RELATIONS BETWEEN TRANSPORT & CHAOS IN HOLOGRAPHIC THEORIES

Richard Davison

Heriot-Watt University

East Asian Strings Webinar, August 2020

based on 1809.01169 (with Mike Blake, Saso Grozdanov, Hong Liu)
1904.12883 (with Mike Blake, David Vegh)

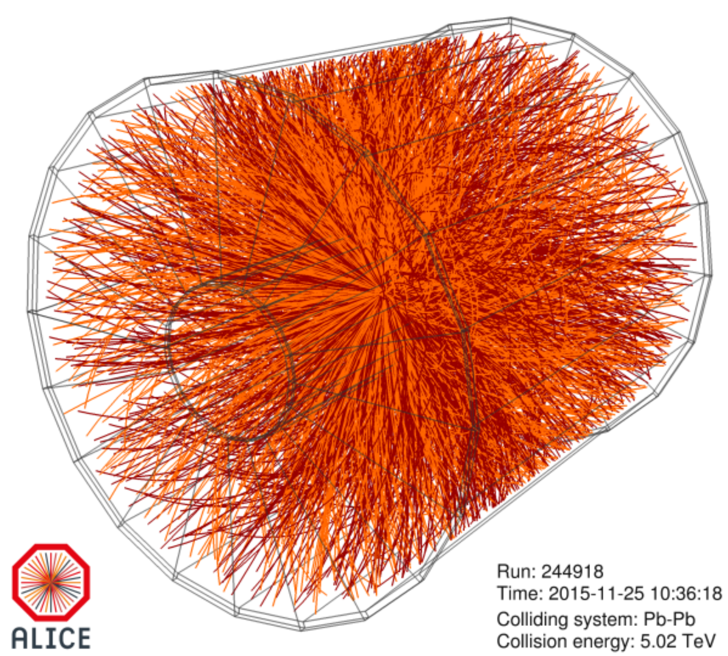
MOTIVATION

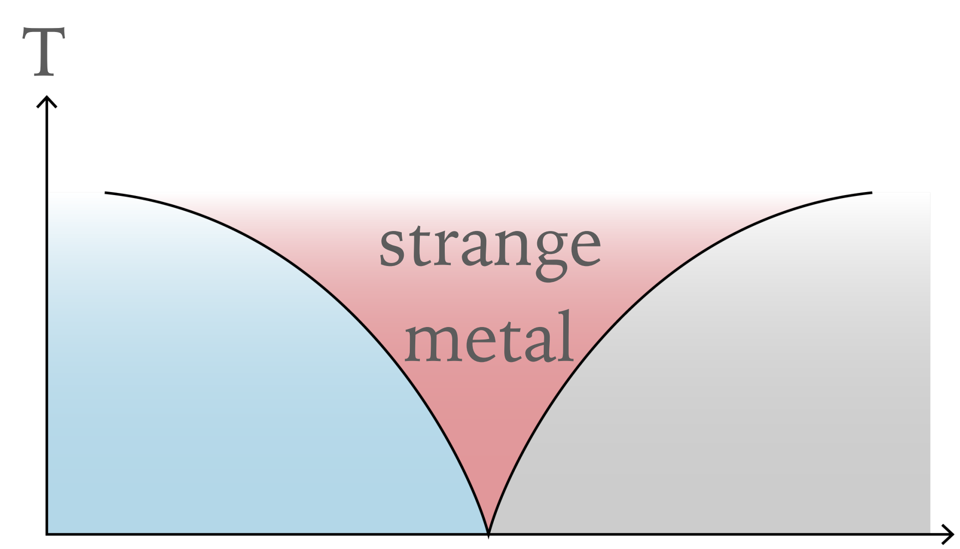
- Quantum field theories with strong interactions are important.

  Significant theoretical role in string theory / quantum gravity.

