§8. Two Scale Problems and TMD Factorization.

§8.1. Two scale problems.

☆ $\sigma(e^+e^- \to \text{hadron})_{tot}$ and $\left(\frac{d\sigma}{dx\, dQ^2}\right)_{DIS}$ at moderate x (not $x \ll 1$)

are examples of single scale problems.

- set $s \gg \Lambda_{QCD}^2$ or $Q^2 \gg \Lambda_{QCD}^2$
- use $\mu_R \sim \sqrt{s}$ or $\sim Q$.

$$\begin{cases} \text{perturbative calculation of OPE coefficients possible} \\ \alpha_s \ln(\sqrt{s}/\mu_R) \text{ is not large.} \end{cases}$$

☆ Many other observables in scattering experiments are functions of two different energy scales (or more).

Even when all the relevant energy scales are $\gg \Lambda_{QCD}$,

we may not be able to avoid $+\alpha_s \ln(E_1/E_2)$ appearing

in theoretical calculations (no matter how we choose μ_R).

- DIS at small x (Q^2 and $(p\cdot q)$ ratio is x)
- two jet production cross section (s and $p_{T.cut}$ for jet def.. ratio θ)
- Drell-Yan transverse mom. dependence

$$\left(\frac{d\sigma}{dy\, dQ^2 d^2\vec{q}_T}\right)(h_1 + h_2 \to V^{(*)} + \text{anything})$$

vector boson $V^{(*)} = \gamma^*, Z^{(*)}, W^{(*)}$ (decay subsequently to leptons)

$q^\mu \Rightarrow \cdot\ Q^2 := q^2$

$= (q^0, q^3, \vec{q}_T)$ $\cdot\ y := \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$

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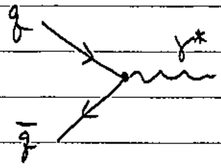
- When are such observables IR safe?
- In which sense do we say that they are safe?
- Even when they are safe, how do we resum perturbative series?

$\hookrightarrow$ Intimately related questions.

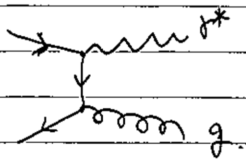
Example: Drell-Yan q_T dependence.

parton model. tree:

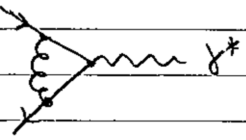


$\Rightarrow \propto \delta^2(\vec{q}_T)$

at NLO.



$\Rightarrow \propto \frac{\alpha_s}{\pi} C_2 \frac{1}{|\vec{q}_T|^2}$



$\Rightarrow \propto \delta^2(\vec{q}_T) \times \left(-\frac{\alpha_s}{\pi} C_2 \times \text{log div.} \right)$

IR divergence expected to cancel after binning in $\vec{q}_T$

- need to resum $\alpha_s^n \ln^{2n}(Q^2/q_T^2)$ if $q_T^2 \ll Q^2$
- partons in a hadron have intrinsic momenta "$f_q(x, \vec{k}_T ; \mu_F^2)$"?

$\longrightarrow$ not that simple actually.

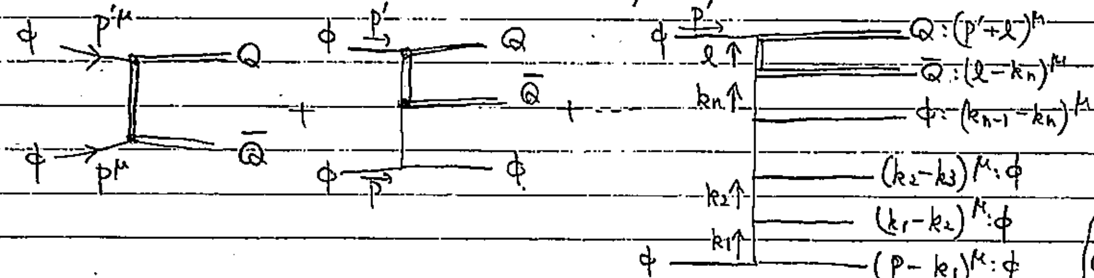
(§8.2)

