

Practical Quantum Gravity

for cosmologists

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What?



Classical Gravity
 $g_{\mu\nu}$: metric

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{gravity}} = \underbrace{T_{\mu\nu}}_{\text{matter}}$$

Classical Gravity

$g_{\mu\nu}$: metric

$$\underbrace{R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu}}_{\text{gravity}} = \underbrace{T_{\mu\nu}}_{\text{matter}}$$



Quantum Gravity

$$\int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]}$$

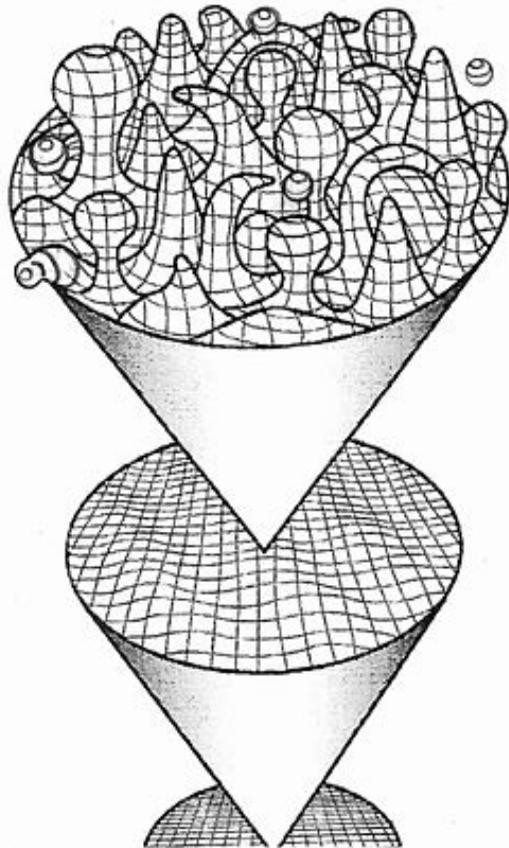
path-integral

↓

$$\sim \sum_{\text{saddle } i} e^{-S[g_{\mu\nu}^{(i)}]} \dots$$

semiclassical

Quantum Geometry ?



Beyond inflation?

relevant everywhere?
(UV/IR mixing)

Dark Matter?

Dark Energy

Dark Energy Accelerated Expansion

Afterglow Light Pattern
375,000 yrs.

Dark Ages

Development of
Galaxies, Planets, etc.

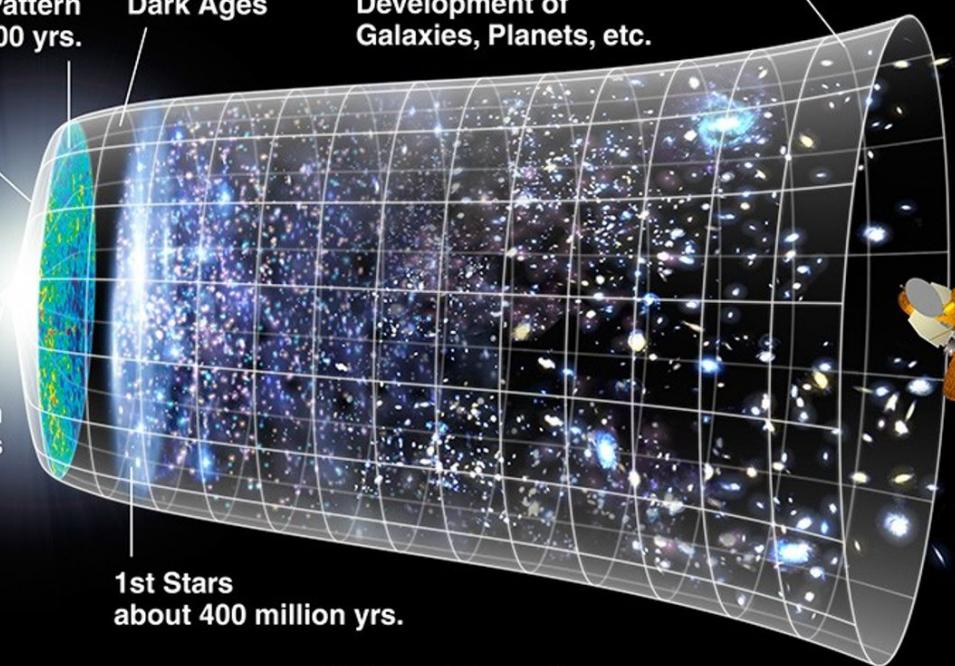
Inflation

Quantum
Fluctuations

1st Stars
about 400 million yrs.

WMAP

Big Bang Expansion
13.77 billion years



Why?

A hand-drawn wavy underline consisting of a single continuous line that starts on the left, dips down, rises up, dips down again, and then rises up towards the right.



- * Fun and very fundamental

- * Source of New Ideas / Techniques

- * New Constraints beyond EFT

- * Necessary e.g. in early universe

Is QG Really Necessary?



- Gravity exists in Nature!

↳ Dark Matter, Dark Energy

- Nature is Quantum - Mechanical

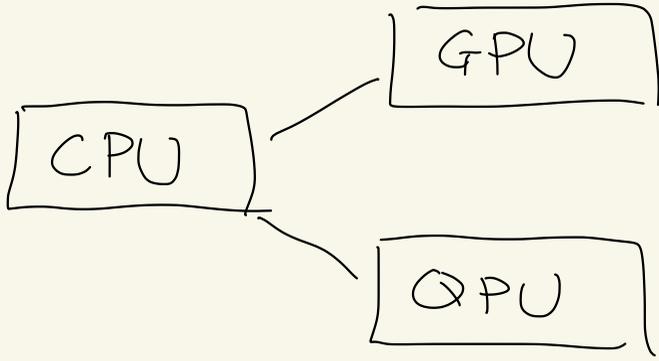
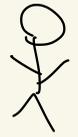
⇒ Quantum Gravity

This is the majority opinion!?

Q: What about GR / non-GR ?
classical / quantum

"hybrid quantum - classical"

cf.



Problem:

gravity

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \langle \hat{T}_{\mu\nu} \rangle$$

matter

classical

Non-linear

expectation value of
quantum operator

[Møller, Rosenfeld]
'62 '63

non-linear QM?

Non-linear QM = "a modified QM"



difficult to formulate

[Weinberg '89]

... e.g. Weinberg's non-linear QM



• Superluminal communication

[Gisin
Polchinski '91]

• "interference between many-worlds"

• solve NP-complete problem ...

[Abrams-Lloyd '98]

... although attempts continue

There are also some uniqueness result of \mathcal{QM}

特集／重力と量子力学

量子力学の数学とは何か

山崎雅人

1. 量子力学の数学とは？*1)

まずは問いからはじめよう：「量子力学の数学」とは何だろうか？ 現在我々が教わる量子力学の基本的な体系は、量子力学の草創期に活躍した巨人たち（ハイゼンベルグ、ポルン、パウリ、シュレーディンガー、ディラックなど）により 100 年ほど前に一旦完成されており、その成果は例えば 1932 年のフォン＝ノイマンの教科書¹⁾に整理されている*2)。この教科書で議論されている量子力学の数学とは主としてヒルベルト空間論であり*3)、現在でも量子力学の数学を学びたい

者の間では関数解析的な問題意識が共有されていないことも多いと思う。例えば、関数解析では作用素は定義域をきちんと定めて議論しないとイケない*5)、非有界作用素の取り扱いはかなり慎重でなければならないはずであるが、物理の本ではこの辺りがいい加減なことが多い。これは物理学者の数学が厳密でないというよくある話になっているかもしれないが、逆にいえば物理学としての量子論の理解の基礎のためにヒルベルト空間論の数学的な詳細がどこまで本質的かどうか考え直す余地があるのかもしれない*6)*7)。

量子力学の数学が何であるのかは深遠な問いであり、

Indirect Evidence for Quantum Gravity

Don N. Page

Department of Physics, The Pennsylvania State University, University Park, Pennsylvania 16802

and

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Department of Physics, William Jewell College, Liberty, Missouri 64068

(Received 9 June 1981)

An experiment gave results inconsistent with the simplest alternative to quantum gravity, the semiclassical Einstein equations. This evidence supports (but does not prove) the hypothesis that a consistent theory of gravity coupled to quantized matter should also have the gravitational field quantized.

PACS numbers: 04.60.+n

Quantum mechanics appears to govern all non-gravitational fields (here called matter), and most people believe it also applies to the gravitational field. However, there has been no explicit experimental test of this. Gravity is so weak that Feynman¹ has questioned whether it must be quantized. As an alternative, Møller² and Rosenfeld³ have proposed a theory in which grav-

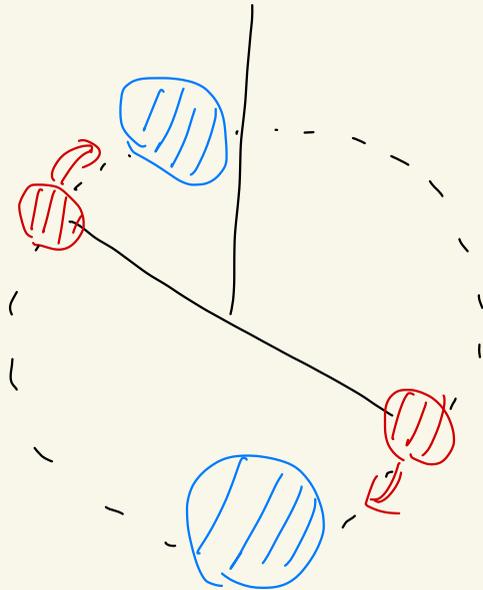
facts have not been ruled out experimentally and thus should not yet be dismissed as unphysical.

A more conclusive argument against the collapse of the wave function in the semiclassical theory is that if ψ collapses, in general the right-hand side of (1) will not be conserved, whereas the left-hand side is automatically conserved.

That is, if $\psi = \sum_i c_i(x^\alpha)\psi_i$ with constant ψ_i 's in

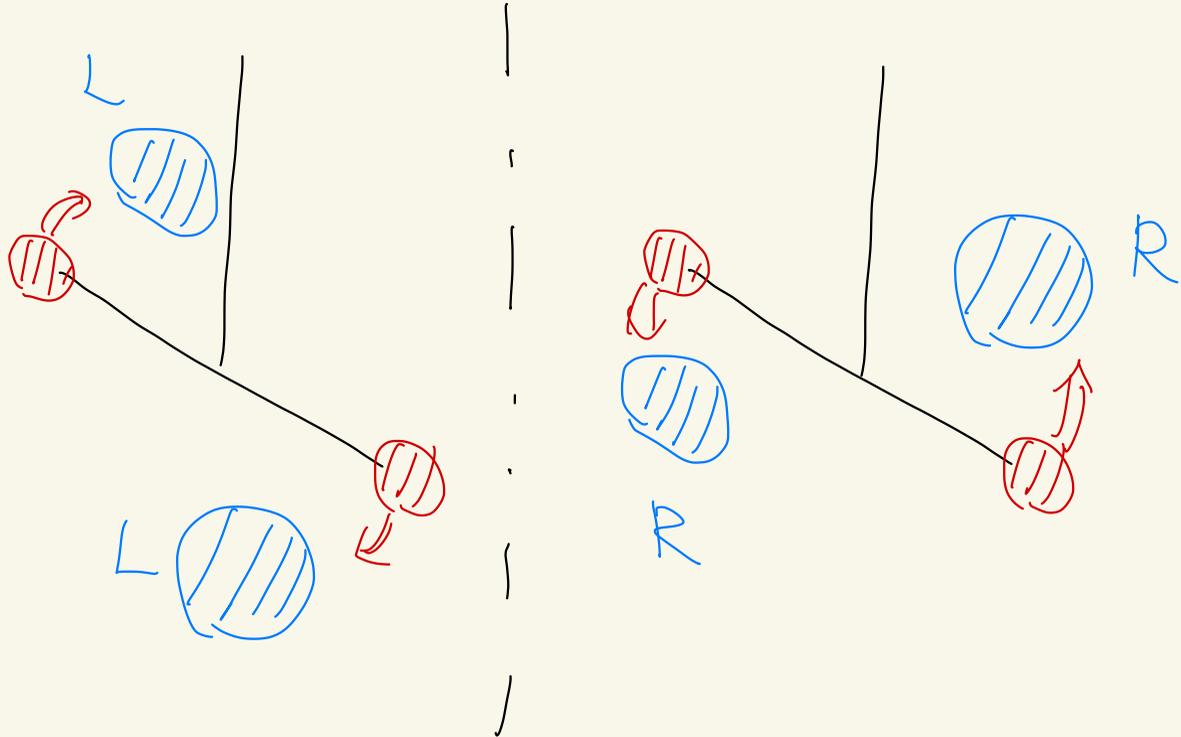
Empirical "Test"

