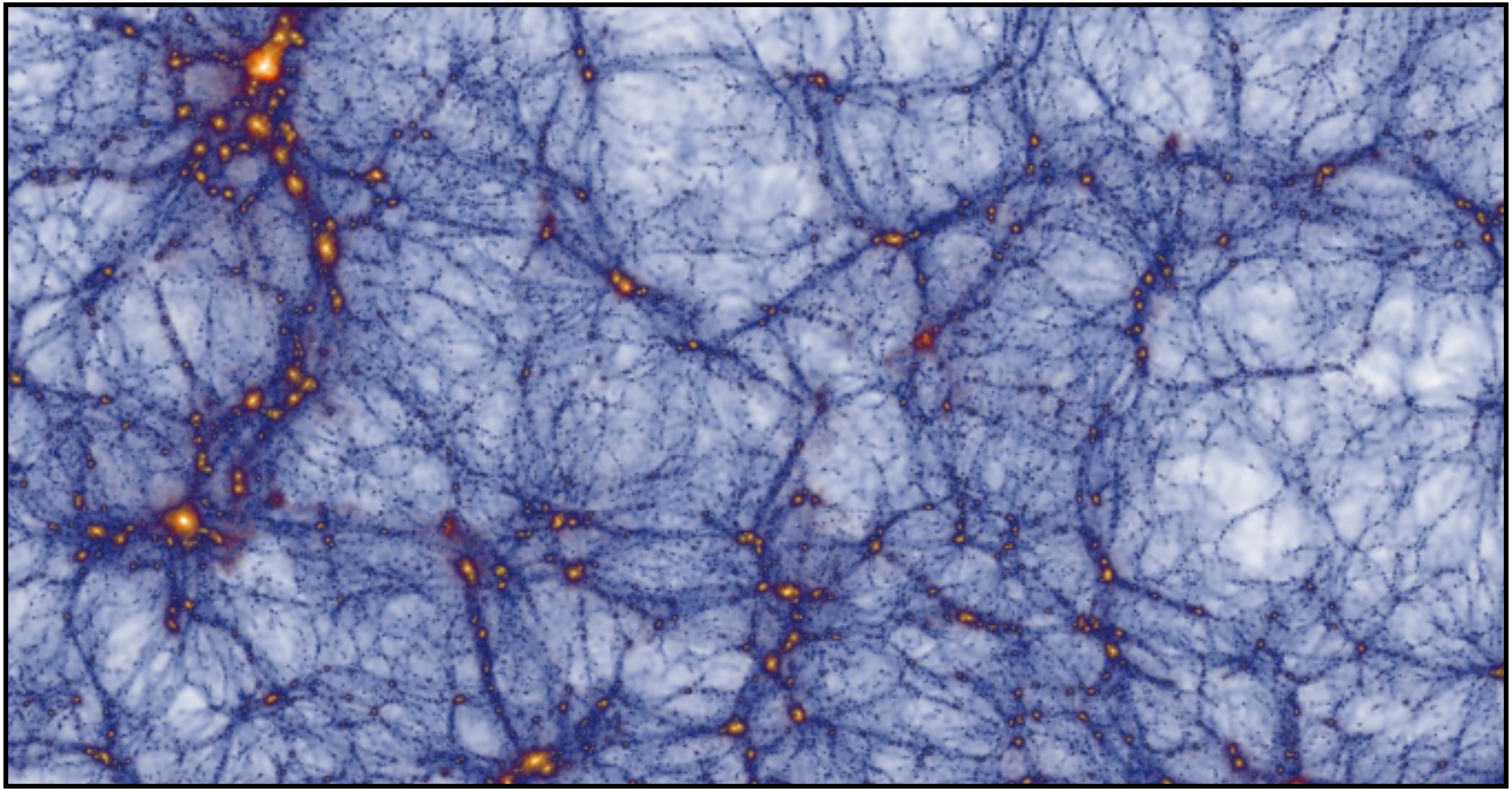
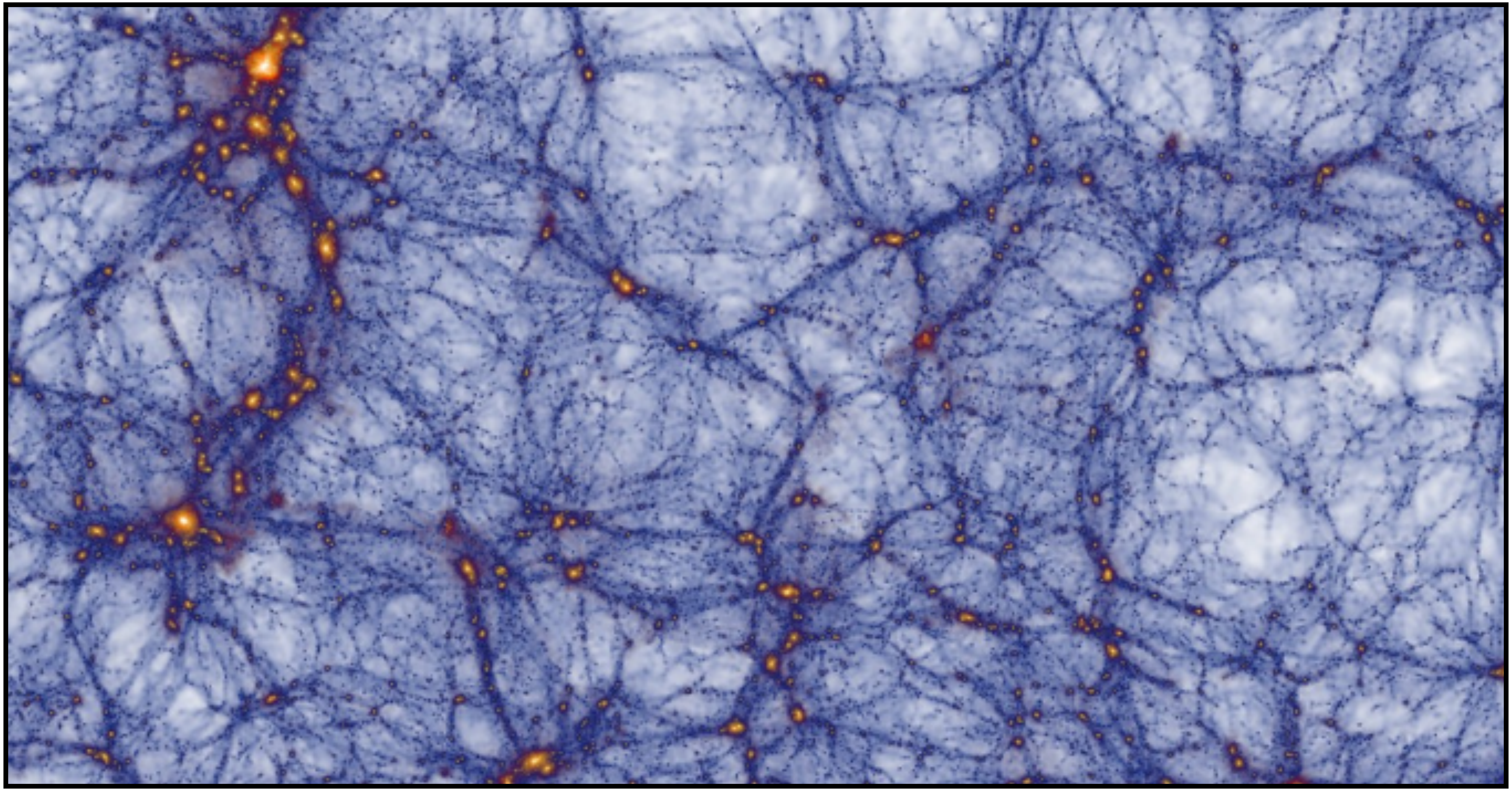


The Cosmological Bootstrap

Daniel Baumann

University of Amsterdam

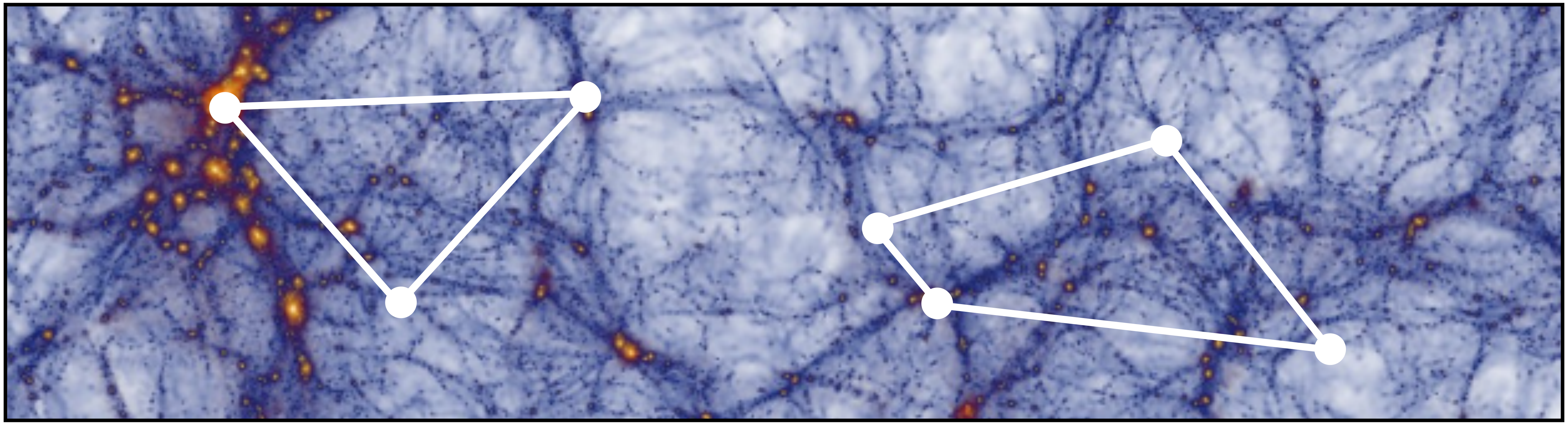
Web Seminar,
April 2020

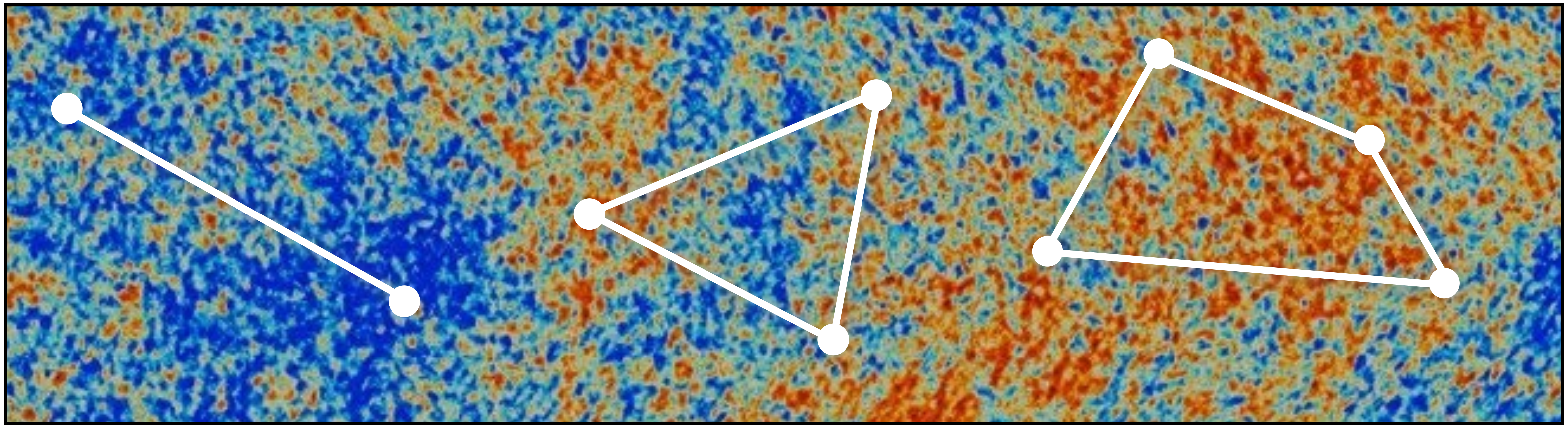


Based on work with

Nima Arkani-Hamed, Hayden Lee, Guilherme Pimentel,
Carlos Duaso Pueyo and Austin Joyce

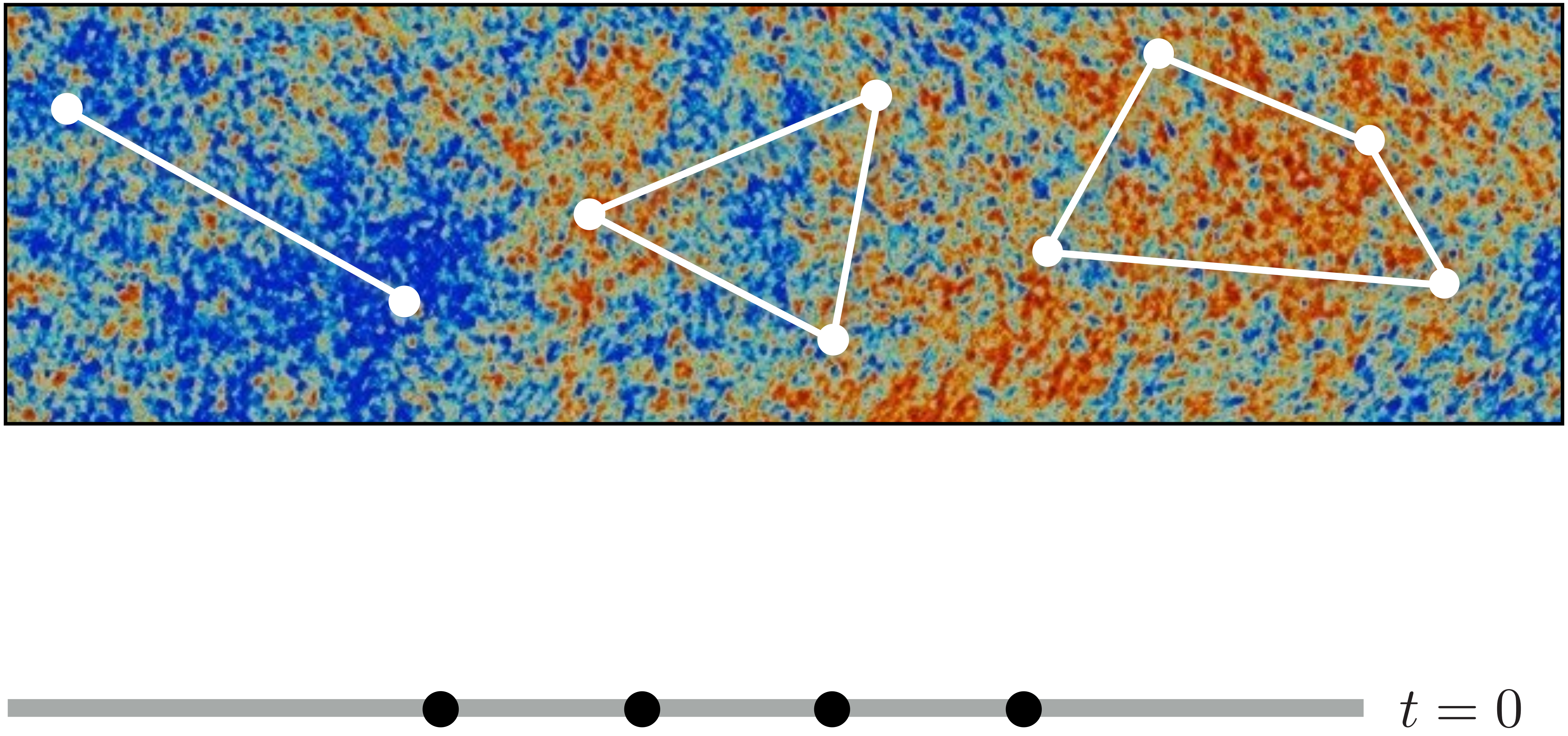
The physics of the early universe is encoded in **spatial correlations** between cosmological structures at late times:





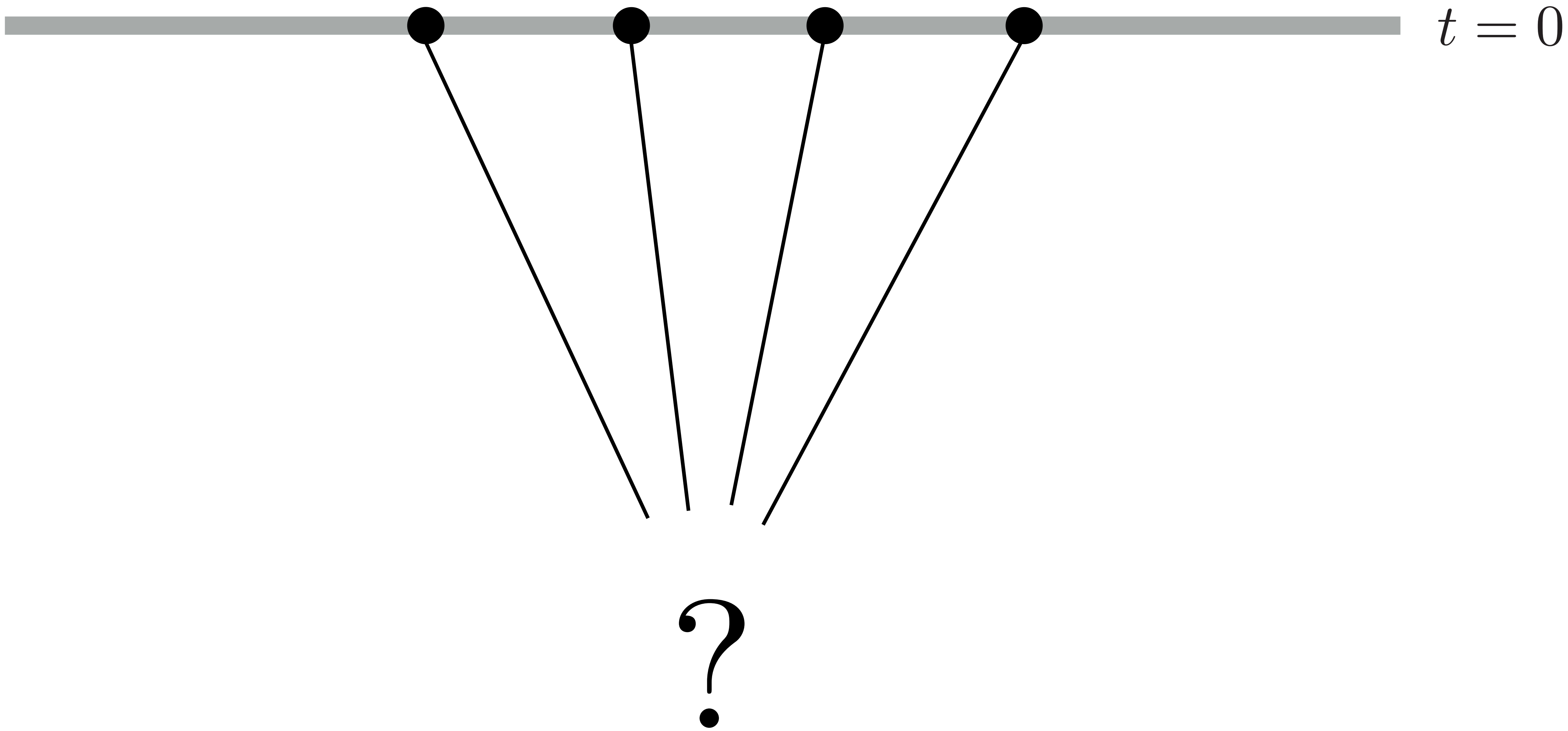
A central challenge of modern cosmology is to construct a **consistent history** of the universe that explains these correlations.

The correlations can be traced back to **primordial correlations** at the beginning of the hot big bang.



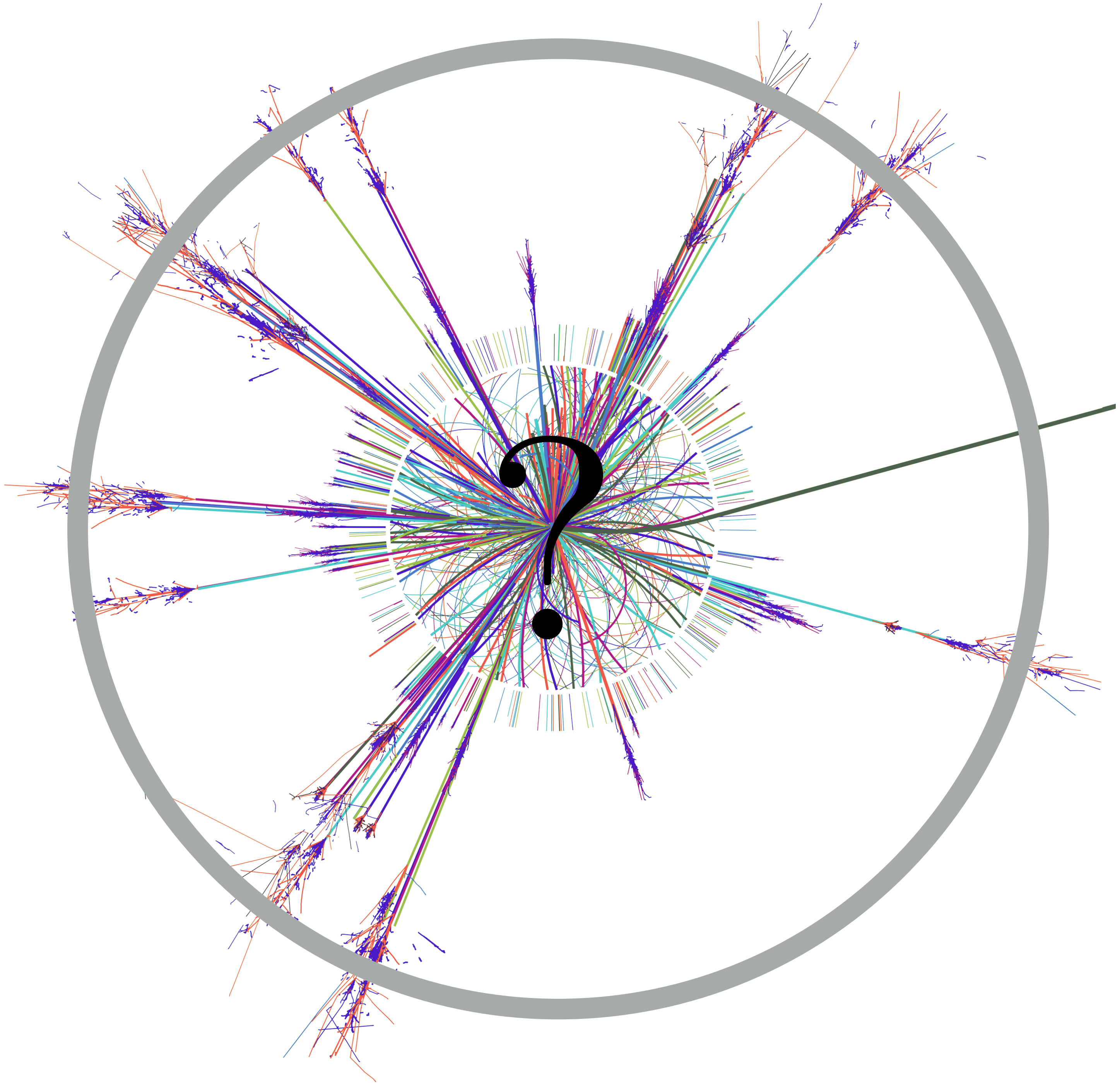
To explain the observed fluctuations in the CMB, these fluctuations must be created **before the hot big bang**!