~~§12-2~~ Double log phase space in 6D ϕ^3 theory

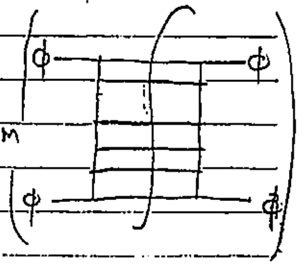
$$\mathcal{L} = \frac{1}{2}(\partial_\mu \phi)^2 - \frac{g}{3!}\phi^3 \qquad \Leftarrow \text{ IR dynamics similar to QCD}$$

$$-\lambda \phi Q \bar{Q} - M^2 |Q|^2 \qquad [\phi] = +2 \qquad [g] = 0$$

At the parton level, think of $\mathcal{M}(\phi^{(*)} + \phi \to Q\bar{Q} + \text{anything})$



ladder diagram. Im



often dominate at higher order.

$$2\,\mathrm{Im}(\underset{\text{forward}}{\mathcal{M}}) = \int \frac{d^6 \ell}{(2\pi)^6} \int \frac{d^6 k_n}{(2\pi)^6} \cdots \int \frac{d^6 k_1}{(2\pi)^6} (2\pi)\delta^+[(p'+\ell)^2 - M^2]\,(2\pi)\delta^+[(k_n - \ell)^2 - M^2]\,(2\pi)\delta^+[(k_{n-1} - k_n)^2]$$

$$\left| \frac{\lambda^2 g^n}{[\ell^2 - M^2][k_n^2][k_{n-1}^2]\cdots[k_2^2][k_1^2]} \right|^2 \quad (2\pi)\delta^+[(p-k_1)^2] \quad (2\pi)\delta^+[(k_1 - k_2)^2]$$

Sudakov parameter

$$\begin{cases} -p'^\mu = 0 \cdot p^\mu + (-1)\,\bar{p}^\mu \\ \ell^\mu = \rho_{n+1} p^\mu + \lambda_{n+1} \bar{p}^\mu + \vec{\ell}_T \\ k_n^\mu = \rho_n p^\mu + \lambda_n \bar{p}^\mu + \vec{p}_{T,n} \\ \quad \vdots \\ k_2^\mu = \rho_2 p^\mu + \lambda_2 \bar{p}^\mu + \vec{p}_{T,2} \\ k_1^\mu = \rho_1 p^\mu + \lambda_1 \bar{p}^\mu + \vec{p}_{T,1} \\ p^\mu = 1 \cdot p^\mu + 0 \cdot \bar{p}^\mu \end{cases}$$

~~$\bar{p}^\mu$ light like.~~

$2p\cdot\bar{p} = s$

- $\delta^+[(k_{i-1} - k_i)^2] = \delta^+[(\rho_{i-1} - \rho_i)(\lambda_{i-1} - \lambda_i)s - |\vec{k}_i - \vec{k}_{i-1}|^2]$

$\Rightarrow (\rho_{i-1} > \rho_i)$ & $(\lambda_{i-1} > \lambda_i)$ & $\delta[f(\cdots)]$

- $\begin{cases} \rho_{n+1} > 0, & \lambda_{n+1} > -1, \\ \rho_n > \rho_{n+1}, & \lambda_n > \lambda_{n+1}, \\ 1 > \rho_1, & 0 > \lambda_1 \end{cases}$

(if $\bar{p}^\mu = p'^\mu$ in DIS,)

$$\Rightarrow \boxed{\begin{array}{l} 1 > \rho_1 > \rho_2 > \cdots > \rho_n > \rho_{n+1} > (x > 0) \\ 0 > \lambda_1 > \lambda_2 > \cdots > \lambda_n > \lambda_{n+1} > -1 \end{array}}$$

assume hierarchical $\Rightarrow \delta[(-\lambda_i)\,\rho_{i-1} s - |\vec{k}_i - \vec{k}_{i-1}|^2]$

$$= \frac{1}{s\rho_{i-1}}\,\delta\left[-\lambda_i - \frac{|\vec{k}_i - \vec{k}_{i-1}|^2}{s\,\rho_{i-1}}\right]$$

