QFT II/QFT "Category E" homeworks (ver. 1)

prepared by TW, Weiguang Cao (TA in 20/21), and Nozomu Kobayashi (TA in 16/17)

- Reports on these homework problems are supposed to be submitted through the U Tokyo LMS. We request that the file name includes the problem number, such as II-1***.pdf or ****-IV-2-IX-1.jpeg.
- Homework problems in category [E] are reading materials. Find one(s) that you are interested in, and read; any kinds of record that you have done so can be submitted as a report to get a credit in this course.
  - "Any kinds of records" include summary written in your own language, and a photo-copy of back-of-the-envelope calculations during the process of reading through these articles.
  - Excerpts from these articles (or their translation) are NOT recognized as the record, however. It is important to try to crystalize your own understanding in your own language, in your own brain. The process of understanding should be something more than swallowing statements in a review article. In particular, please remember to make a clear distinction between statements that you have endorsed (=after full verification efforts and critical thinking processes) and those that you accept temporarily as apparently reliable/widely accepted working knowledge. I believe that students at Tokyo University do not need such a reminder, but just in case.
- It is not necessary to read one article from the beginning all through to the end. If you manage to have understood/digested some part of an article / some subject, I must say that is already a great achievement! I just encourage you to eat as much as you like while you have appetite!!
- This PDF file provides links to the articles referred to here.
- References listed below are only meant to be suggestions, to save your time. If you find a better reading material on your own, you do not have to stick to the references listed below.

[**E-1**] **Recursion relations on scattering amplitudes (using spinor helicity formalism)**: It is certainly the standard and orthodox method in computing scattering amplitedes to use Feynman rules. There is an alternative, however, which uses recursion relations among scattering amplitudes; using amplitudes of scattering m particles with $m < n$ and the recursion relations, one can work out amplitudes of scattering n particles. This alternative method has turned out to be much more powerful and fast, when n is large, than the standard method using Feynman diagram.

A review article [arXiv:1111.5759] "*An Introduction to On-shell Recursion Relations,*" by Bo Feng and Mingxing Luo allows you to catch a glimpse of this "alternative method."
The homework problem D-1 will serve as a side reader to this review article.

[**E-2**] **In-In/Schwinger–Keldysh formalism**: We has had little time in the class to explore formulations known under the name of "In-in formalism," "Shwinger–Keldysh formalism," or any others of that sort. This formalism is suited for solving time evolution of operator correlation functions when an initial state (or density matrix) is prepared, and time-dependent field background is applied to the system. By reading textbooks, review articles or any other suitable materials, you will be able to see what can be done with this formalism. Since I am not an expert on this subject, it is not possible to recommend with confidence which reference to look at. While I refer to the two following references (in addition to a few textbooks referred to in the 2020 course web page or LMS?), I do not necessarily mean that they are my top recommendation.

- [Rev. Mod. Phys. **58**, (1986) 323] "*Quantum field-theoretical methods in transport theory of metals,*" by J. Rammer and H. Smith,
- [hep-th/0506236] "*Quantum contributions to cosmological correlations,*" by S. Weinberg.

[**E-3**] **Out-of-time-ordered correlation functions**: We will discuss mainly time-ordered product correlation functions in the class, and we will (hope to) cover also the in-in formalism correlation functions later in the course (some time in December–January). But they are not all the classes of observables that can be computed in QFT and can also be measured experimentally. Imagine

$$F(t) := \langle \text{state} | \left([A(t), B(0)] \right)^\dagger [A(t), B(0)] | \text{state} \rangle, \tag{1}$$

or more generally,

$$F(t) = \mathrm{Tr}\left[([A(t), B(0)])^\dagger \, [A(t), B(0)] \, \rho \right], \tag{2}$$

where ρ is a density matrix. This class of quantity is interesting, for example, when the operators A and B are associated with different points in space (= differnt lattice sites). At $t = 0$, operators placed at different points commute, and hence $F(t = 0)$ vanishes. As a given quantum system evolves in time, however, $A(t) = e^{iHt}A(0)e^{-iHt}$ ceases to commute with $B(0)$. The quantity $F(t)$ captures how fast this non-commutativity develops between two separate points in space. $F(t)$ written down above is an example of observables called out-of-time-ordered (or out-of-time-order) correlation functions.

- As a homework to the QFT II course, we suggest you to explore literatures proposing (or carrying out) experiments that measure such out-of-time-ordered correlation functions. To find articles, you can Google with such a combination of key words as "out of time order" measure.

