Digging traversable wormholes

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ICREA & UBarcelona

East Asian Strings Webinar

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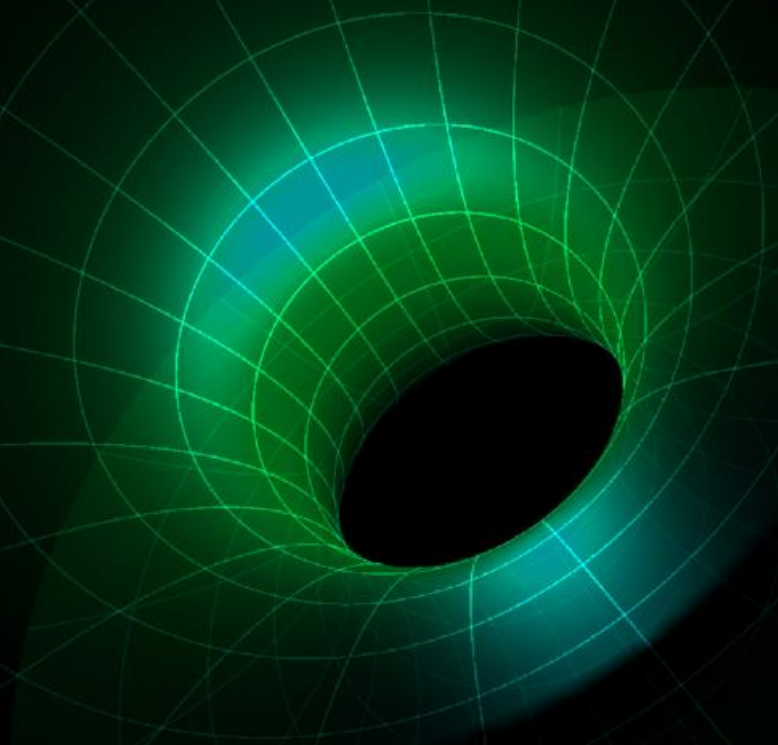


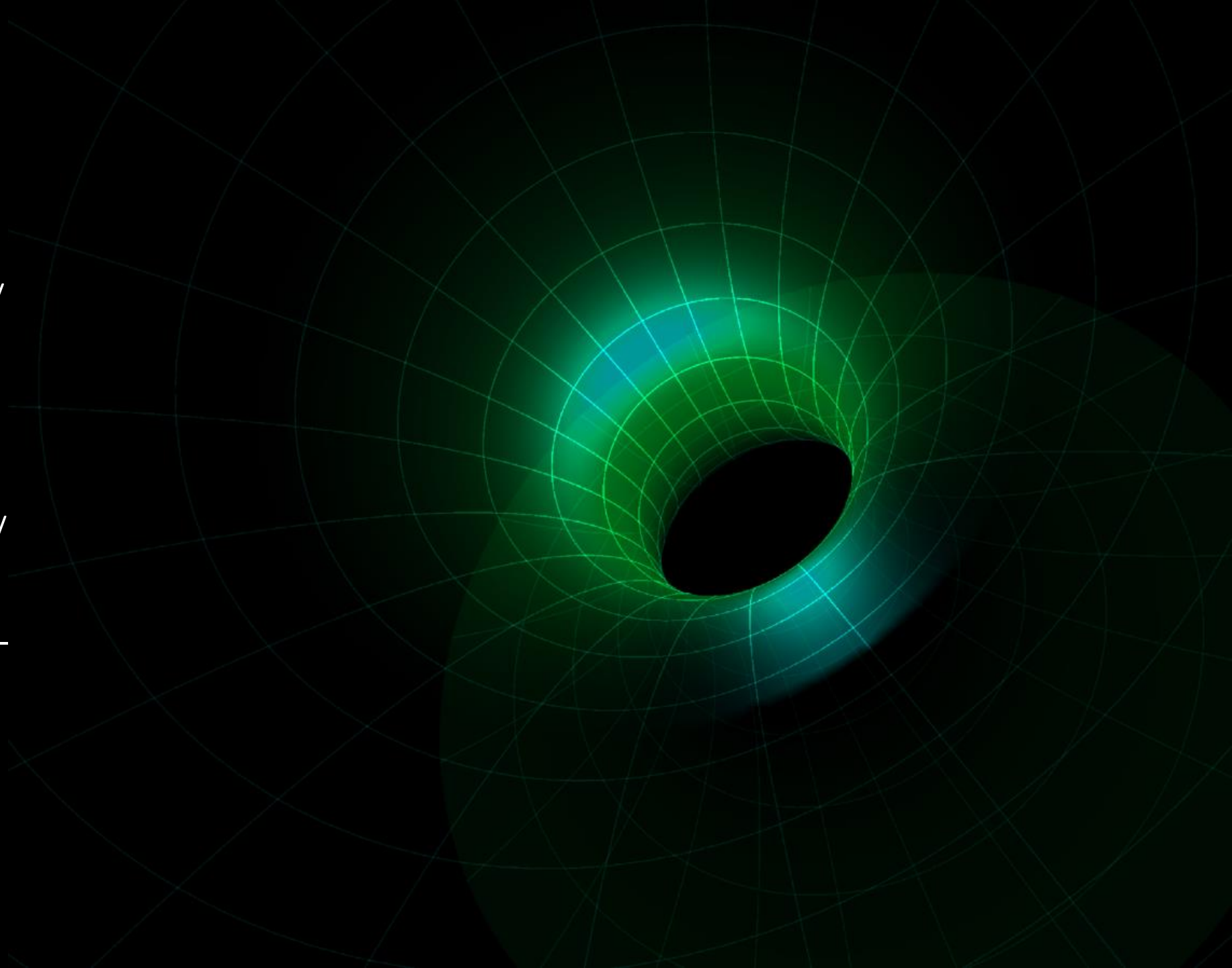
European Research Council
Established by the European Commission

ERC AdG 692951-GravBHs

Based on

2012.07821 [hep-th] w/
Brianna Grado-White
Don Marolf
Marija Tomašević

and work in progress w/
Marija Tomašević



Wormholes in this talk

Lorentzian

Traversable

In a single universe

What can spacetime do?

