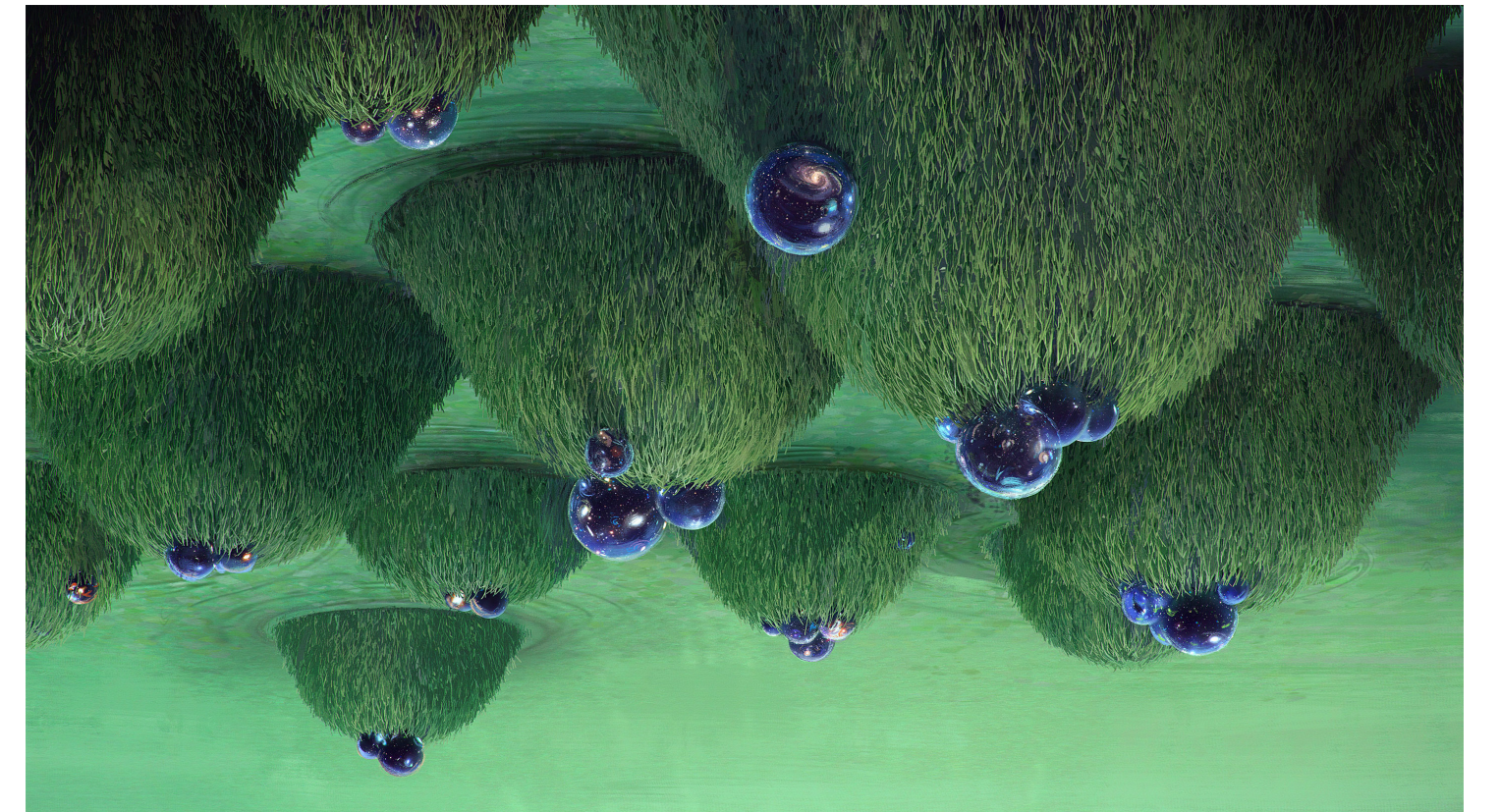
Emergent Sym. in
Ensemble Averages



"Ensemble"