[**E-4**] **Chern–Simons theory / Quantum Hall effects** [E or D]: Chern–Simons theory on 2+1 dimensional space-time has various interesting properties that relativisitic/non-relativisitic QFTs in 3+1-dimensions do not have. It is important also because it is motivated as a low-energy effective theory of some condensed matter systems. (This [E-4] homework problem is regarded as [D-2] (in category [D]) when getting hands only on the small problem "1 and 3" or "2 and 3" below. This is an [E] category problem when getting hands both on the small problem 1 and 2.)

1. Fractional[1] Quantum Hall Effect for $\nu = 1/(\text{odd int})$: read such materials as
   - N. Nagaosa textbook, "Quantum Field Theory in Condensed Matter Physics" '95 Springer (TMP series) [Japanese original version "*busseiron-ni-okeru ba-no-ryoshiron*" Iwanami Publ. Co.], (§6.1 and) §6.2,
   - T. Nakajima and H. Aoki textbook "*bunsu-ryoshi-Hall-kouka*" '98 U. Tokyo Press (*tatai-denshiron* series III), (§2, §3 and) §6.1 (and more from §6).
   - David Tong, a lecture note [arXiv 1606.06687] "*Lectures on the Quantum Hall Effect.*" .

[1] David Tong's lecture note in the main text also contains exposition on the integer quantum Hall effect. A minimum course may be to focus on its §1.2.2, §1.3, §1.4.2, §2.1 and §2.2.

- X.G. Wen textbook "*Quantum Field Theory of Many-Body Systems*" §7, '04 Oxford U. Press.
- any other references you find useful,[2]

and write a report on QFT aspects of the fractional quantum Hall effects in the cases of the filling fraction 1/odd (including which aspects of experimental data are explained).

2. Write a summary on the scalar–vortex duality in 2+1-dimensions ; resources: pp.184–185 (almost the last 2 pages of §6.2) of [Nagaosa] (non-rela system, with derivation), [Tong, §5.3] (relativisitic system, argument), and [Nakajima–Aoki, §6.4.3] (intuition and idea). If possible, also discuss how the duality is used to explain the fractional quantum Hall effects for a ratinoal value of the filling fraction ν different from 1/(odd); resources: [Nakajima–Aoki, §6.4.3] and [Tong, §5.2.4].
3. Knot invariants / link invariants: Explain how abelian Chern–Simons theory on 2+1-dimensions can capture the mutual linking number of two non-mutually-intersecting worldlines.[3]

[**E-5**] **Entanglement entropy**: Correlation functions / matrix elements of <u>local</u> operators are not all the observables we can define theoretically, or measure experimentally. Entanglement entropy and its extensions are such observables, though we did not have enough time to discuss those observables in the QFT II course. The following articles explain how those observables might be useful.

- [cond-mat/0510613] "*Detecting topological order in a ground state wave function*," by M. Levin and X.G. Wen,
- [arXiv:0704.3906] "*Area laws in quantum systems: Mutual Information and Correlators*," by M. Wolf et.al.,

[2] Homework problem VI-1 also deals with canonical quantization of the Chern–Simons action.
On excitation spectra of the relativisitic Higgs–abelian Chern-Simons system on 2+1-dimensional space-time in the Bose–Einstein condensation phase, the following contains a concise summary: G. Dunne [A Les Houches lecture note] "*Aspects of Chern–Simons Theory,*" last paragraph of §2.2.

[3] Ref. [Comm.Math.Phys.**121**(1989) 351-399] "*Quantum field theory and the Jones polynomial,*" by E. Witten says that the "self linking number" is not calculable in a well-defined and theoretically nice way by using the abelian Chern–Simons theory. This article further argues that $G = \mathrm{SU}(2)$ Chern–Simons theory can be used to make sense of the self linking number, and also capture the Jones polynomial of a knot. With a more general choice of G, other knot invariants can also be obtained.

- you might also be interested in the following articles, though the first one involves the notion of renormalization (which the QFT II course did not cover), and the latter two require knowledge on quantum hall effects and Chern-Simons theory.
  - [hep-th/9303048] "*Entropy and area*," by M. Srednicki,
  - [hep-th/0510092] "*Topological entanglement entropy*," by A. Kitaev and J. Preskill,
  - [arXiv:0805.0332] "*Entanglement spectrum as a generalization of entanglement entropy: identification of topological order in non-Abelian fractional quantum hall effect states*," by H. Li and F. Haldane.

[**E-6**] **Any other subjects/topics** that you are interested in, so far as that is remotely relevant to quantum field theory.