What topologies?

What connectivity in space and in time?

Classical or quantum matter?

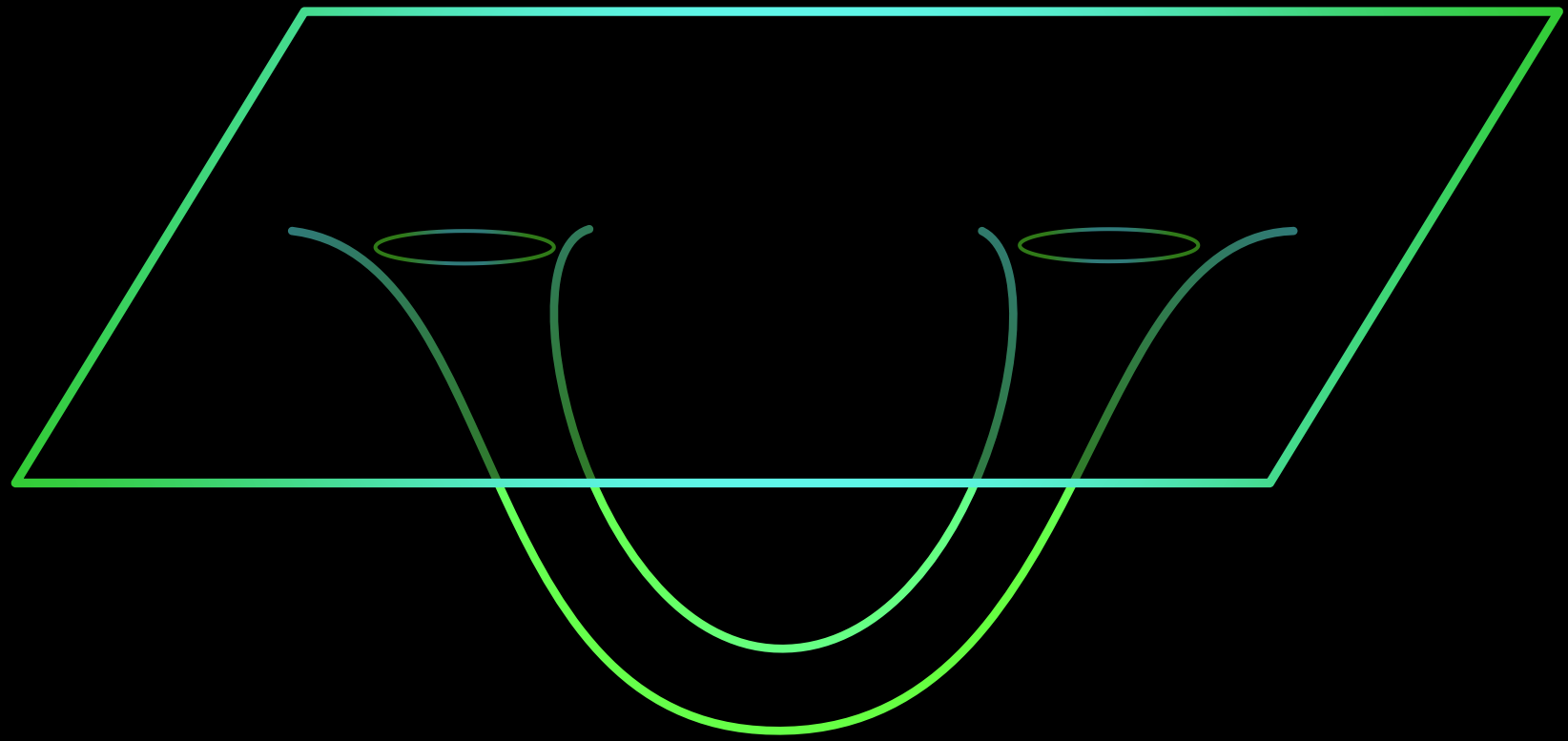
What does it mean?