Recall Cavendish Experiment

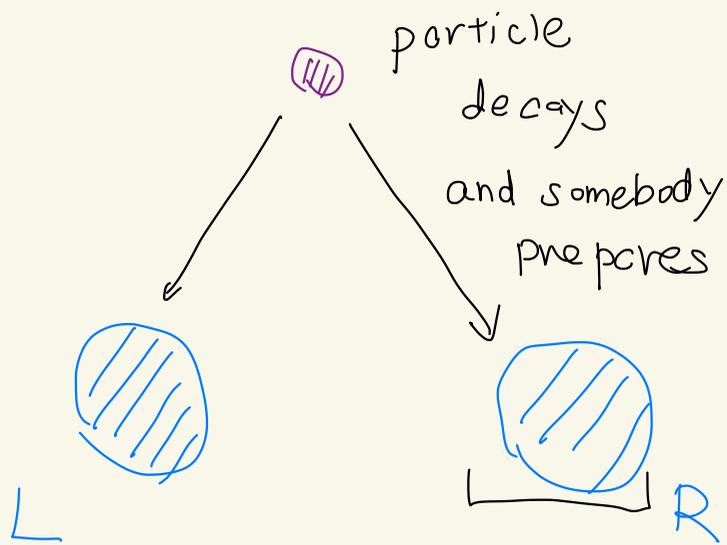


Empirical "Test"

Recall Cavendish Experiment



Empirical "Tests"



$$|\psi\rangle = \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle)$$

X

$$\frac{\langle T_L \rangle + \langle T_R \rangle}{2}$$

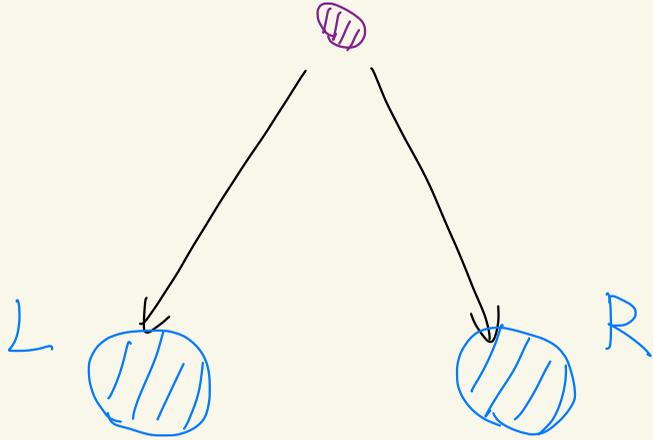
or

○

$$\langle T_L \rangle \quad \& \quad \langle T_R \rangle$$

50% 50%

Empirical "Tests"



$$|4\rangle = \frac{1}{\sqrt{2}} (|L\rangle + |R\rangle)$$

Of course the macro.

state has *decohered*

long ago

\Rightarrow not a test of QG

but inconsistency of

scenario

We can however persist:

stochastic gravity?
~~~~~  
classical  
probability

⇒ We need to modify QM altogether

time evolution, measurement, ...

[e.g. Oppenheim 1811.03116]

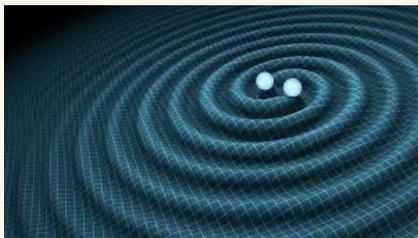
(in a sense more dramatic than QG)

Quantum Signature of Gravity ?

Active Research Area ....

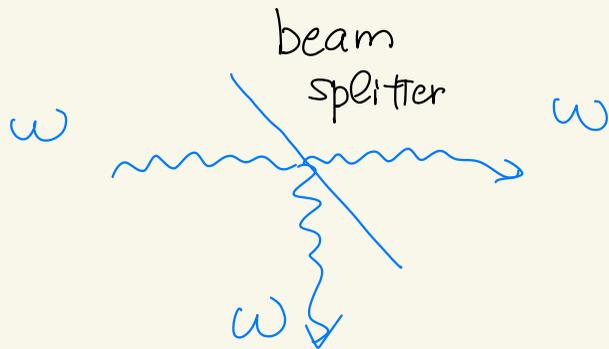
GW ?

( Gravitational Wave  
Interferometer )



GW ?

(Optical Interferometer)



entanglement

second harmonic generation



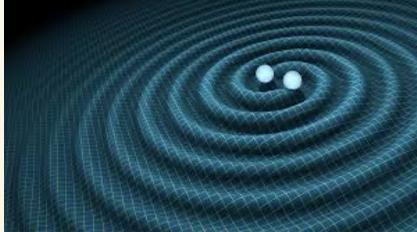
down conversion



non-linear effect

→ squeezed state

( Gravitational Wave  
Interferometer )



Quasi-Normal Modes  $\omega_n$  of BHs

↪ Counterpart of SHG in optics:

non-linearities in BH ringdown

↪ Prove quantumness of gravity?

[e.g. Guerreiro 2508.07348, Manikandan Wilczek 2508.03380]

... however the effects will be very small

[cf. squeezing parameter  $\sim 10^{-4}$  [Kanno-Soda-Taniguchi 2508.17947]]

Signatures in Early Universe?

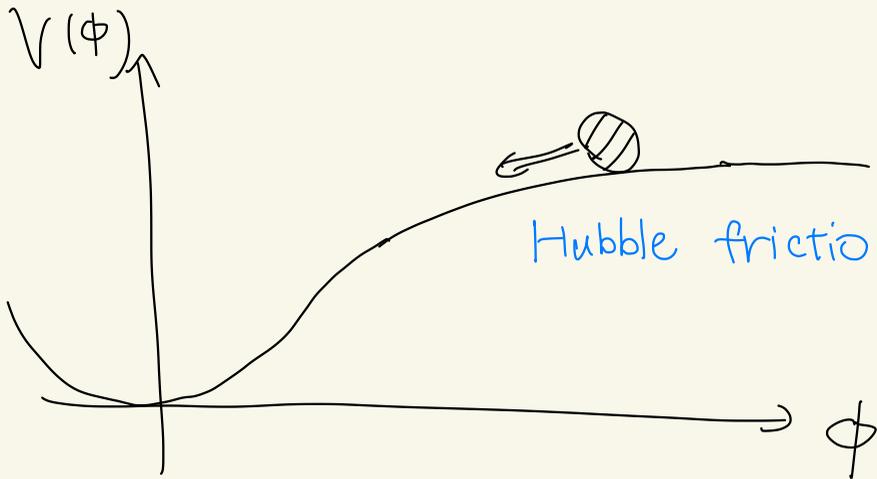
⇒ Inflation

consider an inflaton  $\phi$  in potential  $V(\phi)$

slow-roll inflation

$$\epsilon \equiv \frac{M_{pl}^2}{2} \left( \frac{V'}{V} \right)^2 \ll 1$$

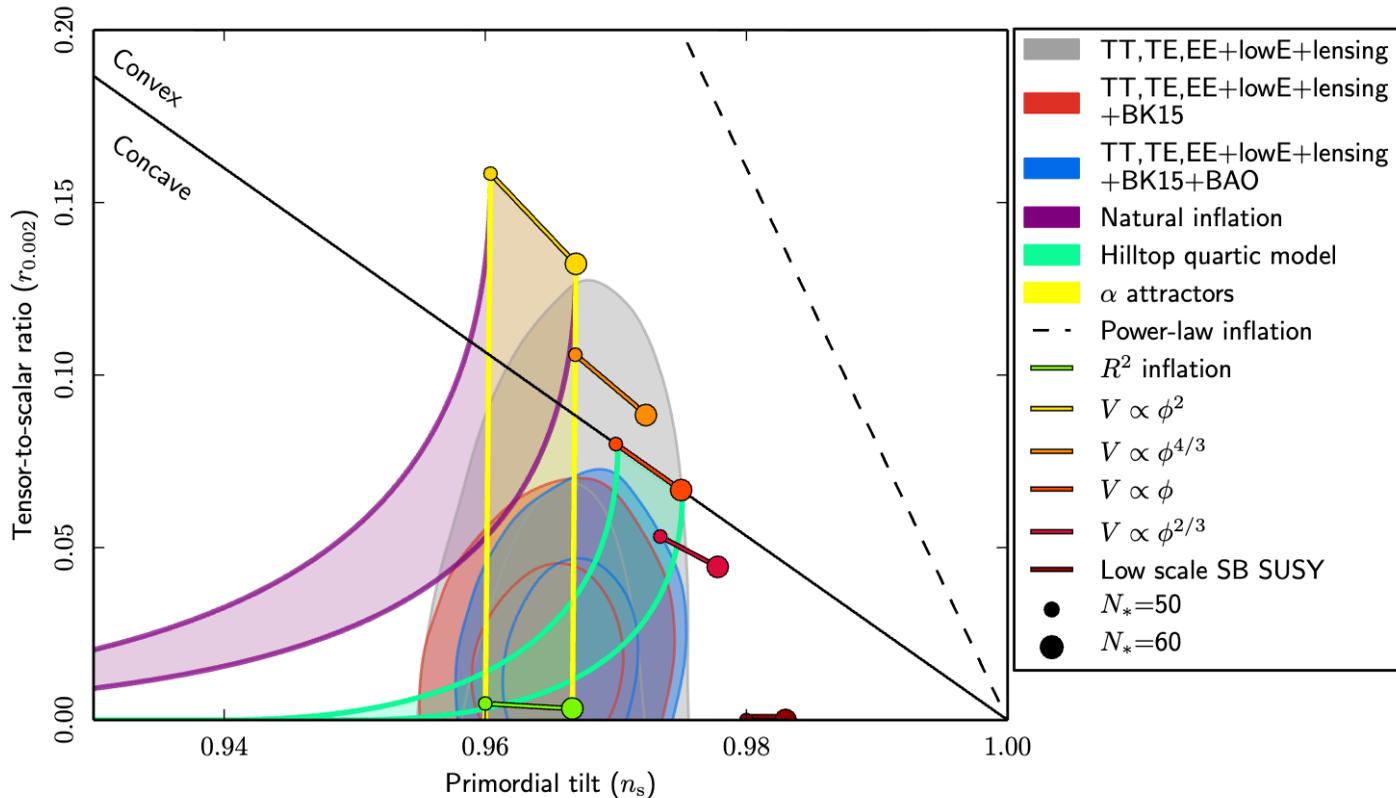
$$\eta \equiv M_{pl}^2 \left| \frac{V''}{V} \right| \ll 1$$



Hubble friction  $H = \frac{\dot{a}}{a}$

We have observational constraints

$$n_s = 1 + 2\eta - 6\epsilon, \quad r = 16\epsilon$$

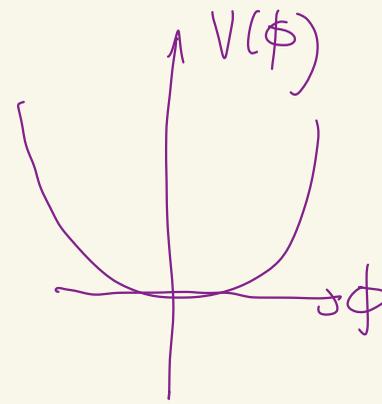


$$V(\phi) = \frac{1}{2} m^2 \phi^2$$

$$V(\phi) = \frac{1}{4} \lambda \phi^4$$

$$V(\phi) = V_0 \left( 1 - \cos \frac{\phi}{f} \right)$$

disfavored  
(convex)

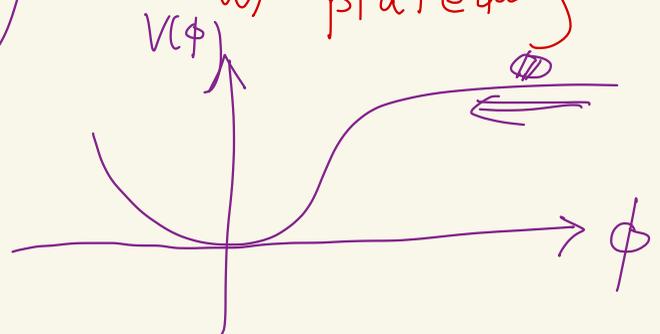


$\alpha$ -attractor [Kallosh-Linde '13]

$$V(\phi) = V_0 \left( 1 - e^{-\sqrt{\frac{2}{3\alpha}} \frac{\phi}{M_{pl}}} \right)^{2n}$$

$$V(\phi) = V_0 \left[ 1 - \left( 1 + \left( \frac{\phi}{f} \right)^2 \right)^{-p} \right]$$

favoured  
(concave  
w/ plateau)



pure natural inflation

[Nomura-Watarai-MY '17]

But fundamentally

$$V(\phi) = \sum_n c_n \phi^n$$

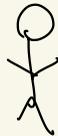
$\infty$ -many parameters

to fit  $n_s, r, \dots$ !



