Supplementary Notes.

Coherence lost by interactions

In Quantum Mechanics, we only have a Hilbert space $\mathcal{H}$ and a unitary time-evolution operator $U(T) = e^{-iHT}$ acting on $\mathcal{H}$. There is no prefered choice of a basis in $\mathcal{H}$ in general. (The principle of superposition)

There are two situations, however, where there is a prefered decomposition of states.

(☆1) where there is a structure $\mathcal{H} \cong \oplus_{i \in I} \mathcal{H}_i$ and $U \sim \mathrm{diag}(U_i)_{i \in I}$.

(i.e. there is negligible transition rate from $\mathcal{H}_i$ to $\mathcal{H}_{j \neq i}$)

[examples]

- $\exists$ a system with a conserved change: $\oplus_{i \in I} \mathcal{H}_i$ $i \in I \Longleftrightarrow$ charge
- a system that is made of $N \gg 1$ copies of a local system $(\mathcal{H}^0, U^0(t))$

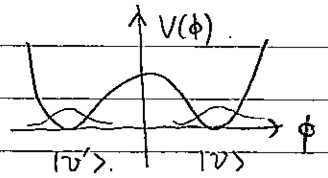
$\mathcal{H} \sim (\mathcal{H}^0)^{\otimes N}$. Even when $\langle v' | U(t) v \rangle \neq 0$ $(v, v' \in \mathcal{H}^0)$,

$\langle v'^{\otimes N} | U(t) v^{\otimes N} \rangle \sim (\langle v' | U^0(t) v \rangle)^N$ will be extremely small.

($N \sim$ number of atoms in a crystal
numbe of points in a large space.)

Then $\mathcal{H} \ni |s\rangle$ has a decomposition $\sum_{i \in I} |s_i\rangle \in \oplus_{i \in I} \mathcal{H}_i$.

It is enough (and natural) to follow the time-evolution of each $|s_i\rangle$.



$\langle v' | U^0(t) v \rangle \neq 0$.

but $\prod_{\vec{x} \in \mathbb{R}^d} \langle v'_{\vec{x}} | U^0(t) v_{\vec{x}} \rangle \simeq 0$

(☆2) where the system of interest $(\mathcal{H}, U(t))$ is a part of a larger system $(\mathcal{H}^{tot}, U^{tot}(t))$ and there is a structure.

$$\mathcal{H} \cong \oplus_{i\in I} \mathcal{H}_i$$

$$\mathcal{H}^{tot} \simeq \mathcal{H} \otimes (\otimes_{i\in I} V_i)$$

$$H^{tot} = \Big(\sum_{i\in I} H_i^\circ \otimes (\mathbb{1}^{\otimes I})\Big) + \sum_{i\in I} \mathbb{1} \otimes (\cdots \mathbb{1} \cdots \otimes H(V_i) \otimes \cdots \mathbb{1})$$

$$+ \sum_{i\in I} \phi_i \otimes (\mathbb{1} \cdots \otimes \Phi_i \otimes \mathbb{1} \cdots) \quad \Leftarrow \text{interaction terms.}$$

$$U^{tot}(t) = e^{-iH^{tot}t} \qquad U(t) = e^{-i(\Sigma_i H_i^\circ)t}$$

H_i°, ϕ_i act on $\mathcal{H}_i \subset \mathcal{H}$.

Φ_i acts on V_i

[examples]

- $(\otimes_{i\in I} V_i)$ as the measurement system (detectors)
- $(\otimes_{i\in I} V_i)$ as the environment / reservoir system whose micro-state details are not probed experimentally
- $(\otimes_{i\in I} V_i)$ as the states of other particles in the Universe

Even if we start from a state $|\Psi\rangle = |s\rangle \otimes \otimes_{i\in I} |\psi_i\rangle \in \mathcal{H} \otimes (\otimes_{i\in I} V_i)$