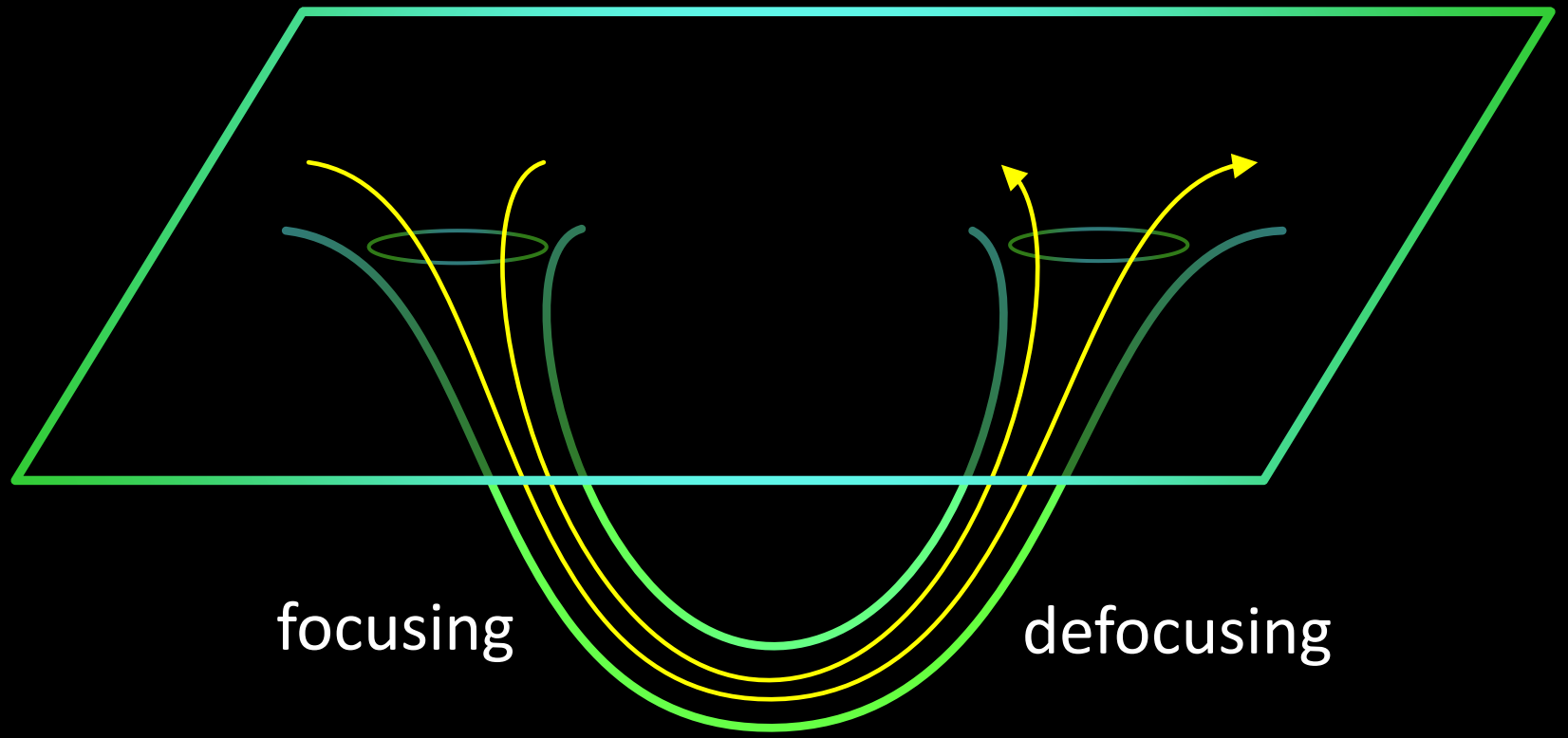
Geometric duals of quantum teleportation

Patterns of holographic entanglement

Are they possible?

Negative null energy required





Repulsive effect on light rays
Negative null energy

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu} < 0$$

Topological censorship

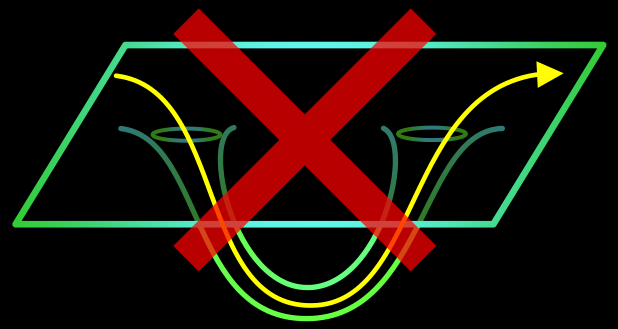
Assume Null Energy Condition (NEC)

$$T_{\mu\nu}\ell^{\mu}\ell^{\nu} \geq 0$$

Then, causal curves are deformable

to boundary of spacetime

$\Rightarrow \nexists$wormholes



Galloway+Schleich+Witt+Woolgar

Weaker assumption

Averaged (integrated) Null Energy Condition (ANEC)

$$\int_{\gamma} d\lambda \, T_{\mu\nu} \ell^{\mu} \ell^{\nu} \geq 0$$

γ: complete null curve

is enough for theorem

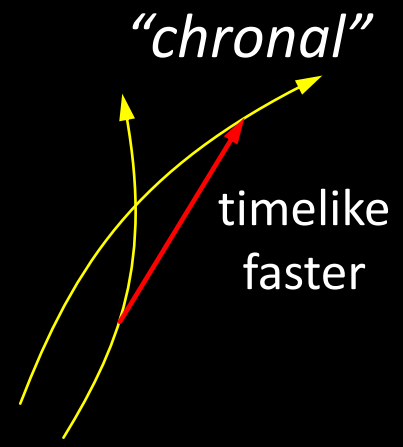
Even weaker assumption

Graham+Olum 2007

Achronal Averaged Null Energy Condition (AANEC)

$$\int_\gamma d\lambda\, T_{\mu\nu}\ell^\mu\ell^\nu \geq 0 \text{ along achronal null lines}$$

achronal light ray = “fastest path”



Classical Matter

Null Energy is positive (NEC)