What is the space of consistent histories?



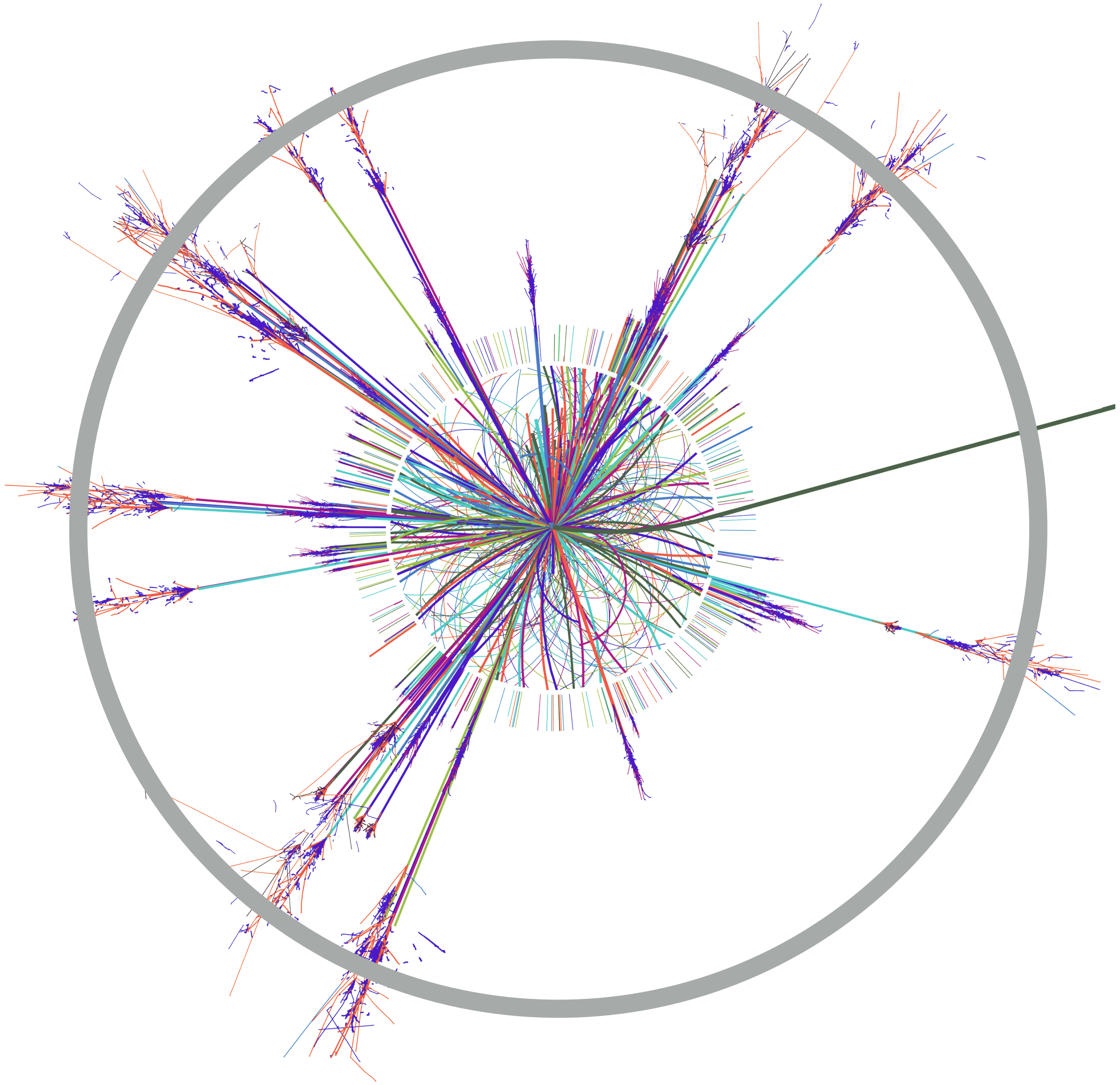
- What are the rules that consistent correlators have to satisfy?
- How are these rules encoded in the boundary observables?

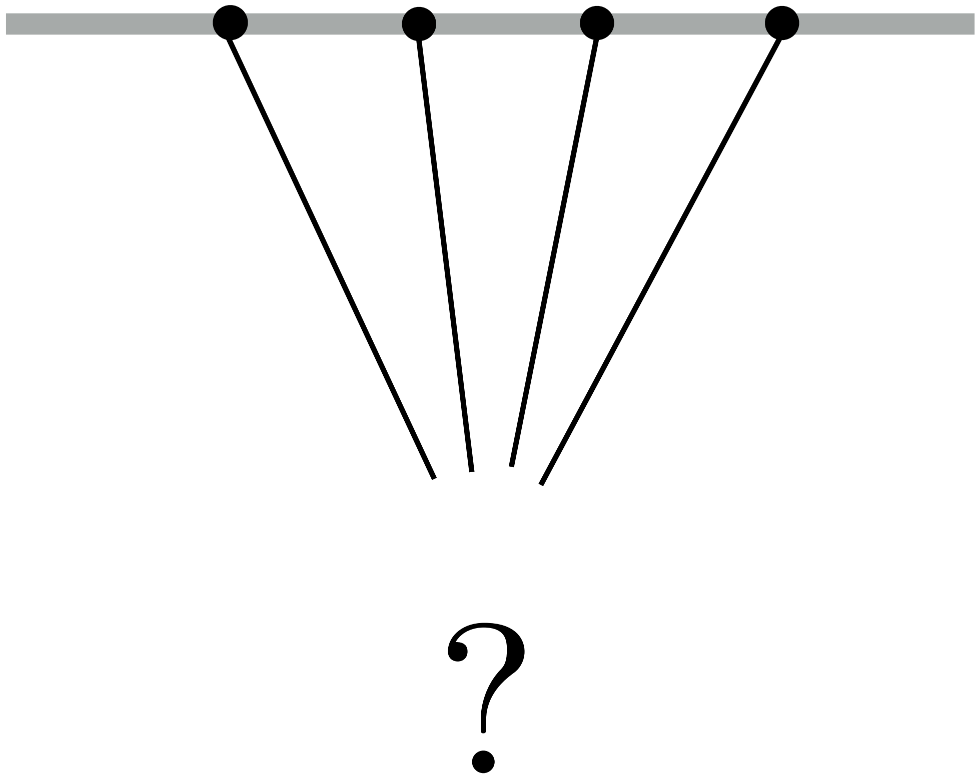
Similar questions have been asked for **scattering amplitudes**:



In that case, the rules of **quantum mechanics** and **relativity** are very constraining.

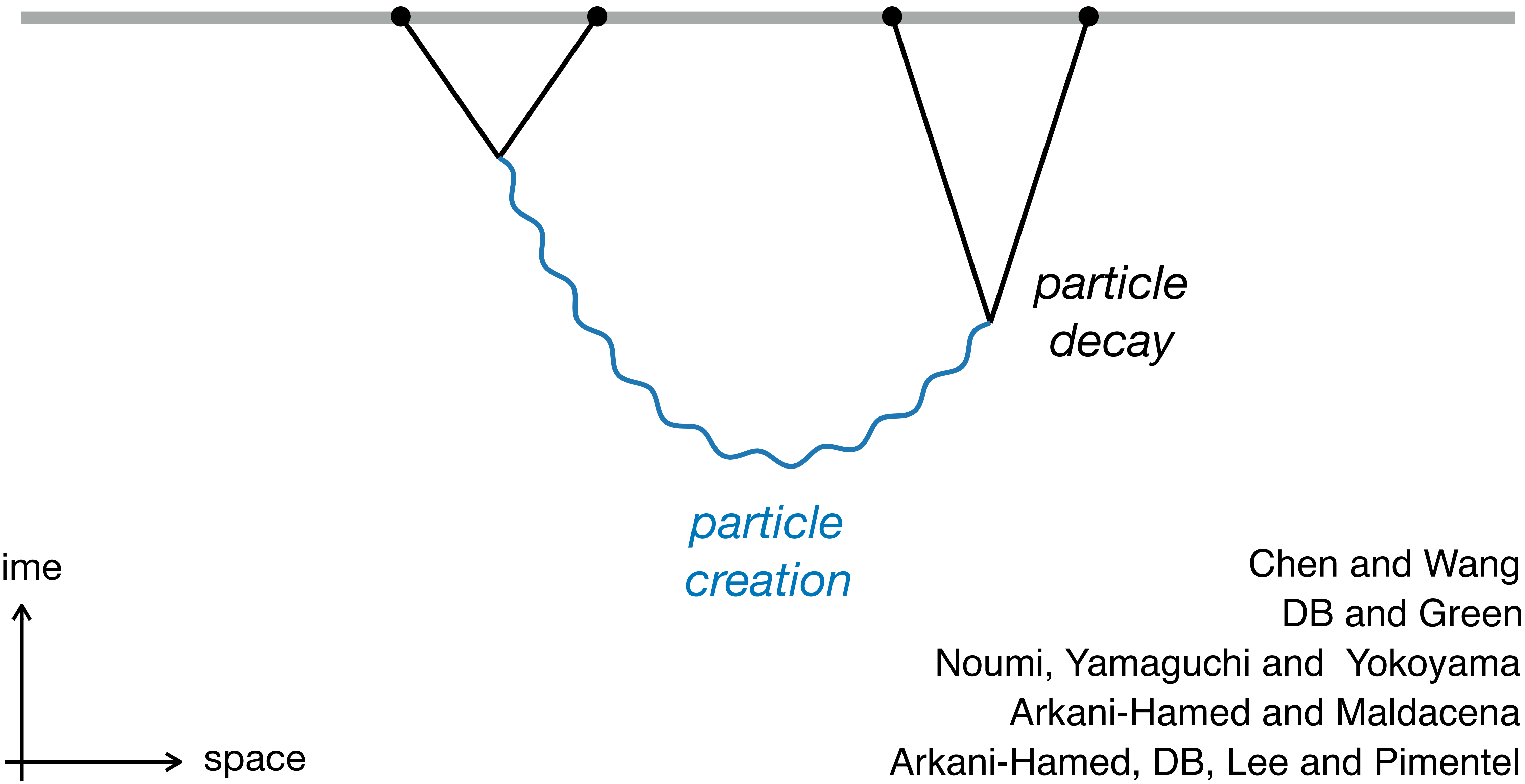
Does a similar **rigidity** exist for cosmological correlators?





Goal: Develop an understanding of cosmological correlators that parallels our understanding of flat-space scattering amplitudes.

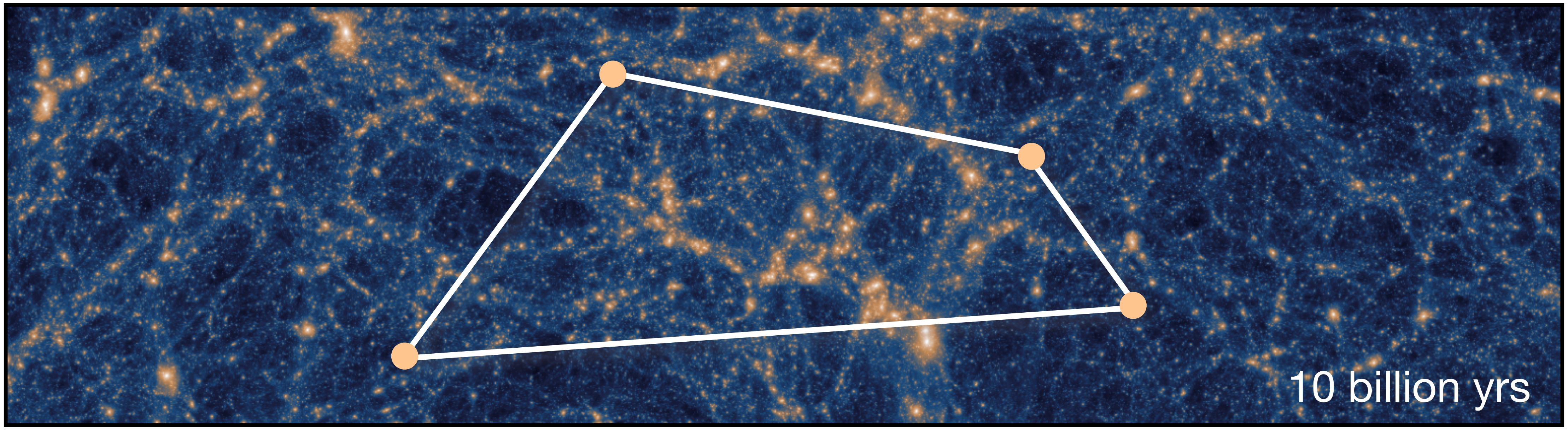
The connection to scattering amplitudes is also relevant because the early universe was like a giant **cosmological collider**:

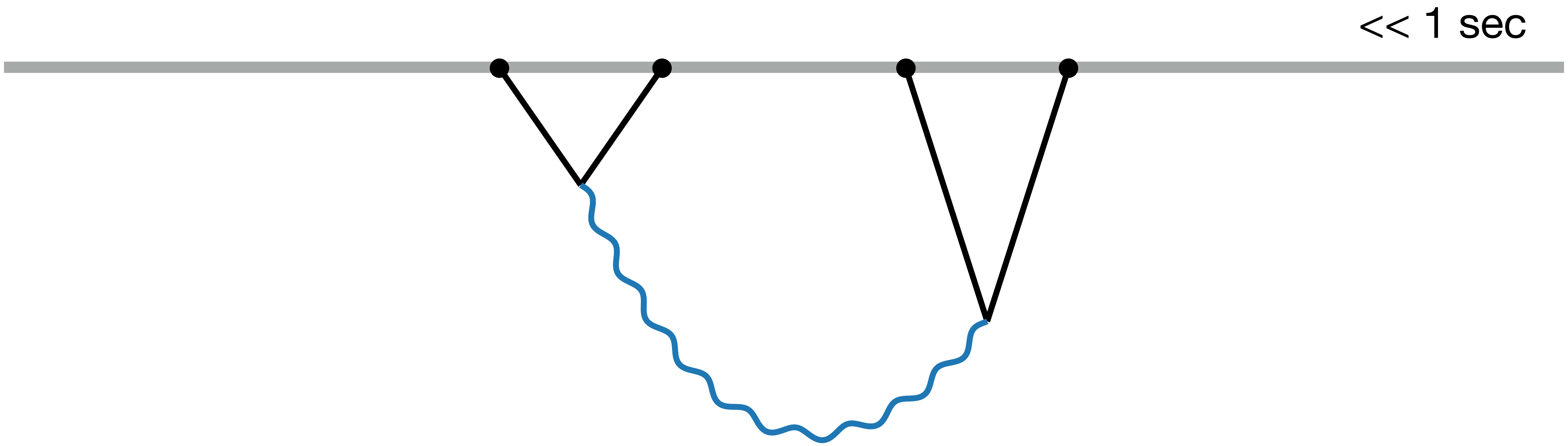


Chen and Wang [2009]
DB and Green [2011]
Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

During inflation, the rapid expansion can produce very **massive particles** ($\sim 10^{14}$ GeV) whose decays lead to nontrivial correlations.

At late times, these correlations will leave an imprint in the distribution of galaxies:





Goal: Develop a systematic way to predict these signals.

Any Questions?

Outline

I. The Cosmological Bootstrap

II. New Developments

I. The Cosmological Bootstrap

Bootstrap Philosophy

Physical Principles →

(Lorentz, locality, …)

Lagrangian

equations of motion

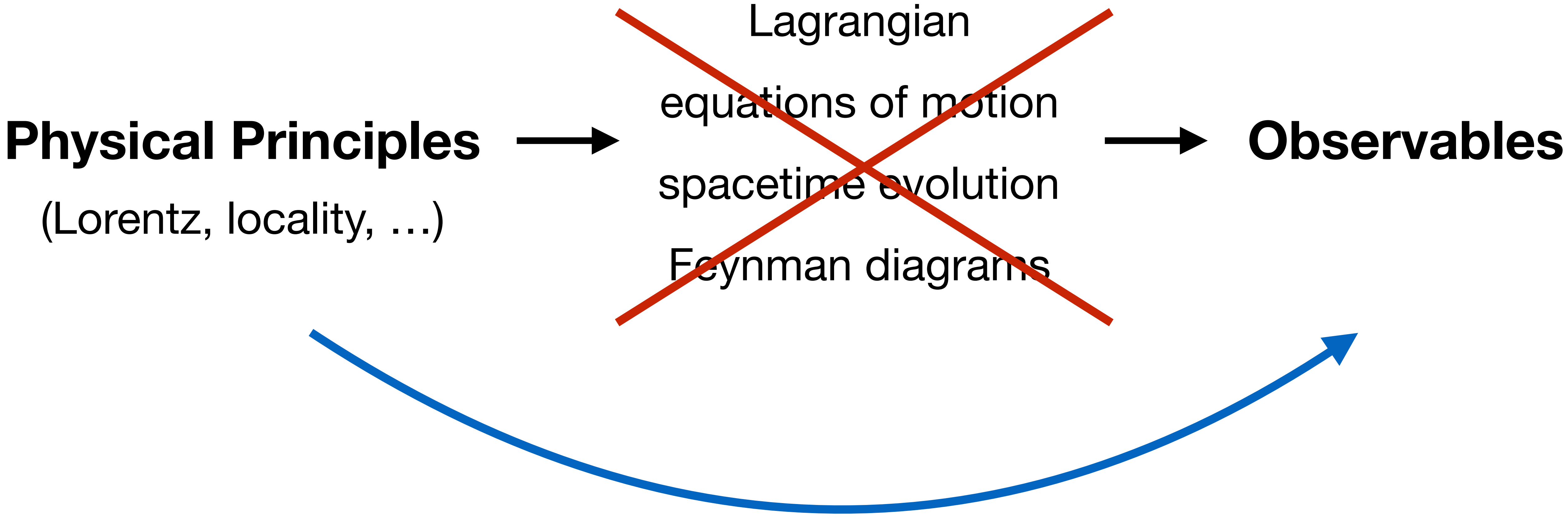
spacetime evolution

Feynman diagrams

→ **Observables**

$$S = \int \underset{\substack{\uparrow \\ \text{locality}}}{\mathrm{d}^4x}\, \mathcal{L}\left[\frac{1}{2}\underset{\substack{\uparrow \\ \text{Lorentz}}}{\partial_\mu}\phi\partial^\mu\phi - \frac{1}{2}\underset{\substack{\uparrow \\ \text{parameters}}}{m^2}\phi^2 + \cdots\right]$$

Bootstrap Philosophy



Physical Principles →

(Lorentz, locality, …)

~~Lagrangian~~

~~equations of motion~~

~~spacetime evolution~~

~~Feynman diagrams~~

→ **Observables**

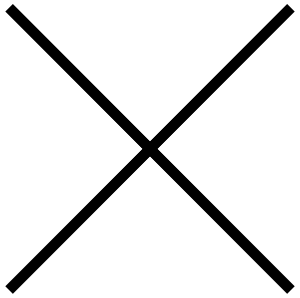
Modern scattering
amplitudes programme

See Yu-tin's book.

