Sharp Boundaries for the Swampland

East Asian String Webinar
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arXiv:2102.08591, with S. Caron-Huot, L. Rastelli, D. Simmons-Duffin

Goal: Constrain Theories of Quantum Gravity

Our universe contains gravity

What is the set of consistent universes?

- Only a few options or a vast landscape?
- Do all quantum gravity theories come from string theory?
- How does the answer depend on the spacetime asymptotics?
  AdS / flat space / dS
- Which low-energy EFTs admit a UV completion?

this talk

Which EFTs Admit a UV Completion?

Powerful constraints on IR from UV consistency:

- no global symmetries
- weak gravity conjecture
- distance conjecture

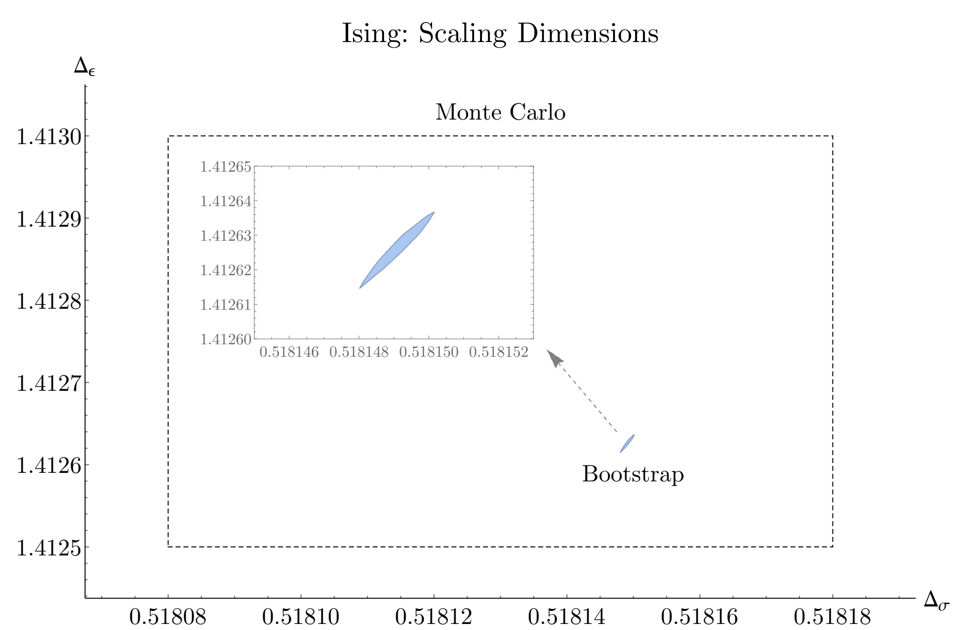
Drawbacks:

- often conjectural, based on examples
- parametric bounds: $A \lesssim B$

Would prefer:

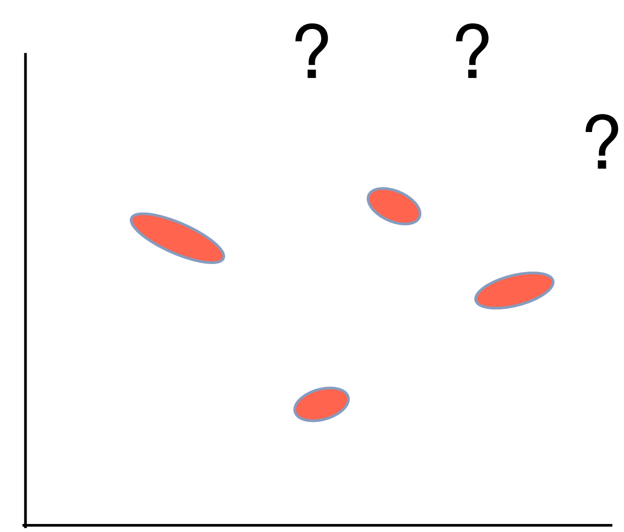
- rigorous, based on general principles
- precise bounds: $A \leq 3B$

a success story: CFT bootstrap



[Kos, Poland, Simmons-Duffin, Vichi '16]

quantum gravity



Roadmap

1. What is the space of a priori parameters?

   ↳ irrelevant couplings in the IR effective field theory

2. What are the fundamental principles / assumptions?

   ↳ causality and unitarity of the S-matrix

3. How do we use 2 to put bounds on 1?

   ↳ use dispersion relations in a novel way

Low-Energy Effective Field Theory of Gravity

$$S = \frac{1}{16\pi G_N} \int dx^D \sqrt{g} \left(R + \alpha_2 R^2 + \alpha_3 R^3 + \ldots \right)$$

all interactions irrelevant! $\qquad G_N = M_p^{2-D}$

EFT works for energies $\qquad E \ll M < M_p$

UV cutoff $\qquad$ Planck scale

expectation: $\quad \alpha_2 = \# \cdot M^{-2} \qquad \alpha_3 = \# \cdot M^{-4}$

[Camanho, Edelstein, Maldacena, Zhiboedov ’14]