Cannot defocus

Traversable wormholes impossible

Quantum Matter

Null Energy can be negative ~~NEC~~

Achronal ANEC holds

GSL⇒AANEC *Wall 2009*

Quantum Physics

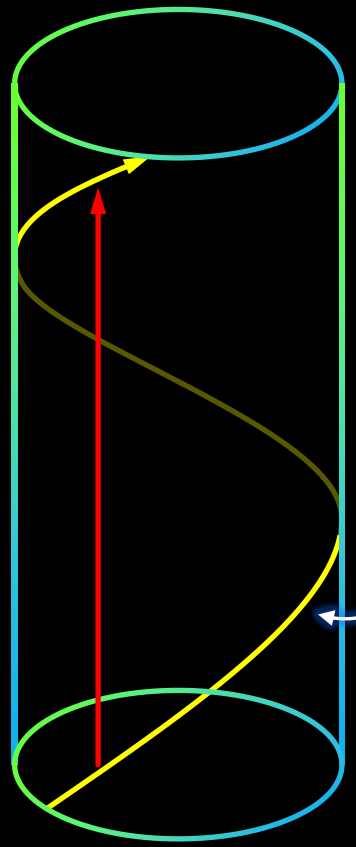
Null Energy can be negative ~~NEC~~

⇒ traversable wormholes become possible

Achronal ANEC holds

⇒ only long wormholes

~~ANEC~~ in Casimir cylinder



Casimir energy <0

$$\int_\gamma d\lambda\, T_{\mu\nu}\ell^\mu\ell^\nu < 0$$

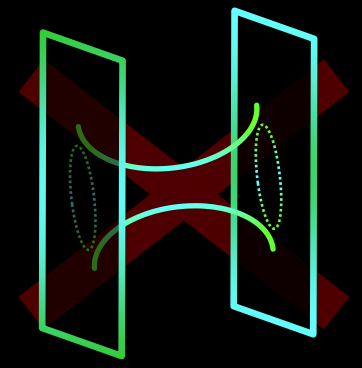
but this null geodesic is not achronal

Achronal ANEC holds

Quantum Physics

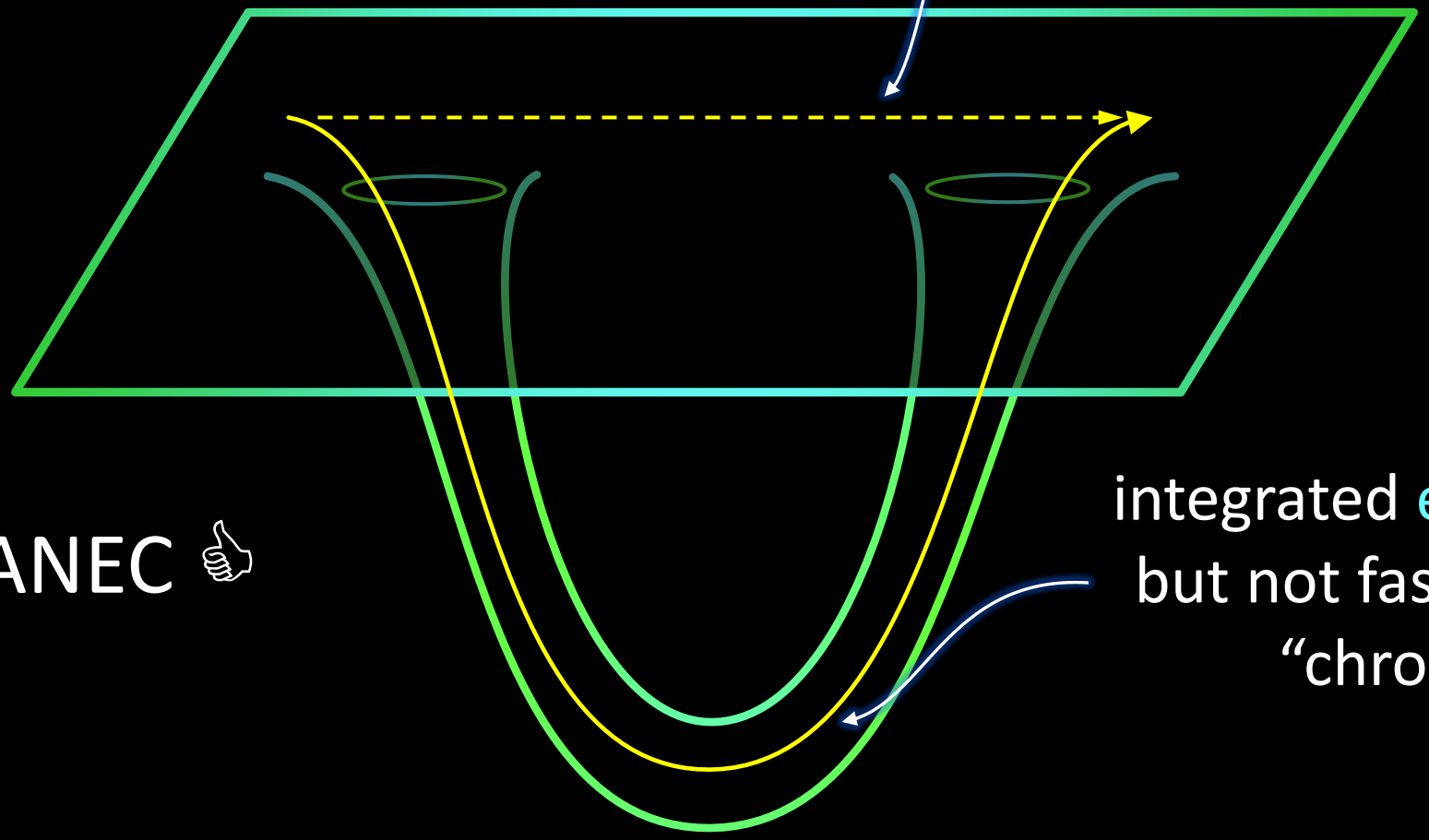
Achronal ANEC implies:

- Short wormholes impossible
- Inter-universe wormholes impossible
- Long wormholes **possible**



Long wormhole

integrated energy > 0
achronal

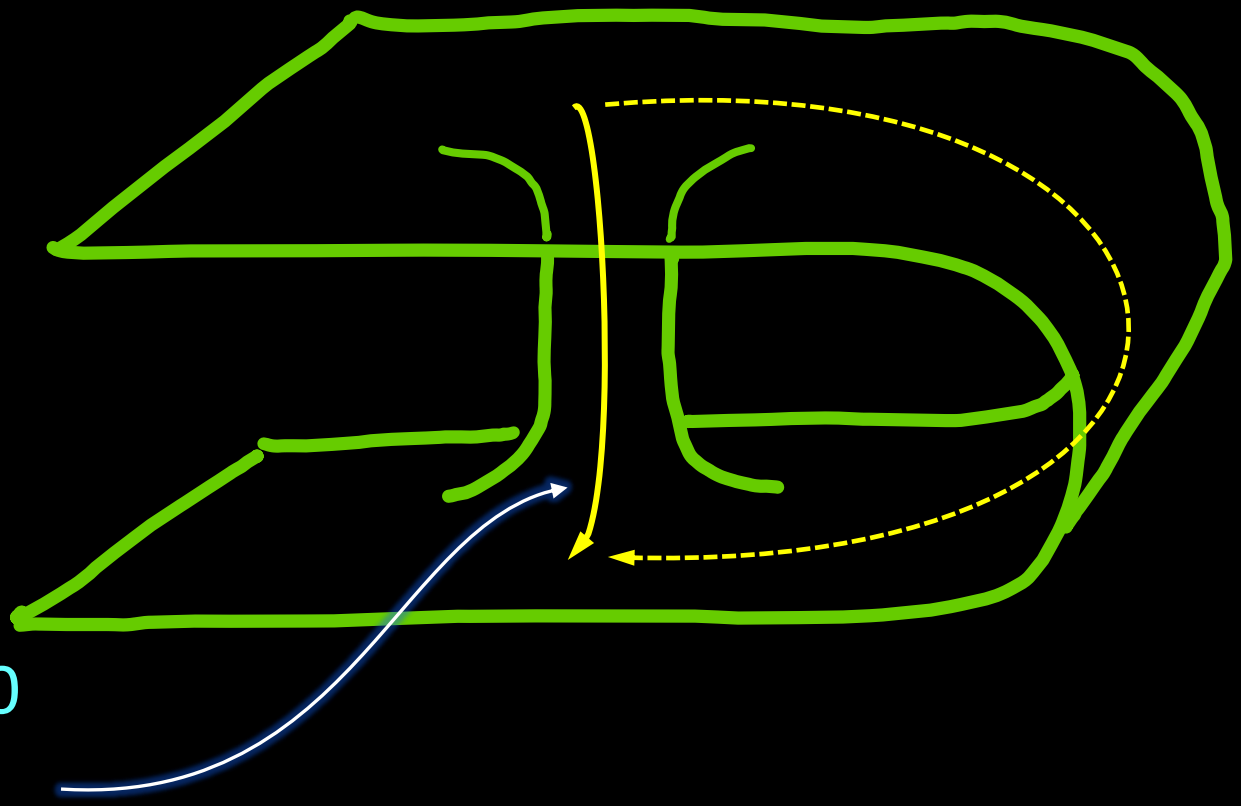


integrated energy < 0
but not fastest path
“chronal”