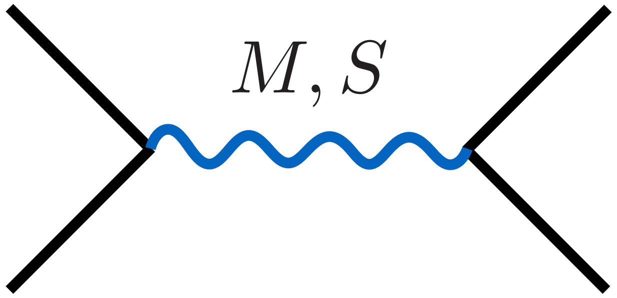
The S-Matrix Bootstrap

The structure of scattering amplitudes at tree level is fixed by **Lorentz invariance**, **locality** and **unitarity**:

$$A(s,t) \quad = \quad \sum a_{nm} s^n t^m \quad + \quad \frac{g^2}{s - M^2} P_S\left(1 + \frac{2t}{M^2}\right)$$



contact interactions



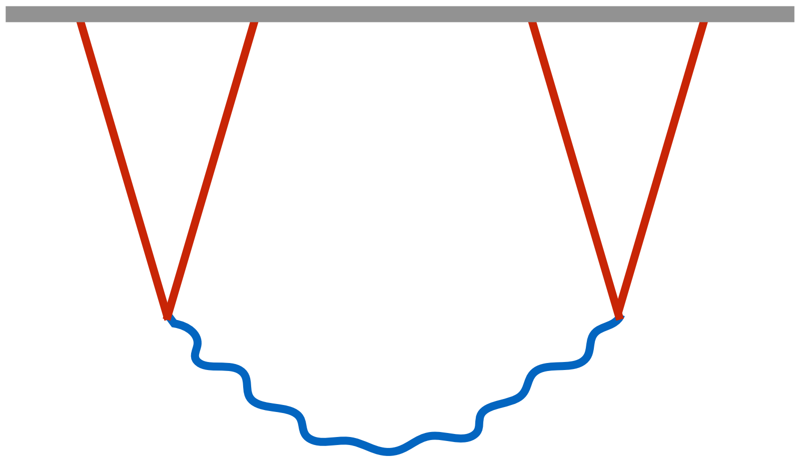
exchange interactions

- No Lagrangian or Feynman diagrams are needed to derive this.
- Basic principles allow only a small menu of possibilities.

The Challenge

Even tree-level processes are hard to compute in cosmology:

$$\langle\phi\phi\phi\phi\rangle \; = $$



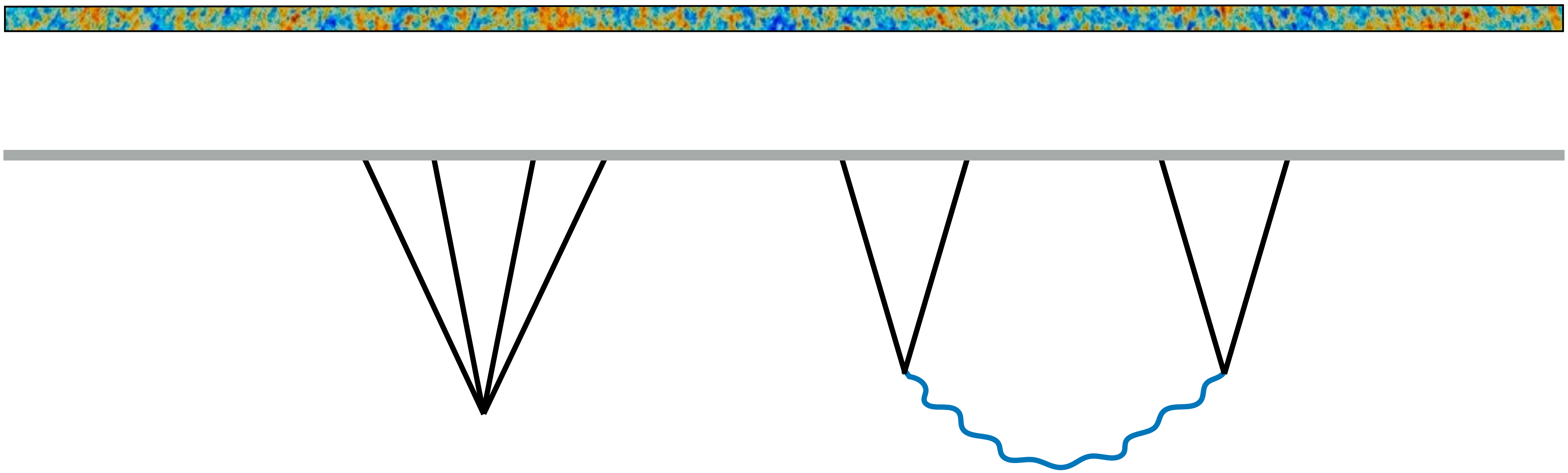
$$\sim \int \mathrm{d}t\,\mathrm{d}\tilde{t}\; e^{i(k_1+k_2)t}\, e^{i(k_3+k_4)\tilde{t}}\, G(|\vec{k}_1+\vec{k}_2|, t, \tilde{t})$$

shown for conformally coupled scalars

complicated function of Hankel functions

The Cosmological Bootstrap

In the cosmological bootstrap, the primordial correlators are determined from consistency conditions alone:

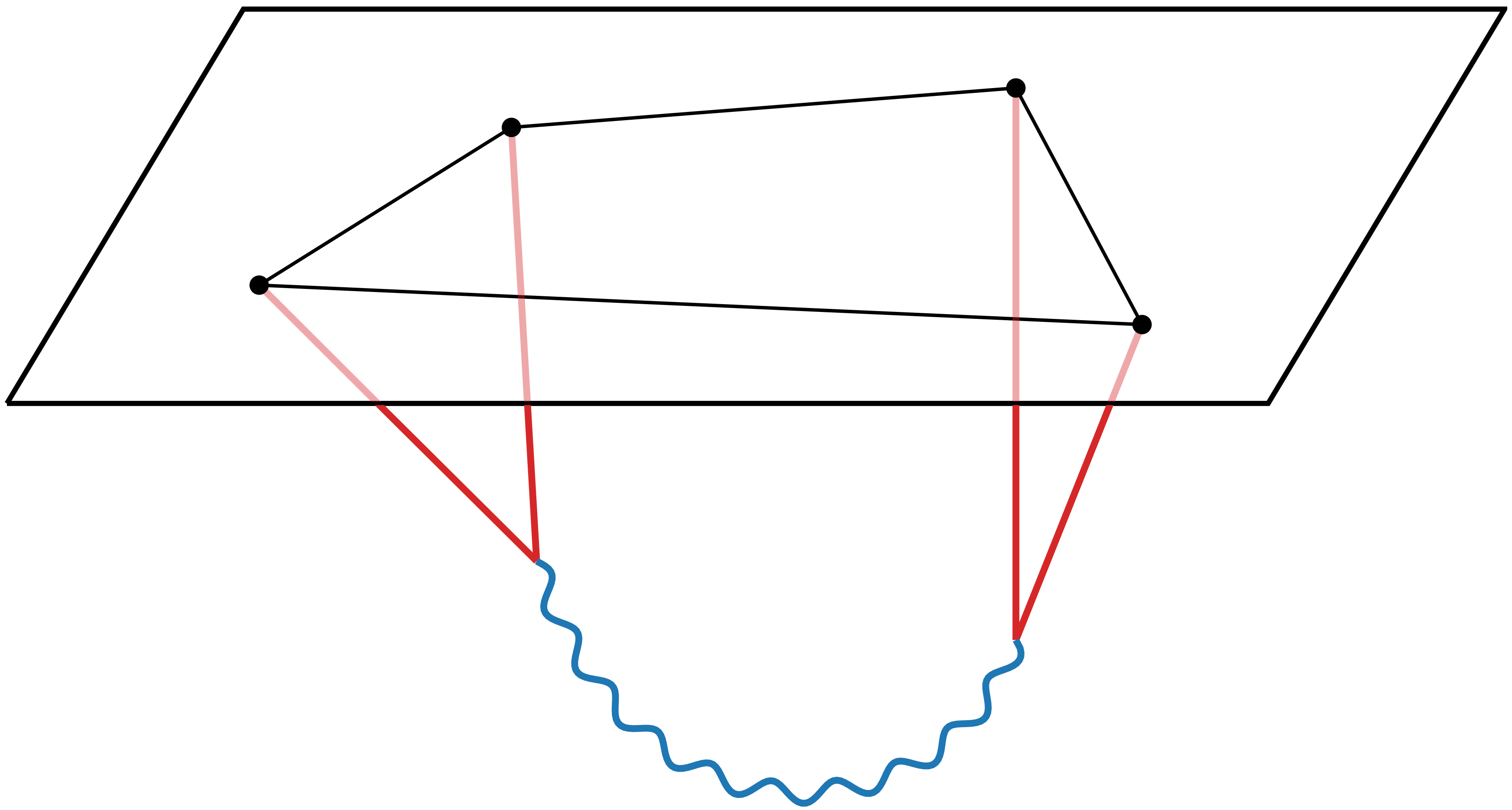


Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2019]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, Benincasa, and Postnikov [2017]
Sleight and Taronna [2019]
Sleight [2019]

Inflation → De Sitter

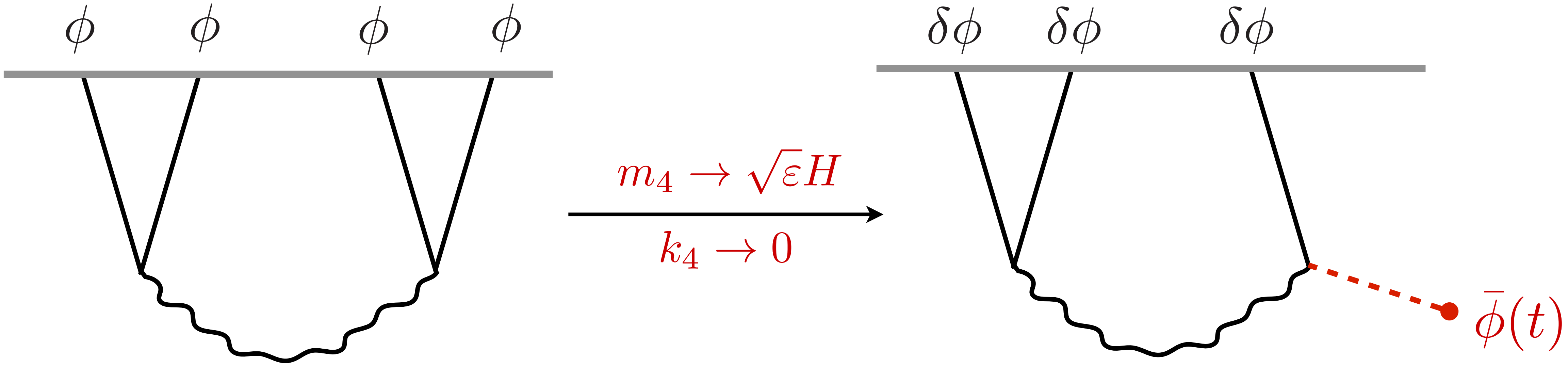
If inflation is correct, then all primordial correlations live on the boundary of an approximate de Sitter spacetime:



- Isometries of dS become conformal symmetries on the boundary.
- This constrains the correlations of weakly interacting particles.

De Sitter → Inflation

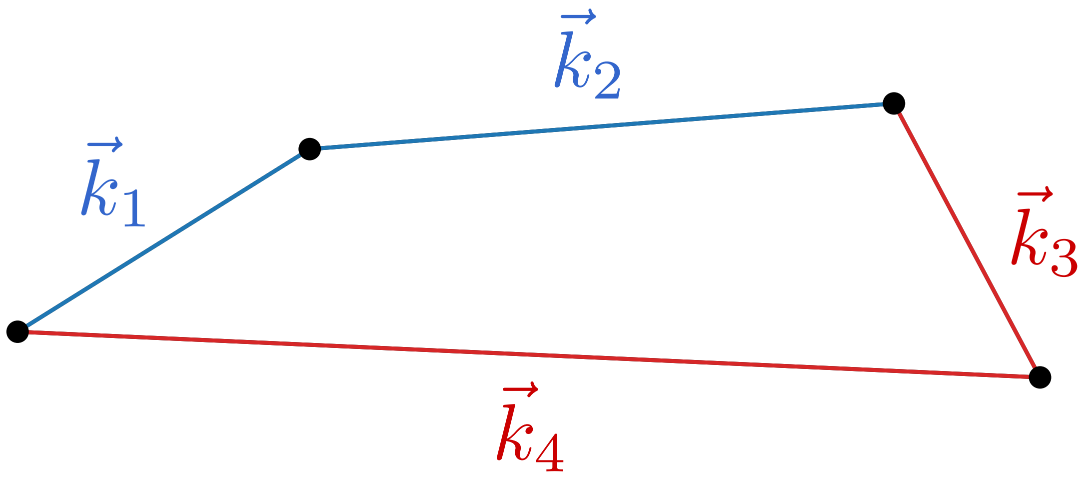
Inflationary three-point functions are obtained from de Sitter four-point functions by evaluating one of the external legs on the background:



We can therefore study de Sitter four-point functions as the fundamental building blocks of inflationary correlators.

Symmetries

If the couplings between particles are weak, then the primordial correlations inherit the symmetries of the quasi-de Sitter spacetime:



- Rotations
- Translations

} Momentum conservation in Fourier space

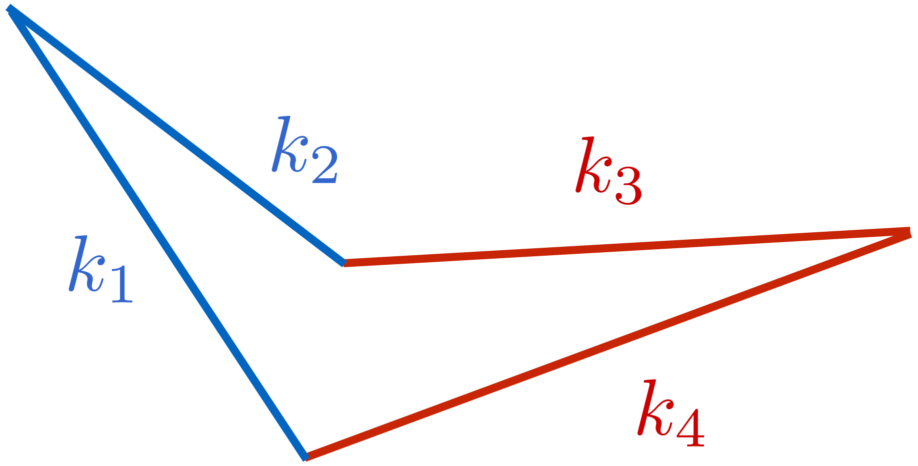
- Dilatation
- Special Conformal

} Determine the allowed deformations (or shapes) of the correlators.

Kinematics

The kinematical data of correlators and amplitudes is similar:

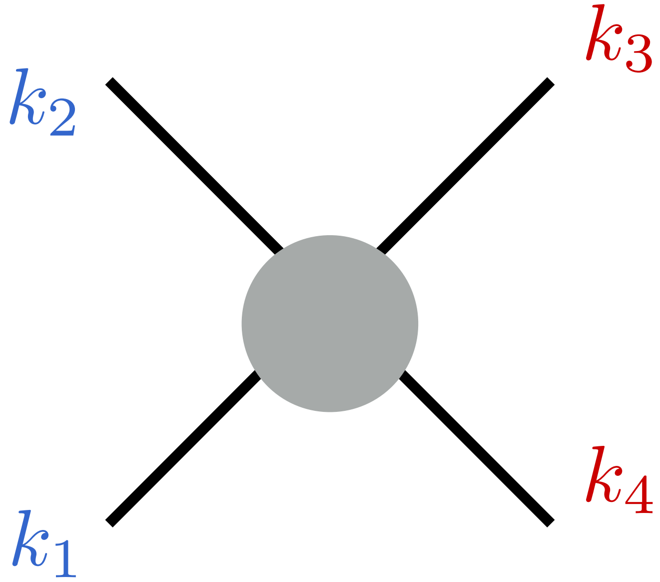
$$F \equiv$$



AMPLITUDE

$$\frac{A}{E^n} + \cdots$$

$$A \equiv$$



$$\delta_D(E \equiv |\vec{k}_1| + \cdots + |\vec{k}_4|)$$

Raju [2012]
Maldacena and Pimentel [2011]

Ward Identities

Invariance under **dilatations** and **SCTs** imply the following **Ward identities**:

$$0 = \left[9 - \sum_{n=1}^{4}\left(\Delta_n - \vec{k}_n \cdot \partial_{\vec{k}_n}\right)\right] F$$

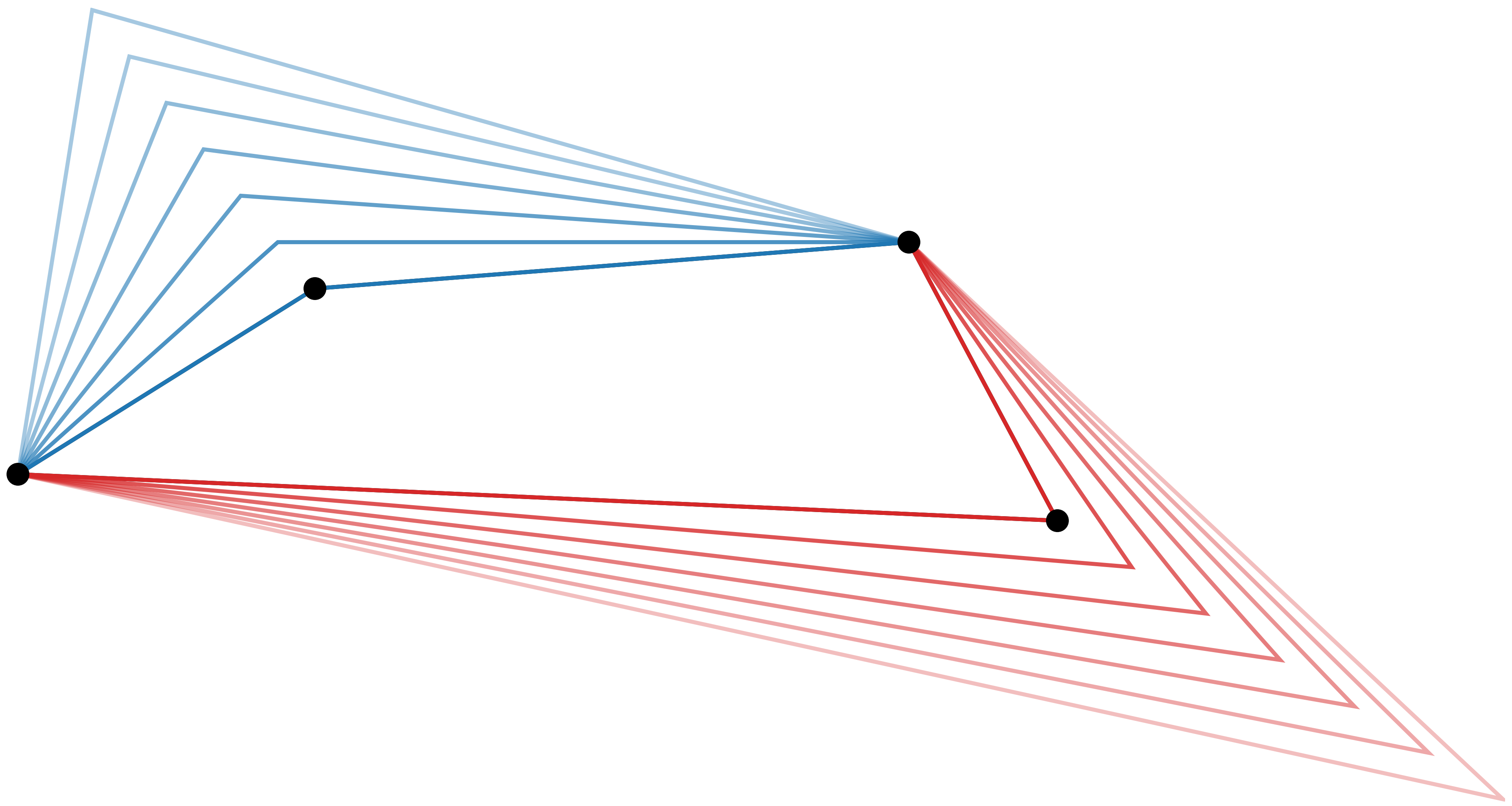
$$0 = \sum_{n=1}^{4}\left[(\Delta_n - 3)\partial_{\vec{k}_n} - (\vec{k}_n \cdot \partial_{\vec{k}_n})\partial_{\vec{k}_n} + \frac{\vec{k}_n}{2}(\partial_{\vec{k}_n} \cdot \partial_{\vec{k}_n})\right] F$$

This is the analog of Lorentz invariance of the amplitude:

$$A(s, t)$$

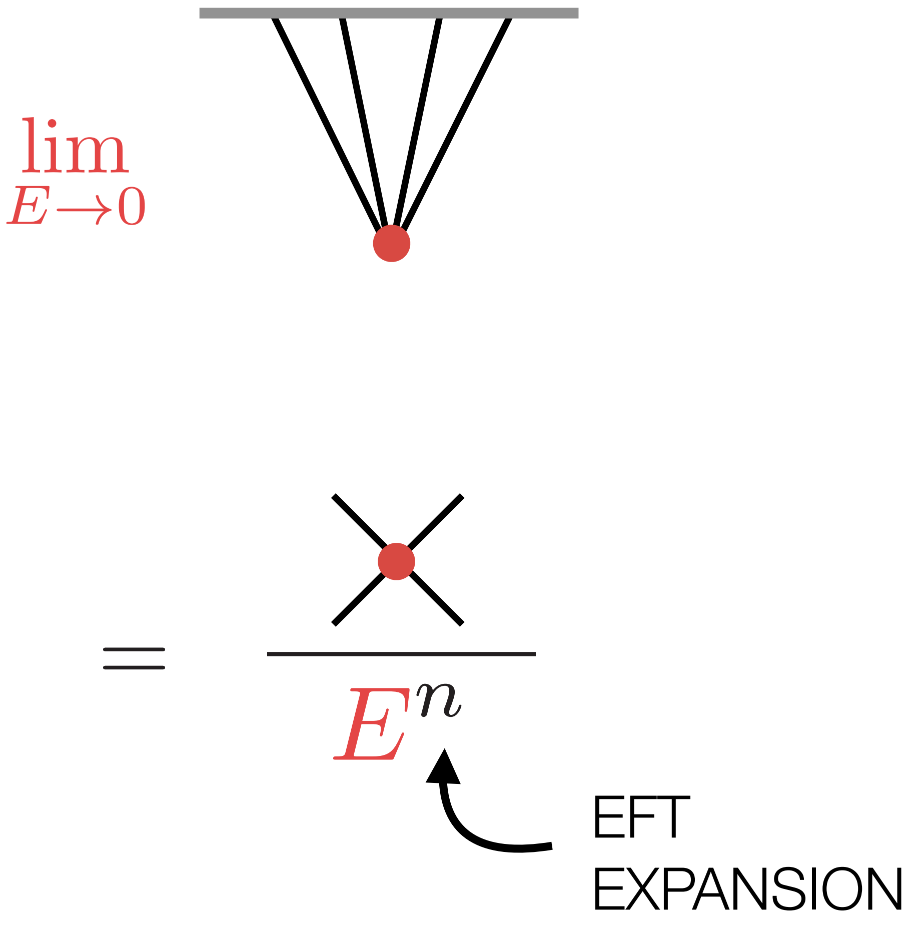
Ward Identities

These Ward identities dictate how the strength of the correlations changes as we change the external momenta:

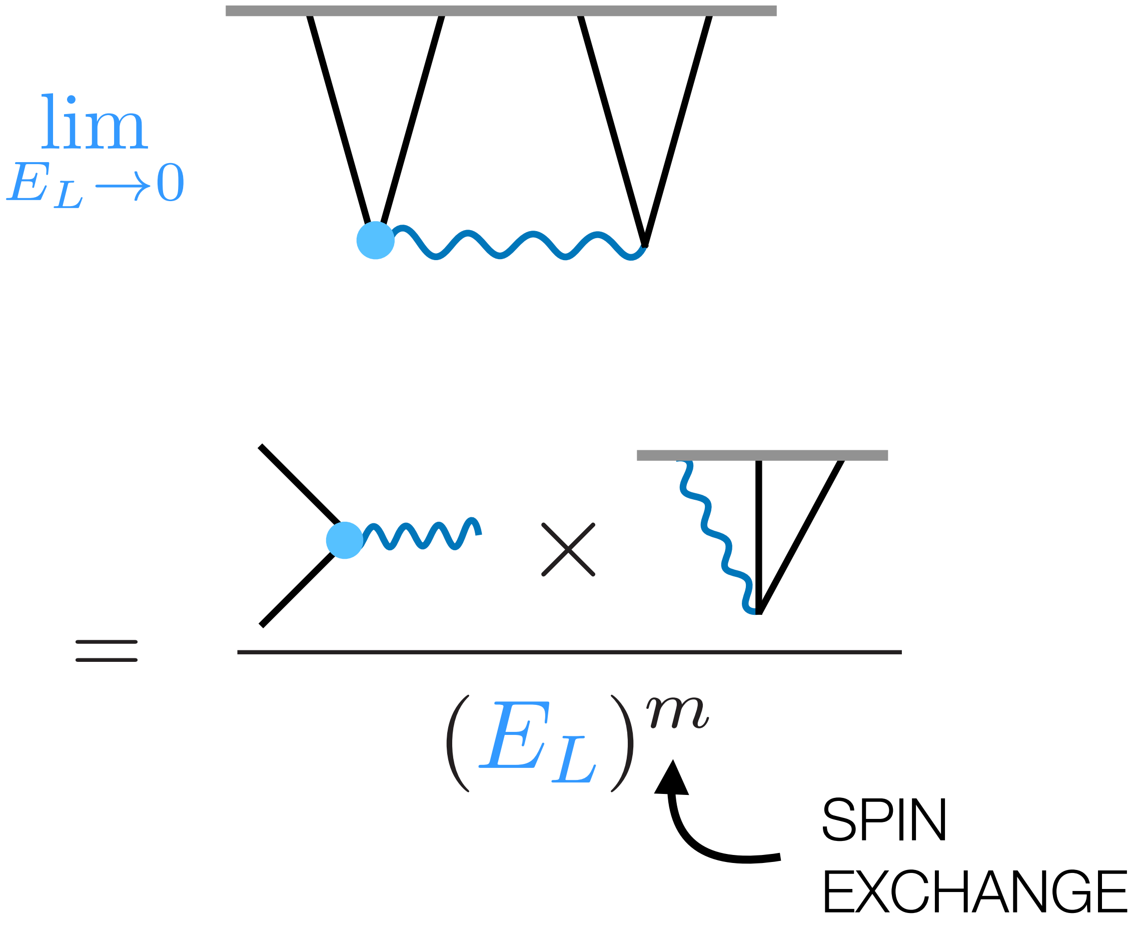


Singularities

The solutions to the Ward identities can be classified by their **singularities**:



Contact solutions only have total-energy poles.

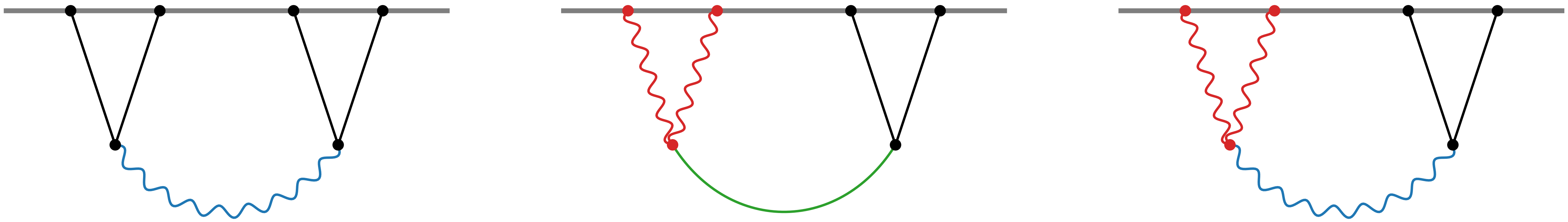


Exchange solutions have additional partial-energy poles.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Exchange Solutions

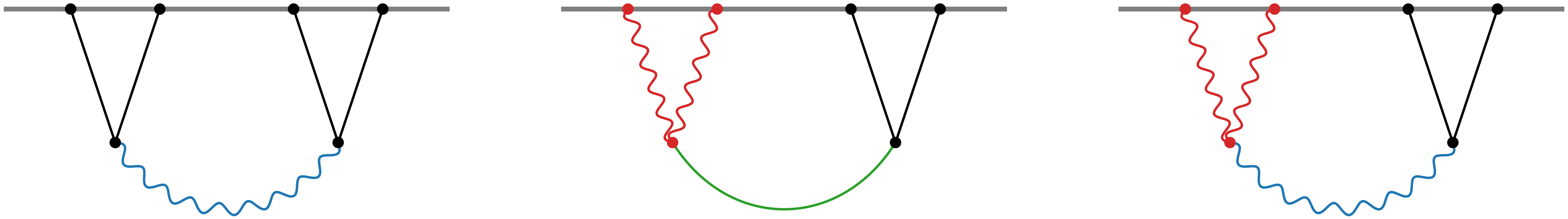
There are **distinct solutions** for distinct microscopic processes during inflation:



Arkani-Hamed, DB, Lee and Pimentel [2018]

Exchange Solutions

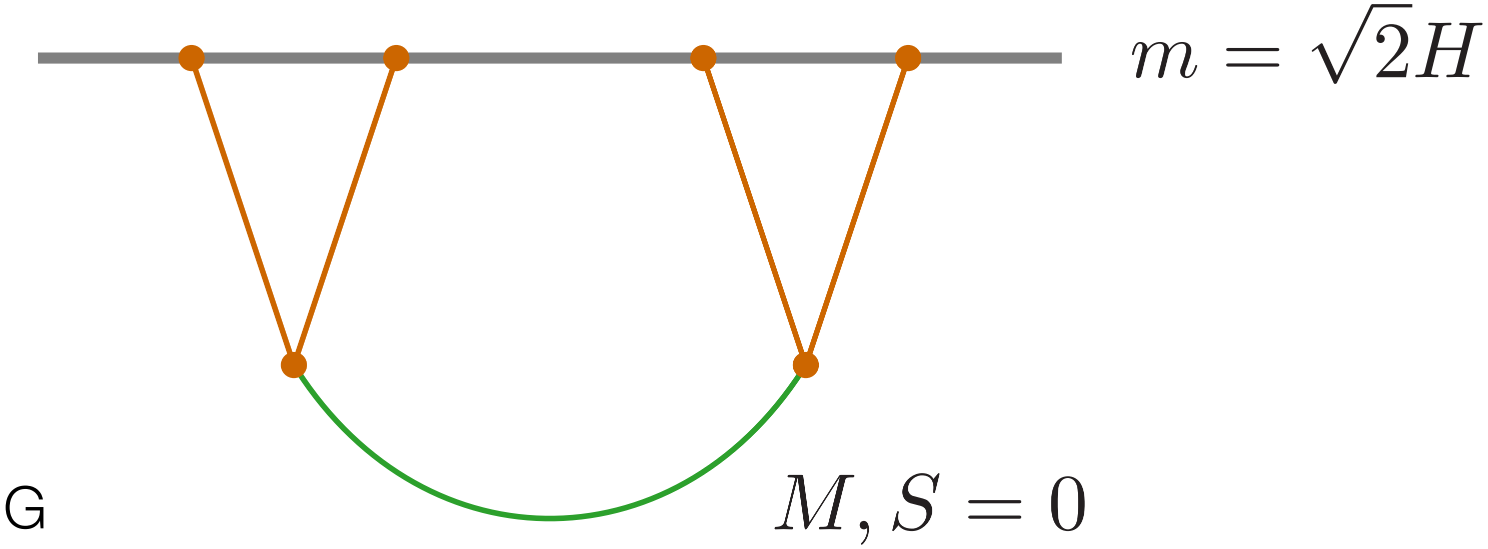
There are **distinct solutions** for distinct microscopic processes during inflation:



$$= \mathcal{D}_n$$

↑

WEIGHT-SHIFTING
OPERATORS



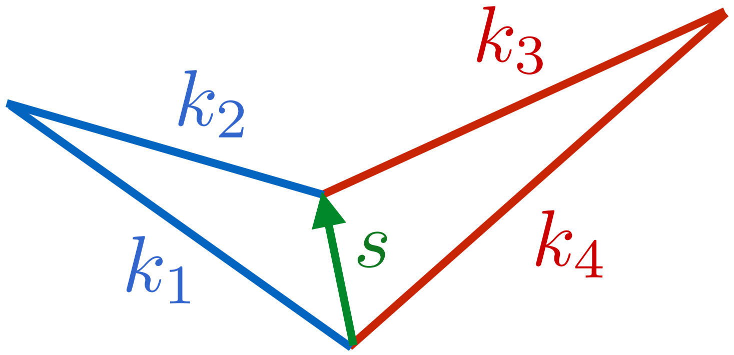
Remarkably, all solutions can be reduced to a **unique building block**.

Arkani-Hamed, DB, Lee and Pimentel [2018]

Seed Solution

- The dilatation Ward identity for the seed is solved if

$$\boxed{F = \frac{1}{s}\hat{F}(u, v)}$$



where we have introduced $u \equiv \dfrac{s}{k_1 + k_2}$ and $v \equiv \dfrac{s}{k_3 + k_4}$.

Seed Solution

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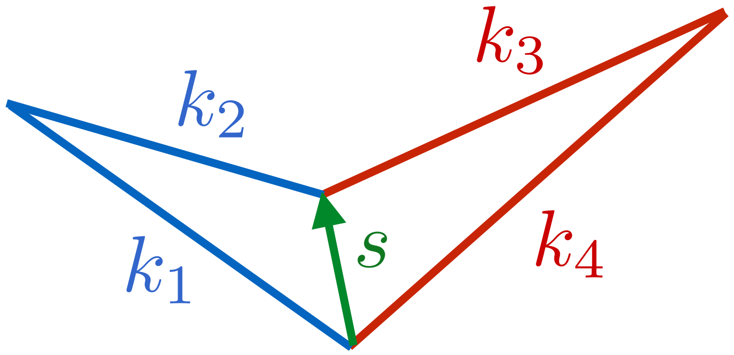


$$\boxed{F = \frac{1}{s}\hat{F}(u, v)}$$

where we have introduced $u \equiv \dfrac{s}{k_1 + k_2}$ and $v \equiv \dfrac{s}{k_3 + k_4}$.

- The conformal Ward identity then becomes

$$\boxed{(\Delta_u - \Delta_v)\hat{F} = 0}$$

where $\Delta_u \equiv u^2(1 - u^2)\partial_u^2 - 2u^3\partial_u$.

Seed Solution

For **tree exchange**, the conformal Ward identity reduces to:

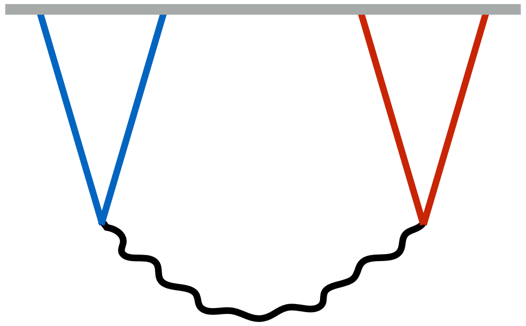
$$\boxed{(\Delta_u + M^2)\hat{F} = \hat{F}_c}$$

CONTACT SOLUTION

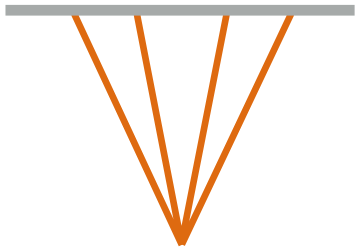
MASS OF THE EXCHANGE PARTICLE

$(\Delta_u + M^2)$

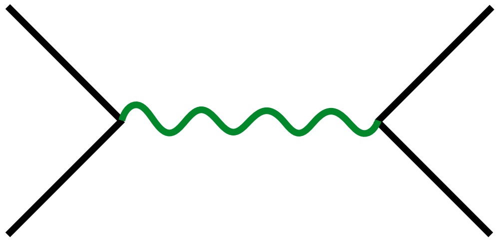
$(\Delta_v + M^2)$



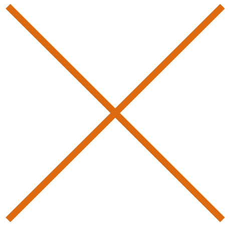
$=$



$(s - M^2)$



$=$



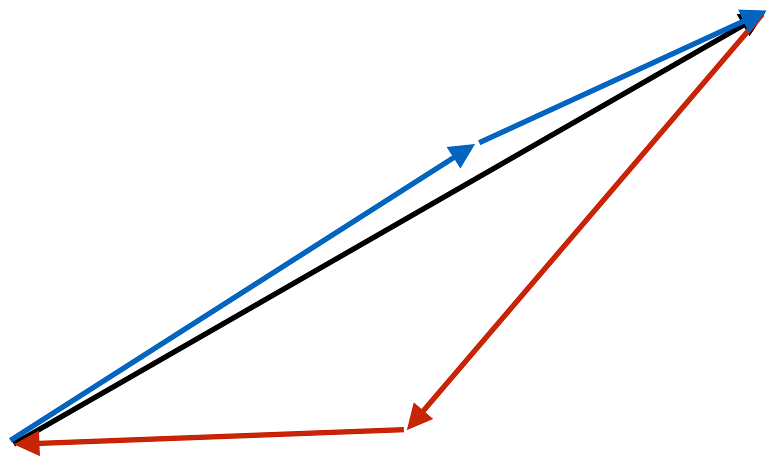
Seed Solution

For **tree exchange**, the conformal Ward identity reduces to:

$$\boxed{(\Delta_u + M^2)\hat{F} = \hat{F}_c}$$

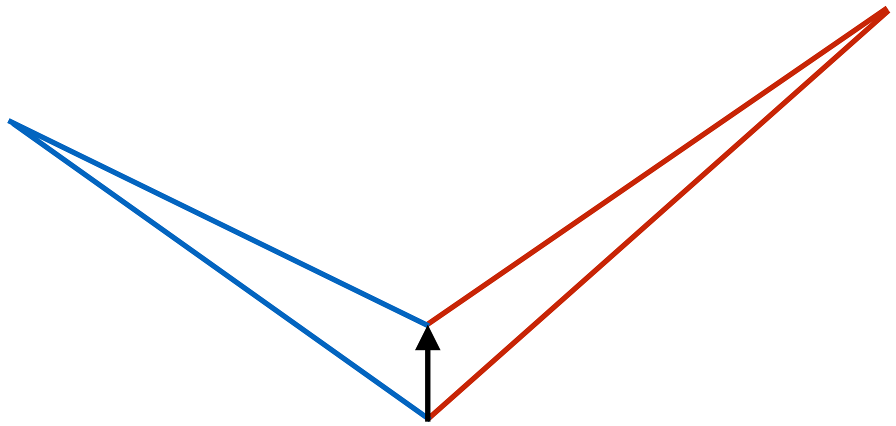
Need **boundary conditions** to solve this ODE:

I. Absence of singularity in the **folded limit:**



= regular

II. Correct normalization in the **collapsed limit:**



= 3pt x 3pt

Seed Solution

The explicit solution for the seed function is

$$(uv)^{\frac{1}{2}\pm iM}\ {}_2F_1\left[\begin{array}{c}\frac{1}{4}\pm iM,\ \frac{3}{4}\pm iM\\ 1\pm iM\end{array}\middle|\,u^2\right]{}_2F_1\left[\begin{array}{c}\frac{1}{4}\pm iM,\ \frac{3}{4}\pm iM\\ 1\pm iM\end{array}\middle|\,v^2\right]$$

$$\downarrow$$

$$\boxed{F=\sum_{m,n}c_{mn}(M)\,u^{2m+1}\left(\frac{u}{v}\right)^{n}+\frac{\pi}{\cosh(\pi M)}\,g(u,v)}$$

$$\uparrow$$

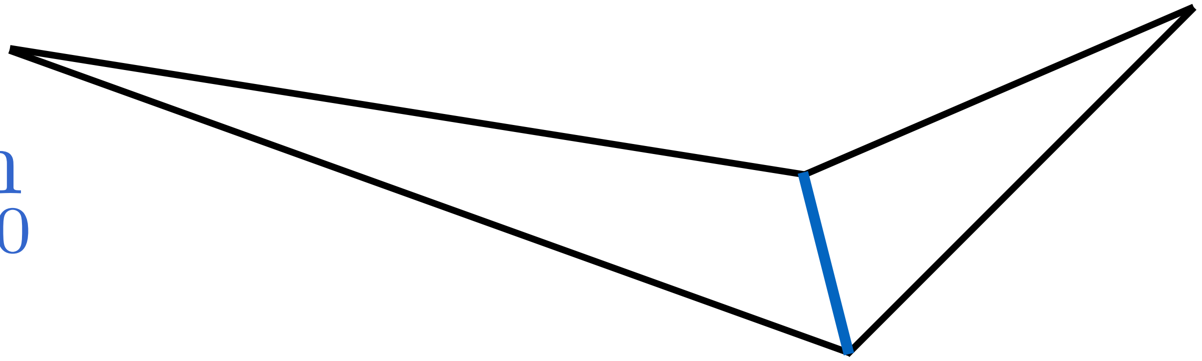
$$F^{2|1|3}_{2|0|1}\left[\begin{array}{c}\frac{1}{2},1\\ \frac{5+2iM}{4},\frac{5-2iM}{4}\end{array}\middle|\begin{array}{c}1\\ -\end{array}\middle|\begin{array}{c}\frac{5+2iM}{4},\frac{5-2iM}{4},\frac{1}{2}+iM\\ \frac{3}{2}+iM\end{array}\middle|\,u^2,\frac{u^2}{v^2}\right]$$

Arkani-Hamed, DB, Lee and Pimentel [2018]

The Collapsed Limit

In the collapsed limit, the solution oscillates:

$$\lim_{s\to 0}$$



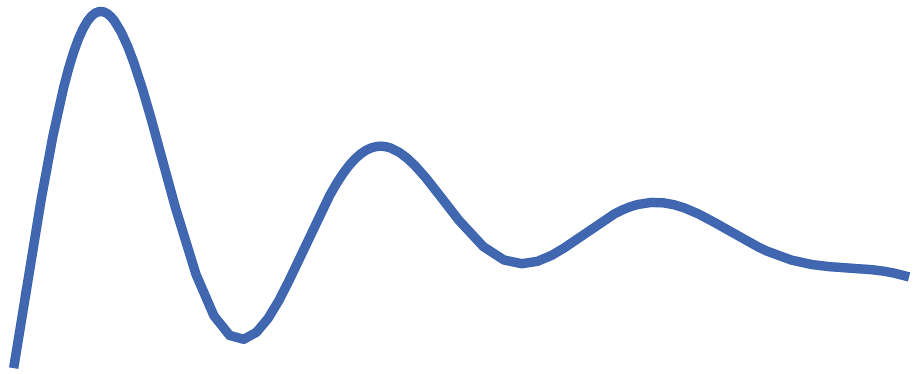
$$= \sin[M \log(s/k_{12})]$$

Noumi, Yamaguchi and Yokoyama [2013]
Arkani-Hamed and Maldacena [2015]
Arkani-Hamed, DB, Lee and Pimentel [2018]

Particle Production

These oscillations are a key signature of **particle production** during inflation:

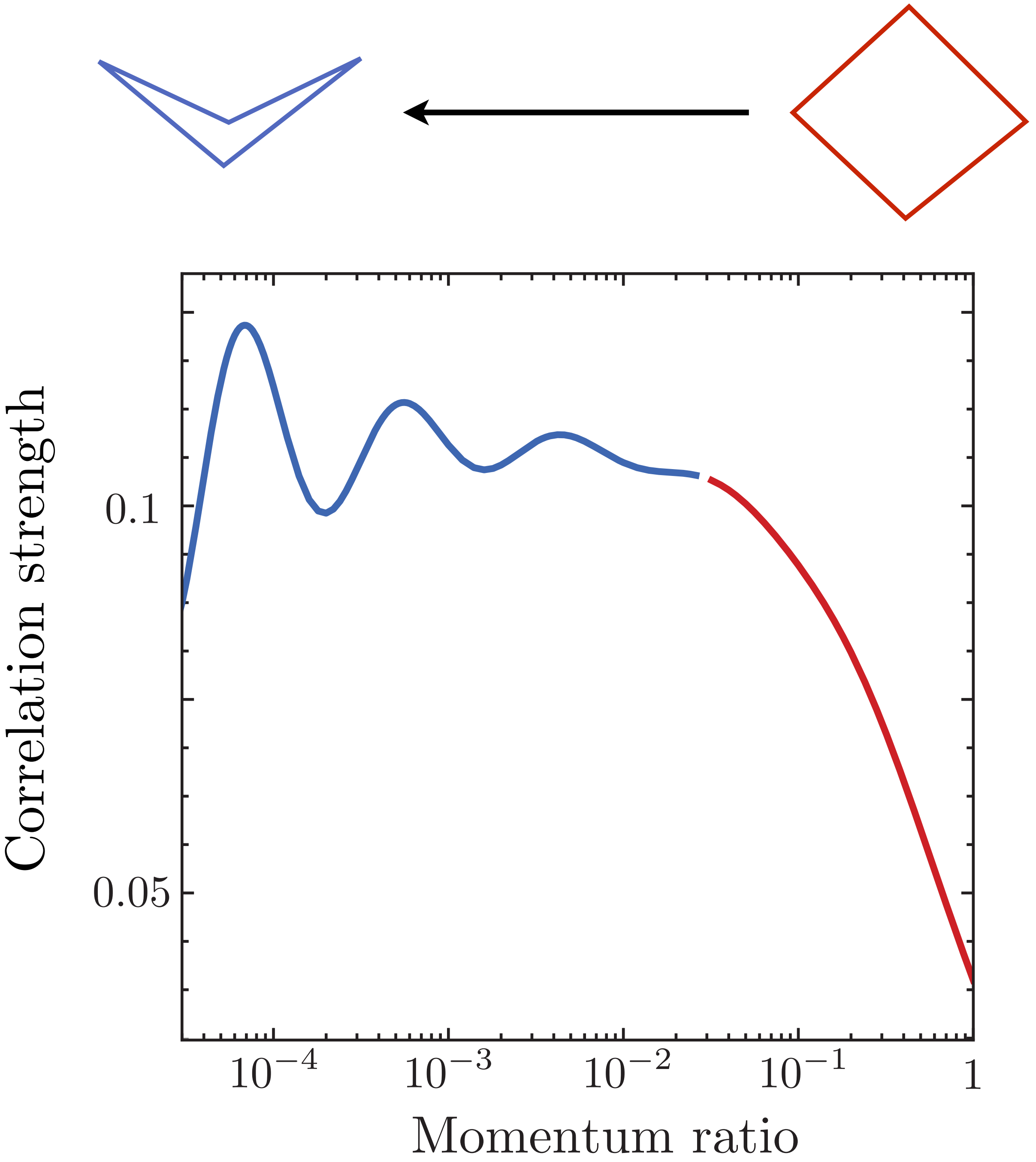
$$e^{iMt} \Rightarrow$$

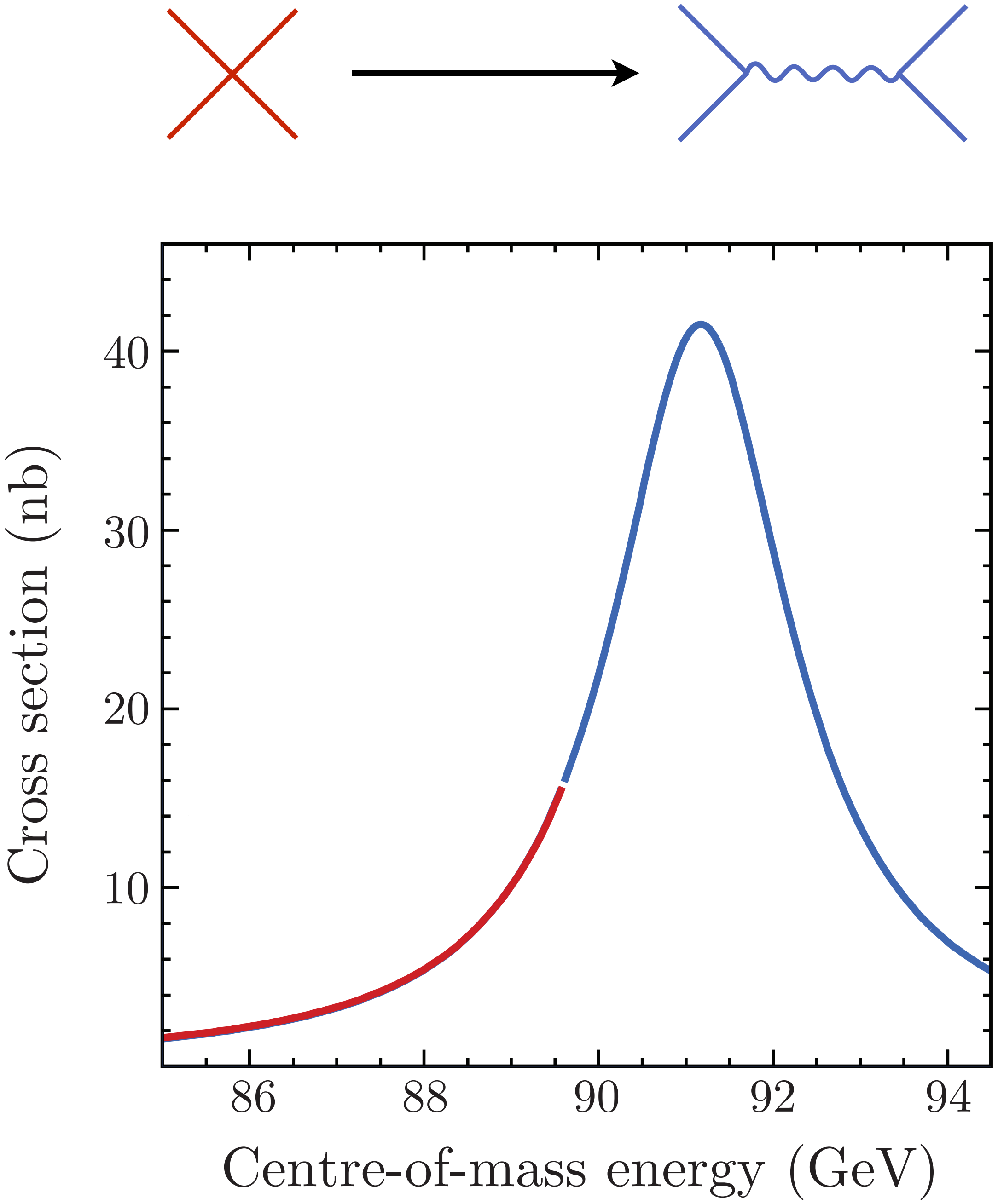


Oscillations in the superhorizon evolution become oscillations in the boundary correlations at late times.

Cosmological Collider Physics

This signal is the analog of **resonances** in collider physics:





Any Questions?

II. New Developments

So far, we have studied the correlations of scalar fields.

Arkani-Hamed, DB, Lee and Pimentel [2018]
DB, Duaso Pueyo, Joyce, Lee and Pimentel [2019]

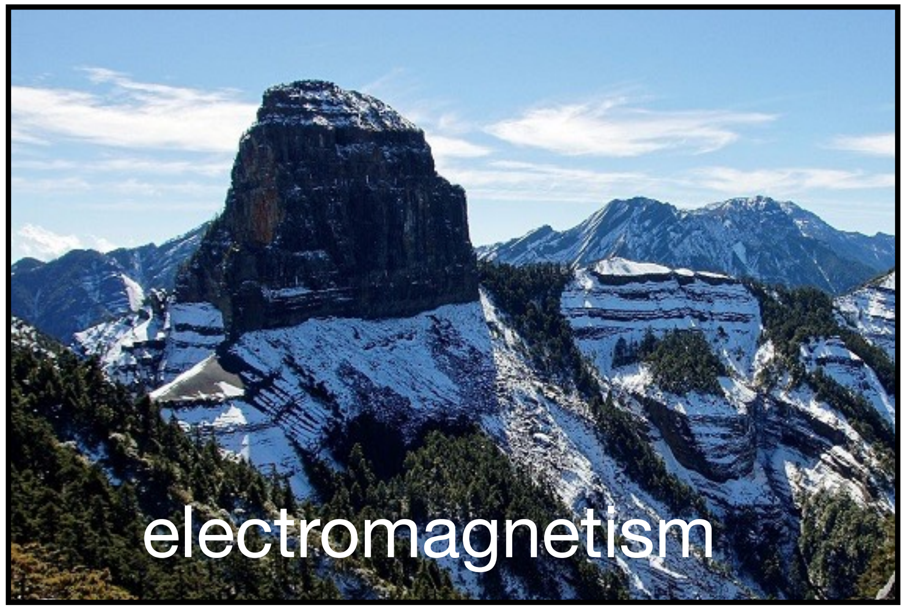
Now, we would like to extend the bootstrap to **spinning correlators**, especially to **massless** fields with spin.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Massless Particles in Flat Space

- Massless bosons mediate long-range forces:

gravity



electromagnetism

- The interactions of massless particles are highly constrained:

spin 2 = **GR**

spin 1 = **YM**

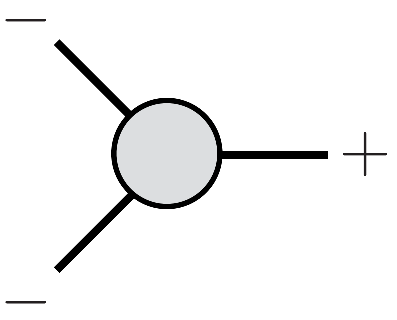
Beyond Feynman Diagrams

- Computations using Feynman diagrams are complicated.
- Physical answers are simple.

Parke and Taylor [1985]
De Witt [1967]

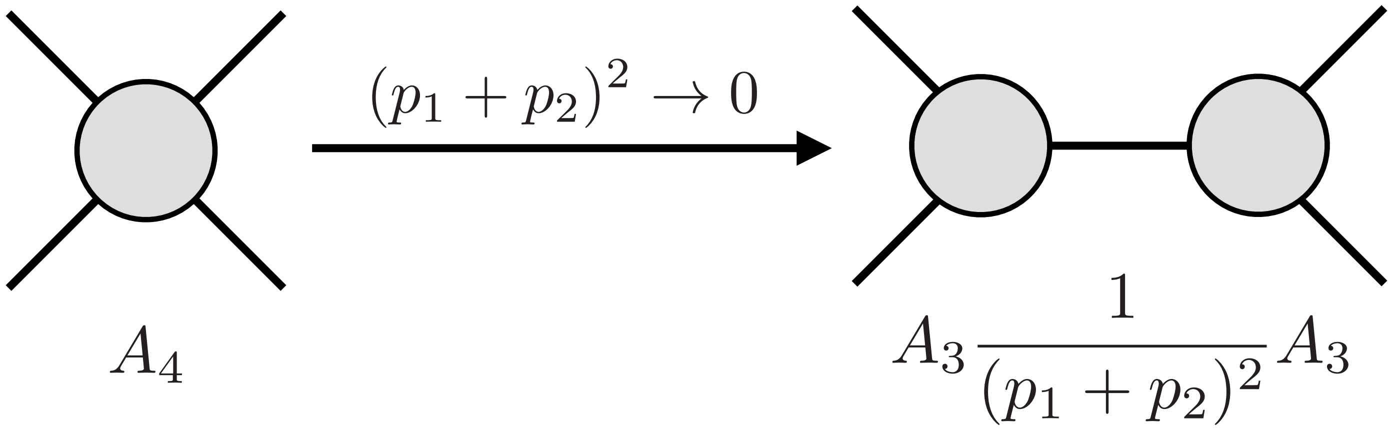
- Bootstrap methods are a necessity, not a luxury:

  - Massless 3pt amplitudes are fixed by Poincare invariance:



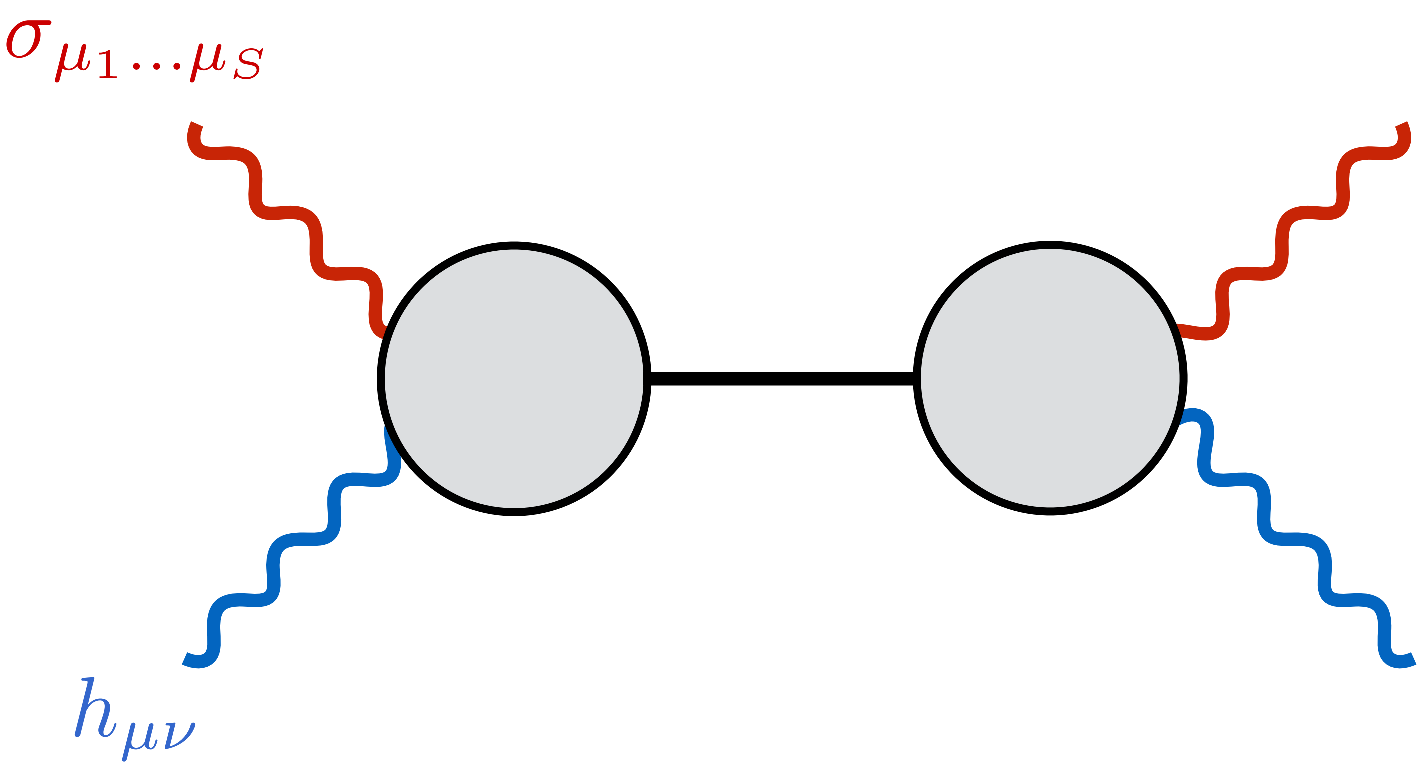
$$= \left(\frac{\langle 12\rangle^3}{\langle 13\rangle\langle 23\rangle}\right)^S$$

  - Higher-point amplitudes are constrained by locality:



The Four-Particle Test

- Consistent factorisation is a nontrivial constraint:



- Only consistent for spins $S = \{\, 0 \,,\, \frac{1}{2} \,,\, \underset{\text{YM}}{\underset{\uparrow}{1}} \,,\, \underset{\text{SUSY}}{\underset{\uparrow}{\frac{3}{2}}} \,,\, \underset{\text{GR}}{\underset{\uparrow}{2}} \,\}$

Benincasa and Cachazo [2007]
McGady and Rodina [2010]

Massless Particles in de Sitter Space

- Fluctuations of all massless fields are amplified during inflation.
- Every inflationary model has two massless modes:

$\boxed{\phi}$ scalar

$\boxed{\gamma_{ij}}$ tensor

- Not much is known about tensor correlators beyond 3pt functions.
- Even less is known about the consistency of partially massless fields:

$\boxed{\Sigma_{i_1 \ldots i_S}}$ partially massless

Beyond Feynman Diagrams

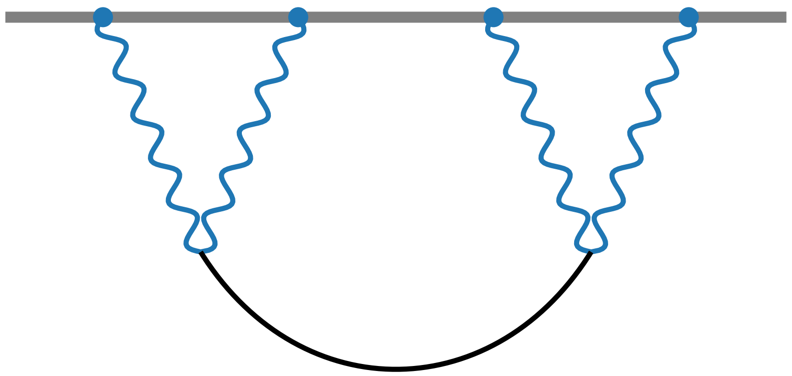
- Direct computations of spinning correlators are very complicated.

- Bootstrap methods are a necessity, not a luxury.

Two Approaches

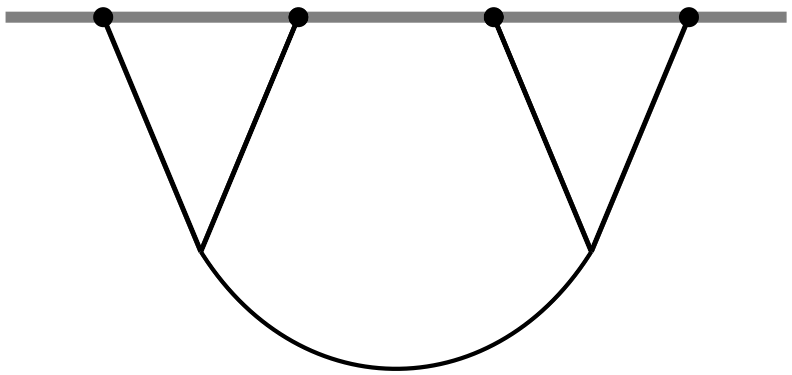
In our new paper, we derive a large class of spinning correlators in de Sitter space. We use two different approaches:

1) **Spin-raising operators**



spinning correlator

$$= \sum_n \mathcal{S}_n$$



scalar seed

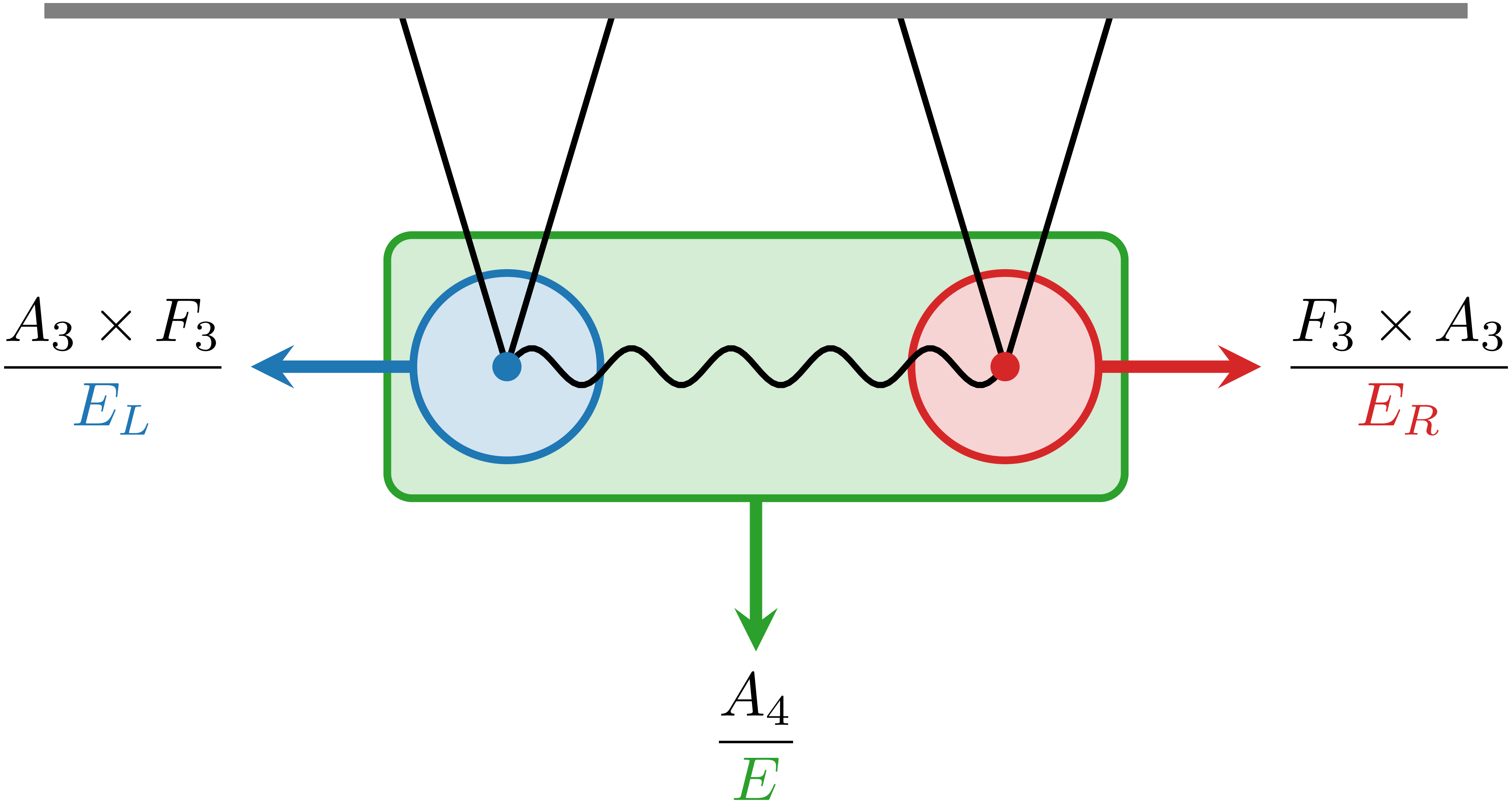
2) **Singularities**

In the following, I will describe the second approach.