We can do whatever we want,  
as long as we fine-tune potential?

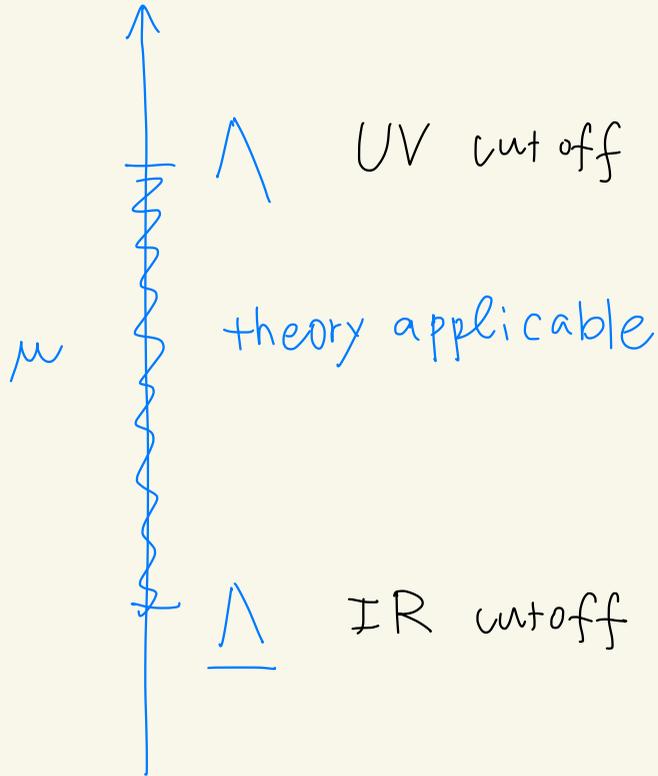
Fine-tuning more subtle in EFT



Effective Field Theory

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A theory (e.g.  $\phi$  w/  $V(\phi)$ ) always has limitations



renormalization:

physics depend on  
energy scale  $\mu$

e.g. scalar field

$$\mathcal{L} = \sqrt{-g} \left[ R + (\partial\phi)^2 - \underbrace{m^2}_{\omega} \phi^2 - \underbrace{\lambda}_{\omega} \phi^4 - \underbrace{\sum_{n=1}^{\infty} c_n \frac{\phi^{4+n}}{\Lambda^n}} \right]$$

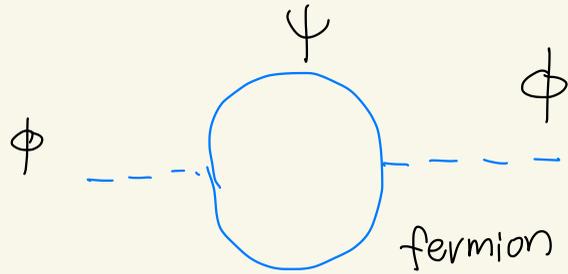
①  $m, \lambda$ : coupling  
running

$$\begin{pmatrix} m = m(\mu) \\ \lambda = \lambda(\mu) \end{pmatrix}$$

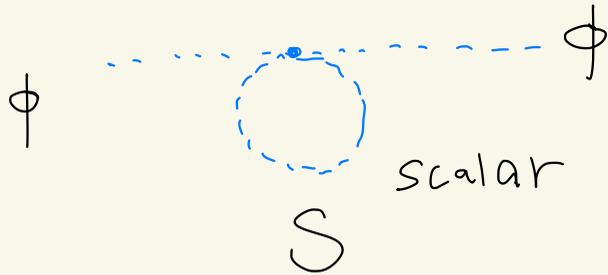
② higher-dim.  
operator

↑  
unknown  
UV physics

⊕ Even if we set  $m^2 \sim 0$ ,  $m^2$  is generated quadratically



$$\int_0^\Lambda \frac{d^4 p}{p^2} \sim \Lambda^2$$



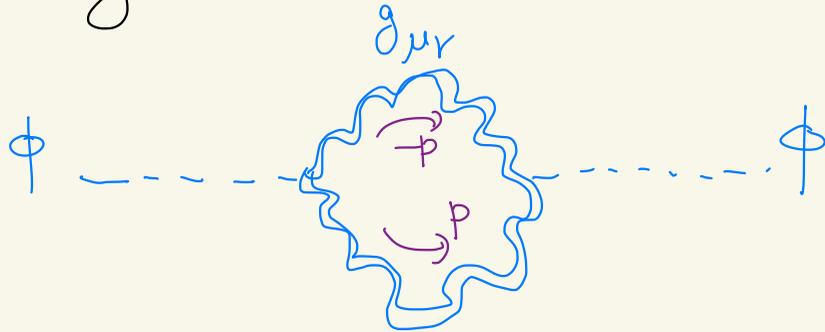
$$\Delta m^2 \sim \left( \frac{\Lambda}{4\pi} \right)^2$$

[ cf. hierarchy problem  
for Higgs ]

This is also true for inflaton

But  $\phi$  inevitably couples to  $g_{\mu\nu}$  (e.g.  $\mathcal{L} \supset \int R\phi^2$ )

and generates a mass for  $\phi$



$$\Delta m^2 \sim \left( \frac{c_{\text{cur}}}{4\pi} \right)^2 \Lambda^2$$

We need  $\Lambda \gg H$  for consistency

$$\rightarrow \eta = M_{\text{pl}}^2 \frac{V''}{V} \sim \frac{m^2}{H^2} \sim \frac{\Lambda^2}{H^2} \gtrsim 1$$

"eta problem"

We can e.g.

- fine-tune by hand
- " by symmetry

e.g. shift symmetry  
 $\phi \rightarrow \phi + \text{const.}$

② In fact, the situation is even worse for higher-dimensional operators

e.g.  $\Delta V = C_n \frac{\phi^{n+4}}{\Lambda^n}$  may break the str.

$\underbrace{\hspace{10em}}_{\text{unknown: UV physics}}$

"small field"

$\bullet \phi \ll \Lambda < M_{pl} \Rightarrow \Delta V \ll V$

but then

$$\Delta \eta \approx \frac{M_{pl}^2}{\phi^2} \sim \frac{M_{pl}^2}{\Lambda^2} \gtrsim 1$$

"large field"

$\bullet \phi \gg M_{pl} \Rightarrow \Delta \eta$  can be small

but we need to control  $\Delta V$  for all  $n$