Achronal ANEC 👍

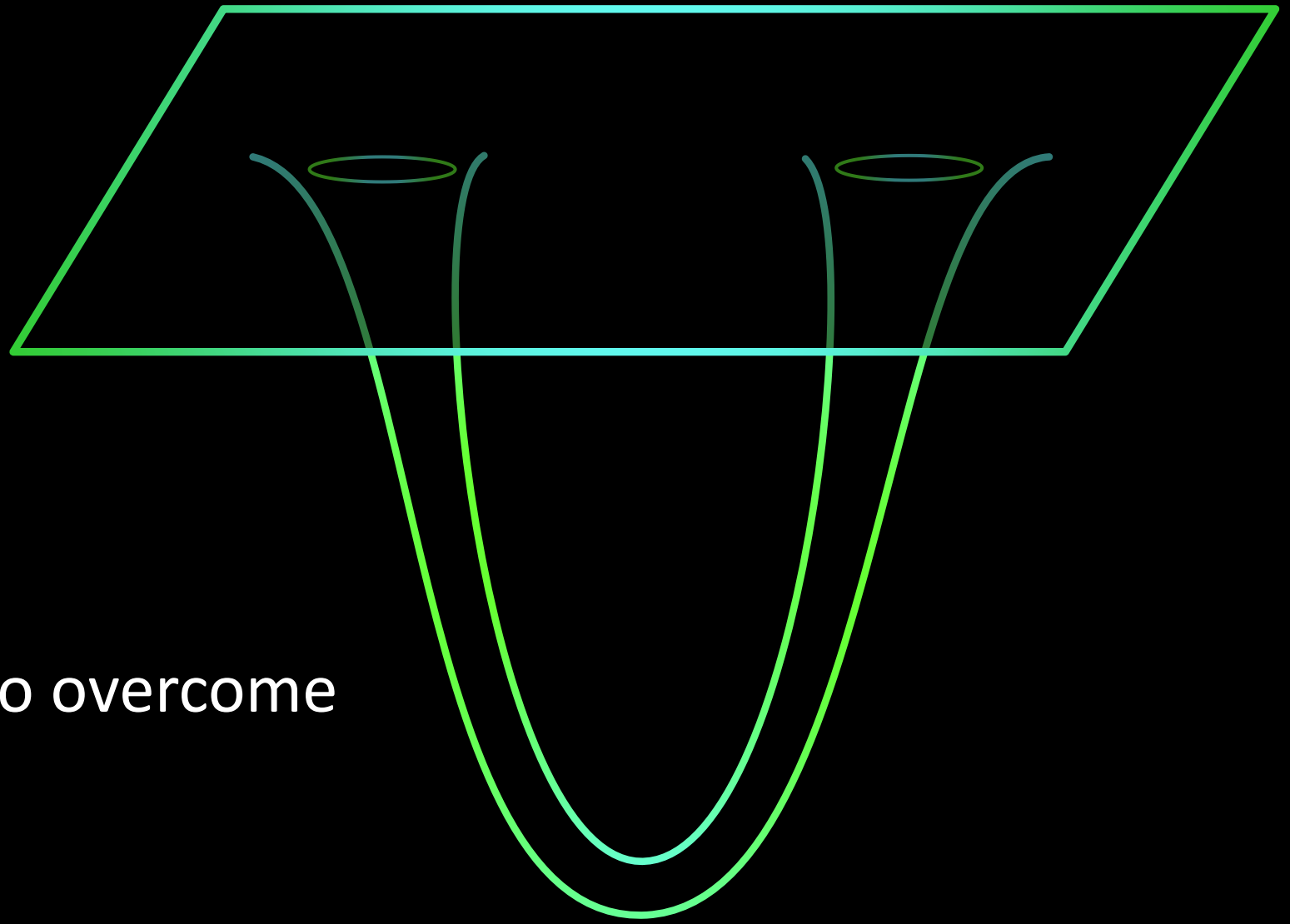
Short wormhole

~~Achronal ANEC~~ 👎



integrated energy < 0
fastest path
achronal

inter-universe wormhole: ultra-short



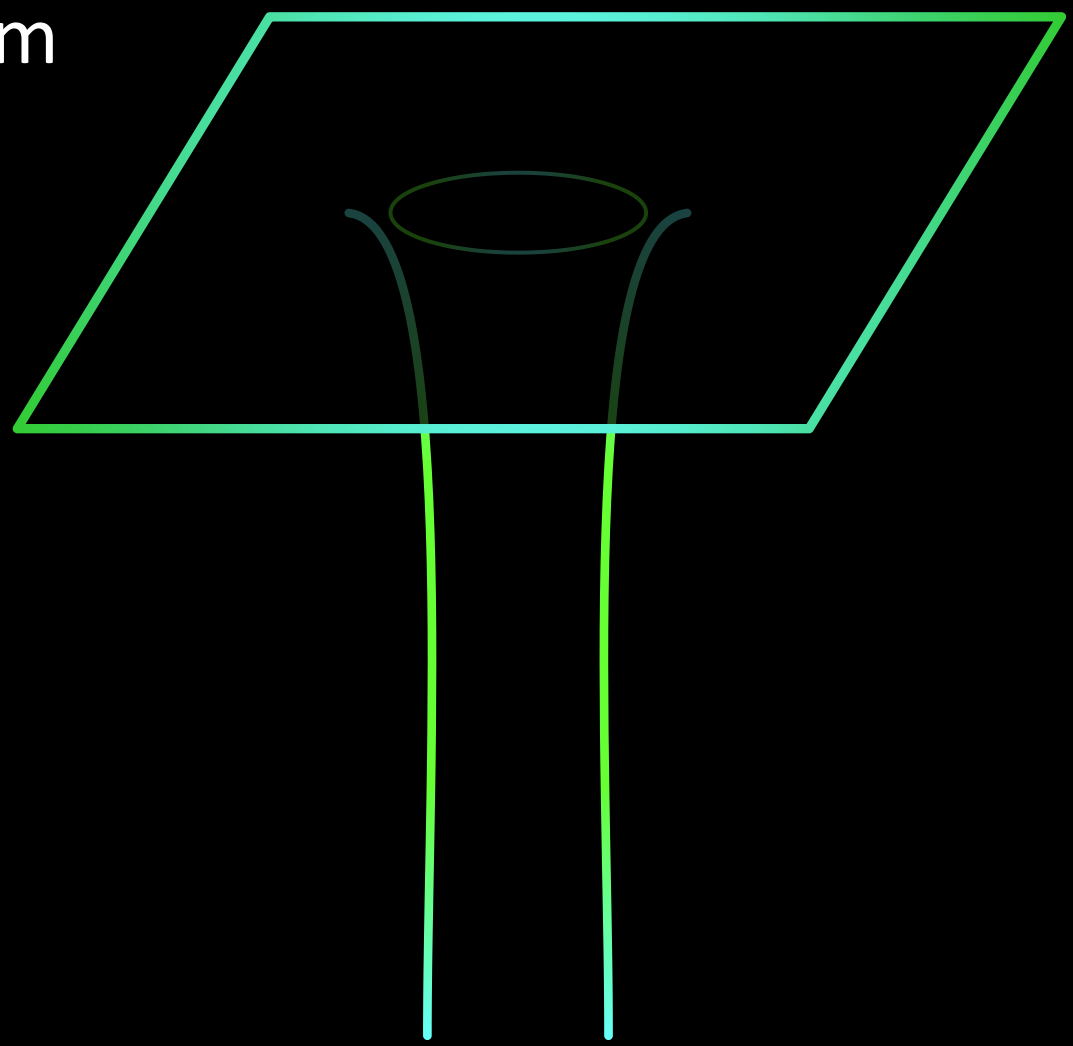
Need

- long throats
- quantum -ve energy to overcome

  +ve classical energy

Near-extremal Reissner-Nordstrom

- long throat