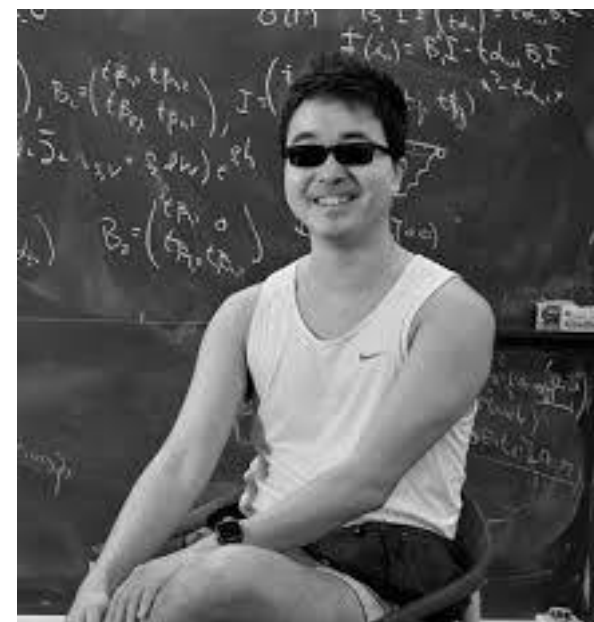
Emergent Sym.
in ∞ -distance



"Swampland"

Strong - to - Weak SSB

[Kawamoto - Tasuki - MY (to appear)]



Q: More general ensemble?

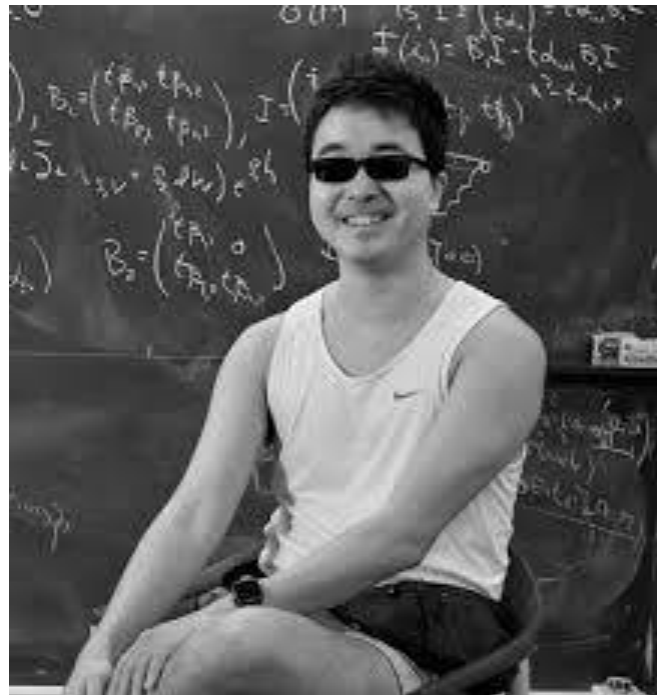
e.g. moduli space of SUSY CFT

connection with Calabi-Yau geometry
&

number theory

Q: More general ensemble?

↑
classical distribution
in quantum theory



What about density matrix?

$$\rho = \sum_i p_i |\psi_i\rangle \langle \psi_i|$$

- ^{weak} symmetry

$$|\psi\rangle \rightarrow U\psi$$

$$\rho = |\psi\rangle\langle\psi| \rightarrow U^\dagger |\psi\rangle\langle\psi| U = U^\dagger \rho U$$

- strong symmetry

$$\rho \rightarrow U\rho$$

Example $G = U(1)$ $|\psi_Q\rangle$: charge Q state

In general

$$\rho = \sum_Q c_Q |\psi_Q\rangle \langle \psi_Q|$$

not strongly symmetric

Fixed charge sector

$$\rho = \rho_Q = |\psi_Q\rangle \langle \psi_Q|$$

specific Q

strongly symmetric

$$\rho \rightarrow e^{iQ\theta} \rho$$

Strong - to - weak
SSB

$G_+ \times G_-$

Strong sym.

[Lee-Jian
- Xu ('23) ...]

G
weak sym.

SSB

no sym.