So, the ladder diagram contributes by

$$\Delta(2\,\mathrm{Im}\,\mathcal{M}) = \frac{\lambda^4}{4\pi}\alpha_g^n \int\frac{d\rho_{n+1}}{\rho_{n+1}}\int\frac{d\rho_n}{\rho_n}\cdots\int\frac{d\rho_1}{\rho_1}(2\pi)\delta\left(-s\rho_{n+1}-|\vec{l}_T|^2-M^2\right)\int\frac{d^4 l_T}{(2\pi)^4}\sim\int\frac{d^4\vec{k}_{1,T}}{(2\pi)^4}$$

$$\left(1\gg\rho_1\gg\rho_2\gg\cdots\gg\rho_{n+1}\gg\frac{M_a^2}{s}\right)\qquad \frac{1}{\left(|\vec{l}_T|^4\right)\left(|\vec{k}_{T,n}|^4\right)\cdots|\vec{k}_{T,2}|^4\,|\vec{k}_{T,1}|^4}$$

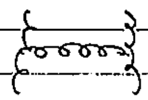
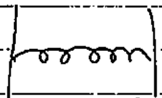
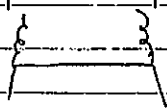
$$\left(k_i^2 = s\rho_i(\lambda_i)-|\vec{k}_{i,T}|^2\ ;\ s\rho_i|\lambda_i| \ll s\rho_i|\lambda_{i+1}|\sim|\vec{k}_{T,i}-\vec{k}_{T,i+1}|^2\right.$$

$$\ll s\rho_{i+1}|\lambda_{i+1}|\sim|\vec{k}_{T,i}-\vec{k}_{T,i+1}|^2$$

$\rightarrow$ ignore $s\rho_i|\lambda_i|$ against $|\vec{k}_{T,i}|^2$.

$$\Rightarrow \frac{\lambda^4}{2}\frac{\ln^{n+1}(s/M_a^2)}{(n+1)!}\alpha_g^n\frac{\ln^{n+1}(Q^2/k_{T,\mathrm{cut}}^2)}{(4\pi)^2}$$

<u>double logarithms</u>

QCD
- gluon ladder : $d^2\vec{k}_T\,\frac{1}{|\vec{k}_T^2|}\,\frac{1}{|\vec{k}_T^2|}\times\left((\partial_\kappa)^2\leftarrow\text{vertex}\right)$ 
- fermion + glue ladder : $d^2\vec{k}_T\,\frac{1}{k_T}\,\frac{1}{k_T}$ 
- $d^2k_T\,\frac{1}{k_T^2}\,\frac{1}{k_T^2}\,\cancel{K}$. 

What is <u>$k_{\mathrm{cut},T}$</u>? (necessary for a sensible results)

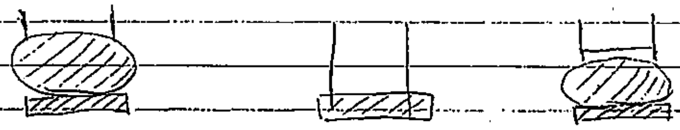
- Make it a rule not to include $|\vec{k}_T| < \mu_I$ DOF in the loop
- Replace the initial single parton state $\phi(p^\mu)$ by a distribution $f(p, \vec{k}_T; \mu_I)$

Bethe-Salpeter equation / Balitsky Fadin Kuraev Lipatov eq

Let $F(\rho, \lambda, \vec{k}_T; \mu_I) \overset{?}{=} \int \langle h(\vec{p}) | \ \phi(-x/2)\,\phi(x/2)\, | h(\vec{p}) \rangle\, e^{+i(p^- x \rho + \vec{p}^- x\lambda - \vec{k}_T \cdot \vec{x}_T)}\, d^6x$