Natural task: prove rigorous bounds on the O(1) coefficients

$$|\alpha_2| \leq \# \cdot M^{-2} \qquad |\alpha_3| \leq \# \cdot M^{-4}$$

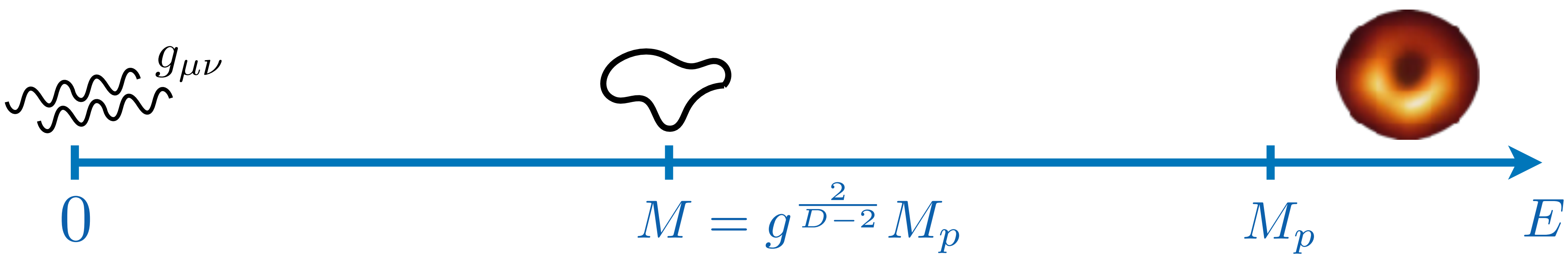
Measurable in inflationary correlators if Hubble scale $\sim M$

First Step: Weakly Coupled Theories

$$S = \frac{1}{16\pi G_N}\int dx^D \sqrt{g}\,\left(R + \alpha_2 R^2 + \alpha_3 R^3 + \ldots\right)$$

overly ambitious goal: show that $|\alpha_2| \leq \# \cdot M_p^{-2}$

counterexample: weakly-coupled string theory $g \ll 1$



$\alpha_2 \sim M^{-2} \gg M_p^{-2}$

a natural first goal: prove bounds when coupling is weak

$|\alpha_2| \leq \# \cdot M^{-2}$ for $M \ll M_p$

$\Longrightarrow$ can ignore loop effects at scale M and below

String Universality?

More generally, classify weakly-coupled theories of gravity

Only 3 known answers in flat-space:

- Einstein
- heterotic string
- type II string

[Arkani-Hamed, Huang]

[Chowdhury, Gadde, Gopalka, Halder, Janagal, Minwalla '19]

[Chakraborty, Chowdhury, Gopalka, Kundu, Minwalla, Mishra '20]

Is this the complete list?

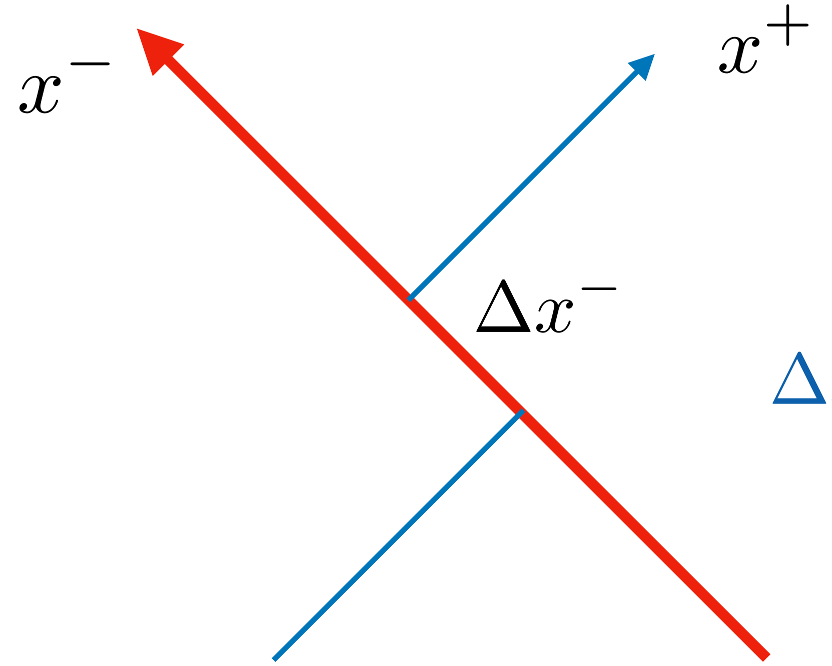
Related question without gravity: classify theories of weakly-coupled massive higher-spin particles.

Large N gauge theories

[Caron-Huot, Komargodski, Sever, Zhiboedov '14]

Bounds from Causality [Camanho, Edelstein, Maldacena, Zhiboedov '14]

Idea: too large irrelevant couplings lead to superluminal propagation

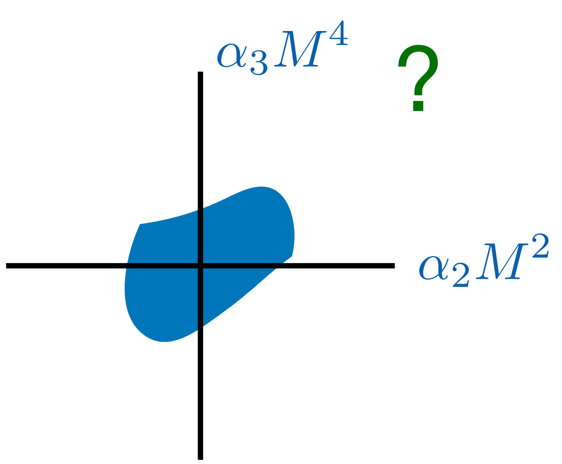


$$\Delta x^- = \left(1 \pm \frac{\alpha_2}{b^2} \pm \frac{\alpha_3}{b^4}\right) \frac{G_N E}{b^{D-4}}$$

Superluminal propagation unless $|\alpha_2| \lesssim b_{\rm min}^2$ where $b_{\rm min} \sim M^{-1}$