DB, Duaso Pueyo, Joyce, Lee and Pimentel [2020]

Singularities of Cosmological Correlators

The four-point function is controlled by **three** singularities:



Singularities occur when energies add up to zero.

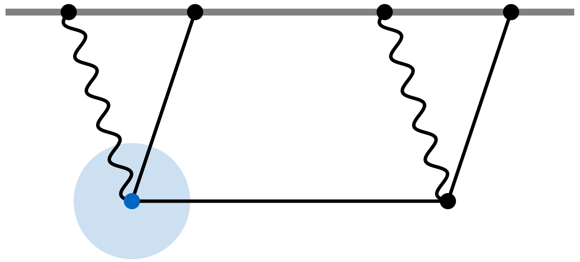
Raju [2012]
Maldacena and Pimentel [2011]
Arkani-Hamed, Benincasa, and Postnikov [2017]

A Simple Example

Consider **Compton scattering** in de Sitter space.

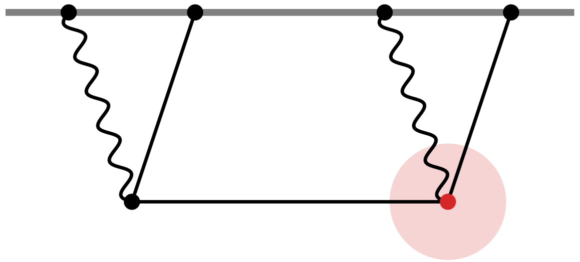
- The factorisation limits of the s-channel are

$$\lim_{E_L\to 0}$$

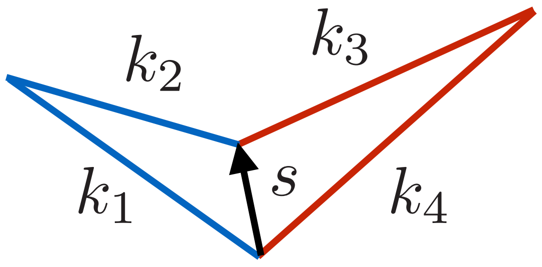


$$= \frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L}\,\frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R(k_{34}-s)}$$

$$\lim_{E_R\to 0}$$



$$= \frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R}\,\frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L(k_{12}-s)}$$



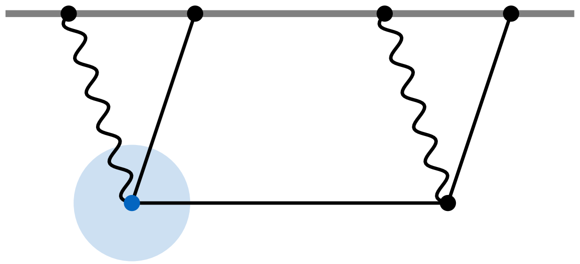
$$E_L \equiv k_{12}+s$$
$$E_R \equiv k_{34}+s$$
$$E \equiv k_{12}+k_{34}$$

A Simple Example

Consider **Compton scattering** in de Sitter space.

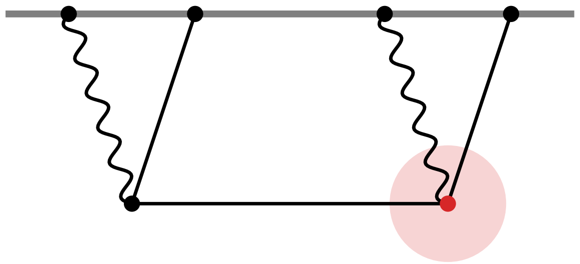
- The factorisation limits of the s-channel are

$$\lim_{E_L\to 0}$$

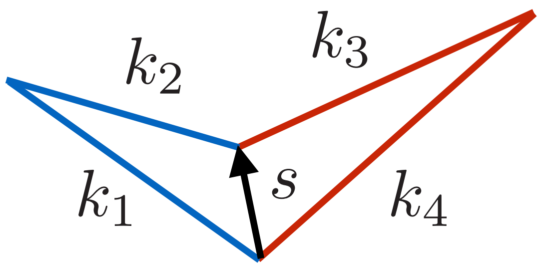


$$= \frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L}\,\frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R(k_{34}-s)}$$

$$\lim_{E_R\to 0}$$



$$= \frac{\vec{\xi}_3\cdot\vec{k}_4}{E_R}\,\frac{\vec{\xi}_1\cdot\vec{k}_2}{E_L(k_{12}-s)}$$



$$E_L \equiv k_{12}+s$$
$$E_R \equiv k_{34}+s$$
$$E \equiv k_{12}+k_{34}$$

- The unique solution that is consistent with these limits is

$$\boxed{\langle J\phi J\phi\rangle_s = \frac{(\vec{\xi}_1\cdot\vec{k}_2)(\vec{\xi}_3\cdot\vec{k}_4)}{E_L E_R E}}$$

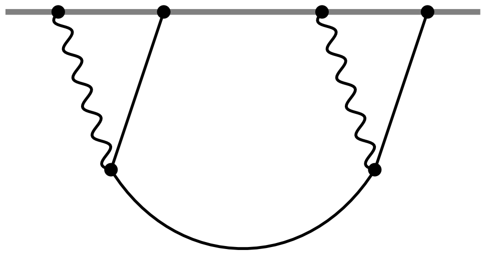
- The total energy singularity has the correct residue.

$$E_L E_R \xrightarrow{E\to 0} S$$

A More Complicated Example

Consider Compton scattering of **gravitons**.

- The solution in the s-channel is



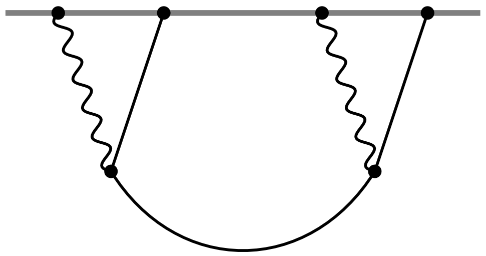
$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \Bigg[\frac{1}{E_L^2 E_R^2} \overbrace{\left(\frac{2sk_1k_3}{E^2} + \frac{2k_1k_3 + E_L k_3 + E_R k_1}{E} \right)}^{\text{fixed by factorisation}}$$

$$\frac{1}{E_L E_R} \left(\underbrace{\frac{2k_1k_3}{E^3}}_{\text{fixed by total energy singularity}} + \underbrace{\frac{k_{13}}{E^2} + \frac{1}{E}}_{\text{fixed by conformal symmetry}} \right) \Bigg]$$

A More Complicated Example

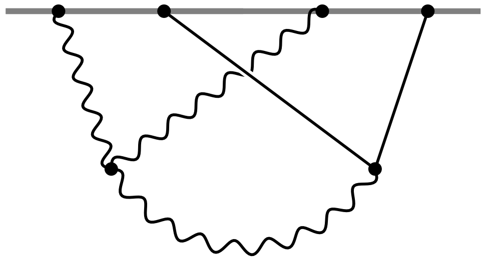
Consider Compton scattering of **gravitons**.

- The solution in the s-channel is



$$= (\vec{\xi}_1 \cdot \vec{k}_2)^2 (\vec{\xi}_3 \cdot \vec{k}_4)^2 \left[\frac{1}{E_L^2 E_R^2} \left(\frac{2sk_1k_3}{E^2} + \frac{2k_1k_3 + E_Lk_3 + E_Rk_1}{E} \right) \right.$$
$$\left. \frac{1}{E_LE_R} \left(\frac{2k_1k_3}{E^3} + \frac{k_{13}}{E^2} + \frac{1}{E} \right) \right]$$

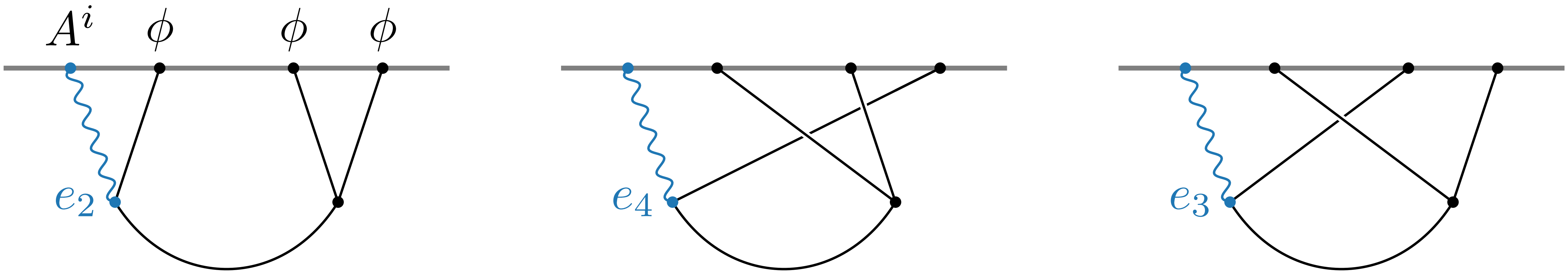
- The solution in the u-channel is



$$= \frac{1}{E_L^2 E_R^2} \overbrace{\left(\frac{2k_1k_3}{E^2} + \frac{E_L}{E} \right) \mathcal{N}(\vec{\xi}_1, \vec{\xi}_3, \vec{k}_2, \vec{k}_4)}^{\text{fixed by factorisation}}$$
$$+ \frac{1}{E_LE_R} \left(\underbrace{\frac{2k_1k_3}{E^3}}_{\text{fixed by total energy singularity}} + \underbrace{\frac{k_{13}}{E^2} + \frac{1}{E}}_{\text{fixed by conformal symmetry}} \right) \mathcal{M}(\vec{\xi}_1, \vec{\xi}_3, \vec{k}_2, \vec{k}_4)$$

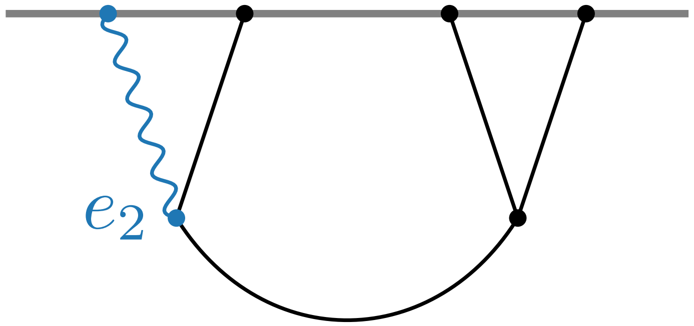
One Channel Is Not Enough

Consider the correlator of one photon and three scalars:



s-channel t-channel u-channel

- Can we have $e_3 = e_4 = 0$?



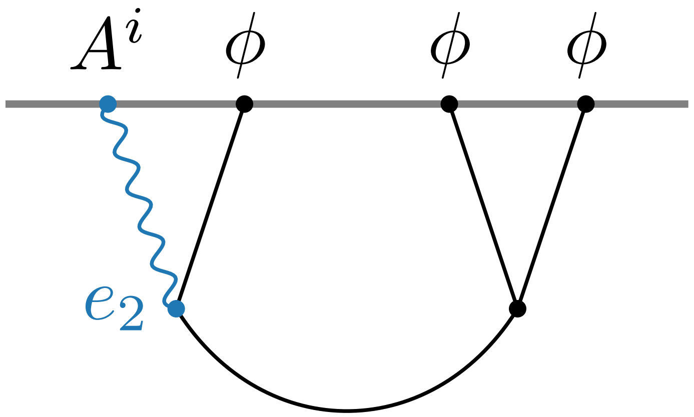
$$\xrightarrow{E\to 0} \frac{e_2}{E}\left(\frac{\langle 12\rangle\langle\bar{2}\bar{4}\rangle\langle 41\rangle}{ST} - \frac{\langle 14\rangle\langle\bar{4}1\rangle}{2k_1}\frac{1}{T}\right)$$

flat-space amplitude

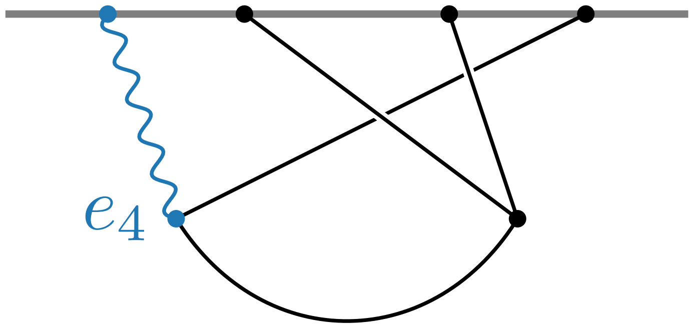
not Lorentz-invariant

One Channel Is Not Enough

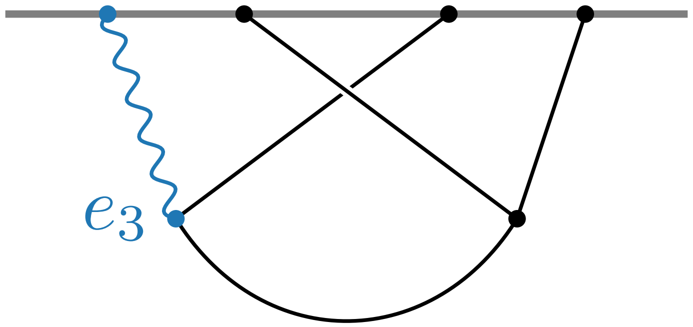
Consider the correlator of one photon and three scalars:



s-channel



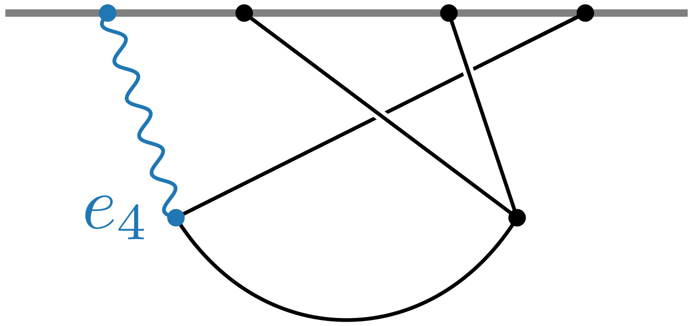
t-channel



u-channel

- Let $e_4 \neq 0$:

$+$

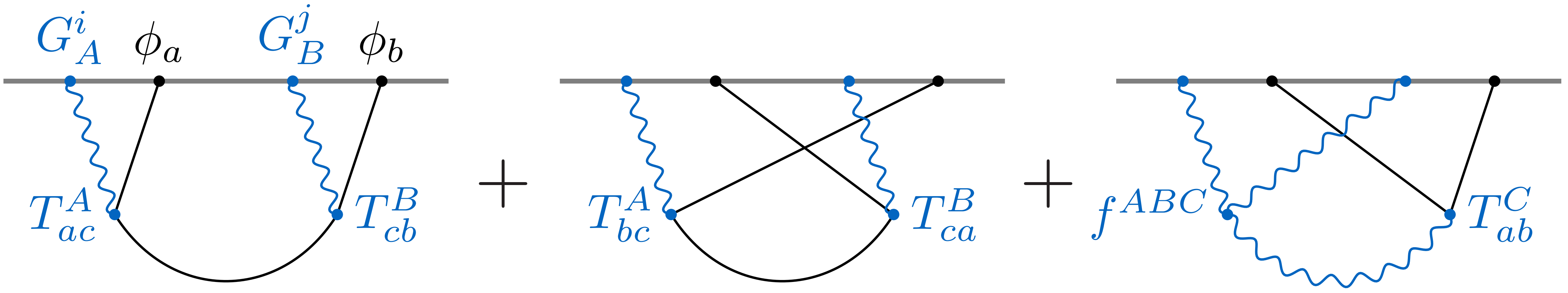


$$\xrightarrow{E\to 0} \frac{1}{E}\left(e_2\frac{\langle 12\rangle\langle\bar{2}\bar{4}\rangle\langle 41\rangle}{ST} - (e_2+e_4)\frac{\langle 14\rangle\langle\bar{4}1\rangle}{2k_1}\frac{1}{T}\right)$$

Lorentz-violation disappears when $\boxed{e_2 + e_4 = 0}$ **charge conservation**

Discovering Yang-Mills

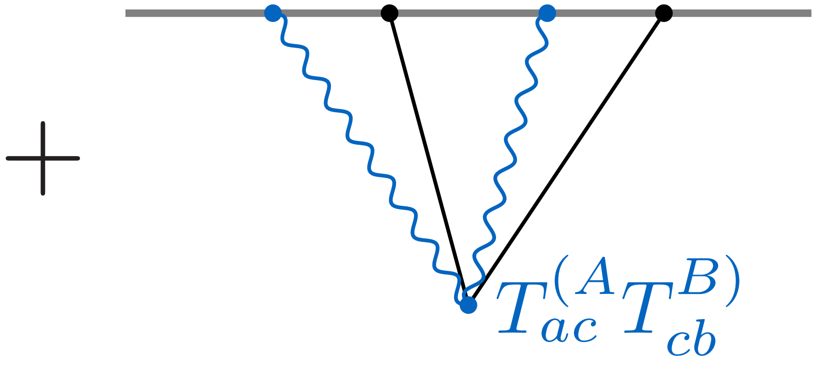
Consider two gluons and two scalars:



s-channel t-channel u-channel

- The sum of all channels is only consistent if the couplings satisfy the **Lie algebra**:

$$\boxed{[T^A, T^B]_{ab} = f^{ABC}T^C_{ab}}$$

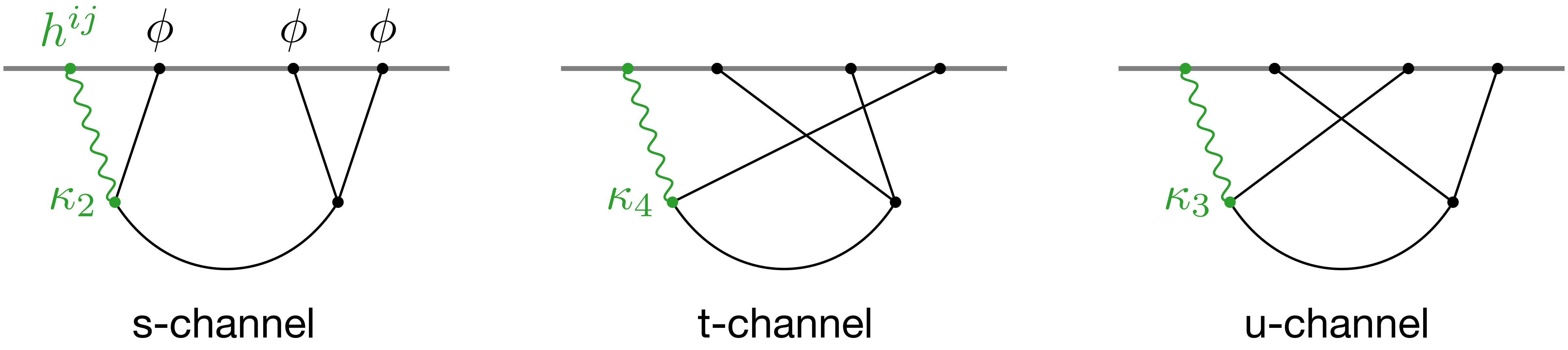


contact

- Consistency also fixes the contact term required by gauge invariance.