Strong SSB

Doubling / Choi

$$\rho = \sum_Q c_Q |\psi_Q\rangle \langle \psi_Q| \rightsquigarrow |p\rangle\rangle = \sum_Q c_Q |\psi_Q\rangle \otimes |\psi_Q^*\rangle$$

$$\uparrow$$
$$\mathcal{H}^+ \otimes \mathcal{H}^-$$

Schwinger - Keldysh

$$\int_p D\phi_+ D\phi_- e^{iS_+[\phi_+, J_+] - iS_-[\phi_-, J_-]}$$

$G_+ \times G_- \text{ sym.}$

$$= \langle\langle I | U(J_+) \otimes U(J_-)^* | p \rangle\rangle$$

$$|4\rangle = \sum_Q c_Q |4_Q\rangle \langle 4_Q|$$

$$\langle I | U(J_+) \oplus U(J_-)^* | p \rangle \rangle$$

$$= \sum_{Q, Q'} c_Q \underbrace{\langle Q' | U(J_+) | Q \rangle}_{\text{no sym for } Q \neq Q'} \underbrace{\langle Q | U(J_-)^* | Q \rangle}_{\text{no sym for } Q \neq Q'}$$

ensemble
average

SWSSB : Emergence of Sym in Ensemble Av.

How general is SWSSB?

Consider $G = U(1)$ & $\mathcal{O}$ charged op.

strong-to-weak SSB: [Sala, Gopalakrishnan, Oshikawa, You ('25)]

$$\lim_{|x-y| \rightarrow \infty} \frac{\langle \mathcal{O}(x) \mathcal{O}(y)^\dagger \rho \mathcal{O}(y) \mathcal{O}^\dagger(x) \rho \rangle}{\langle \rho^2 \rangle} = \mathcal{O}(1)$$

&

$$\lim_{|x-y| \rightarrow \infty} \langle \mathcal{O}(x) \mathcal{O}(y)^\dagger \rho \rangle = 0$$

Renyi-2 correlator

Consider $p \propto \underbrace{\pi_Q}_{\text{projector}} e^{-\beta H}$

Conjecture: [Sala - Gopakrishnan - Oshikawa - Yon ('24)]

We inevitably have $SWSSB$ or SSB

for any temperature $\beta \in (0, \infty)$

Idea

$$\langle O(x) O(y)^\dagger \rho O(y) O(x)^\dagger \rho \rangle$$
$$= \langle O(x; i\beta) O(y, i\beta)^\dagger O(y) O(x)^\dagger \rangle e^2_\alpha \pi_\alpha e^{-2\beta H}$$

$$\approx \underbrace{\langle O(x; i\beta) O(x)^\dagger \rangle}_{\rho^2} \langle O(y, i\beta)^\dagger O(y) \rangle_{\rho^2}$$

$$\downarrow$$
$$\langle O(x; \frac{i\beta}{2}) O(x; -\frac{i\beta}{2})^\dagger \rangle_{\text{grand can.}}$$