$|\alpha_3| \lesssim b_{\rm min}^4$

Want to replace $\lesssim \;\to\; \leq$

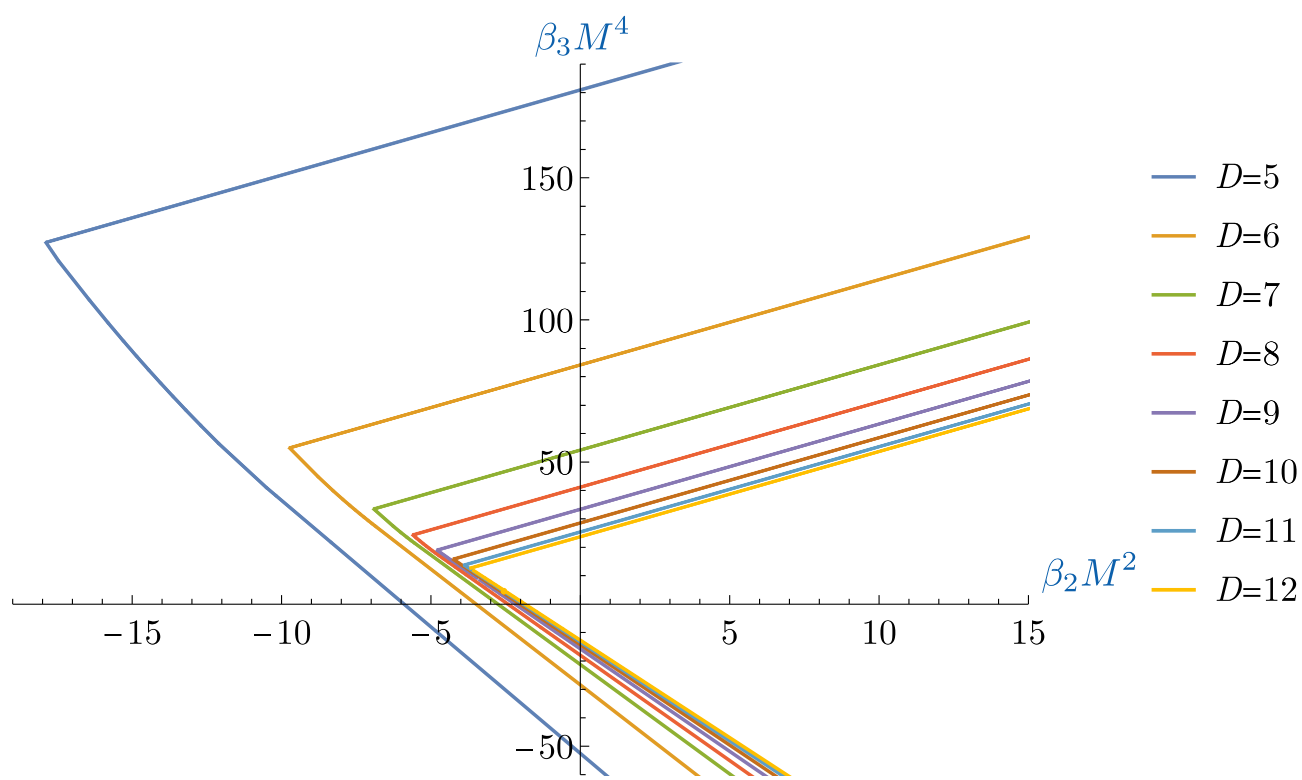


Our Main Result: Bounds for Gravity + Scalar

[Caron-Huot, DM, Rastelli, Simmons-Duffin ’21]

$$S = \frac{1}{16\pi G_N}\int d^Dx\sqrt{g}\left[R + (\nabla\phi)^2 + \beta_0\phi^4 + \beta_2\nabla^4\phi^4 + \beta_3\nabla^6\phi^4 + \ldots\right]$$

- assume the UV cutoff exactly at M
- and that the theory is weakly coupled, i.e. $M \ll M_p$



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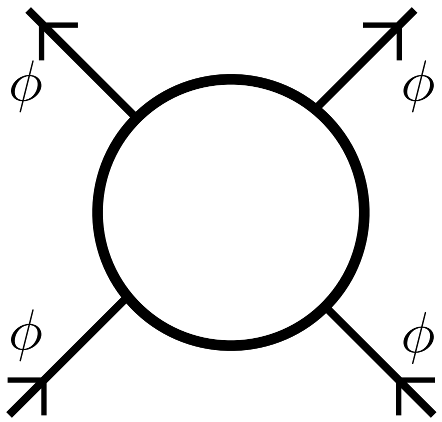
The S-Matrix Axioms