Equivalence Principle (without falling elevators)

Consider one graviton and three scalars:

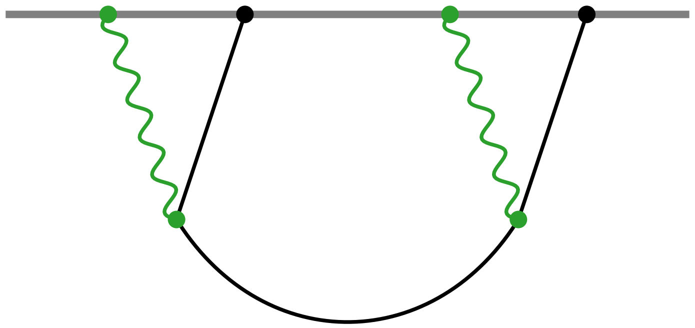


- The individual channels are not consistent.
- The sum of all channels is consistent if and only if

$$\boxed{\kappa_2 = \kappa_3 = \kappa_4}$$

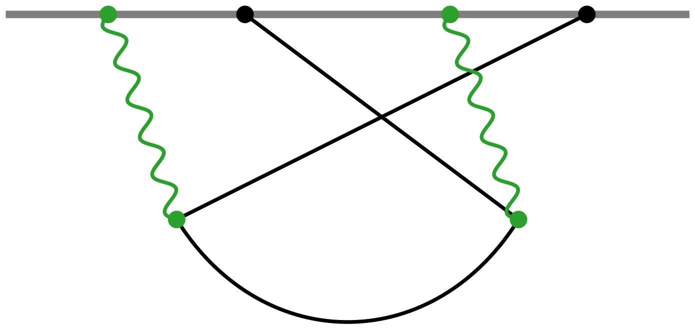
Equivalence Principle (without falling elevators)

Consider two gravitons and two scalars:



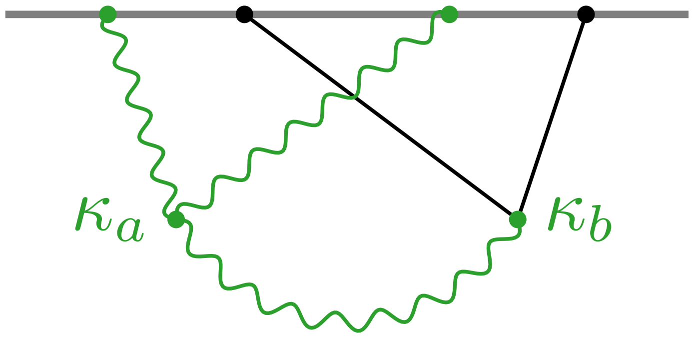
s-channel

+



t-channel

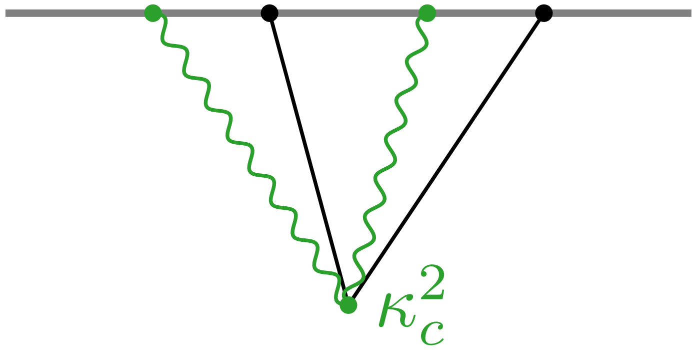
+



u-channel

- The sum of all channels is only consistent if all gravitational couplings are **universal**:

+



contact

$$\boxed{\kappa_a = \kappa_b = \kappa_c}$$

Ruling Out Theories

The bootstrap approach will also allow us to rule out theories:

- Couplings of massless gravitinos will **not** be supersymmetric.
- Couplings of higher-spin particles will **not** be local.
- Multiple gravitons must be decoupled.
- Interactions of partially massless particles will be highly constrained.
- …

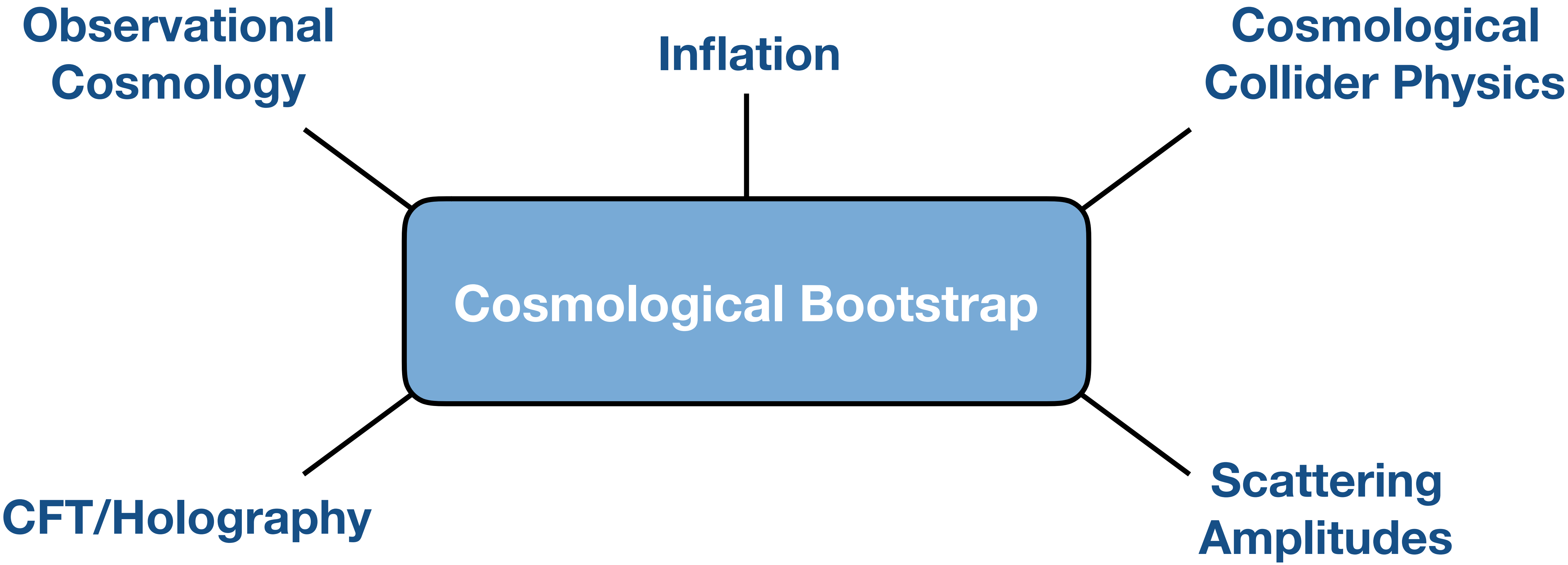
We hope to report on such no-go results in the future.

Any Questions?

III.

Future Challenges

We have only scratched the surface of a fascinating subject:



Observational Cosmology

Inflation

Cosmological Collider Physics

Cosmological Bootstrap

CFT/Holography

Scattering Amplitudes

Much more remains to be discovered.

Open Problems

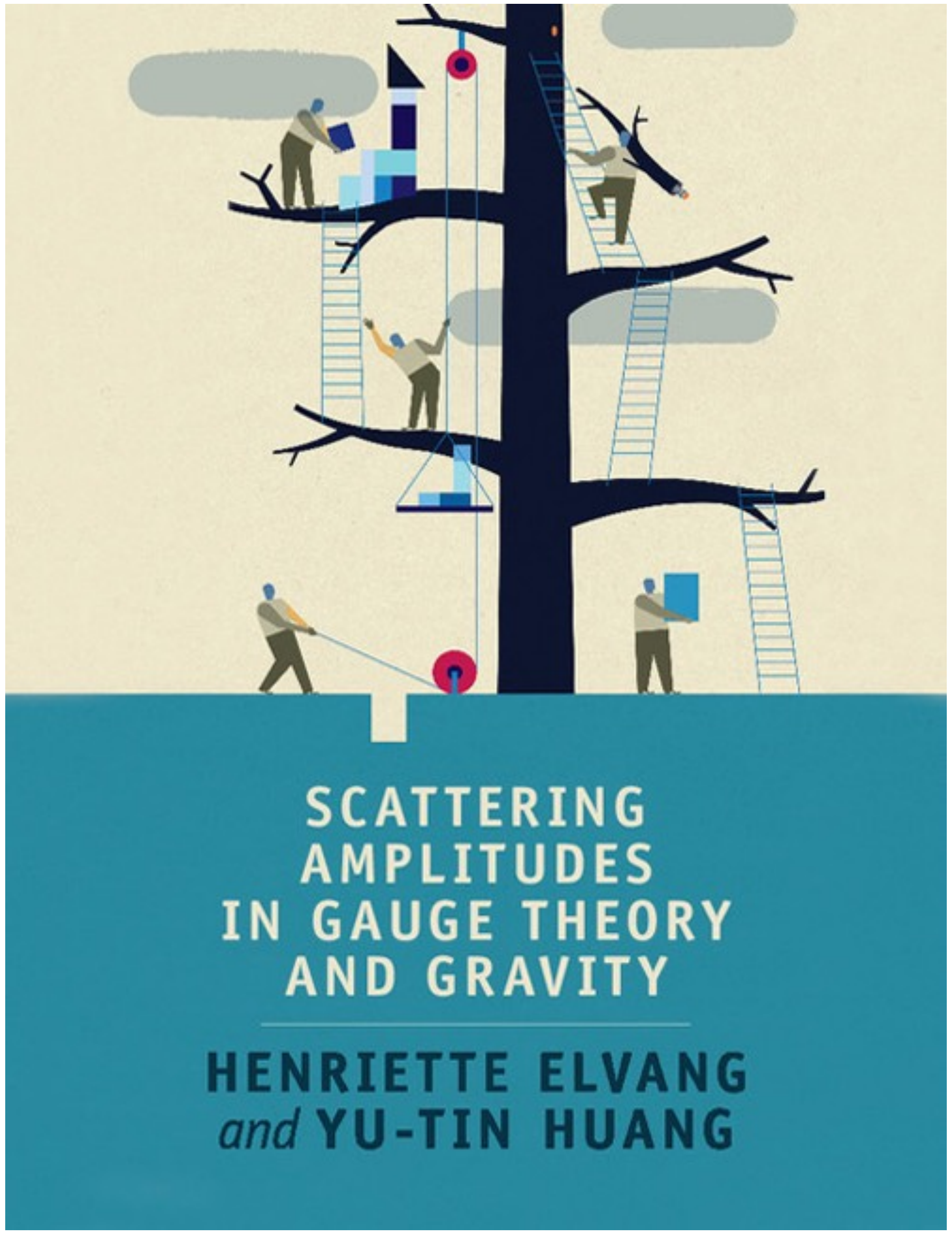
- **Beyond Feynman Diagrams**
    - What is the on-shell formulation of cosmological correlators?
    - What are the fundamental building blocks?
    - How are these building blocks connected?
    - Is there a cosmological analog of Parke-Taylor?
    - Where is the hidden simplicity?

- **Ultraviolet Completion**
    - What are the rules?
    - How is unitarity encoded in the boundary correlators?
    - Are there interesting positivity constraints?
    - How does this constrain the space of consistent correlators?
    - Does this motivate new observational targets?

Thank you for your attention!

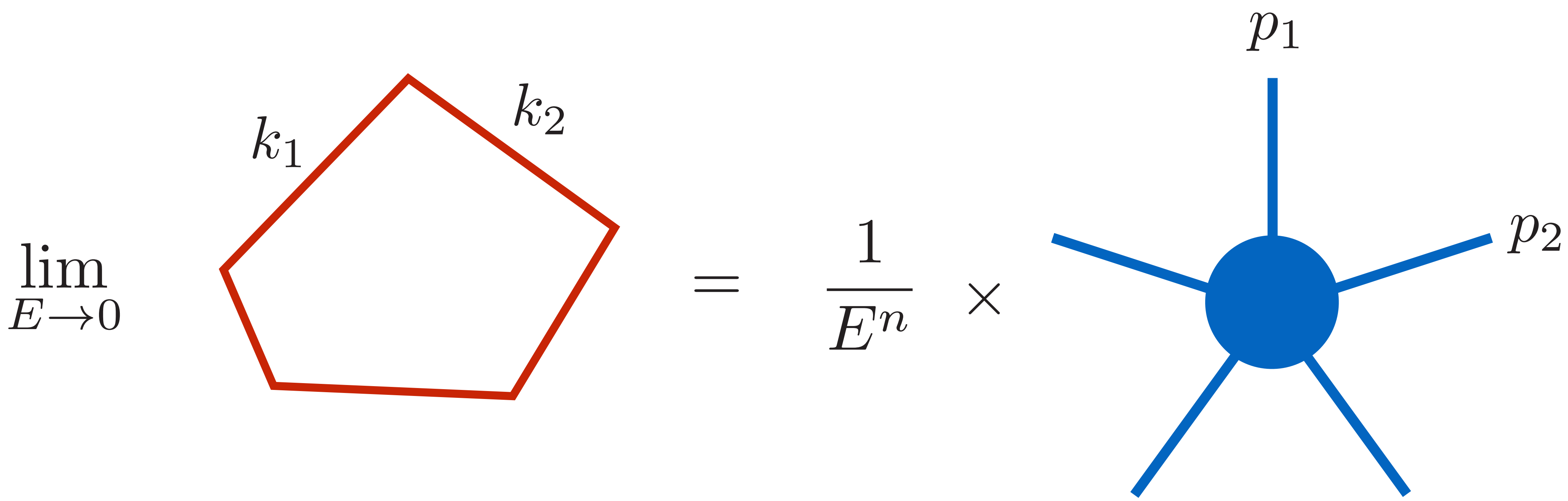
Dramatic progress in the study of **scattering amplitudes**:

- on-shell recursion relations
- color-kinematics duality
- soft theorems
- hidden positivities
- spinor helicity formalism
- generalised unitarity
- momentum twistors
- …



Do these insights translate to **cosmological correlators**?

Amplitudes *live inside* correlators.

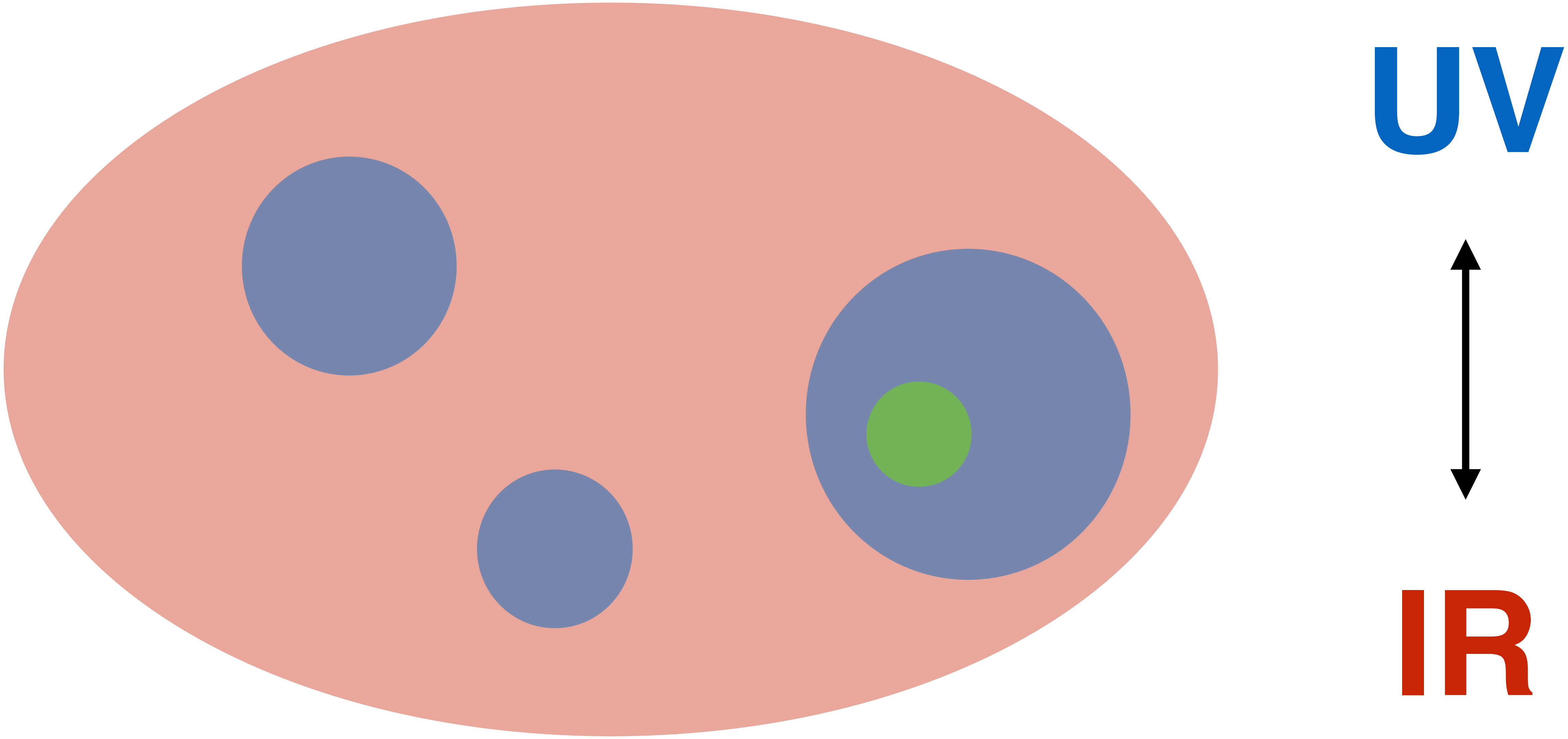


Raju [2012]
Maldacena and Pimentel [2011]

Insights from the modern scattering amplitudes programme must therefore be relevant for cosmology.

What is the goal?

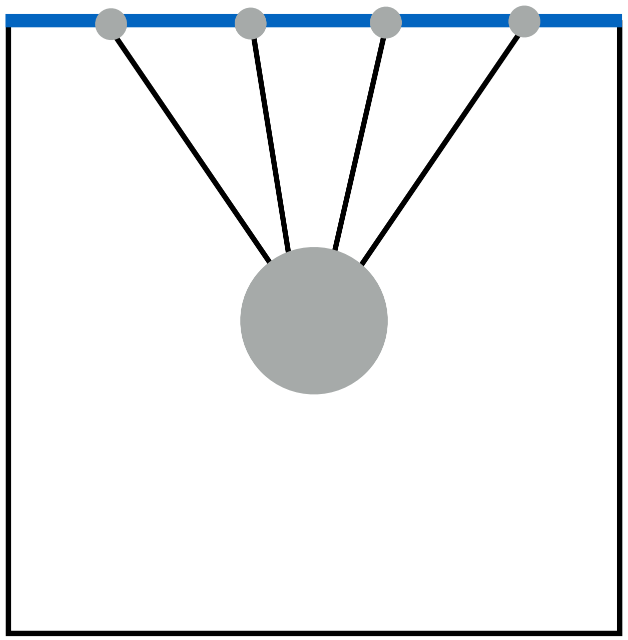
Relate low-energy predictions to high-energy physics:



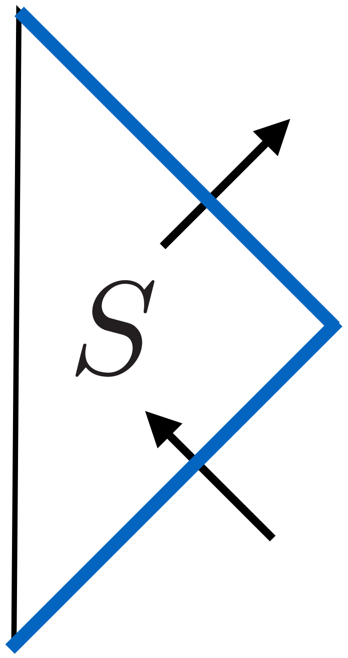
Define observational targets.

What are the rules?

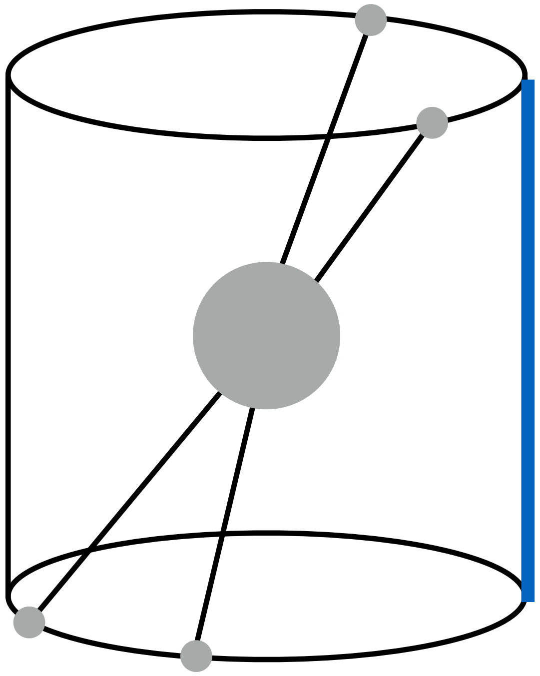
The closer we get to the real world, the less we understand:



de Sitter space



flat space



anti-de Sitter space

under-standing

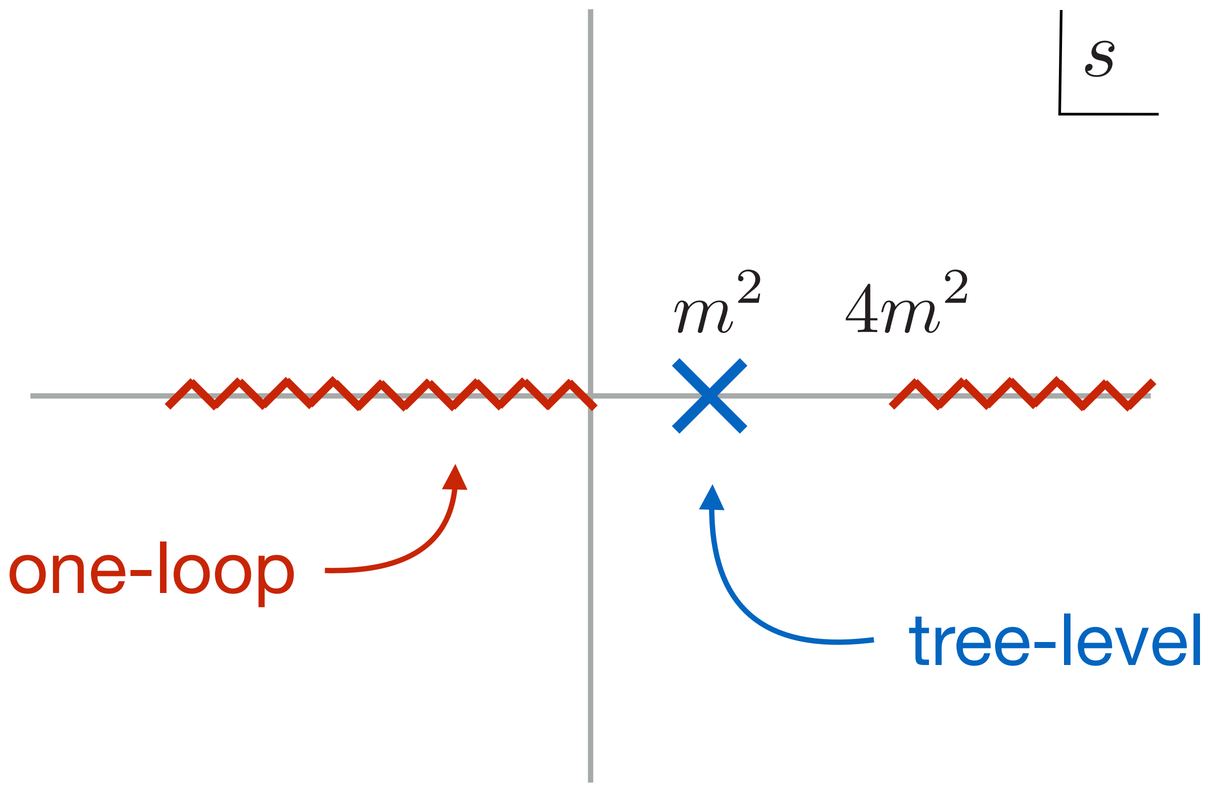
?