Then we say that

$$\boxed{F(\rho, \lambda, \vec{k}_T; \mu_I) \simeq F(\rho, \lambda, \vec{k}_T; \mu_0) + \frac{g^2}{\vec{k}_T^{\,2}\, \vec{k}_T^{\,2}} \frac{S}{2} \int \frac{d\rho'}{2\pi} \int \frac{d\lambda'}{2\pi} \int_{\mu_I} \frac{d^2 \vec{k}_T'}{(2\pi)^2} (2\pi)\, \delta[S\rho'\lambda - |\vec{k}_T - \vec{k}_T'|^2]\, F(\rho', \lambda', \vec{k}_T'; \mu_I)}$$



$d\lambda$

$$\boxed{\mathcal{F}(\rho, \vec{k}_T; \mu_I) \simeq \mathcal{F}(\rho, \vec{k}_T; \mu_0) + \frac{\alpha_g}{k_T^2\, k_T^2} \int_\rho^1 \frac{d\rho'}{\rho'} \int_{\mu_0}^{\mu} \frac{d^2 \vec{k}_T'}{(2\pi)^2}\, \mathcal{F}(\rho', \vec{k}_T'; \mu_I)}$$

~~Assume that~~ $\mathcal{F}(\rho, \vec{k}_T; \mu_I) \sim \frac{f(\rho; \mu_I)}{|k_T|^x}$ $\quad \mathcal{F}(\rho, \vec{k}_T; \mu_0) = \frac{f(\rho; \mu_0)}{|k_T|^x}$

Then $\int_0^1 d\rho\, \rho^{j-1}$ (Mellin transformation)

$$\tilde{f}(j; \mu_I) \simeq \tilde{f}(j; \mu_0) + \frac{\alpha_g \ln(\mu_I^2/\mu_0^2)}{(4\pi)^2} \frac{1}{j}\, \tilde{f}(j; \mu_I)$$

$$\Rightarrow \boxed{\tilde{f}(j; \mu_I) \simeq \frac{j\, \tilde{f}(j; \mu_0)}{j - \frac{\alpha_g \ln(\mu_I^2/\mu_0^2)}{(4\pi)^2}}}$$

Take the inverse Mellin transformation

$$\boxed{\mathcal{F}(\rho, \vec{k}_T; \mu_I) \sim \rho^{-\left[\frac{\alpha_g \ln(\mu_I^2/\mu_0^2)}{(4\pi)^2}\right]} \Big/ \left(\vec{k}_T^{\,2}\, \vec{k}_T^{\,2}\right) \times \tilde{f}\left(\frac{\alpha_g}{(4\pi)^2} \ln\left(\frac{\mu_I^2}{\mu_0^2}\right), \mu_0\right)}$$

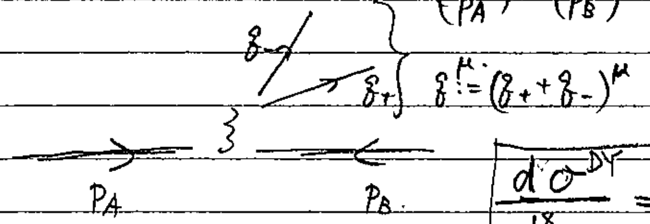
by picking up the pole $j = \frac{\alpha_g}{(4\pi)^2} \ln(\mu_I^2/\mu_0^2)$

§8.3

Drell-Yan Diff. cross sect'n. & TMD-factorizat'n

$$\boxed{\text{Drell-Yan process}: h_A + h_B \longrightarrow V^{(*)} + (\text{anything})}$$

$(p_A^\mu)\quad (p_B^\mu) \qquad \hookrightarrow e^+e^-,\ \mu^+\mu^- \cdots$



$q^\mu = (q_+ + q_-)^\mu \qquad q_+^\mu,\ q_-^\mu$

$$\boxed{\frac{d\sigma^{DY}}{d^4q} = \frac{d\sigma^{DY}}{dQ^2\, dy\, d^2\vec{q}_T} \times 2}$$

$$q^\mu =: (q^0, q^3, \vec{q}_T) \qquad Q^2 := q^2 \qquad y := \frac{1}{2}\ln\left(\frac{q^0+q^3}{q^0-q^3}\right)$$