- They are also relevant to some experimentally accessible systems:

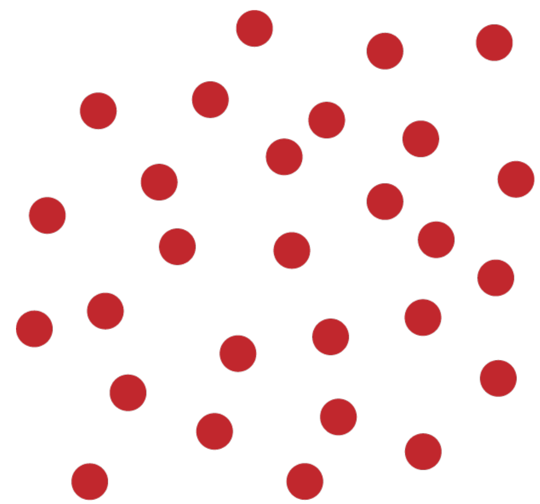
  e.g. quark-gluon plasma ‘strange’ metals





NON-QUASIPARTICLE STATES

- Cartoon of a normal metal:
  - ✳ electron-like excitations with charge e, mass m, speed v_F , lifetime τ
  - ✳ properties of these quasiparticles govern the properties of the metal



- Strange metals have properties that seem inconsistent with a quasiparticle-based theory.

- Strongly interacting QFT is a framework for describing non-quasiparticle states.

  But it is poorly understood.

INSIGHT FROM BLACK HOLES

- Holographic duality gives us a handle on some strongly interacting QFTs

  Black holes have proven to be useful toy models of strange metals

- Main reason: black holes exhibit some universal properties

  → help to identify general features of strongly interacting QFTs

- I will describe a new universal property of black holes, and its implications
  - ✳ Certain features of black hole excitation spectrum depend only on near-horizon physics
  - ✳ QFT transport properties are related to underlying chaotic dynamics

TRANSPORT PROPERTIES

- Transport properties characterize the dynamics of a system's conserved charges over long distances and timescales.

  i.e. the properties of $T^{\mu\nu}$ and J^{μ} at small (ω, k)

- Examples: electrical resistivity, thermal resistivity, shear viscosity, diffusivity of energy,….

- Transport properties are important experimental observables
  - They are relatively easy to measure
  - They exhibit universality across different systems

TRANSPORT PROPERTIES

- There are also two theoretical reasons that transport properties are privileged.

(1) The dynamics of $T^{\mu\nu}$ and J^{μ} are constrained by symmetries

$\longrightarrow$ governed by a simple effective theory over long distances and timescales: **hydrodynamics**

For a given QFT, we just need to determine the parameters of the effective theory.

(2) Transport is directly related to the dynamics of the basic gravitational variables:

$$T^{\mu\nu} \longleftrightarrow g_{\mu\nu}$$

$\longrightarrow$ there is a degree of universality to transport in holographic theories

TRANSPORT PROPERTIES: AN EXAMPLE

- **Example**: system whose only conserved charge is the total energy.
- Local thermodynamic equilibrium $\longrightarrow$ state characterized by slowly-varying energy density: $\varepsilon \equiv T^{00}(t, \underline{x})$ $\partial\varepsilon \ll 1$
- Equations of motion: $\partial_t \varepsilon + \nabla \cdot j = 0$ $j = -D \nabla \varepsilon - \Gamma \nabla^3 \varepsilon + O(\nabla^5)$

$$\longrightarrow \quad \partial_t \varepsilon = D \nabla^2 \varepsilon + \Gamma \nabla^4 \varepsilon + O(\nabla^6)$$

or

$$\omega = -iDk^2 - i\Gamma k^4 + O(k^6)$$

- Hydrodynamics: energy diffuses over long distances.

  What sets the values of the transport parameters D, Γ, etc ?

CHAOTIC PROPERTIES

- Chaotic dynamics are seemingly something very different from transport.

$$C(t,\underline{x}) = -\left\langle [V(t,\underline{x}), W(0,\underline{0})]^2 \right\rangle_T$$

- In theories with a classical gravity dual, these correlations have the form

$$C(t,\underline{x}) \sim e^{\tau_L^{-1}\left(t - |\underline{x}|/v_B\right)}$$

  - The timescale is always $\tau_L = (2\pi T)^{-1}$ Shenker, Stanford (1306.0622)
  - But the "butterfly velocity" v_B depends on the particular theory.