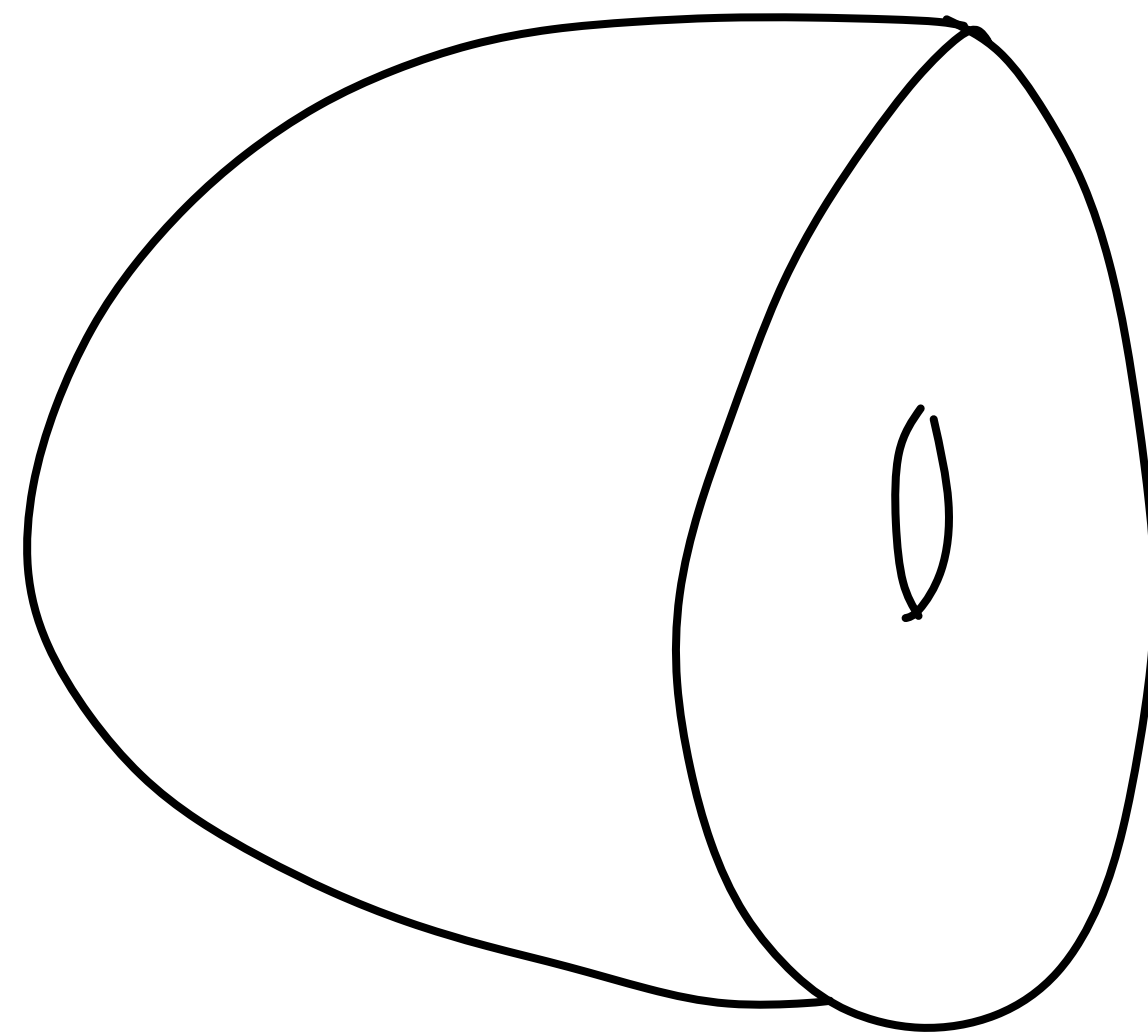
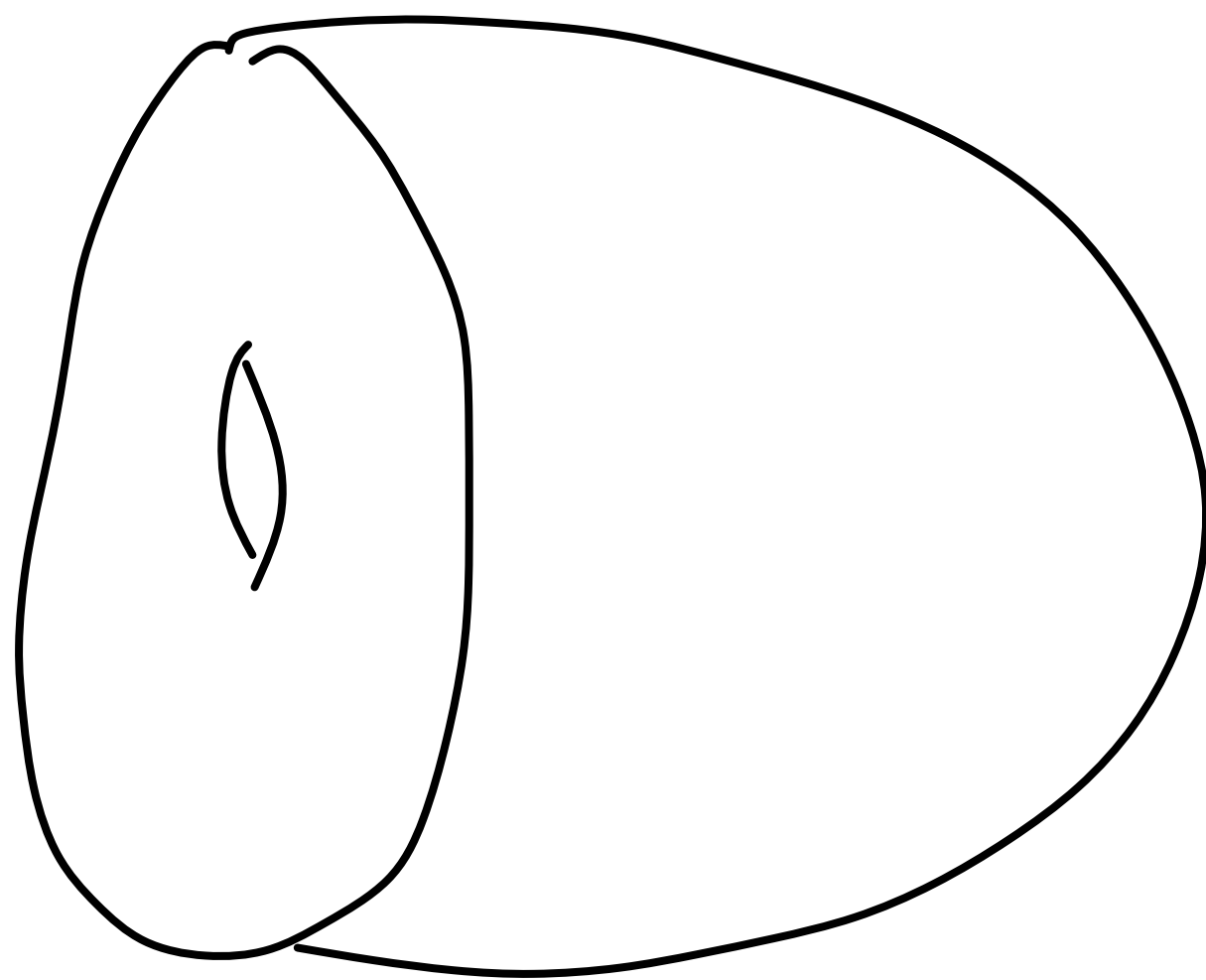
|
V
O