$$d\sigma^{DY} = \frac{d^4q}{(2\pi)^4}\int \frac{d^3\vec{q}_+}{(2\pi)^3}\frac{1}{2E_{\vec{q}_+}}\frac{d^3\vec{q}_-}{(2\pi)^3}\frac{1}{2E_{\vec{q}_-}}(2\pi)^4\delta^4(q_++q_--q)\ \frac{e^4}{Q^4}\times\underbrace{\left(q_+^\mu q_-^\nu + q_-^\mu q_+^\nu - \eta^{\mu\nu} q_+\cdot q_-\right)}_{\text{Tr}_{spin}\left[\not{q}_+\gamma^\mu\not{q}_-\gamma^\nu\right]}\times$$

$$\frac{1}{2S_{AB}}\times(*)_{\mu\nu}\cdot Q_f^2$$

$\text{Tr}_{spin}\left[\not{q}_+\gamma^\mu\not{q}_-\gamma^\nu\right] \leftarrow$ (spin of the final states summed up)

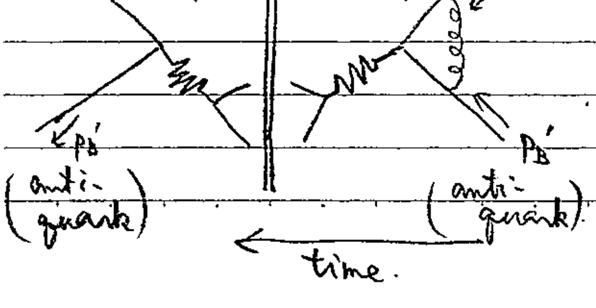
$$(*)^{\mu\nu} = \int d\Pi_X \int d^4x\ \langle h_A(\vec{p}_A)h_B(\vec{p}_B)|J^\mu(0)|X\rangle\langle X|J^\nu(x)|h_A(\vec{p}_A)h_B(\vec{p}_B)\rangle\, e^{iq\cdot x}$$

$$J^\mu(x) := (\bar{\Psi}_f\gamma^\mu\Psi_f)(x)$$

- Two hadrons in the initial state.
- $\vec{q}_T$-dep. diff. cross section: transverse momentum of partons?

  - Does IR divergence cancel?
  - What are the non-perturbative informat'n to describe $d\sigma^{DY}$?

(Intuition behind factorization of soft gluon dynamics)



soft gluon: vertex correction, real emission.

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soft modes

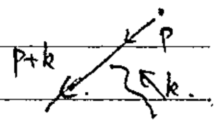
reasoning ① (based on amplitude calculation)

one soft gluon line $\frac{-i\eta_{\nu\lambda}}{(k^2+i\varepsilon)}$ joining nearly on-shell (energetic) lines.

always acompanied by $\begin{pmatrix}\text{an extra vertex}\\ \text{an extra propagator}\end{pmatrix} \times \begin{pmatrix}\text{an extra vertex}\\ \text{an extra propagator}\end{pmatrix}$

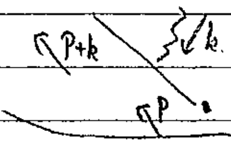
- k^μ flows into a nearly on-shell incoming $q(\vec{p})$

$$\Rightarrow \frac{i(\not{p}+\not{k})}{(p+k)^2+i\varepsilon}(-igt^a\gamma^\lambda) \simeq \frac{2ip^\lambda}{(2p\cdot k+i\varepsilon)}(-igt^a)$$



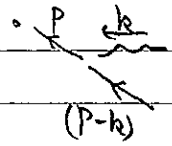
- k^μ into incoming $\bar{q}(\vec{p})$

$$\Rightarrow (-igt^a\gamma^\lambda)\frac{i(-\not{p}-\not{k})}{(-p-k)^2+i\varepsilon} \simeq \frac{-2ip^\lambda}{(2p\cdot k+i\varepsilon)}(-igt^a)$$



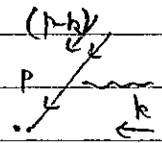
- k^μ flows into a nearly on-shell outgoing $q(\vec{p})$

$$\Rightarrow (-igt^a\gamma^\lambda)\frac{i(\not{p}-\not{k})}{(p-k)^2+i\varepsilon} \simeq (-igt^a)\frac{2ip^\lambda}{-2p\cdot k+i\varepsilon}$$