Roberts, Stanford, Susskind (1409.8180)

MAIN RESULTS

- In QFTs with a gravity dual, the transport properties are constrained by v_B, τ_L

- The collective modes that transport energy are characterized by their dispersion relations $\omega(k)$.

  There is always a mode with $\omega(k_*) = i\tau_L$ where $k_*^2 = -(v_B\tau_L)^{-2}$.

- Under appropriate conditions, the diffusivity of energy is set by

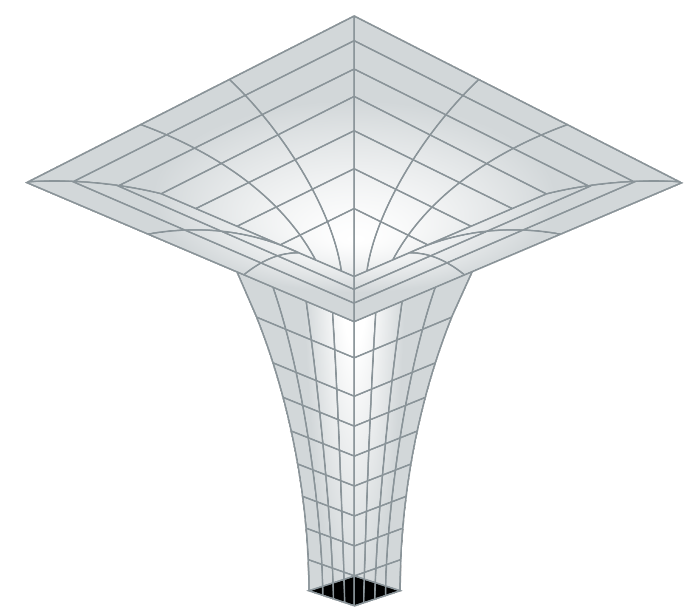
$$D \sim v_B^2 \tau_L$$

(In a normal metal, $D \sim v_F^2 \tau$)

THE GRAVITATIONAL THEORIES

- I will discuss asymptotically AdS_{d+2} black branes supported by matter fields:

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + h(r)d\underline{x}_d^2$$



In ingoing co-ordinates

$$ds^2 = -f(r)dv^2 + 2dvdr + h(r)d\underline{x}_d^2$$

- For definiteness: $S = \int d^{d+2}x\sqrt{-g}\left(R - Z(\phi)F^2 - \frac{1}{2}(\partial\phi)^2 + V(\phi)\right)$

- Matter fields induce an RG flow from the UV CFT : $F_{vr}(r) \neq 0 \quad \& \quad \phi(r) \neq 0$

Numerical solution of equations of motion yield $f(r), h(r)$ etc.

QUASI-NORMAL MODES OF BLACK HOLES

- Focus on one aspect of these spacetimes: **quasi-normal modes**.

  i.e. solutions to linearized perturbation equations, obeying appropriate BCs

  * regularity (in ingoing coordinates) at the horizon $r = r_0$
  * normalizability near the AdS boundary $r \to \infty$

- e.g. probe scalar field $\partial_a(\sqrt{-g}\,\partial^a \delta\varphi) - m^2\sqrt{-g}\,\delta\varphi = 0$

  * 2 independent solutions: $\delta\varphi_{norm}(r, \omega, k)$ and $\delta\varphi_{non-norm}(r, \omega, k)$
  * If $\delta\varphi_{norm}$ is regular at the horizon $\longrightarrow$ quasi-normal mode.