"Large field"  $\phi \gtrsim M_{\text{pl}}$  : e.g. chaotic inflation

$$V \sim \phi^2, \phi^4$$

but disfavored by  $r \lesssim 0.05$ .

(cf. Lyth bound  
↑

$$N_e \gtrsim 50$$

$$\frac{\Delta\phi}{M_{\text{pl}}} \gtrsim 0.25 \times \sqrt{\frac{r}{\underbrace{0.01}_p}}$$

future observations

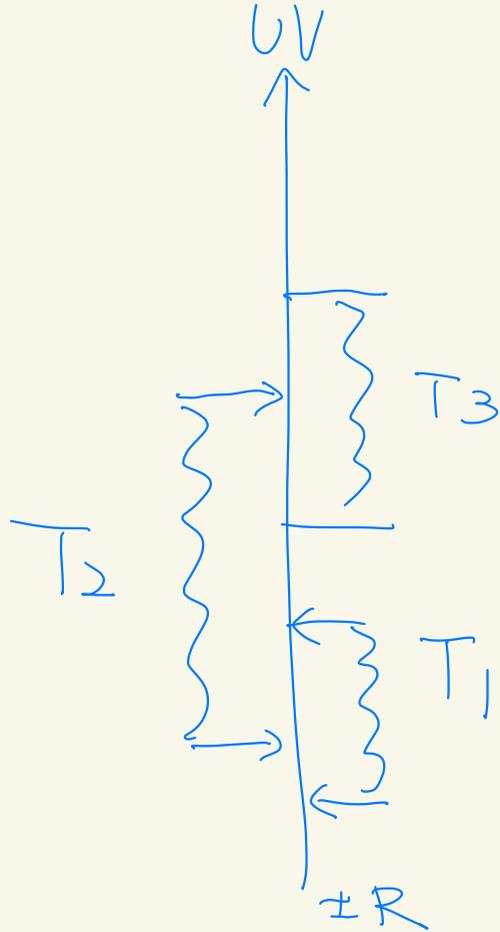
Beyond EFT:

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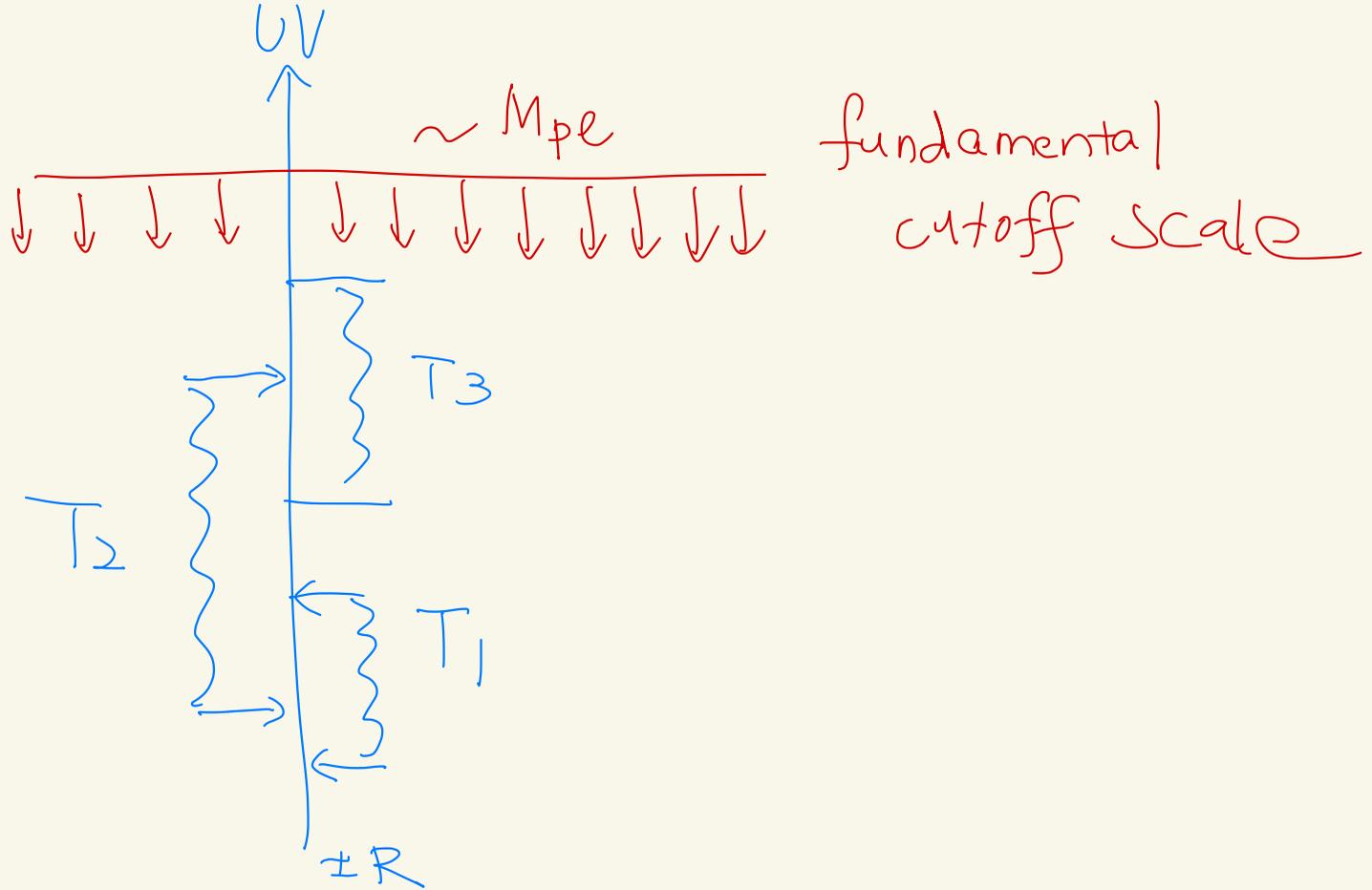
String Theory

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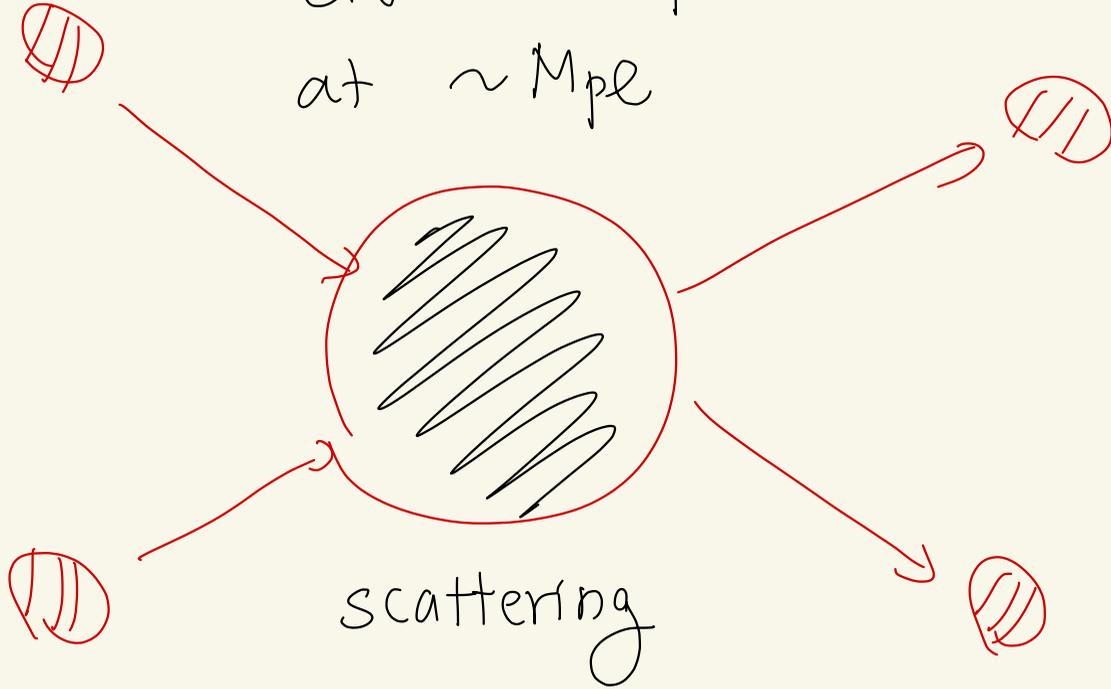
End to EFT paradigm?



End to EFT paradigm?

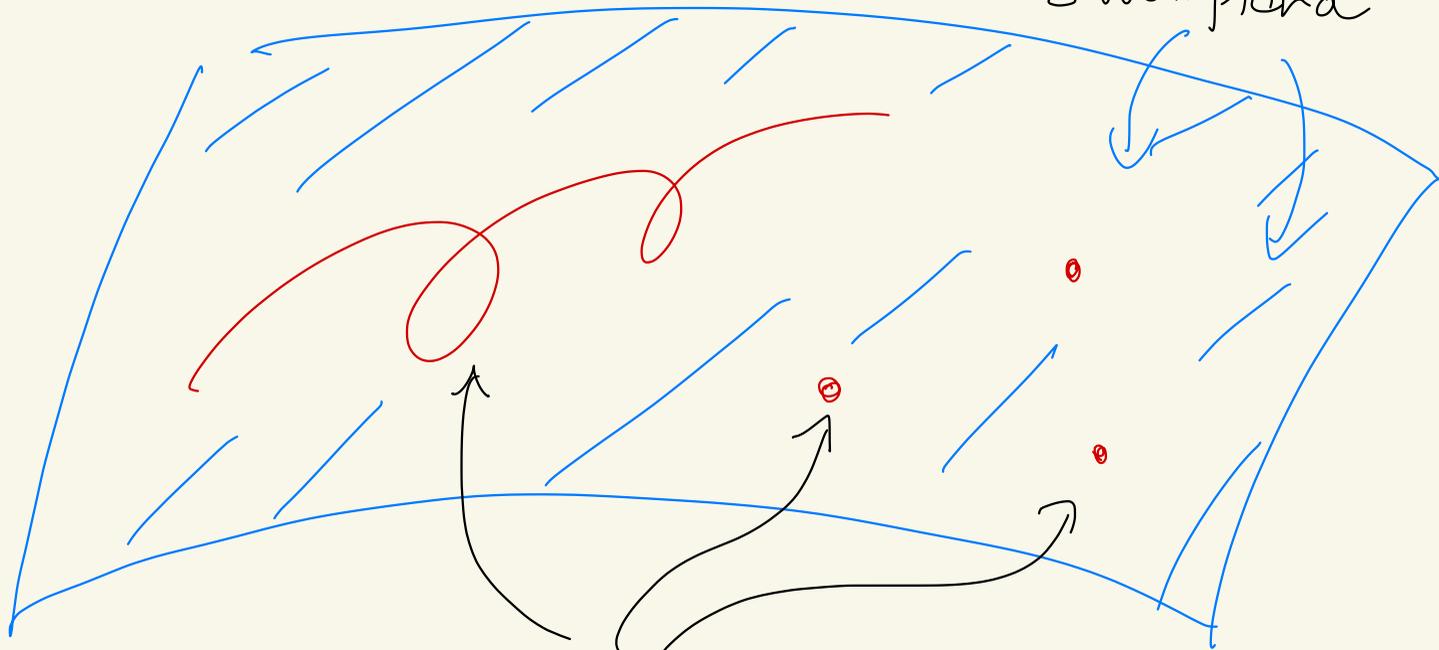


creates BH w/ horizon  
at  $\sim M_{pl}$



consistent EFT

SWampland



landscape

# String Landscape



# Swampland



Example of

Swampland Constraints

Swampland distance conjecture

$$\Delta\phi \lesssim M_{\text{pl}}$$

cf. Lyth bound

$$\frac{\Delta\phi}{M_{\text{pl}}} \gtrsim 0.25 \times \sqrt{\frac{r}{\underbrace{0.01}_{\uparrow}}}$$

future observations

String theory seems to indicate  $r \lesssim 0.01$

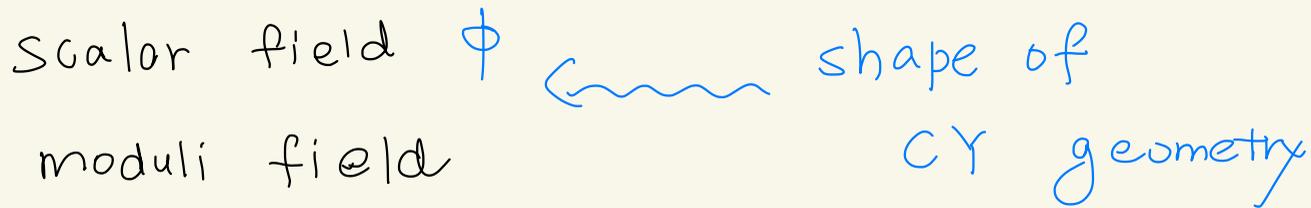
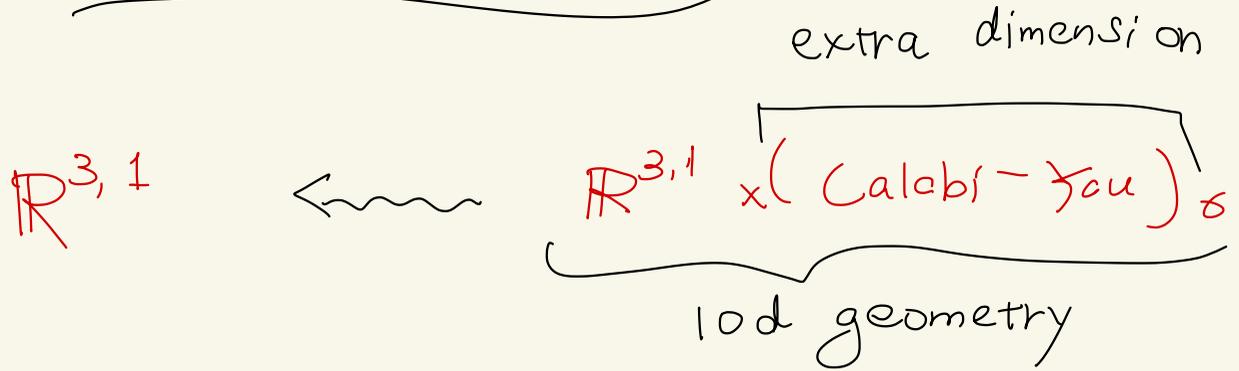
modulo important caveats

Why  $\Delta\phi \lesssim M_{pl}$ ?

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(meaning?, loophole?, ...)

# Calabi-Yau moduli

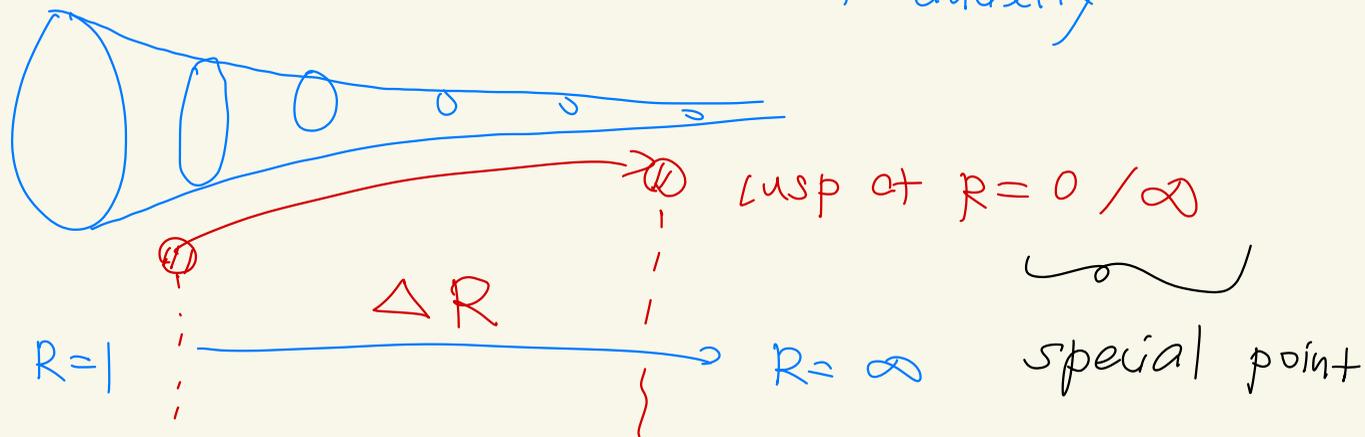


Consider  $S^1$  with radius  $R \in (0, \infty)$

metric:  $ds^2 = \left(\frac{dR}{R}\right)^2$  : invariant under

$$R \leftrightarrow 1/R$$

T-duality



# Kaluza-Klein theory:

$\Phi$ : scalar in  $(4+1)D$

$$\left. \begin{array}{c} \underbrace{\mathbb{R}^{3,1}}_x \times \underbrace{S^1}_\theta \\ \downarrow \end{array} \right\}$$

Fourier decomposition

$$\Phi(x, \theta) = \sum_n \Phi_n(x) e^{i 2\pi \frac{\theta}{R} n}$$

$\Phi_n$ : Kaluza-Klein modes

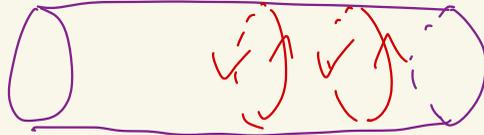
with mass  $m_n = \frac{|n|}{R}$  ( $n = \pm 1, \pm 2, \dots$ )

•  $R \rightarrow \infty$ : Kaluza-Klein mode  $m_n \rightarrow 0$   
 a tower of massless

[ $\infty$ -many species of  
 new particles!]