- k^μ into outgoing $\bar{q}(\vec{p})$

$$\Rightarrow \frac{i(-\not{p}+\not{k})}{(-p+k)^2+i\varepsilon}(-igt^a\gamma^\lambda) \simeq (-igt^a)\frac{-2ip^\lambda}{(-2p\cdot k+i\varepsilon)}$$



This approx "extra vertex & extra propagator" is the same as the gluon absorption amplitude by the following inserted operator:

$$-ig\int_{-\infty}^{0} da\, p\cdot A(pa),\quad -ig\int_{0}^{-\infty} da\, p\cdot A(pa),\quad -ig\int_{0}^{+\infty} da\, p\cdot A(pa),\quad -ig\int_{+\infty}^{0} da\, p\cdot A(pa)$$

e.g. $\int_{-\infty}^{0} da\, e^{-ik\cdot(pa)} \approx \int_{-\infty}^{0} da\, e^{-i(k\cdot p+i\varepsilon)a} = \frac{i}{k\cdot p+i\varepsilon}$

converge @ $a\sim-\infty$

$\dim[a] = (\text{length})^2$

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So, (the $\overset{\text{Drell-Yan}}{\vee}$ amplitude w/ soft gluons) $\simeq$

(the amplitude w/o a soft gluon propagating) $\times$

$$\langle 0| P\exp\left[-ig\int_0^{-\infty} da\, p_A\cdot A^{soft}(p_A a)\right] P\exp\left[-ig\int_{-\infty}^{0} db\, p_B\cdot A^{soft}(p_B b)\right]$$

$$P\exp\left[-ig\int_0^{-\infty} db\, p_B\cdot A^{soft}(x+p_B b)\right] P\exp\left[-ig\int_{-\infty}^{0} da\, p_A\cdot A^{soft}(x+p_A a)\right]|0\rangle$$

$\rightarrow$ called the soft function. (two vectors n_A^μ, n_B^μ. two transverse coordinates $\vec{x}_T$)

(intuition)

for a gluon field w/ soft momentum k^μ

an energetic quark line looks classical (unaffected)
($p^\mu + k^\mu \simeq p^\mu$)

(idnt)

quark $-ig\int dt\, d\vec{x}\left(\bar{\Psi}\gamma^\mu A_\mu \Psi\right)(t,\vec{x}) \Rightarrow -ig\int da\, p^\mu A_\mu(pa)$

$\hookrightarrow \delta^3(\vec{x}-\frac{\vec{p}}{E}t)\frac{p^\mu}{E}$, set $\frac{t}{E} =: a$.

anti-quark $-ig\int dt\, d\vec{x}\left(\bar{\Psi}\gamma^\mu \quad \Psi\right)(t,\vec{x}) \Rightarrow +ig\int db\, p^\mu A_\mu(pb)$

$\hookrightarrow -\delta^3(\vec{x}-\frac{\vec{p}}{E}t)\frac{p^\mu}{E}$ set $\frac{t}{E} =: b$.

reasoning ② (field redefinition) $\leftarrow$ not a precise enough logic.
(intuition)

$$\Psi(x) = P\exp\left[-ig\int_{-\infty}^{0} da\, p_A\cdot A^{soft}(x+p_A a)\right] \underbrace{P\exp\left[-ig\int_0^{-\infty} da\, p_A\cdot A^{soft}(x+p_A a)\right]\Psi(x)}_{=\,\tilde{\Psi}(x)}$$

$\tilde{\Psi}(x)$ does not couple to soft gluons.

(the covariant derivative on $\tilde{\Psi}(x)$ does not contain A_μ^{soft}.)

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after cut

cancellation of IR divergence should be seen within the soft function.