- Quasi-normal modes are characterized by their dispersion relations $\omega(k)$

QUASI-NORMAL MODES OF BLACK HOLES

- Collective excitations of the dual QFT are encoded in the quasi-normal modes.

quasi-normal modes $\omega(k)$ of a field ⟷ poles $\omega(k)$ of retarded Green's function of dual operator

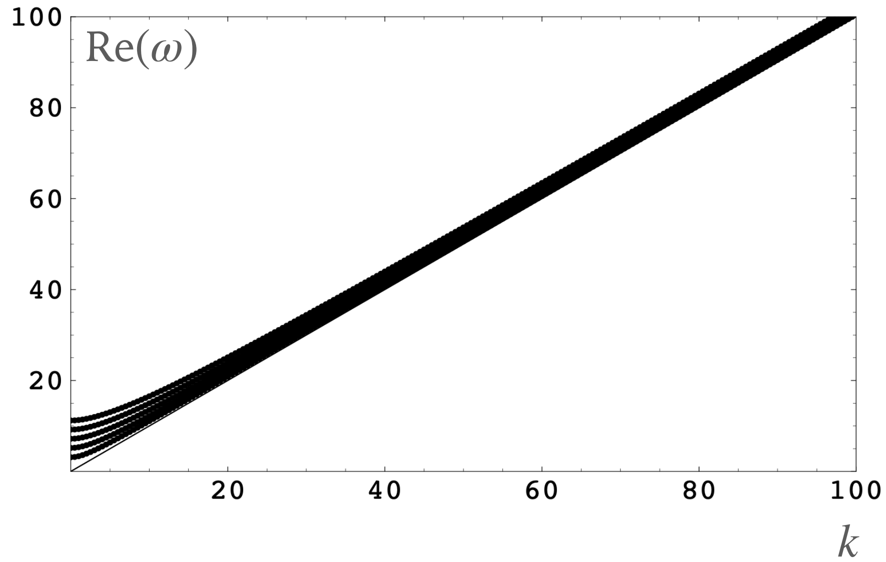
Horowitz, Hubeny (hep-th/9909056)
Son, Starinets (hep-th/0205051)

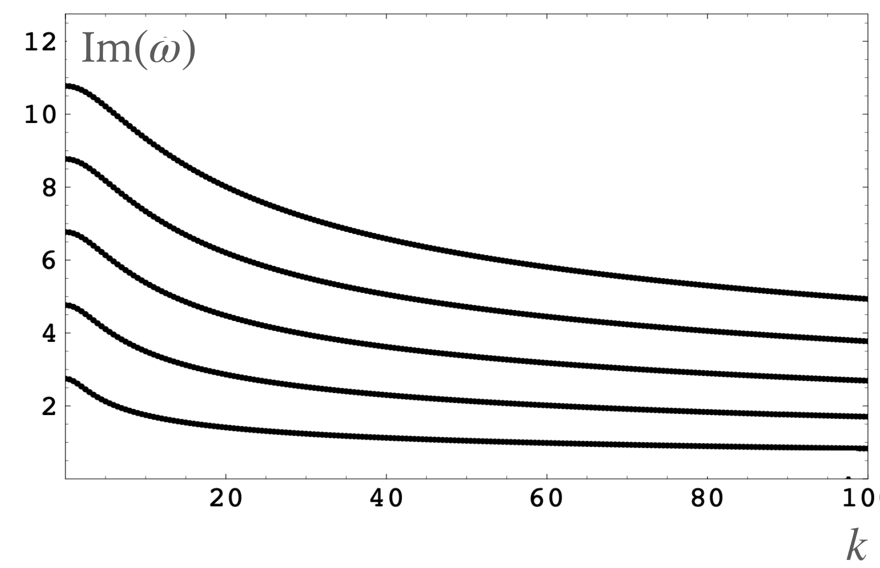
- The spectrum depends in detail on the particular theory, spacetime, field, etc

Numerical computation is required even in very simple cases.

e.g. massless scalar field in Schwarzschild-AdS_5

Plots from hep-th/0207133 by A. Starinets





HORIZON CONSTRAINTS ON THE SPECTRUM

- Certain features of the spectrum depend only on the near-horizon dynamics.