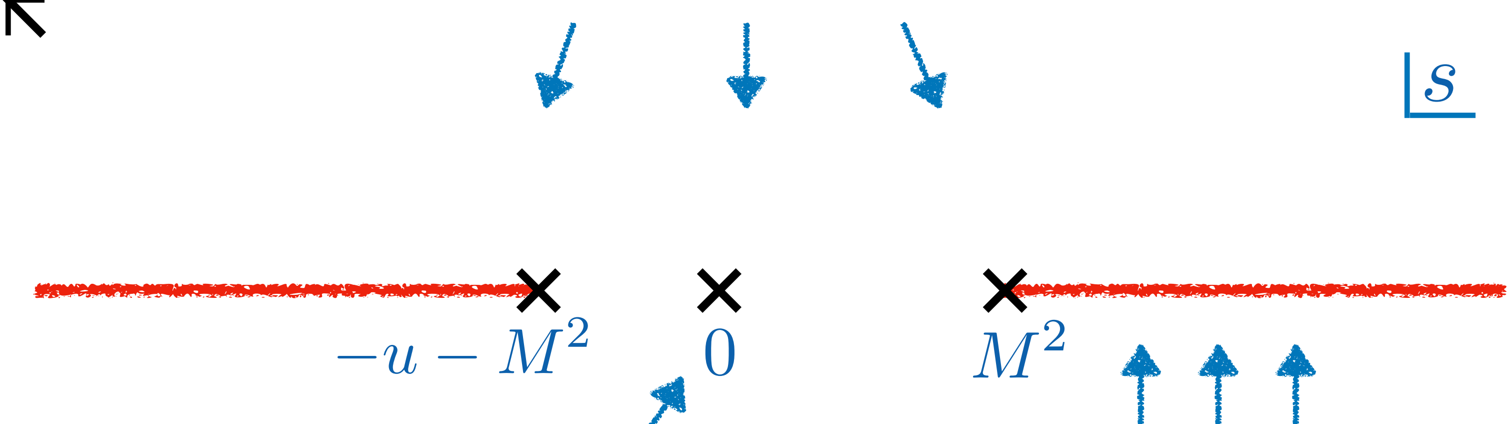


$= A(s,t)$

genuinely Lorentzian observable

1. causality

$\Longrightarrow$ analyticity in the complex s plane



2. weak coupling in the IR

$$A(s,t) = 8\pi G_N \left[\frac{tu}{s} + \frac{us}{t} + \frac{st}{u} + \beta_0 + \right.$$
$$\left. + \beta_2 (s^2 + t^2 + u^2) + \beta_3 (stu) + \ldots \right]$$

3. unitarity

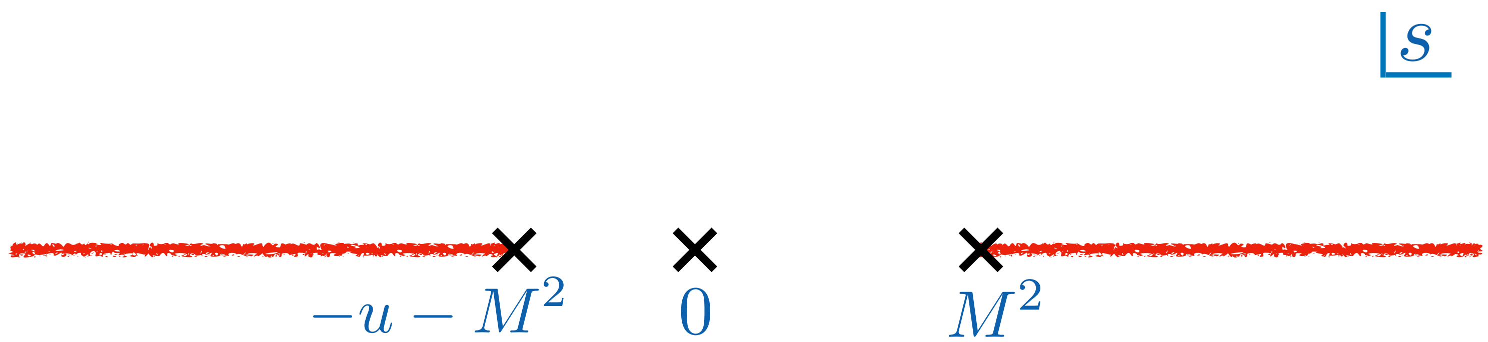
$\Longrightarrow$ positivity of the UV contributions

$$\mathrm{Im}[A(s,t)] = \sum_{J=0}^{\infty} \underbrace{\rho_J(s)}_{\geq 0} P_J(\cos\theta)$$

4. bound on Regge growth $\qquad \dfrac{A(s,t)}{s^2} \to 0$ as $s \to \infty$

Bound on the Regge Growth

$$\frac{A(s,t)}{s^2} \to 0 \quad \text{as} \quad s \to \infty$$



Satisfied in weakly coupled string theory $\qquad A(s,t) = O(s^{2+\alpha' u})$

Classical Regge growth conjecture $\quad A(s,t) = O(s^2)$

[Camanho, Edelstein, Maldacena, Zhiboedov ’14]
[Maldacena, Shenker, Stanford ’15]
[Chandorkar, Chowdhury, Kundu, Minwalla, ’21]

We need a little more!

Rigorously provable for scattering in AdS $\qquad |\mathcal{G}^{\circlearrowleft}| \leq |\mathcal{G}|$

[Caron-Huot, ’17]

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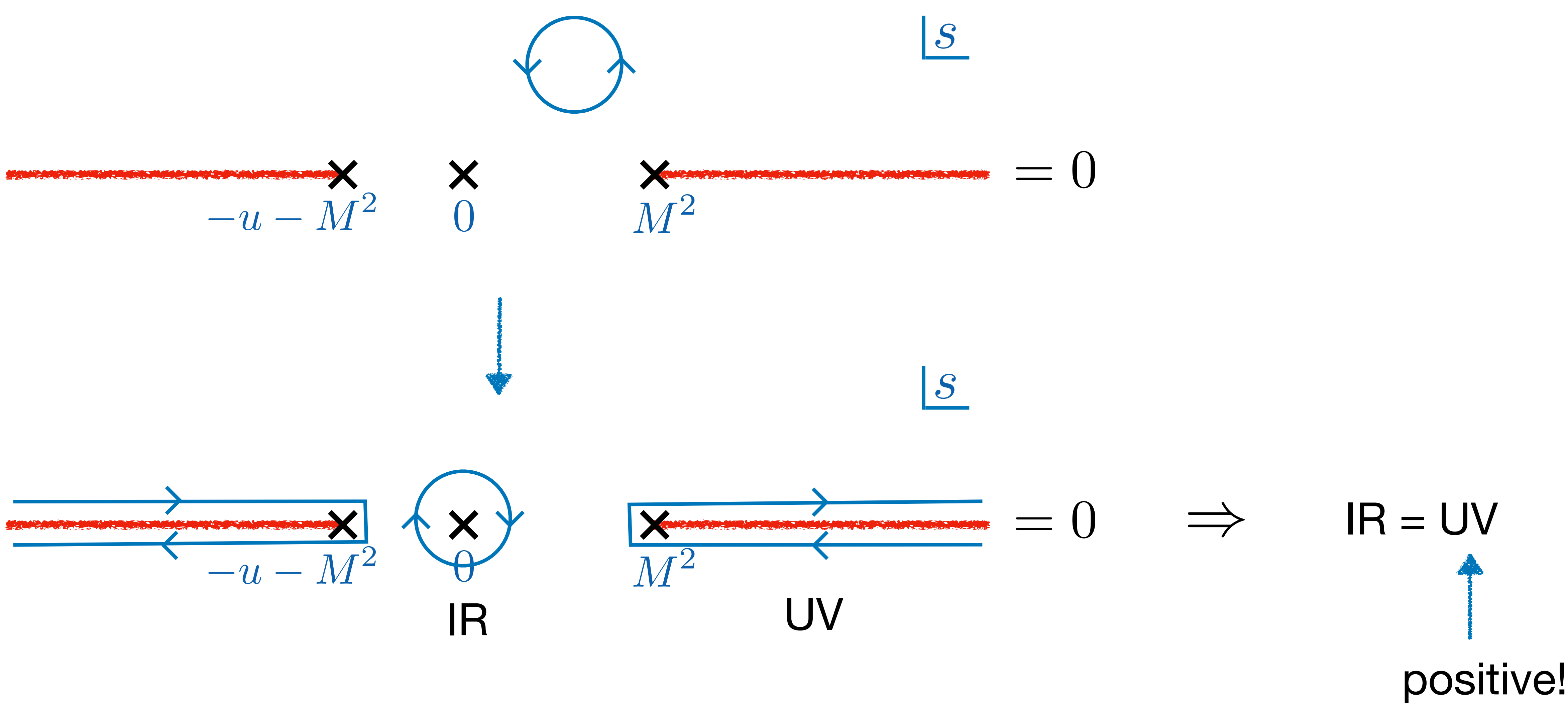
Dispersion Relations