$|s\rangle = \sum_{i\in I} |s_i\rangle \in \oplus_{i\in I} \mathcal{H}_i = \mathcal{H}$,

we will have

$$U^{tot}(t)|\Psi\rangle \simeq \sum_{i\in I} \Big(\underset{\substack{\Uparrow \\ \mathcal{H}_i}}{e^{-i(H_i^\circ+\phi_i)t}|s_i\rangle}\Big) \otimes \Big[\cdots \underset{\substack{\Uparrow \\ V_{j\neq i}}}{e^{-iH(V_j)t}|\psi_j\rangle} \cdots \otimes \Big(\underset{\substack{\Uparrow \\ V_i}}{e^{-i(H(V_i)+\Phi_i)t}|\psi_i\rangle}\Big)\Big].$$

If each of the systems $\{V_i\}_{i\in I}$ is complex enough so that

$$\langle e^{-iH(V_i)t}\psi_i | e^{-i(H(V_i)+\Phi_i)t}\psi_i\rangle \simeq 0 \quad \text{for large enough } t.$$

then

$$\mathrm{Tr}_{\otimes_{i\in I}V_I}\left[U^{\mathrm{tot}}(t)\,|\Psi\rangle\langle\Psi|\,U^{\mathrm{tot}}(t)^{-1}\right]\sim\sum_i e^{-i(H_i^0+\phi_i)t}\,|s_i\rangle\langle s_i|\,e^{i(H_i^0+\phi_i^\dagger)t}.$$

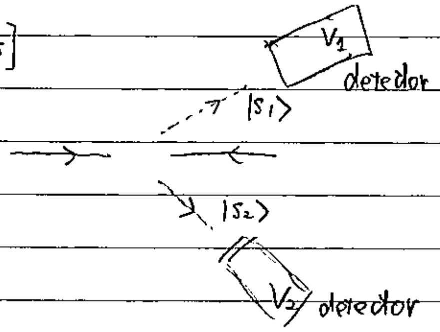
The off-diagonal terms are negligible.

The coherence among $|s_i\rangle$ $i\in I$ is lost over time due to the interactions with the systems $\otimes_{i\in I}V_i$.

How the coherence is lost (how $|s\rangle$ is decomposed (into $\sum_i|s_i\rangle$)) is governed by the structure of both

$$\sum_{i\in I}\underbrace{\phi_i}_{\text{on }\mathcal{H}_i\subset\mathcal{H}}\otimes(\cdots\underbrace{1}_{\text{on }V_j}\cdots\otimes\underbrace{\Phi_i}_{\text{on }V_i})\quad\text{and}\quad\sum_{i\in I}\underbrace{1}_{\text{on }\mathcal{H}}\otimes(\underbrace{\cdots}_{\text{on }V_j}\otimes\underbrace{H(V_i)}_{\text{on }V_i}).$$

[examples]



$|s_1\rangle, |s_2\rangle$: a particl coming out in a scattering experiment, but with different momentum (scattering angle)

ϕ_1, ϕ_2: annihilation (if γ)
$(\psi_k^\dagger\psi_k)$ deflection (if $\mu^\pm$)

Φ_1, Φ_2: ionization, electromagnetic shower...

(a harmonic oscillator system) $\otimes$ (reservoir system)

$|s\rangle$: mix of multiple energy eigenstates.

$H \supset (a+a^\dagger)\,O \sim \int d\phi_*\, P_{\phi_*}\, O\, \phi_*$ $\quad$ P_{ϕ_*}: projection to "the" state $|\phi_*\rangle$ s.t. $(a+a^\dagger)|\phi_*\rangle \sim \phi_*|\phi_*\rangle$

(no structure $(\otimes_{i\in I}V_i, \{\Phi_i\})$ given here; one common V but ϕ_*-dependent $\Phi = \phi_* O$)

$\Rightarrow$ decomposition $|s\rangle = \int d\phi_*\, c_s(\phi_*)\, |\phi_*\rangle$

not into energy eigenstates.

(quantum fluctuation of a scalar field (inflaton) turns into the origin of the galaxy structure of the universe)

(a free scalar field $\Leftrightarrow$ harmonic oscillators labeled by $\vec{k}\in\mathbb{R}^3$) $\otimes$ (other fields)