Blake, RD, Vegh (1904.12883)
see also Kovtun et al (1904.12862)

- **Example**: probe scalar field

  ✳ Ansatz: solution that is regular at the horizon $\delta\varphi(r) = \sum_{n=0}^{\infty} \varphi_n (r - r_0)^n$

  ✳ Solve iteratively for $\varphi_{n>0}$:

$$2h(r_0)(2\pi T - i\omega)\varphi_1 = \left(k^2 + m^2 h(r_0) + i\omega \frac{dh'(r_0)}{2}\right)\varphi_0 \qquad \text{etc.}$$

  ✳ At (ω_1, k_1) both solutions are regular at the horizon !

$$\omega_1 = -i2\pi T, \qquad k_1^2 = -\left(m^2 h(r_0) + d\pi T h'(r_0)\right)$$

HORIZON CONSTRAINTS ON THE SPECTRUM

- Moving infinitesimally away from (ω_1, k_1) yields one regular solution:

$$\begin{aligned} \omega &= \omega_1 + i\delta\omega \\ k &= k_1 + i\delta k \end{aligned} \longrightarrow \frac{\varphi_1}{\varphi_0} = \frac{1}{4h(r_0)}\left(4ik_1\frac{\delta k}{\delta\omega} - dh'(r_0)\right)$$

But this regular solution depends on the arbitrary slope $\delta k/\delta\omega$.

- Can obtain an arbitrary combination of φ_{norm} and $\varphi_{non-norm}$ by tuning $\delta k/\delta\omega$:

$$\varphi_{ingoing}(\omega_1 + i\delta\omega, k_1 + i\delta k) = \left(1 - v_z\frac{\delta k}{\delta\omega}\right)\varphi_{norm} + C\left(1 - v_p\frac{\delta k}{\delta\omega}\right)\varphi_{non-norm}$$

For an appropriate choice of slope $(\delta\omega = v_p\delta k)$, there is a quasi-normal mode.

⟶ there must be a dispersion relation obeying $\omega(k_1) = \omega_1$

HORIZON CONSTRAINTS ON THE SPECTRUM

- This feature of the spectrum is independent of the rest of the spacetime.

Near-horizon dynamics yield exact constraints on the dispersion relations $\omega(k)$

- A more complete analysis of this type yields infinitely many constraints

$$\omega = -i2\pi T n, \qquad k = k_n \qquad n = 1,2,3,\dots$$

for appropriate values k_n .

- These points in complex Fourier space are called **pole-skipping points**.

Intersection of a line of poles with a line of zeroes in the dual QFT 2-point function

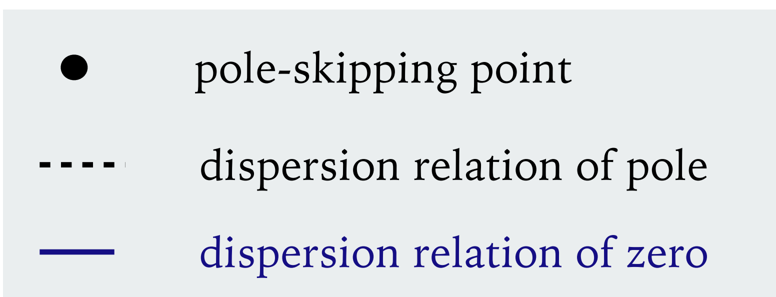
$$G = C\frac{\delta\omega - v_z\delta k}{\delta\omega - v_p\delta k}$$