$$\Delta R \sim \int_0^R \frac{dr}{r} \sim \ln R \Rightarrow m_n \sim \frac{n}{R} \sim e^{-\Delta R \cdot n}$$

•  $R \rightarrow 0$ : string winding mode  $\tilde{m}_n \rightarrow \infty$

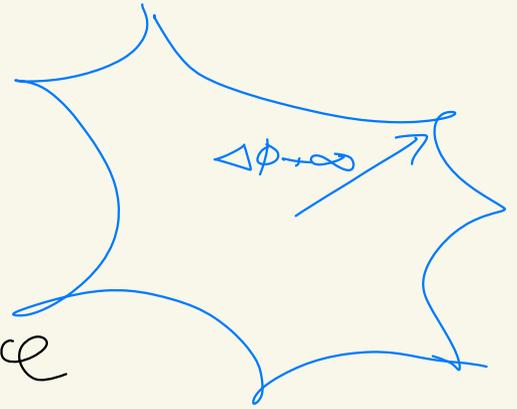


$$\text{mass} \sim (\text{tension}) \times (\text{length}) \propto R \rightarrow \infty$$

Conjecture: In general:

•  $\Delta\phi \rightarrow \infty$ :

singular pt in moduli space



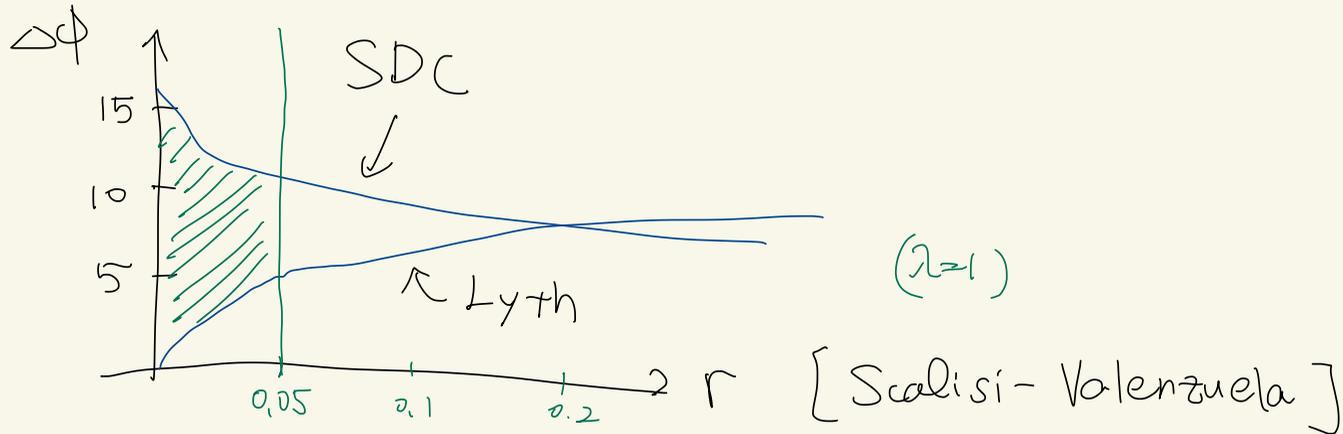
• as we approach  $\Delta\phi \rightarrow \infty$   
a tower of massless states of mass

$$m_n \sim e^{-n\Delta\phi}$$

\* constraints on large-field inflation models

SDC  $H \leq \Lambda \quad \rightsquigarrow \quad \Delta\phi \lesssim - \left( \log \frac{\pi^2 A_s}{2} + \log r \right)$

Lyth bound  $\rightsquigarrow \quad \Delta\phi \geq 3 \sqrt{\frac{r}{0.01}} \text{ Mpl}$



What if we find

$$r \approx 0,01 \tau$$

Can we get longer  $r$ ?

One idea:

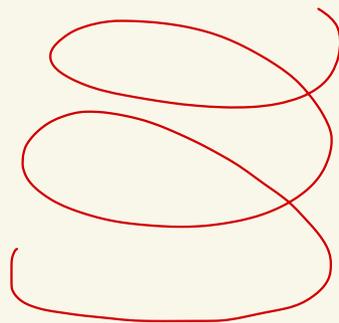
monodromy inflation

in string theory

[Silverstein, Westphal]

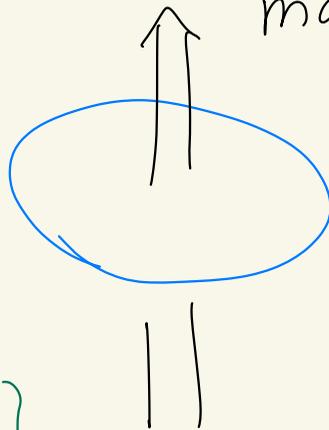
in gauge theory

[Kaloper, Lawrence, Sorbo]  
[Nomura, Watari, MY]



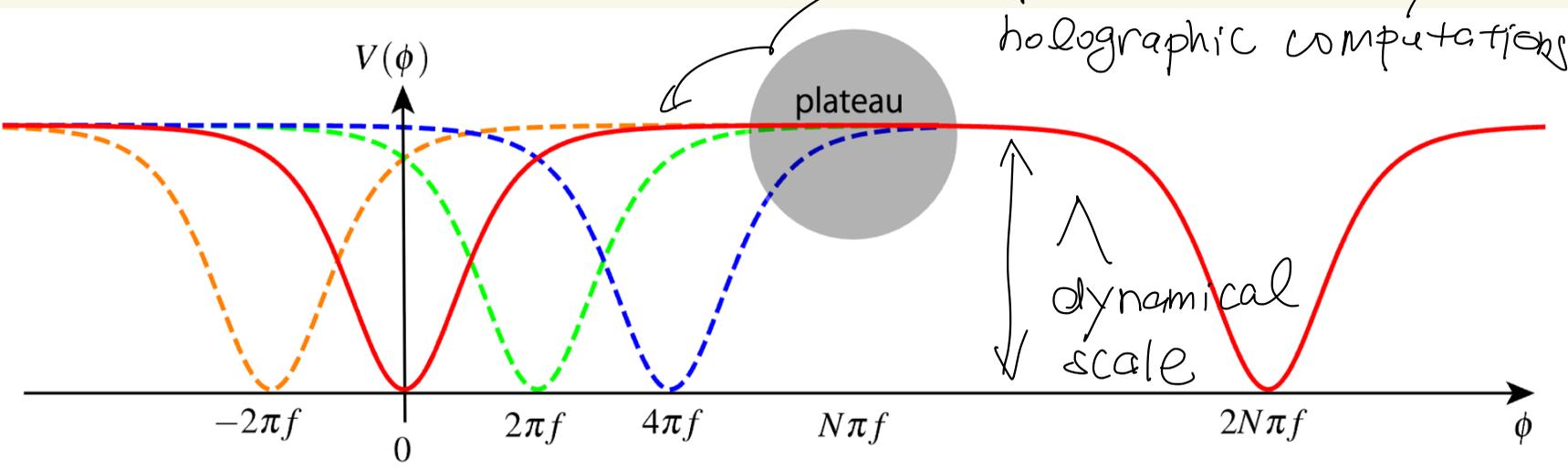
multi-valued  
function

magnetic flux



$$\mathcal{L} = \frac{1}{32\pi^2} \frac{\phi}{f} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu} \quad \text{SU(N) pure Yang-Mills}$$

axion
shape: determined by holographic computations



multi-valued potential in large  $N$

[Witten, ..., Yonekura-Y, Nomura-Y, ...]

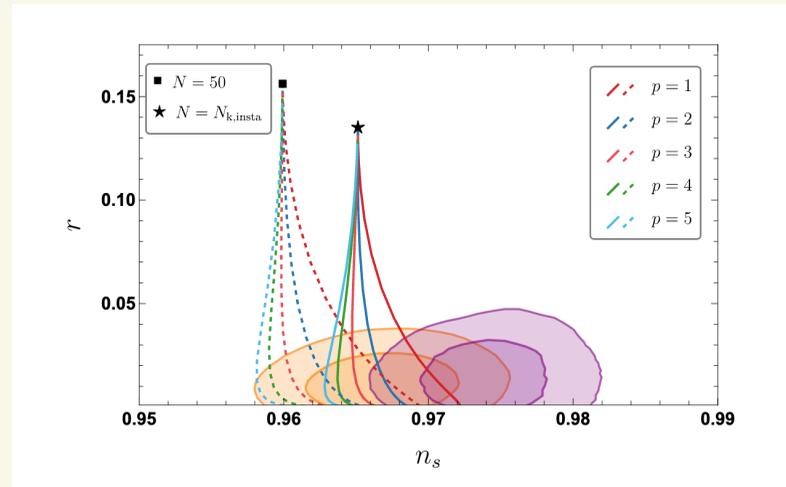
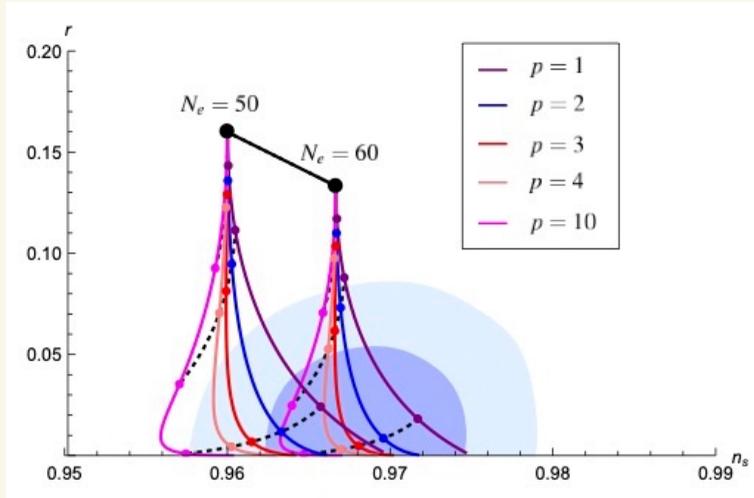
Pure Natural Inflation  
well and alive

[Nomura-Watarai-MY '17]

$$V(\phi) = V_0 \left[ 1 - \left( 1 + \left( \frac{\phi}{F} \right)^2 \right)^{\frac{1}{2}} \right]$$

↑ motivated by large  $N$  QCD

[Zenteno Gatica - Papageorgiou  
- Fastello '16]

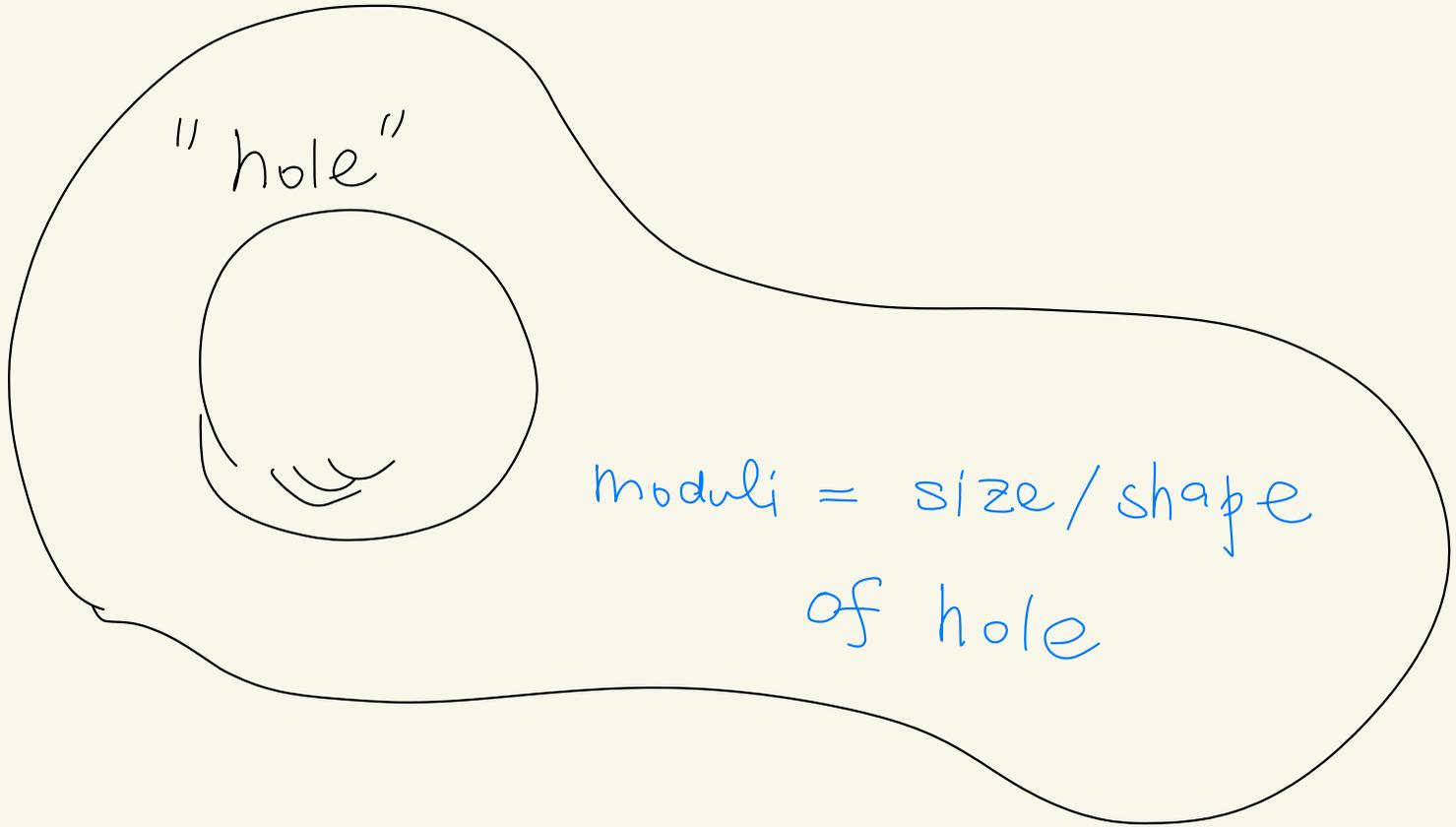


# String moduli

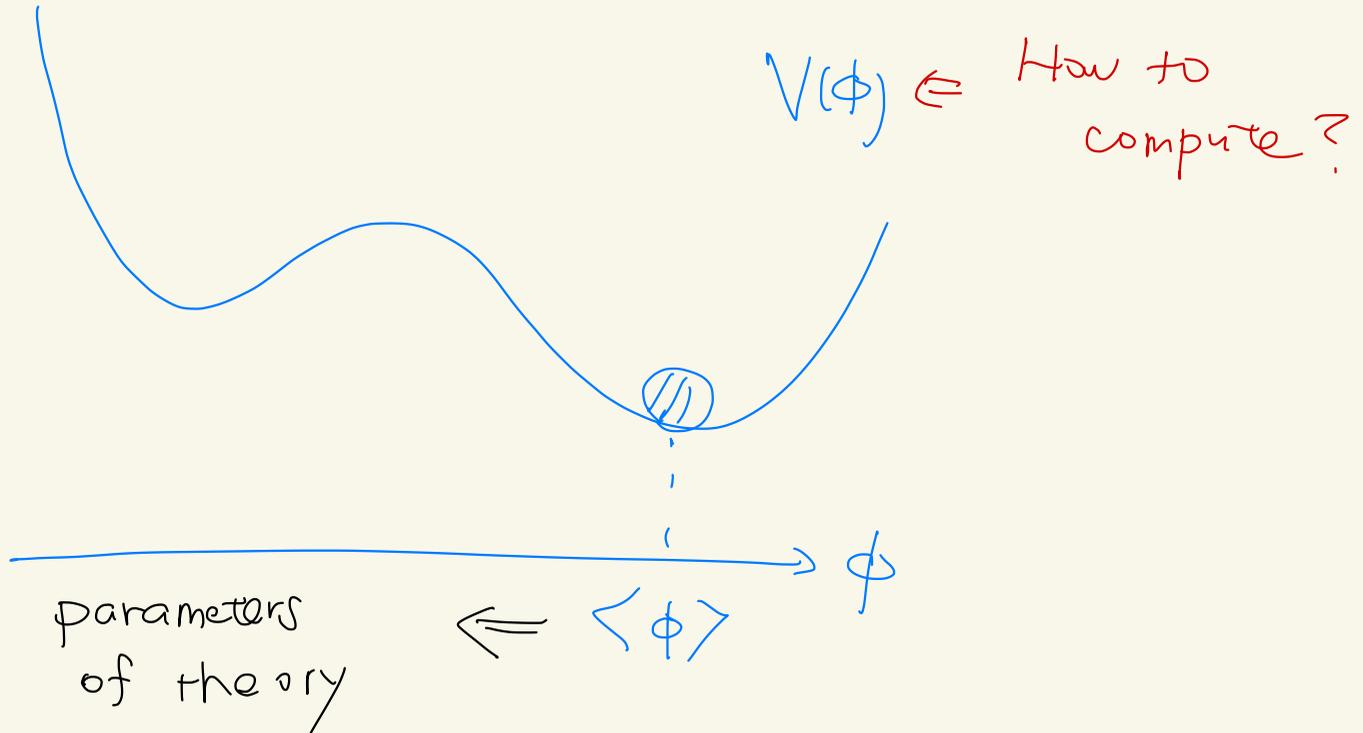
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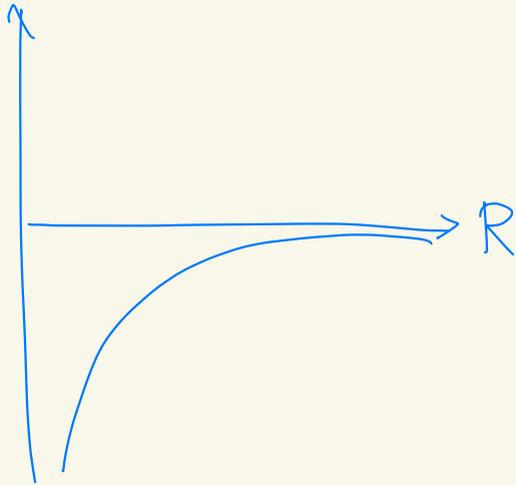
# Calabi-Yau



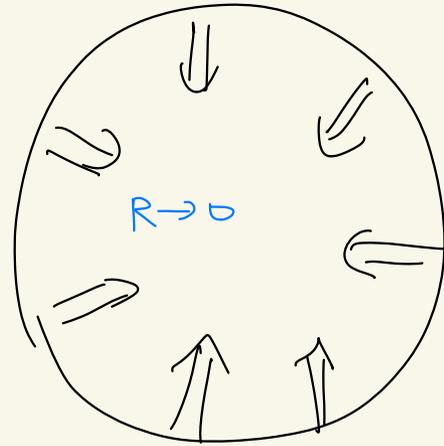
string theory : only one dimensionful parameter  
 $l_s$ : string scale