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi ’06]
[Arkani-Hamed, Huang, Huang ’20] ...

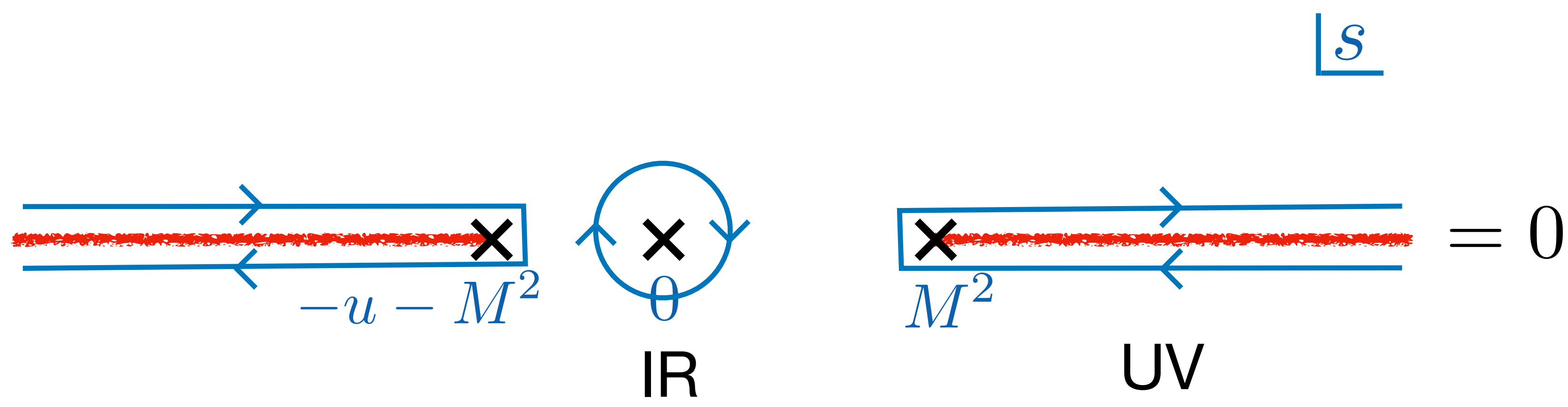
Causality requires IR and UV to talk to each other!

$$\oint ds f(s) = 0$$



Dispersive Sum Rules

[Arkani-Hamed, Huang, Huang '20]
[Caron-Huot, DM, Rastelli, Simmons-Duffin '21]



integrate $A(s,t)$ against $\dfrac{s-t}{(st)^{\frac{k}{2}+1}}$ with $k = 2, 4, \ldots$

IR UV

$$\mathcal{C}_{k,u} = \mathrm{Res}_{s'=0}\left[\frac{2s'+u}{s'(s'+u)}\frac{A(s',u)}{\left[s'(s'+u)\right]^{k/2}}\right] - \left\langle \frac{2m^2+u}{m^2+u}\frac{P_J\left(1+\frac{2u}{m^2}\right)}{\left[m^2(m^2+u)\right]^{k/2}}\right\rangle = 0$$

where $$\langle(\cdots)\rangle \equiv \frac{1}{\pi}\sum_{J\text{ even}} n_J^{(D)}\int_{M^2}^{\infty}\frac{dm^2}{m^2}m^{4-D}\rho_J(m^2)\,(\cdots)$$

Dispersive Sum Rules

only sensitive to terms growing in the UV: gravity, β_2 , β_3 , ...

$$8\pi G_N \left(-\frac{1}{t} + 2\beta_2 - \beta_3 t + 8\beta_4 t^2 + \dots \right) = \left\langle \frac{(2m^2+t)P_J\left(1+\frac{2t}{m^2}\right)}{m^2(m^2+t)} \right\rangle_{UV}$$

$t \to 0 \quad \Rightarrow \quad G_N > 0$ gravity is attractive

Suppose no gravity $\Longrightarrow$ can expand around the forward limit $t=0$

$$\beta_2 = \left\langle \frac{1}{m^2} \right\rangle_{UV} > 0 \qquad \beta_3 = \left\langle \frac{3-\#}{m^4} \right\rangle_{UV} \leq \frac{3\beta_2}{M^2}$$

[Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06]
[de Rham, Melville, Tolley, Zhou '17]
[Arkani-Hamed, Huang, Huang '20]
[Caron-Huot, Van Duong '20], [Bellazzini, Miro, Rattazzi, Riembau, Riva '20], [Tolley, Wang, Zhou '20], [Sinha, Zahed '20], [Gopakumar, Sinha, Zahed '20]

Problem: Would like to isolate individual couplings in the presence of gravity but can not expand around the forward limit.