- large redshift: small energies

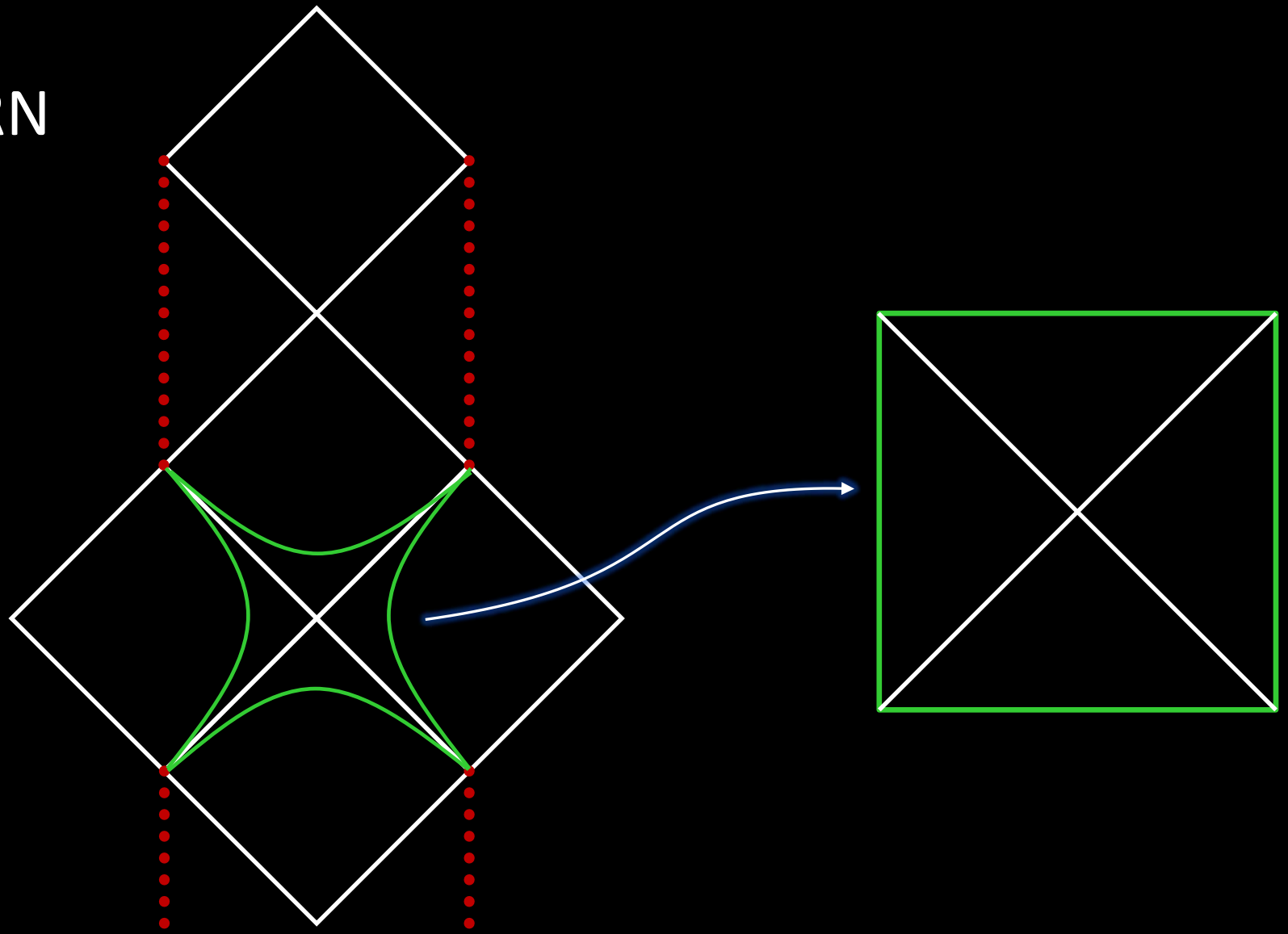


Maldacena+Milekhin+Popov 2018

Near-extremal RN

near horizon

$AdS_2 \times S^2$

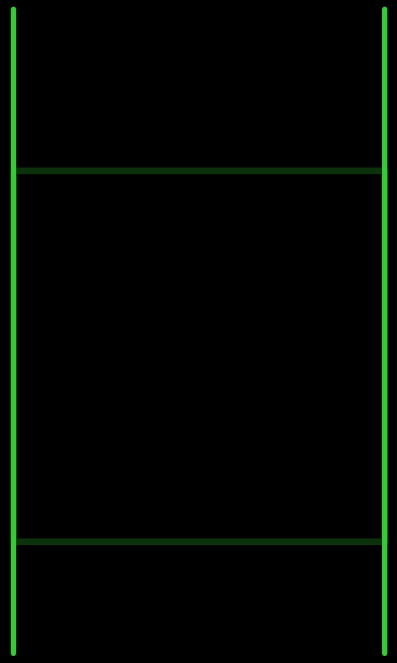


non-traversable wormhole

ER bridge

Rindler-AdS$_2$

$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{r^2 - 1}$$



traversable wormhole