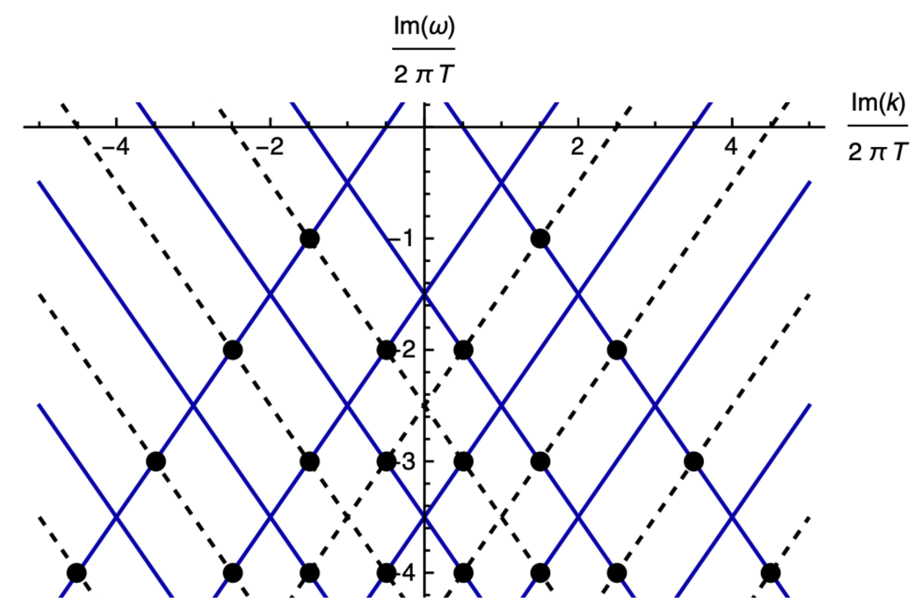
POLE-SKIPPING EXAMPLES

- The argument can be generalized to other spacetimes

  e.g. BTZ black hole / CFT_2 at non-zero T

  $\Delta = 5/2$





- And it can be generalized to non-scalar fields/operators, e.g.
  - ✳ U(1) Maxwell field: some k_n are real
  - ✳ Fermionic fields: frequencies shifted to $\omega = -i2\pi T(n + 1/2)$

Ceplak, Ramdial, Vegh (1910.02975)

CONSTRAINTS ON ENERGY DENSITY MODES

- Usually very complicated to determine the collective modes of energy density

  δg_{vv} couples to other metric perturbations and to matter field perturbations

- But in this case, near-horizon Einstein equations yield a simple constraint

$$\omega(k_*) = +i2\pi T \qquad\qquad k_*^2 = -d\pi T h'(r_0)$$

Independent of the matter field profiles.

Blake, RD, Grozdanov, Liu (1809.01169)

- Universal constraint on the collective modes of energy density:

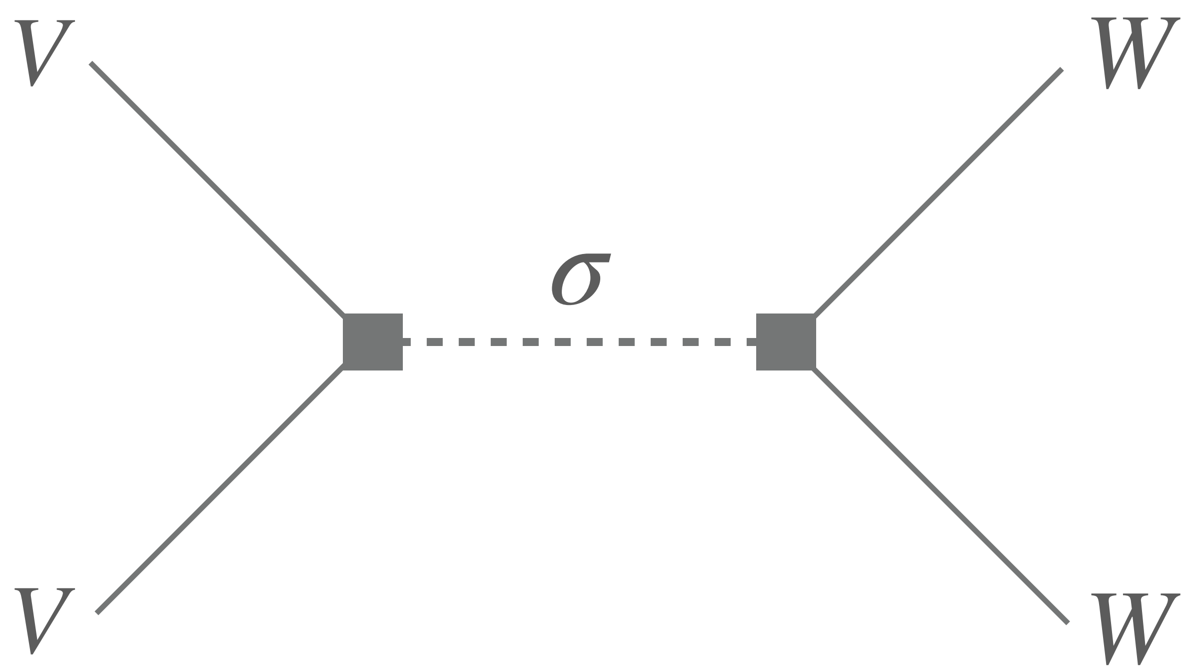
$$\omega(k_*) = +i\tau_L^{-1} \qquad\qquad k_*^2 = -(v_B\tau_L)^{-2}$$