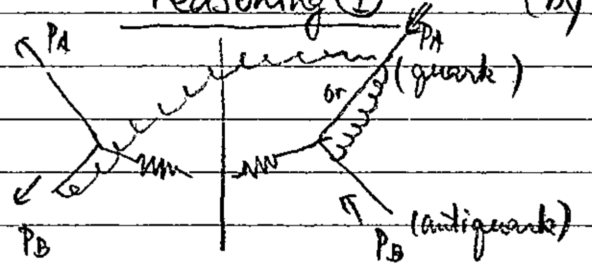
$$\langle 0|\, P\exp\left[-ig\int_0^{-\infty} da\, p_A\cdot A(p_A a)\right] P\exp\left[-ig\int_{-\infty}^{0} db\, p_B\cdot A(p_B b)\right] P\exp\left[-ig\int_0^{-\infty} db\, p_B\cdot A\right] P\exp\left[ig\int_{-\infty}^{0} da\, p_A\cdot A\right] |0\rangle$$

$S_0(\vec{x}_T)$ // real soft gluon emission — vertex correct'n.

Collinear modes

(when $\exists$ massless quark (approximately))

reasoning ① (by amplitudes)



virtual / real gluon propagator $\frac{-i\eta_{\kappa\lambda}}{k^2+i\varepsilon}$ w/ an approx'ly light-like $k^\mu \simeq p_A^\mu$.

attached to a pair of

- a quark line ($m_f=0$) w/ momentum (light-like + soft)
- an anti-quark line w/ momentum ($\underbrace{\text{on-shell}^{(p_B)} + \text{soft} + \text{collinear} \propto p_A^\mu}_{p_B'}$)

$\Rightarrow$ (an extr vertex / an extra propagator) from the anti-quark line is still simple.

$$(-ig t^a\gamma^\lambda)\frac{i[-(\not p_B'-\not k)+M]}{(-p_B'-k)^2-M^2+i\varepsilon} \Rightarrow \frac{(-igt^a)\left[(-2ip_B')^\lambda \text{ or } (-2ip_B^\lambda)\right]}{(2p_B'\cdot k+i\varepsilon)\simeq(2p_B\cdot k+i\varepsilon)}$$

[as expect $\gamma^\lambda(\propto p_A)_\lambda \Rightarrow \not p_A \not k \simeq 0$ (use $k\cdot p_A\simeq 0$)]

$$\boxed{\tilde{\Psi}^{(p_B)}(x) =: \tilde{\tilde{\Psi}}^{(p_B)}(x)\, P\exp\left[-ig\int_0^{-\infty} db\, p_B\cdot A^{(\text{collin } p_A)}(x+p_B b)\right].}$$

$\tilde{\tilde{\Psi}}^{(p_B)}(x)$ does not couple to a gluon w/ momentum approximately light-like $\propto p_A^\mu \neq p_B^\mu$.

So the cancellation of IR divergence $\begin{pmatrix}\text{vertex corrcredn} \\ \text{real emission}\end{pmatrix}$
should be seen within.

- $\langle h_A(\vec{p}_A) | \left(\tilde{\bar{\psi}}(0)\right)_a \; P\exp\left[-ig\int_{+\infty}^{0} db'\, p_B\cdot \overset{(\propto p_A)}{A}(p_B b')\right] \; P\exp\left[-ig\int_0^{-\infty} db\, p_B\cdot \overset{(\propto p_A)}{A}(x+p_B b)\right]_b \left(\tilde{\psi}(x)\right) | h_A(\vec{p}_A)\rangle$
- $\langle h_B(\vec{p}_B) | \; P\exp\left[-ig\int_0^{+\infty} da'\, p_A\cdot \overset{(\propto p_B)}{A}(p_A a')\right]_c \left(\tilde{\bar{\psi}}(0)\right)\left(\tilde{\psi}(x)\right)_d \; P\exp\left[-ig\int_{-\infty}^{0} da\, p_A\cdot \overset{(\propto p_B)}{A}(x+p_A a)\right] | h_B(\vec{p}_B)\rangle$

(end pt)