Holography ?

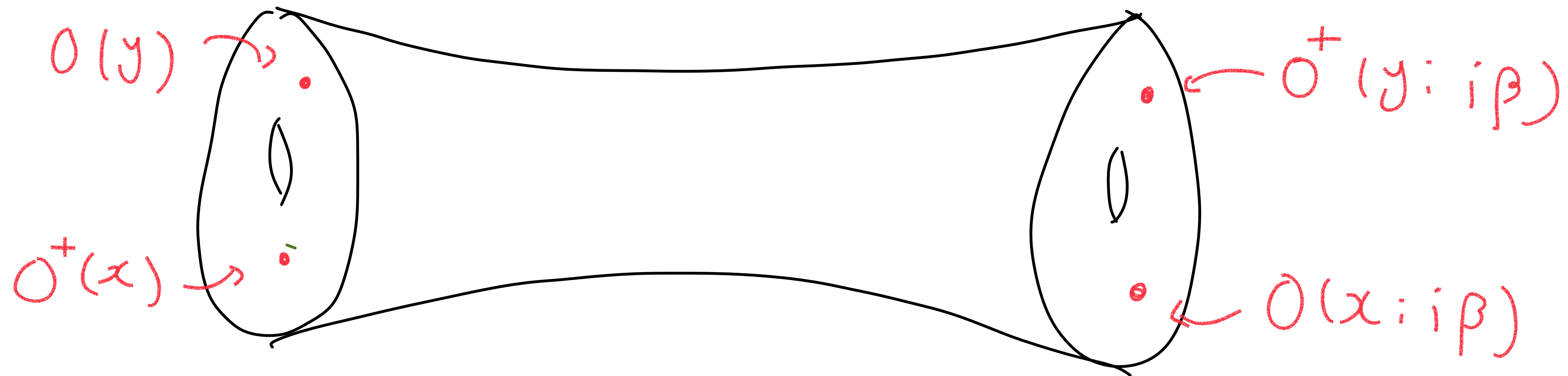
$$\text{AdS}_3 / \text{CFT}_2 \quad \left(\begin{array}{c} \text{Einstein} \\ \text{gravity} \end{array} \right) + \left(\begin{array}{c} \text{U(1)}_k \\ \text{CS} \end{array} \right)$$