First observed numerically in Schwarzschild-AdS_5: Grozdanov, Schalm, Scopelliti (1710.00921)

HYDRODYNAMIC INTERPRETATION

- An interpretation: chaotic behavior has hydrodynamic origin

Blake, Lee, Liu (1801.00010)



σ : hydrodynamic mode of energy conservation

See also: Gu, Qi, Stanford (1609.07832), Haehl, Rozali (1808.02898),...

- Conversely, the chaotic behavior constrains the hydrodynamic parameters of theories with holographic duals.

IMPLICATIONS FOR HYDRODYNAMICS

- There is typically a collective mode of energy density with dispersion relation

$$\omega_{hydro}(k) = -iDk^2 - i\Gamma k^4 + O(k^6)$$

At long distances, this is the hydrodynamic diffusion of energy density.

- $\omega_{hydro}(k_*) = +i\tau_L^{-1}$ ⟶ constraint on hydrodynamic parameters.

- Make an additional assumption:

If diffusive approximation $\omega_{hydro}(k) \approx -iDk^2$ is good up to $\omega = i\tau_L^{-1}, k = k_*$

⟶ $D \approx -k_*^{-2}\tau_L^{-1}$ ⟶ $D \approx v_B^2 \tau_L$

LOW TEMPERATURE DIFFUSIVITIES

- Consistent with the diffusivity of energy density at low temperatures
  - ✳ For a large class of theories with $AdS_2 x R^d$ IR fixed points

as $T \to 0$

$$D = v_B^2 \tau_L$$

Blake, Donos (1611.09380)

  - ✳ Generic IR fixed point has symmetry $t \to \Lambda^z t, \quad \underline{x} \to \Lambda \underline{x}$

as $T \to 0$

$$D = \frac{z}{2(z-1)} v_B^2 \tau_L$$

Blake, RD, Sachdev (1705.07896)

  - ✳ When $z = 1$, diffusive approximation breaks down at $\omega \ll \tau_L^{-1}$.

RD, Gentle, Goutéraux (1808.05659)

SUMMARY

- Near-horizon dynamics yield exact constraints on the dispersion relations of collective modes.

- There is a universal constraint for collective modes of energy density

$$\omega(k_*) = +\, i\tau_L^{-1} \qquad\qquad k_*^2 = -\,(v_B\tau_L)^{-2}$$

- Under appropriate conditions, this constrains the diffusivity of energy density

$$D \sim v_B^2\tau_L$$

i.e. transport is related to underlying chaotic properties.

OPEN QUESTIONS

- How robust is the universal constraint on the energy density collective modes?

  More direct evidence of the chaos/hydrodynamics link?

  Grozdanov (1811.09641), Ahn et al (1907.08030, 2006.00974), Natsuume, Okamura (1909.09168), Abbasi & Tabatabaei (1910.13696), Liu, Raju (2005.08508), …

- Regime of validity of (diffusive) hydrodynamics in holographic theories?

  Withers (1803.08058), Kovtun et al (1904.01018), …

- What generalizes to other (non-holographic) strongly interacting systems?

  Gu, Qi, Stanford (1609.07832), Patel, Sachdev (1611.00003), …

- Precise restrictions on transport parameters from near-horizon constraints.

  Grozdanov (2008.00888)

- Pole-skipping points in more general spacetimes / field theories ?

  Ahn et al (2006.00974)

THANK YOU!