Those two functions
$\left(\text{argument}: \begin{array}{l} n_B^\mu,\ \vec{x}_T,\ (\text{non-}p_B^\mu)\text{ direct'n of } x^\mu \\ n_A^\mu,\ \vec{x}_T,\ (\text{non-}p_A^\mu)\text{ direct'n of } x^\mu \end{array}\right)$

p_A^μ direct'n of x^μ absorbed by redef. of $\mathcal{S}_0$

called <u>jet functions</u> $J_A(n_B; \vec{x}_T, x_A)$ $J_B(n_A; \vec{x}_T, x_B)$

generalizat'n of the PDF

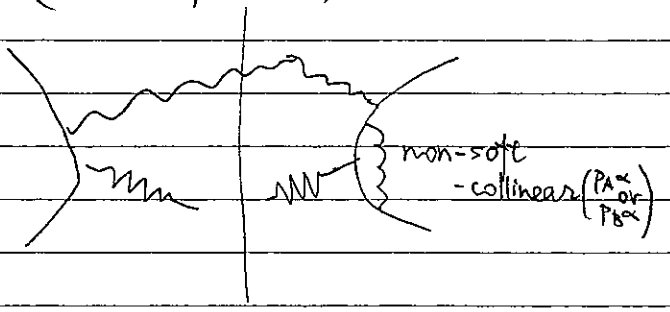
Quantum effects of
Gluons & quarks w/ [momentum $\propto p_A^\mu$] (matrix elements)
$\Rightarrow$ should be taken into account in $J_A(n_B; \vec{x}_T, x_A)$
(while forgetting propagator/vertex etc. of anti-quarks / other DOFs)

story similar to the quantum effects
redef. of gluons w/ soft momentum. (in $\mathcal{S}_0(n_A, n_B; \vec{x}_T)$)

$\boxed{\text{non-perturbative information } J_A,\ J_B,\ \mathcal{S}_0 \text{ necessary.}}$

J_A: Fourier transform w.r.p.t. x_A $\Rightarrow$ Bjorken x.
(long. t. mom. fract'n)
Fourier transform w.r.p.t. $\vec{x}_T$ $\Rightarrow$ transv. mom.
as in §7.X

(hard part)



quantum effects
(involving DOF's whose momentum is neither soft or collinear to p_A^μ or p_B^μ)

$\Rightarrow$ integrate out
take into account in the OPE coefficients.

$$\left\{ J^\mu(0)\, J^\nu(x) \right\} e^{iq\cdot x} d^4x \Rightarrow (\text{coefficient}) \times \left(\tilde{\tilde{\bar{\Psi}}}^{(p_A)} P\exp\left[\int A^{\text{soft}\,(p_A)}\right] P\exp\left[\int A^{(\text{soft})\,(p_B)}\right] \tilde{\tilde{\Psi}}^{(p_B)} \right)$$

$$\underset{H(x^+, x^-, \vec{x}_T)}{\overset{\parallel}{}} \quad \left(\tilde{\tilde{\bar{\Psi}}}^{(p_B)}(x)\, P\exp\left[\int A^{(\text{soft})\,(p_B)}\right] P\exp\left[\int A^{(\text{soft})\,p_A}\right] \tilde{\tilde{\Psi}}^{(p_A)} \right).$$

$\boxed{\text{factorizat'n formula}}$

$$(*)^{\mu\nu} = \int d^4x\, e^{iq\cdot x}\, H(x^+, x^-, \vec{x}_T)\, J_A(x^+, \cancel{x^-}, \vec{x}_T; n_B)\, J_B(\cancel{x^+}, x^-, \vec{x}_T; n_A) \times \mathcal{S}_0(\cancel{x^+}, \cancel{x^-}, \vec{x}_T; n_A, n_B)$$

$\vec{q}_T$ in the Drell-Yan process.

- recoil of transverse momenta $\left\{ \begin{matrix} \text{hard} \\ \text{soft} \end{matrix} \right\}$
- intrinsic transverse momenta of partons in h_A & h_B

$\boxed{\text{Analogue of the DGLAP eq.}}$

need to draw a line between.

(hard DOF's vs collinear DOFs.
collinear DOFs vs soft DOF's.)

evolution equat'n (like DGLAP)

($\uparrow$ determined by the anomalous dim. of the local operators)

$\boxed{\text{soft-collinear effective theory}}$