Saddle: BTZ / thermal AdS_3

$$\langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{\mathbb{P}^2}$$

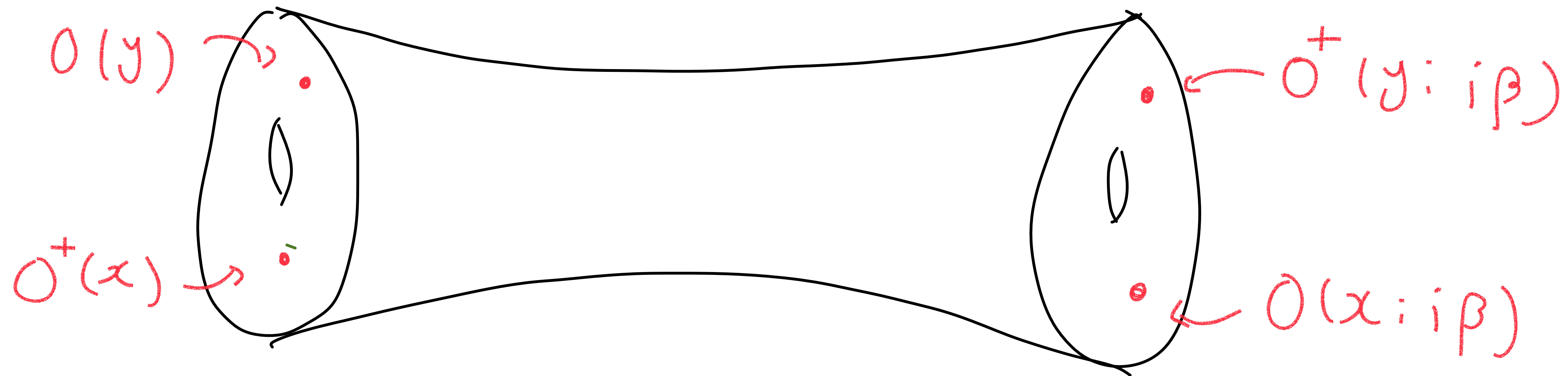


$$\langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{\rho^2}$$



→ imaginary time

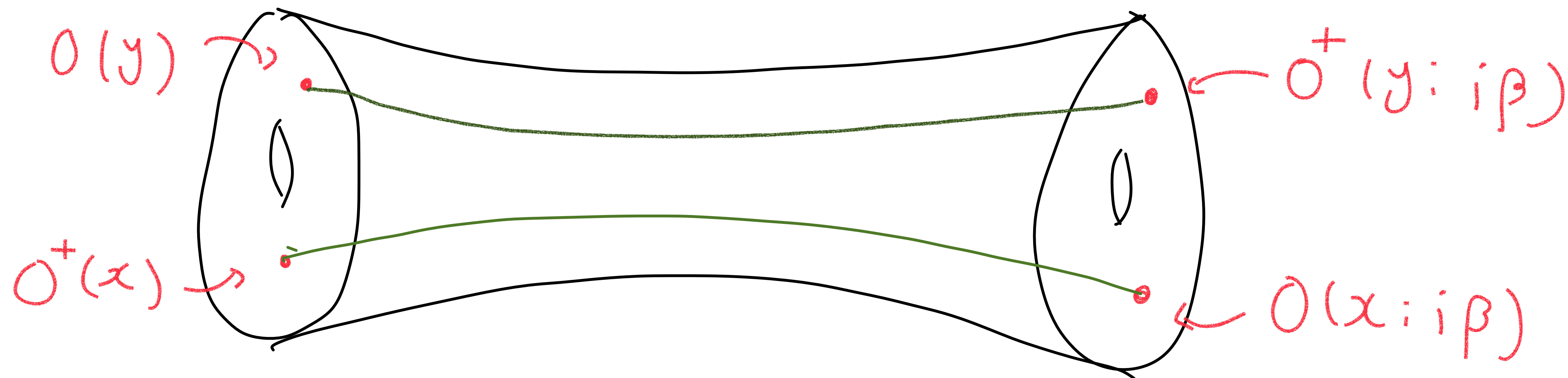
$$R^{(2)} = \langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{p^2}$$