Solution of the Problem [Caron-Huot, DM, Rastelli, Simmons-Duffin '21]

$$8\pi G_N \left(-\frac{1}{t} + 2\beta_2 - \beta_3 t + 8\beta_4 t^2 + \dots\right) = \left\langle \frac{(2m^2+t)P_J\left(1+\frac{2t}{m^2}\right)}{m^2(m^2+t)} \right\rangle_{UV}$$

Part 1: Use crossing symmetry to eliminate $\beta_4,\ \beta_5, \dots$

$$8\pi G_N \left(-\frac{1}{t} + 2\beta_2 - \beta_3 t\right) = \left\langle \frac{(2m^2+t)P_J\left(1+\frac{2t}{m^2}\right)}{m^2(m^2+t)} + \text{corrections} \right\rangle_{UV}$$

Part 2: Do not use wavepackets localized around the forward limit.

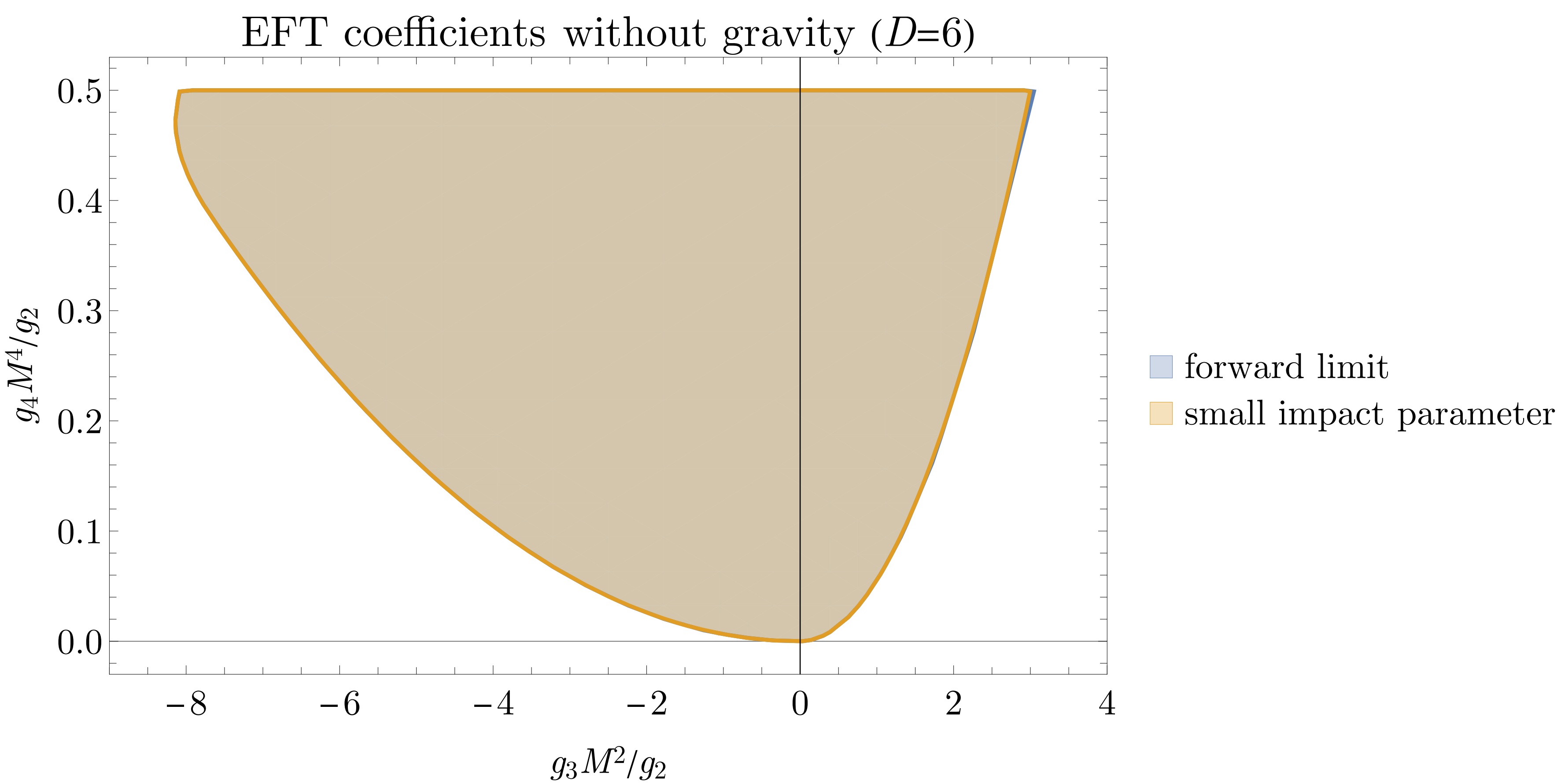
Instead, localize around small impact parameter!