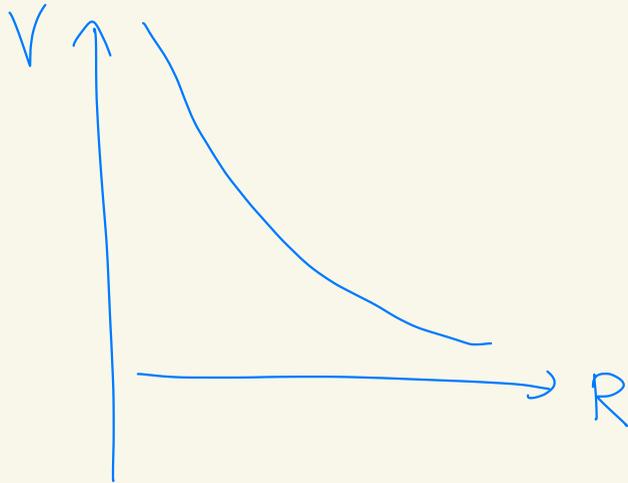
$$V(\phi) \sim -\frac{c}{R^2}$$



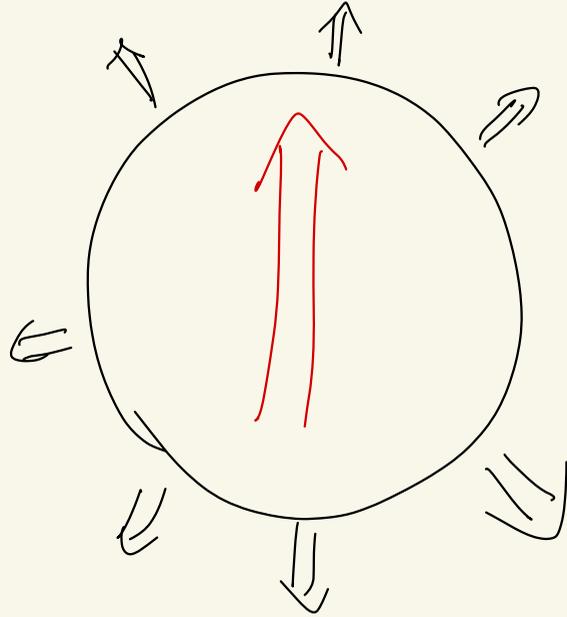
gravity



$$V \sim \frac{c(\gamma_0)}{R^\#}$$

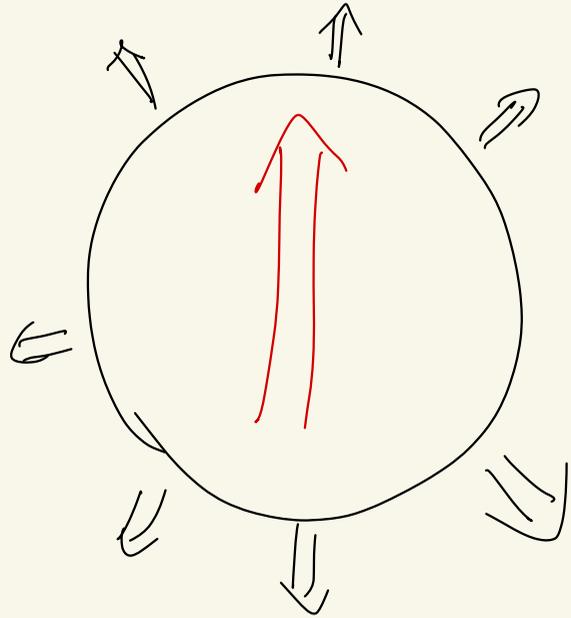


electric/magnetic



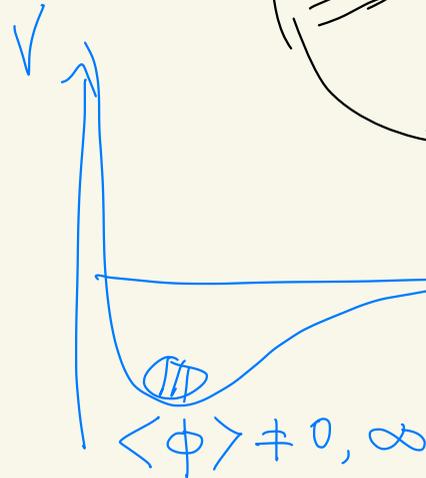
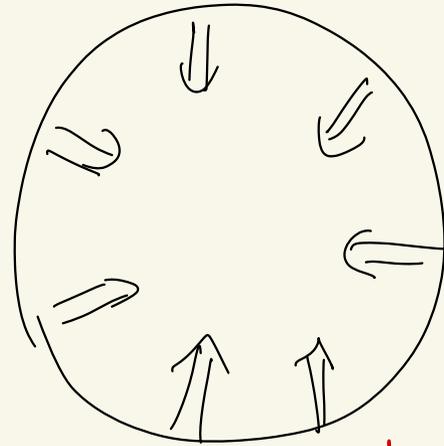
flux  $ne \mathbb{Z}$

electric/magnetic



flux  $h \in \mathbb{Z}$

gravity



decompactify

$V=0$   
(SUSY)



In general **model**: stabilization is subtle

\* difficult to control corrections

$$V \sim \underbrace{\oplus \phi^{-n} - \oplus \phi^{-n'} + \oplus \phi^{n''} + \dots}_{\text{all similar order}}$$

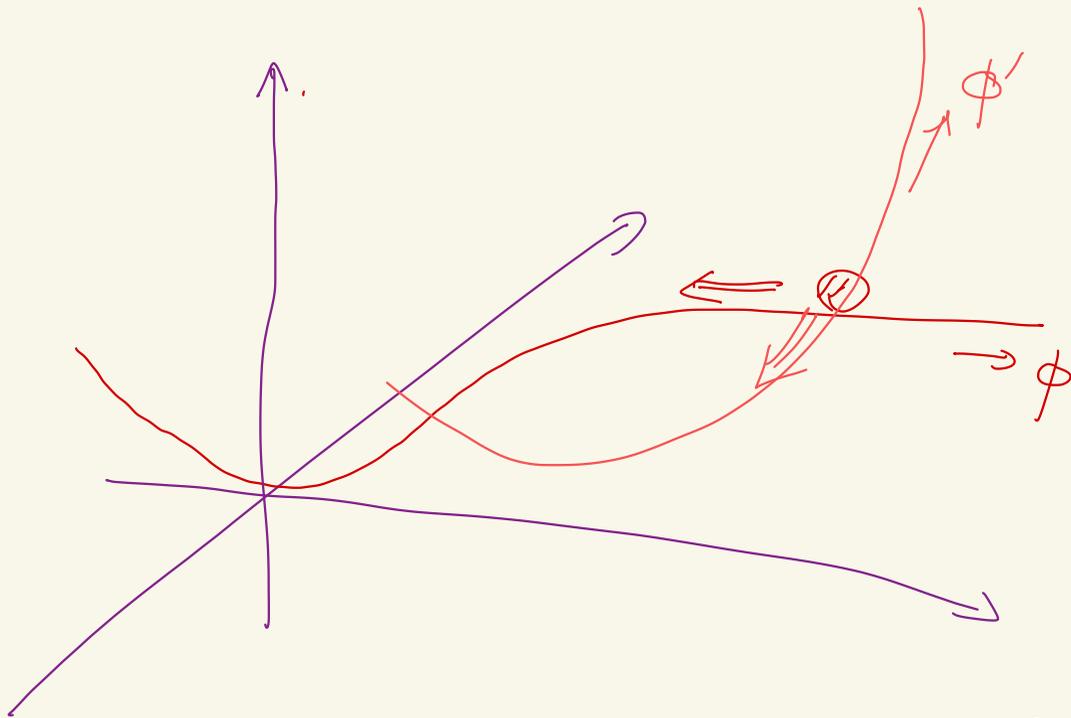
all similar order  
near minimum?

[cf. Dine-Selberg]

\* model dependent

e.g. which geometry

\* need to be careful in multi-field dynamics



not easy to obtain hierarchy of scales

$$\Lambda > 0 ?$$

(Dark Energy)

## Flux vacua

- # cycles  $\sim \mathcal{O}(100)$
  - # 10 fluxes in each cycle
- $10^{100}$  vacua?

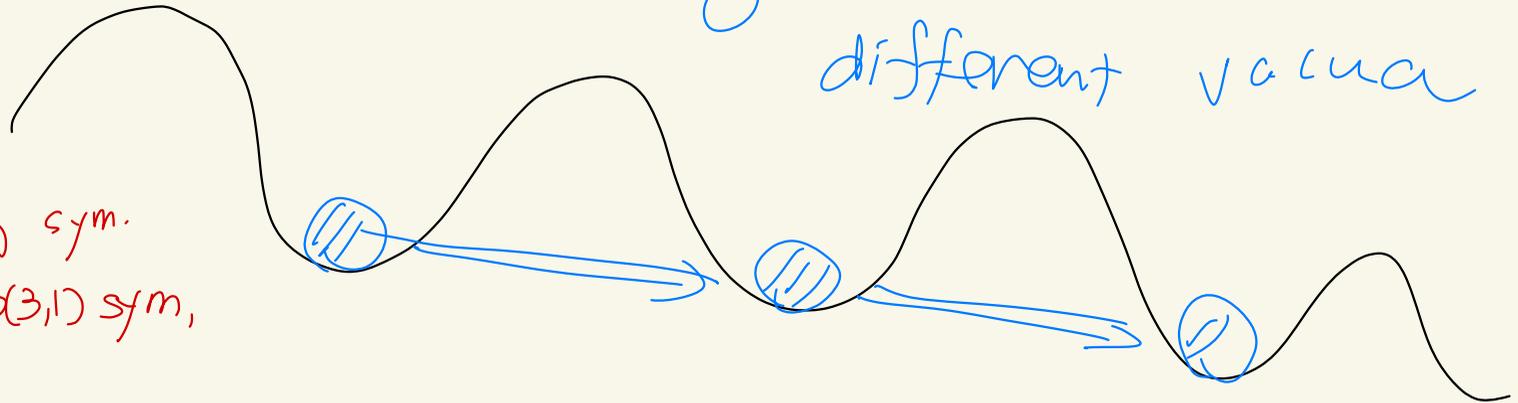
cf.  $10^{27200}$  vacua in F-theory?

[Taylor-Wong]

Multiverse? Anthropic Principle?

tunneling between  
different vacua

$SO(4)$  sym.  
 $\rightarrow SO(3,1)$  sym.



$\dot{\chi}$  positive curvature  $\Omega_K \lesssim -10^{-4}$  disfavors false-vac. eternal  
inflation

[Guth-Nomura 1203.6876; Kleban-Schillo 1202.5037]



Huge vacua  $\rightsquigarrow$  vacua with  
small  $\Lambda > 0$

[ anthropic solution of  
cosmological const. problem ]

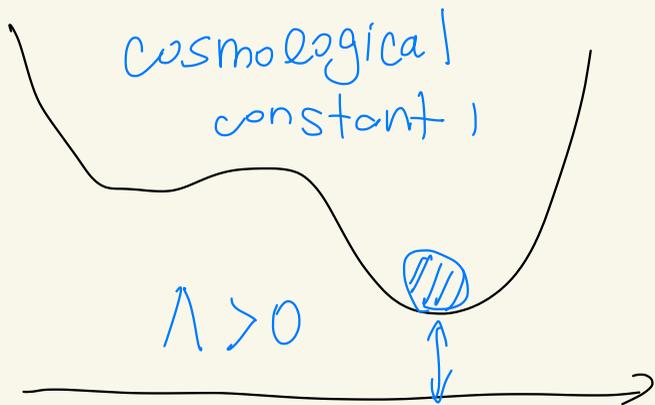


Loss predictability? NP-hard?

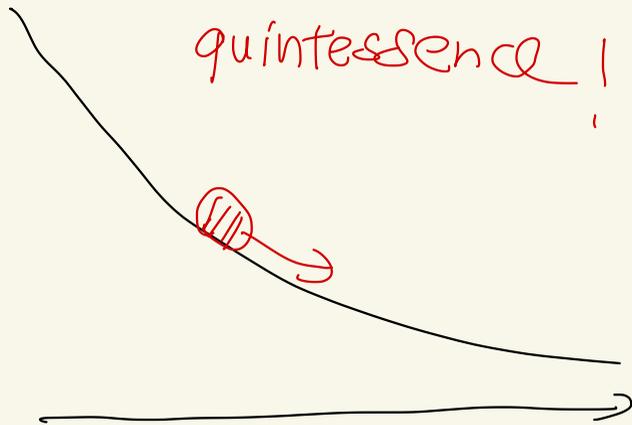
[ machine-learning? ]  
[ Hamada - Uematsu - MY ]

# Two viewpoints

Landscape!  
 $10^{\mathcal{O}(100)}$  vacua!  
multiverse!  
anthropic!



dS vacua very hard!  
Swampland constraints!  
Dynamical Dark Energy!



Inspiration from

---

QFT / Swampland / String?

---

In QG we have sum over geometries:

$$\int \mathcal{D}g_{\mu\nu} e^{-S[g_{\mu\nu}]} \sim \sum_i e^{-S[g_{\mu\nu}^i]}$$

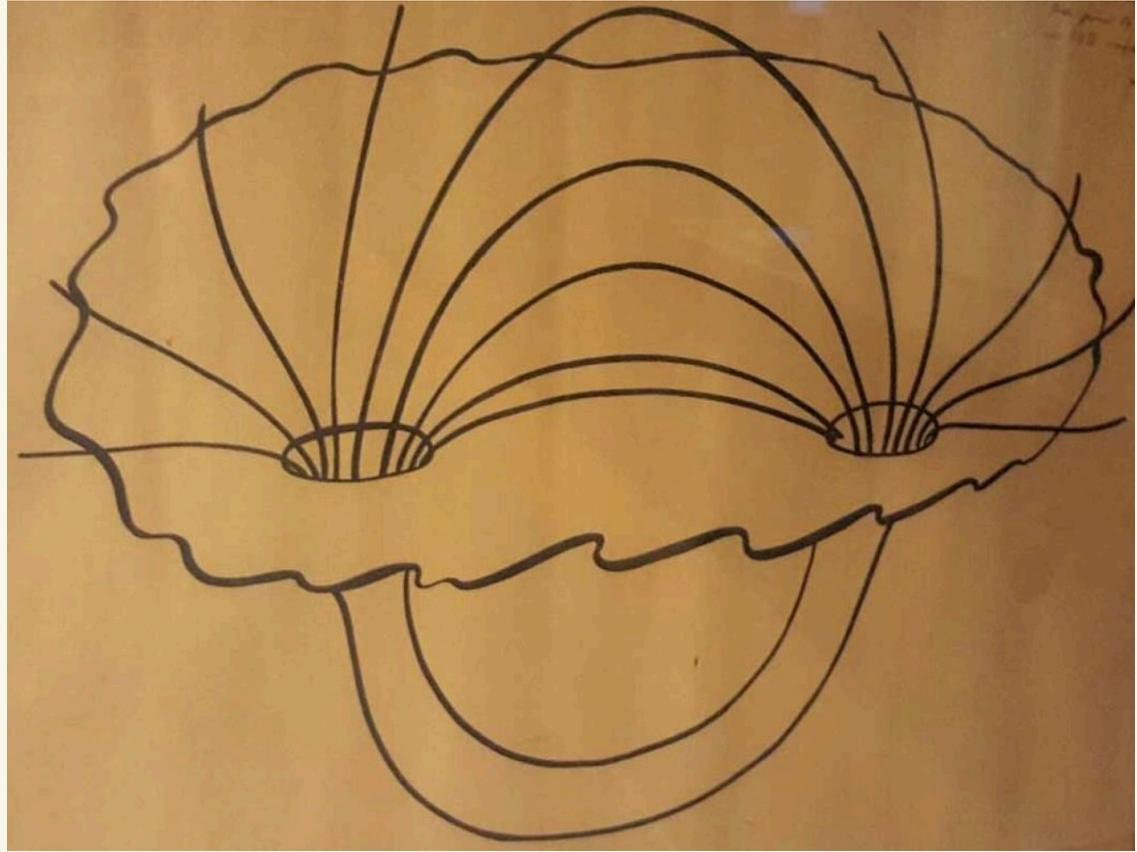
saddle  
i



different spacetime

crazy geometry?