large N factorization

$$\begin{aligned} R^{(2)} \approx & \langle O_+(x) O_-(x) \rangle \langle O_+(y) O_-(y) \rangle \\ & + \langle O_+(x) O_+(y) \rangle \langle O_-(x) O_-(y) \rangle \\ & + \langle O_+(x) O_-(y) \rangle \langle O_-(x) O_+(y) \rangle \end{aligned}$$

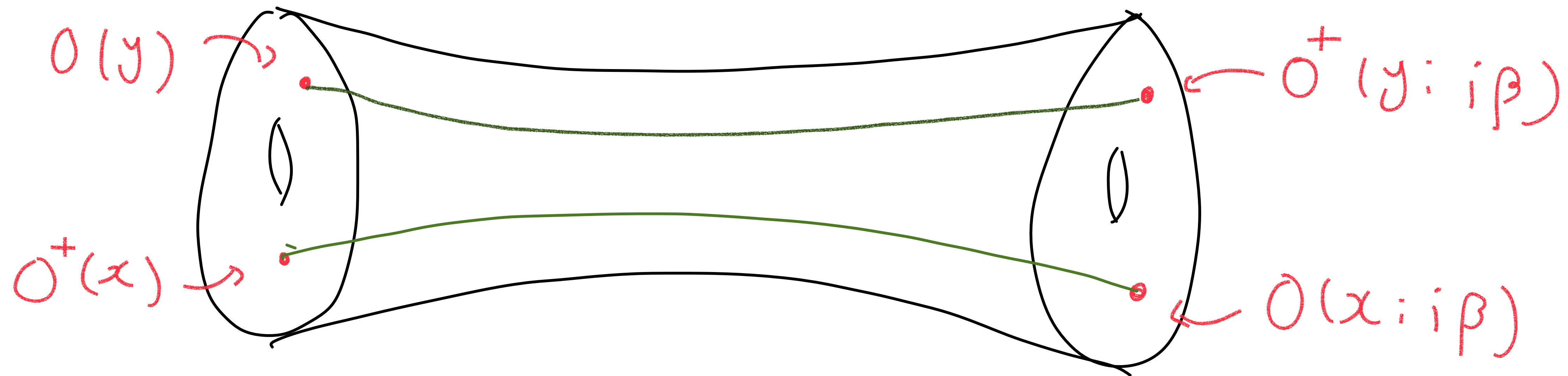
$$R^{(2)} = \langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{\mathcal{P}^2}$$