$$|b| \lesssim M^{-1}$$

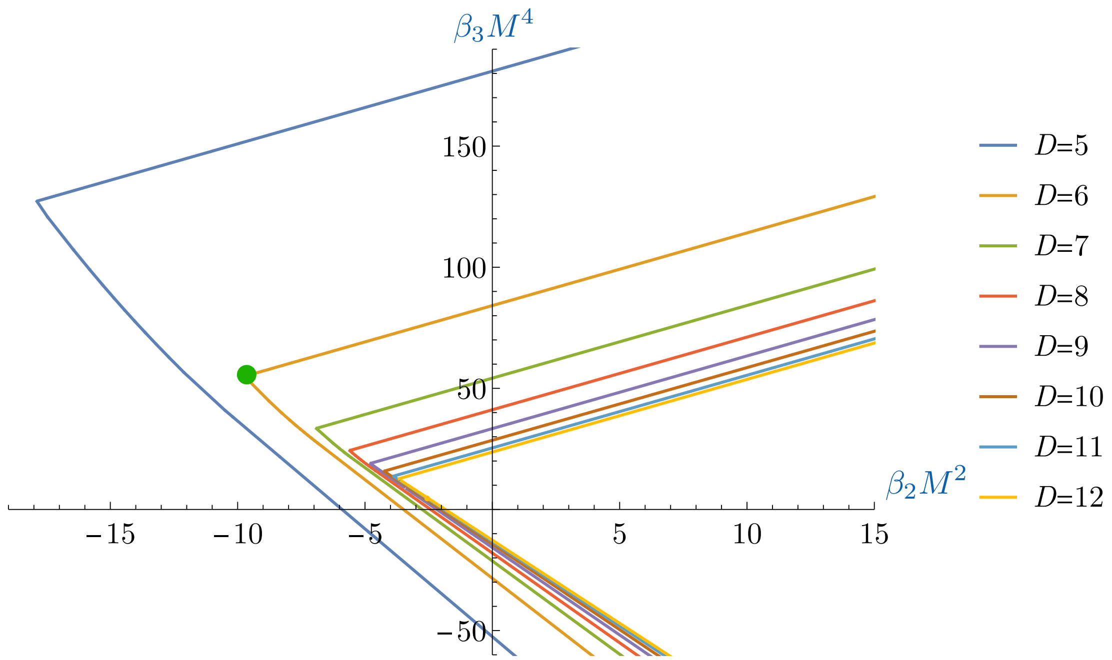
requires going far from the forward limit $|t| \sim M^2$

effectively combines [Adams, Arkani-Hamed, Dubovsky, Nicolis, Rattazzi '06] + [Camanho, Edelstein, Maldacena, Zhiboedov '14]