tree-level
(some loops)

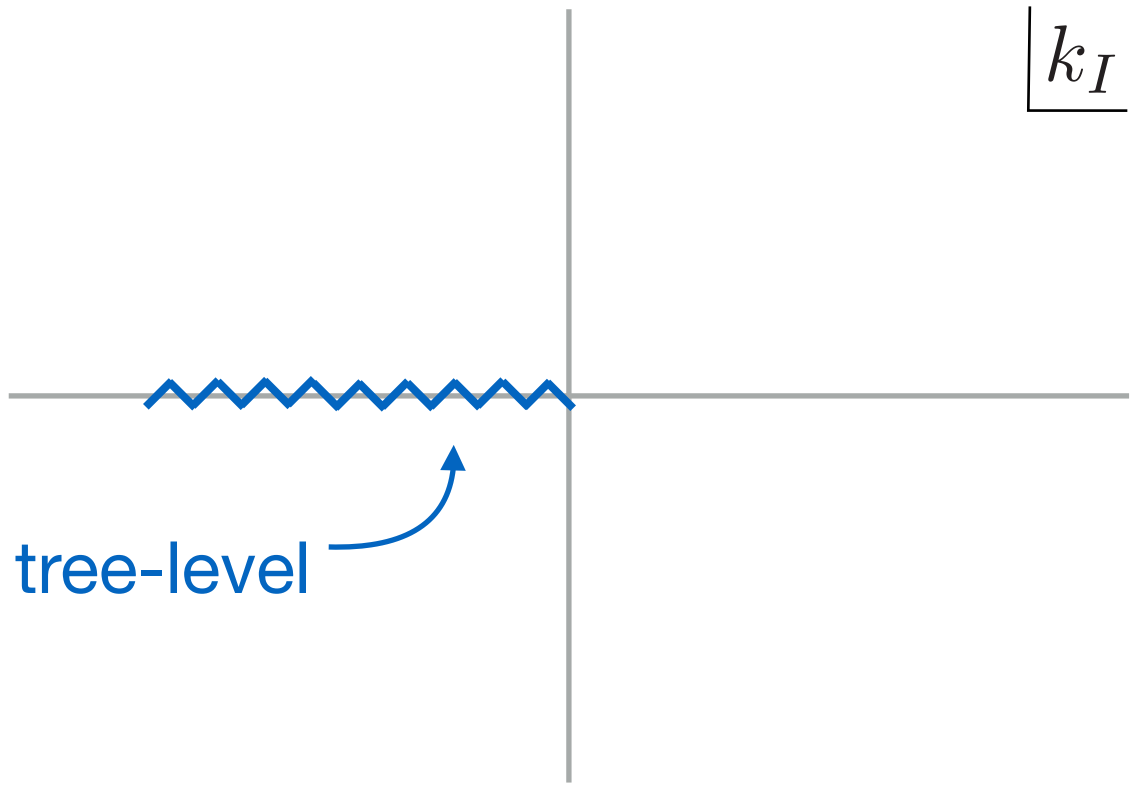
non-perturbative

Causality

Consistent time evolution is encoded in the **analytic structure** (poles and branch cuts) of amplitudes and correlators:



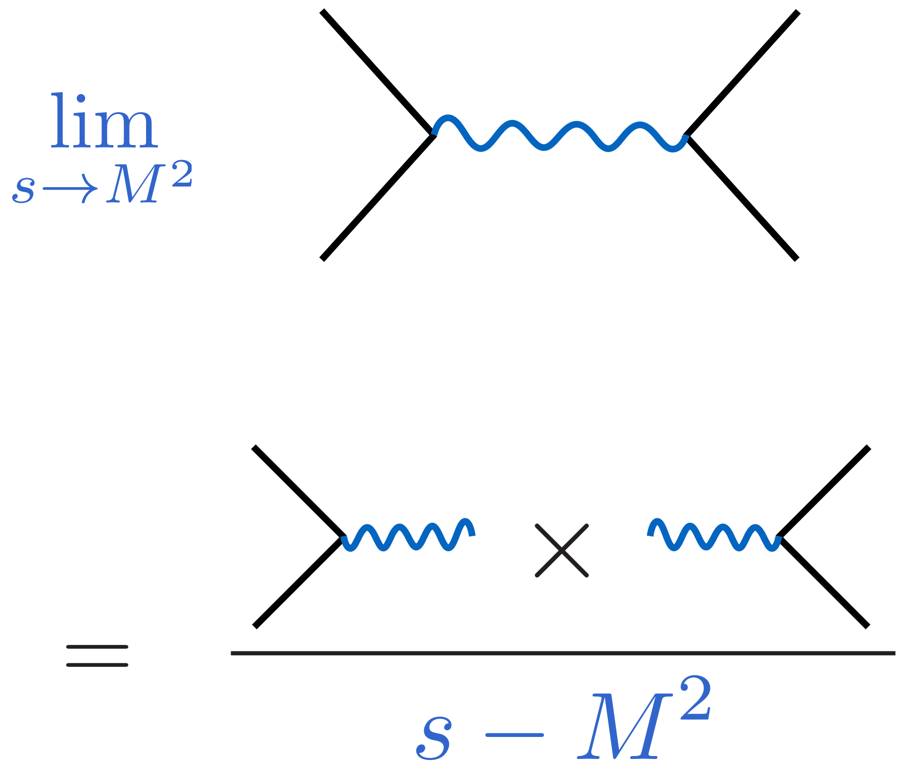
Scattering amplitudes



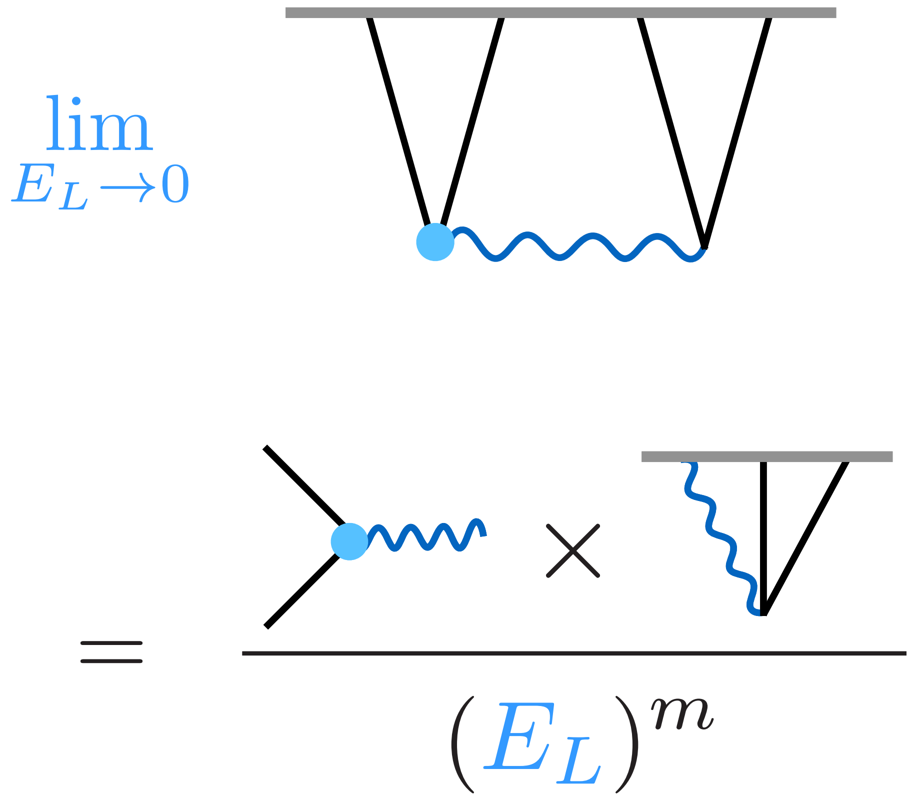
Cosmological correlators

Locality

Locality is encoded in **factorization**:



Scattering amplitudes



Cosmological correlators

Unitarity

Unitarity is encoded in **positivity**:

$$A(s, t \to 0) = \sum_n a_n s^n$$

$$a_n > 0$$

Scattering amplitudes

?

Cosmological correlators

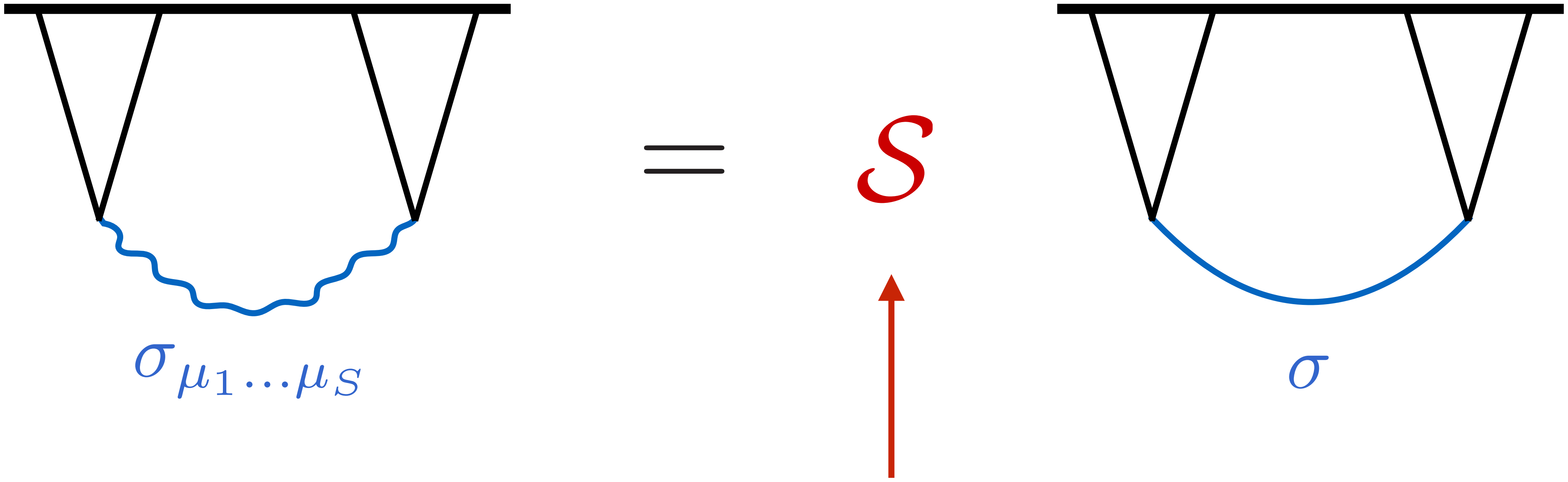
Landscape vs Swampland

The ultraviolet completion of scattering amplitudes is highly constrained by these basic physical principles:

Exchange of Spinning Particles

Strategy

We wish to find differential operators that relate scalar exchange to spin exchange:

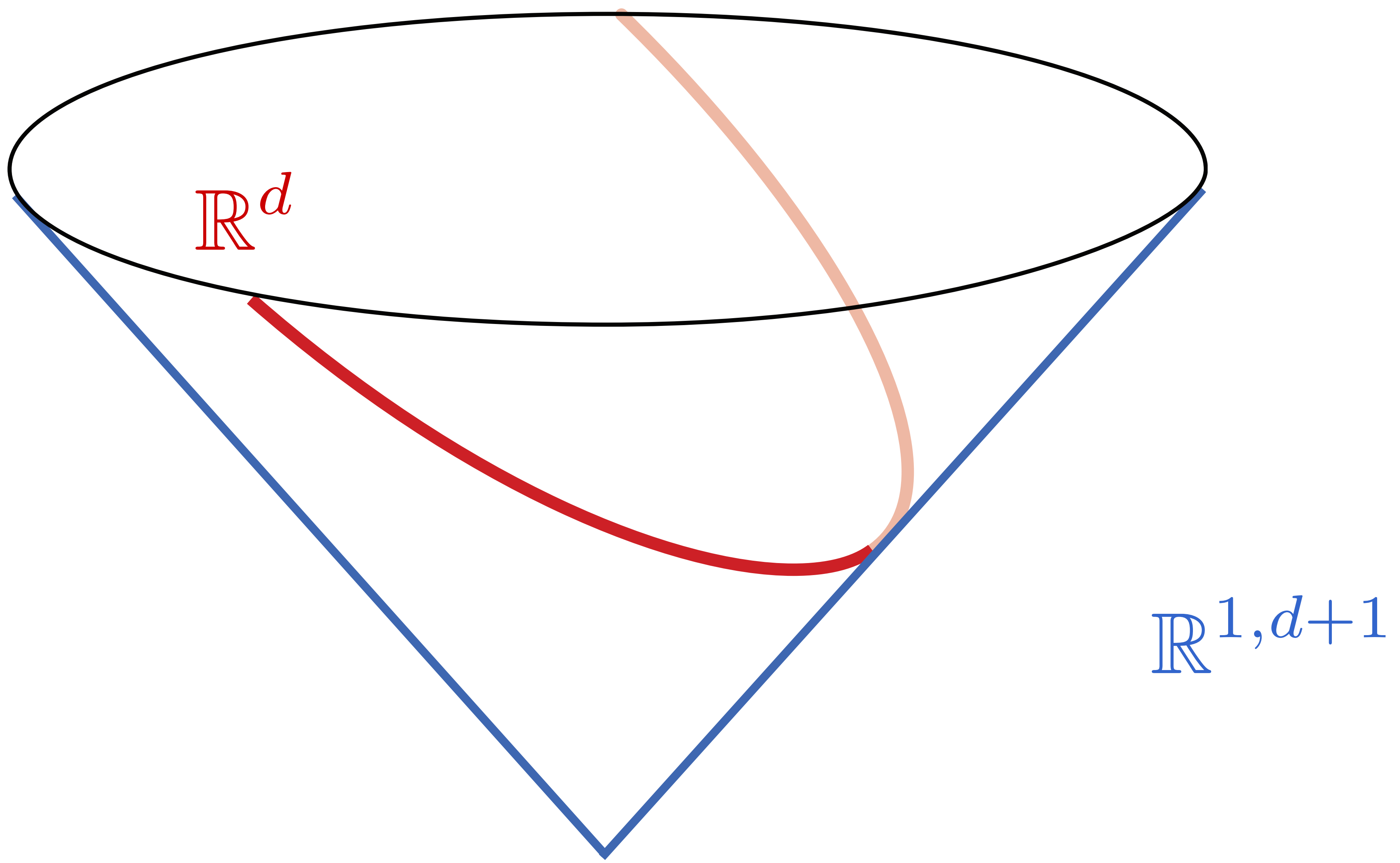


spin-raising operator

It turns out that the spin raising is best implemented in embedding space and then Fourier transformed.

CFTs in Embedding Space

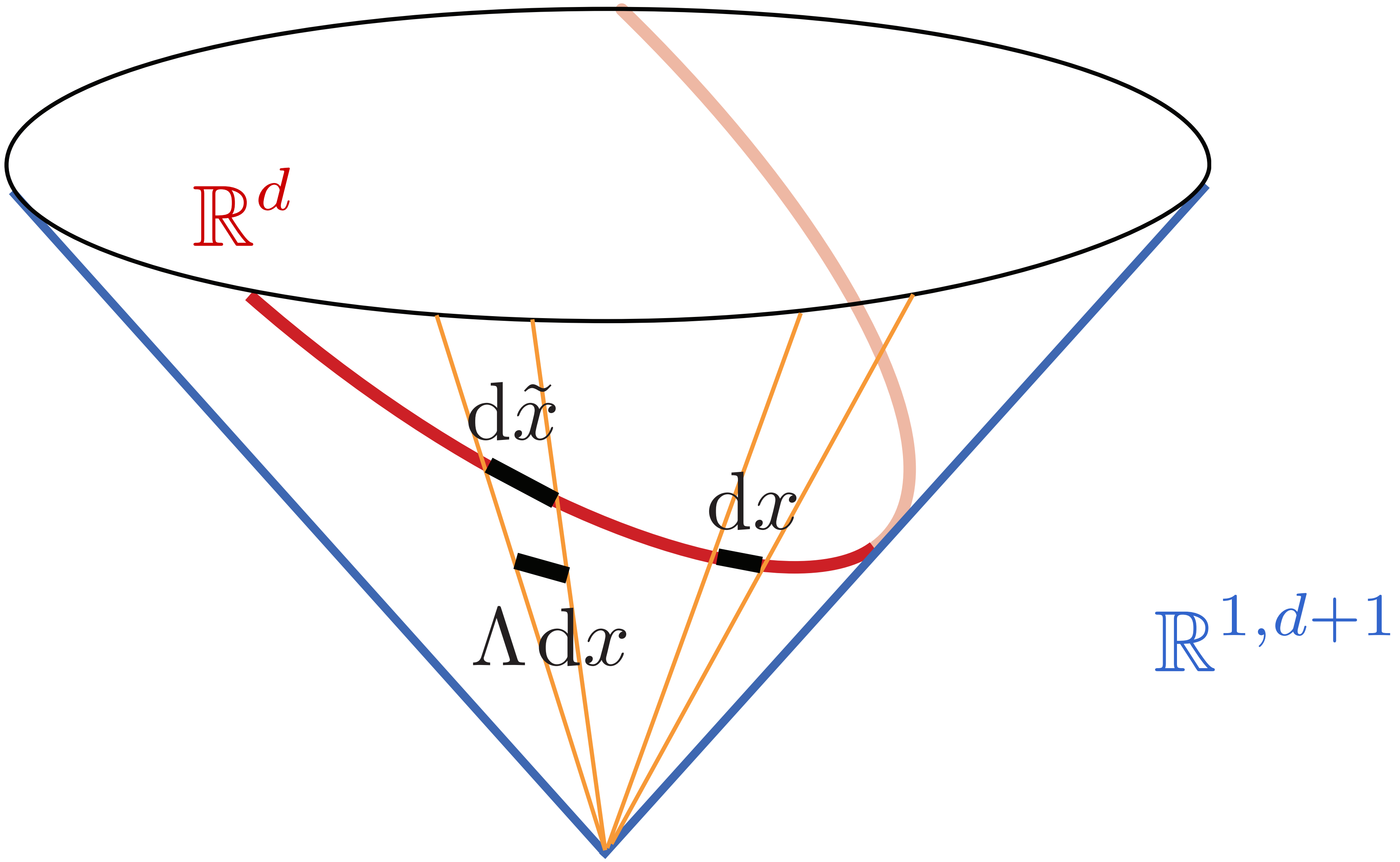
Consider the following embedding of d-dimensional Euclidean space into (d+2)-dimensional Minkowski space:



Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

CFTs in Embedding Space

Lorentz transformations in embedding space become conformal transformations on the Euclidean section:



Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

CFTs in Embedding Space

Conformal correlators in embedding space are simply the most general Lorentz-invariant expressions with the correct scaling behavior:

$$\langle \phi_1 \phi_2 \rangle = \frac{1}{X_{12}^{\Delta_1}} \, ,$$

$$\langle \phi_1 \phi_2 \phi_3 \rangle = \frac{1}{X_{12}^{(\Delta_1+\Delta_2-\Delta_3)/2} X_{23}^{(\Delta_2+\Delta_3-\Delta_1)/2} X_{31}^{(\Delta_3+\Delta_1-\Delta_2)/2}} \, ,$$

$$\langle \phi_1 \phi_2 \phi_3 \phi_4 \rangle = f(u,v) \prod_{n<m}^{4} \frac{1}{X_{nm}^{\Delta_n+\Delta_m-\Delta_t/3}} \, ,$$

where $X_{mn} \equiv X_n \cdot X_m \;\rightarrow\; (x_n - x_m)^2$.

Dirac [1936]
Costa, Penedones, Poland and Rychkov [2011]

Spin-Raising Operator

Correlators of spinning fields can be written in terms of scalar seeds. For example:

$$\langle\phi\tilde{\phi}\Sigma^M\rangle = \frac{X_1^M X_{23} - X_2^M X_{13}}{(X_{12}X_{23}X_{31})^{1/2}}\langle\phi\tilde{\phi}\Sigma\rangle \;=\; \mathcal{S}^M\langle\phi\tilde{\phi}\Sigma\rangle\,,$$

where $\boxed{\mathcal{S}^M \equiv (X_3\cdot X_2)\frac{\partial}{\partial X_3^M} - X_2^M X_3\cdot\frac{\partial}{\partial X_3}}\,.$

In Fourier space, this becomes

$$\boxed{\mathcal{S}^i \equiv (\partial_{k_3^i} - \partial_{k_2^i}) + \frac{k_3^i}{2}(\partial_{k_3^j} - \partial_{k_2^j})(\partial_{k_3^j} - \partial_{k_2^j})}\,.$$

Karateev, Kravchuk and Simmons-Duffin [2018]
Costa, Penedones, Poland and Rychkov [2011]

Bootstrapping Spin Exchange

Using this spin-raising operator, we have

$$\hat{F}_S = \sum_{\lambda=0}^{S} P^{(\lambda)}_{i_1\ldots i_S j_1\ldots j_S}\,(\mathcal{S}_L^{i_1}\cdots\mathcal{S}_L^{i_S})(\mathcal{S}_R^{j_1}\cdots\mathcal{S}_R^{j_S})\,\hat{F}_0\ ,$$

spin-exchange solution

polarization tensor

spin-raising operator

scalar-exchange solution

which can be written as

$$\boxed{\hat{F}_S = \sum_{\lambda=0}^{S} \Pi_{S,\lambda}(\text{angles})\,\mathcal{D}^{(S,\lambda)}_{uv}\,\hat{F}_0}$$

e.g. $\mathcal{D}^{(S,S)}_{uv} \equiv [(uv)^2\partial_u\partial_v]^S$

Arkani-Hamed, DB, Lee and Pimentel [2018]