large N factorization

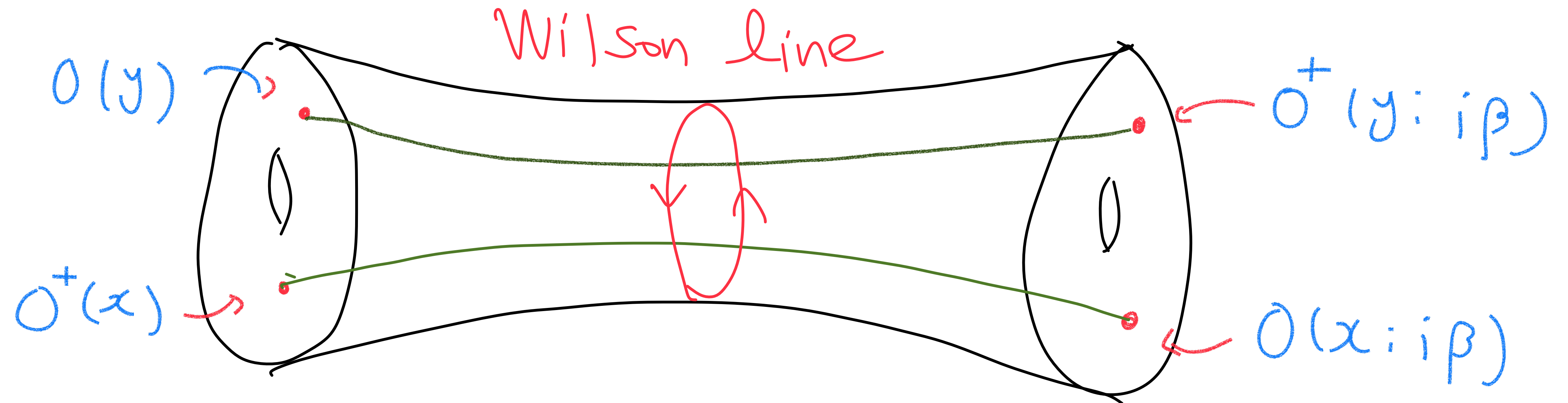
$$R^{(2)} \approx \left(\begin{aligned} &\langle O_+(x) O_-(x) \rangle \langle O_+(y) O_-(y) \rangle \\ &+ \langle O_+(x) O_+(y) \rangle \langle O_-(x) O_-(y) \rangle \\ &+ \langle O_+(x) O_-(y) \rangle \langle O_-(x) O_+(y) \rangle \end{aligned} \right)$$

$$\langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{p^2}$$



$$\langle O(x; i\beta) O^{\dagger}(x) \rangle \neq 0?$$

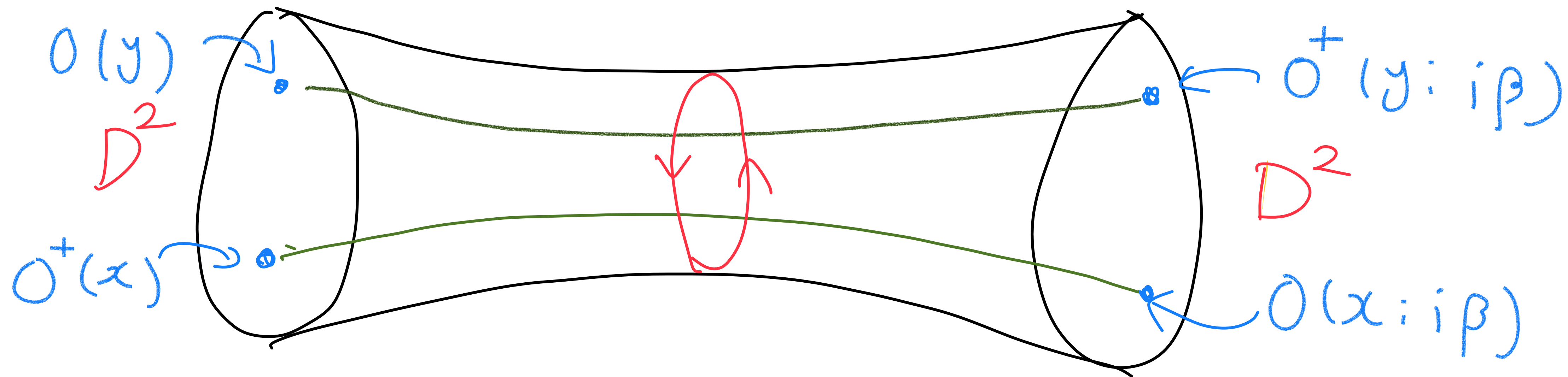
$$\langle O(x; i\beta) O(y; i\beta)^{\dagger} O(x)^{\dagger} O(y) \rangle_{P^2}$$



subtlety:

$$P = \underbrace{\prod_Q}_{\propto \sum_{m \in \mathbb{Z}_k}} e^{-\beta H} e^{im(\hat{Q} - Q)/k}$$

Thermal AdS_3 ?



No winding cycle ?

microscopic wormholes?

Summary

Global symmetries:

(T-) duality

- "forbidden in QG"
- emerges in ensemble average
- emerges in ∞ -distance
- result of SWSSB

Symmetry : guiding principle of QG