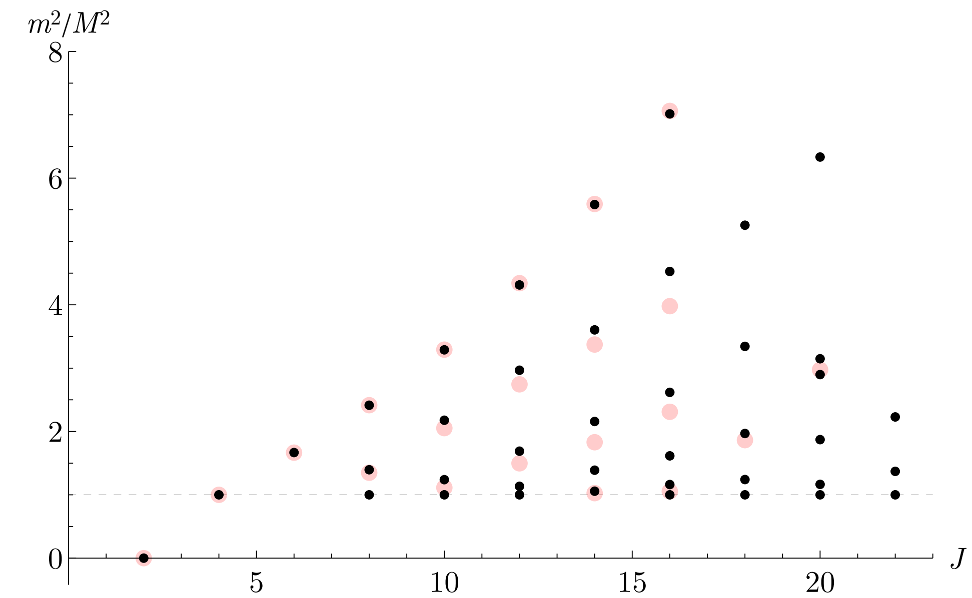
A Comparison with the Forward Limit



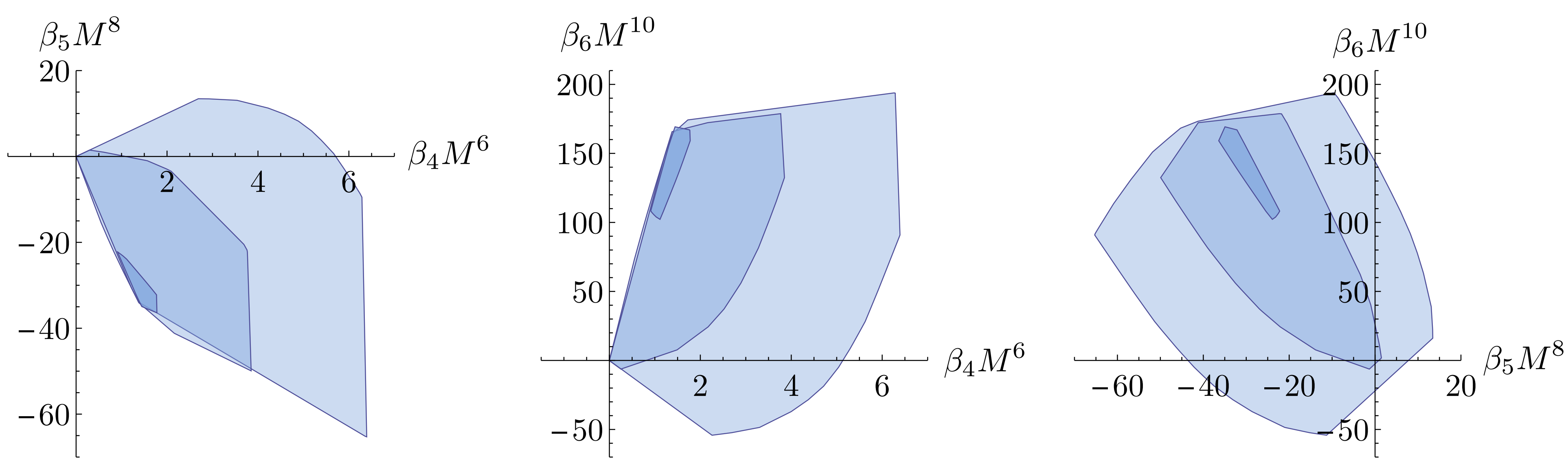
Results with Gravity



spectrum at the corner



bounds on higher dimension couplings for $\beta_2 M^2 = -9, -5, 0$



A Bound in Maximal Supergravity

$$A_{\text{sugra}}(s,t) = 8\pi G_N \left[\frac{1}{stu} + \beta_0 + \beta_2(s^2+t^2+u^2) + \ldots \right]$$

$$0 \leq \beta_0 M^2 \leq 3.000\ldots$$

consistent with type II string theory: $\beta_0 M^2 = 2\zeta(3) \approx 2.40.$

[Guerrieri, Penedones, Vieira '21] : a lower bound on $\beta_0 M_{\text{pl}}^2$

saturated by strongly-coupled string theory?

Summary

Derived rigorous bounds on EFT couplings in consistent weakly coupled theories of gravity.

The bounds follow from unitarity and causality, implemented by dispersion relations.

The key new idea is to localize the dispersion relation in the transverse impact parameter space.

This resolves the notorius difficulties presented by the t-channel graviton pole.

Note: only used weak coupling of the EFT, not the UV.

can incorporate EFT loops

Looking Ahead

The technique generalizes to arbitrary S-matrix elements

a systematic bootstrap program for classifying weakly-coupled gravity theories from first principles

uniqueness of string theory or new possibilities?

non-gravitational S-matrices with massive higher-spin particles

explore the space of large N gauge theories

More Fascinating Questions

Relationship to flat space holography

Cosmology: sum rules for de Sitter gravity?

AdS/CFT

Conjecture: [Heemskerk, Penedones, Polchinski, Sully '09]

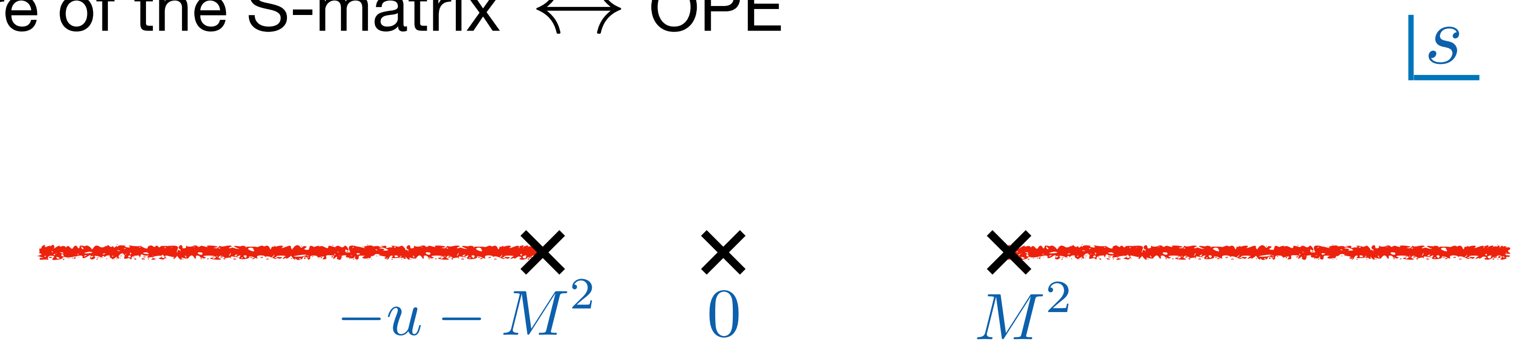
Large N and large gap implies local description in the bulk.

[Hartman, Jain, Kundu]
Caron-Huot, Alday, Bissi, Kravchuk, Meltzer, Simmons-Duffin, Penedones, Perlmutter, Zhiboedov, ...

Dispersive Sum Rules in AdS/CFT

[Caron-Huot, DM, Rastelli, Simmons-Duffin]

1. Structure of the S-matrix $\leftrightarrow$ OPE



$$\phi \times \phi = 1 + \underbrace{\sum_{n,\ell} [\phi\phi]_{n,\ell} + \sum \mathcal{C}_{\text{light}} + [\text{composites}]}_{\tau<\Delta_{\text{gap}}} + \underbrace{\sum \mathcal{C}_{\text{heavy}}}_{\tau>\Delta_{\text{gap}}}$$

2. $\text{Im}[A(s,t)] \leftrightarrow \langle\Omega|[\phi(x_4),\phi(x_1)][\phi(x_3),\phi(x_2)]|\Omega\rangle$

3. Causality $\leftrightarrow$ crossing equation

$$\langle\Omega|\phi(x_4)[\phi(x_1),\phi(x_3)]\phi(x_2)|\Omega\rangle = 0$$

4. Dispersive sum rules (= analytic functionals) come from integrating along null rays.

Dispersive Sum Rules in AdS/CFT

Can essentially repeat the flat space argument.

Conjecture: **[Heemskerk, Penedones, Polchinski, Sully '09]**

Large N and large gap implies local description in the bulk.

✓ for 4pt functions of scalar primaries

Show that Einstein gravity in AdS emerges from these assumptions.