Wormholes! [Einstein-Rosen '35, Wheeler '55]



## Perturbative Gravity

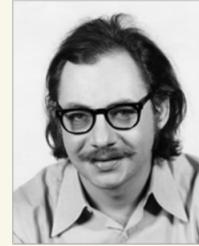
$$\mathcal{L} \supset \underbrace{\lambda}_{\mathcal{O}(1)} \frac{\phi^{4+n}}{M_{pl}^n}$$

## Non-Perturbative Gravity

$$\mathcal{L} \supset \underbrace{e^{-S}}_{\text{can be } \ll 1} \frac{\phi^{4+n}}{M_{pl}^4}$$

# Wormholes -> Global Symmetry Breaking

“The charge escapes to  
Baby Universe”



[Coleman ('88)]

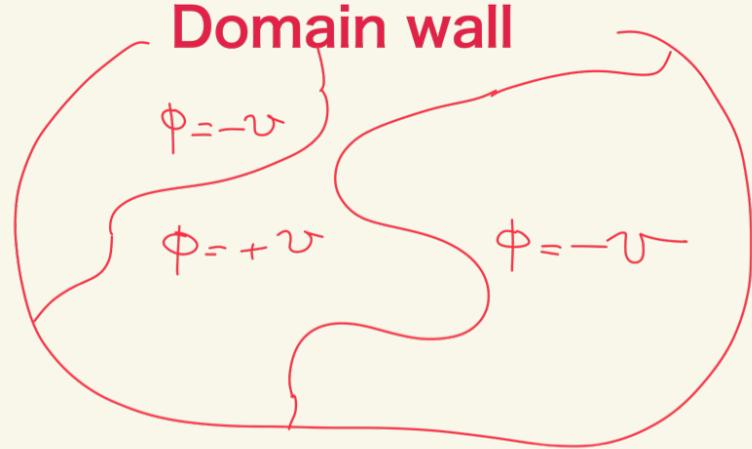
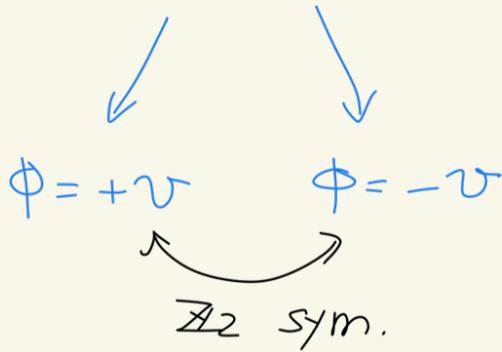
  
(Swampland conjecture:  
no global symmetry in QG)



[Nambu]

# Symmetry may be spontaneously broken

$$V = \lambda (\phi^2 - v^2)^2$$



## Domain wall fills the Universe

[Zeldovich-Kobzarev-Okun ('74)]



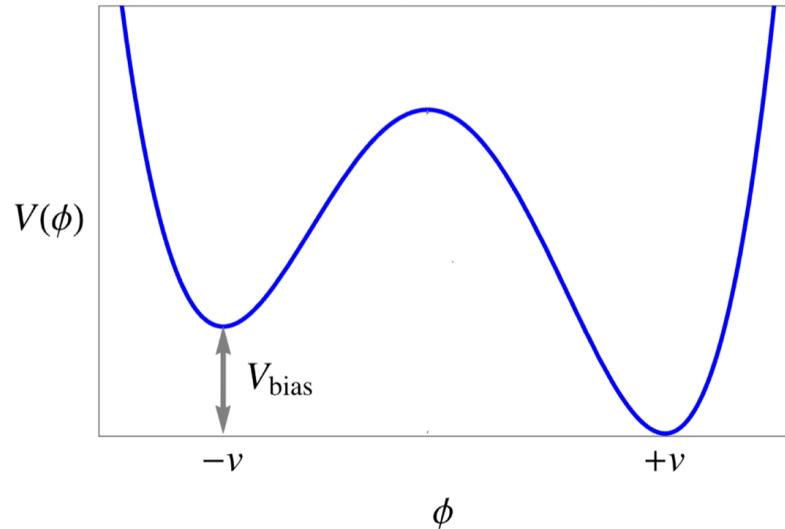
# Symmetry may be slightly broken: Wormhole effect?

[Gelmini-Gleiser-Kolb ('89)]

$$V = \lambda (\phi^2 - v^2)^2$$

+

$$V_{\text{bias}} = \frac{\phi^5}{\Lambda}$$



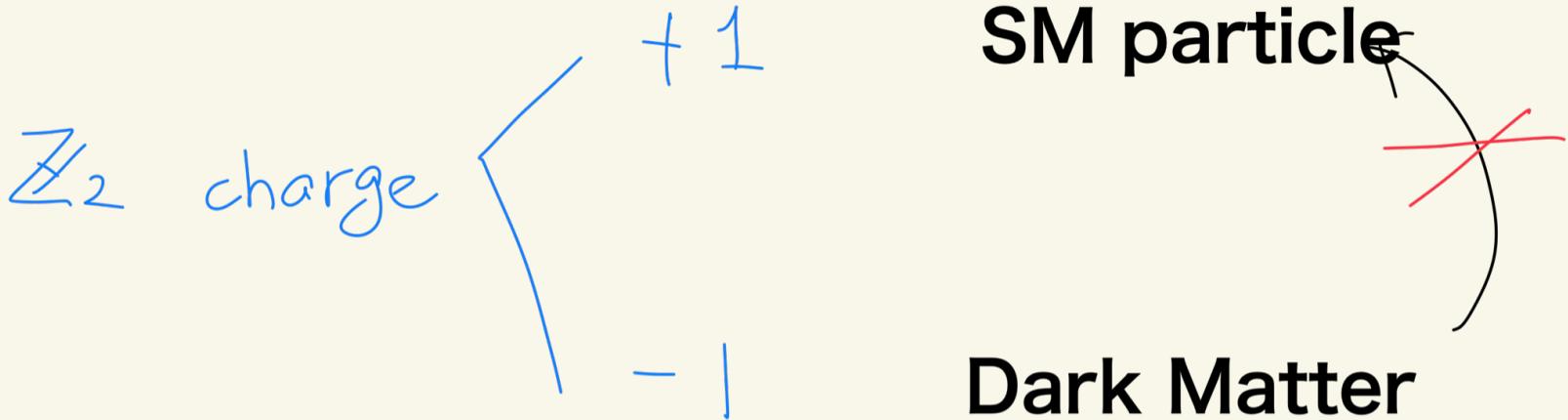
$$V = \frac{\phi^5}{\Lambda} \quad \Lambda \gg M_{pl} \sim 10^{18} \text{ GeV}$$

$$= \frac{e^{-S}}{M_{pl}} \phi^5 \quad S \gg 1$$

**Wormhole natural explains  
large scale**

**GW originating from QG?**

# Global Symmetry explains Stability of DM



# Domain wall

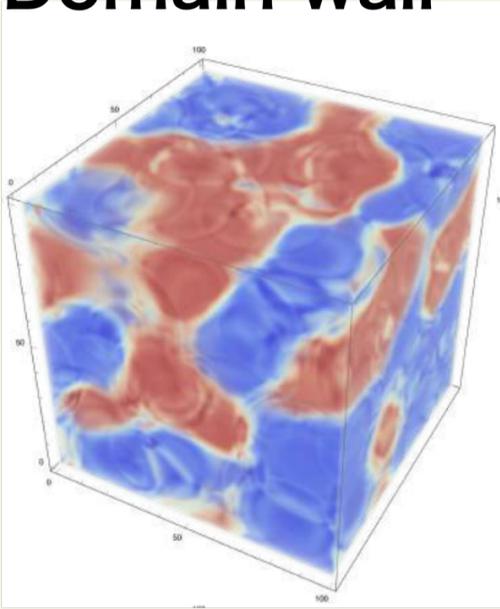
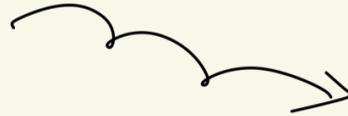
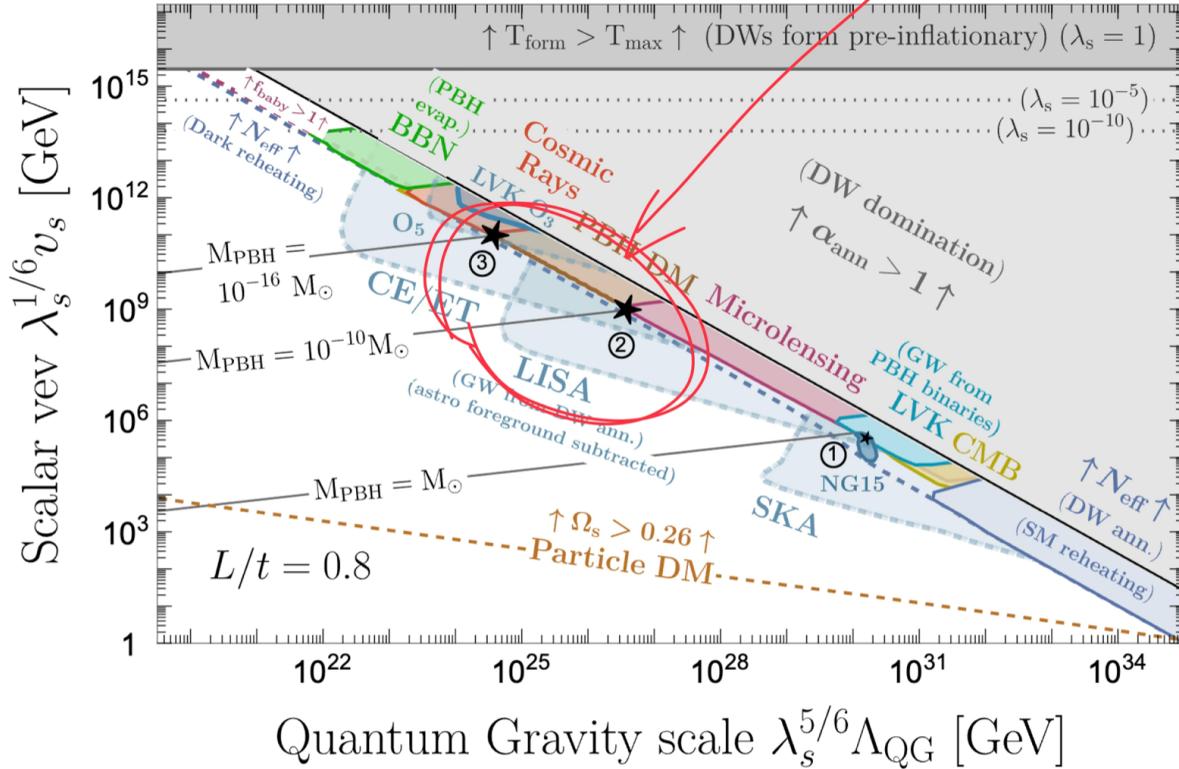


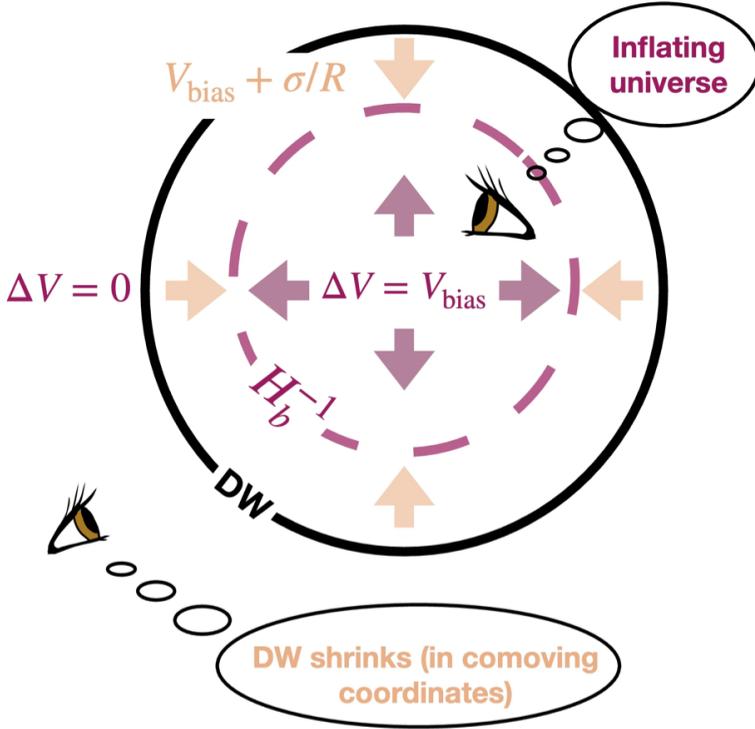
figure credit: arXiv: 2208.07186

# Primordial Black Hole



# Dark matter candidate

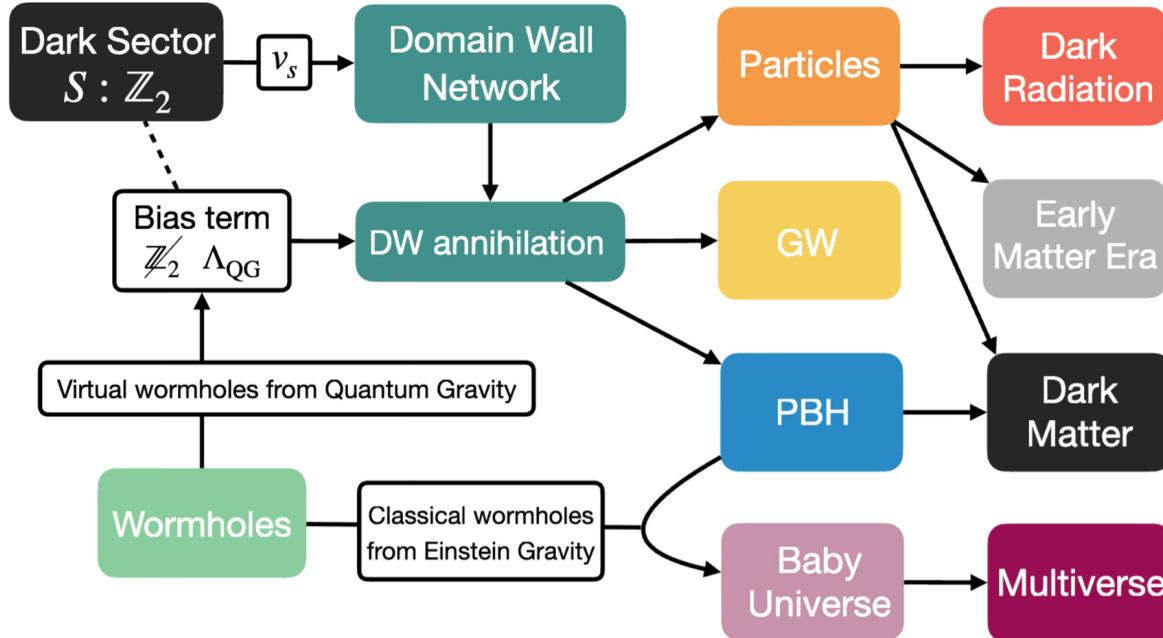




# Baby universe from PBH, leading to Multiverse

[Blau-Guendelma-Guth ('87),  
Gouttenoire-Vatagliano ('23)]

# Universe starts and ends with Wormholes?



[Gouttenoire et al. (incl. MY) ('23)]

# Summary

- **Cosmology** is a perfect place for exploring **QG**  
in both **theory & experiment**

- **QG** is a prerequisite for  
serious attempt for **Cosmology**

↑  
many questions are **intrinsically string theory**

"Listen to what string actually says"

Cosmology : Cosmos (order) in the Universe



hidden harmony of strings

2050?

2100?

String cosmology as Precision Science