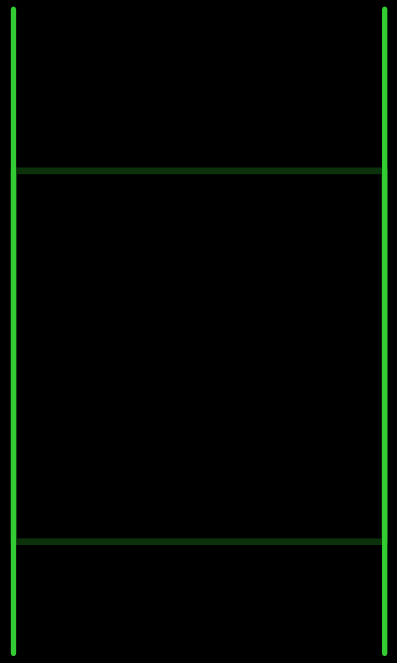
Global AdS$_2$

$$ds^2 = -(r^2 + 1)dt^2 + \frac{dr^2}{r^2 + 1}$$

non-traversable wormhole

ER bridge

Rindler-AdS$_2$



traversable wormhole

Global AdS$_2$

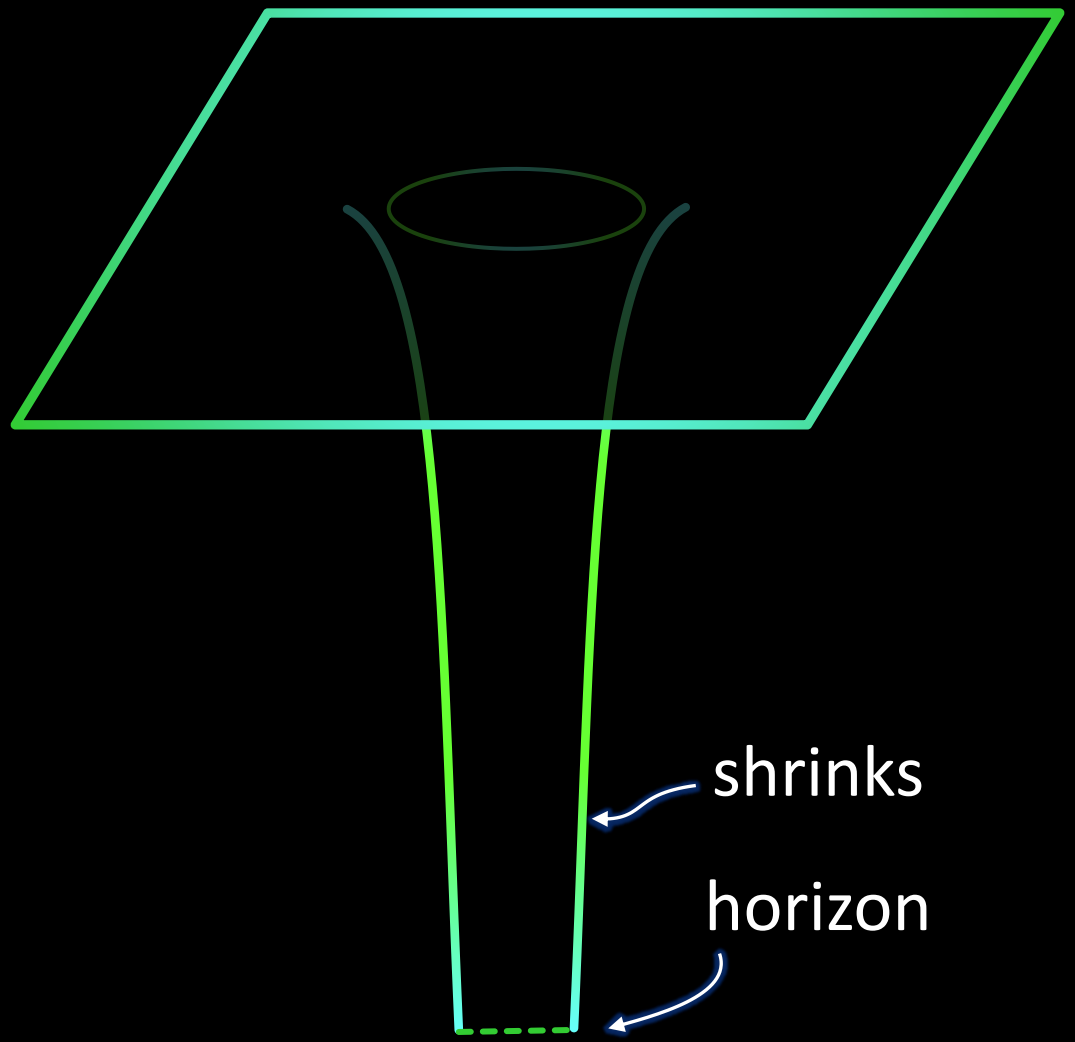
$$ds^2 = -(r^2 - 1)dt^2 + \frac{dr^2}{r^2 - 1}$$

positive energy above extremality

$$ds^2 = -(r^2 + 1)dt^2 + \frac{dr^2}{r^2 + 1}$$

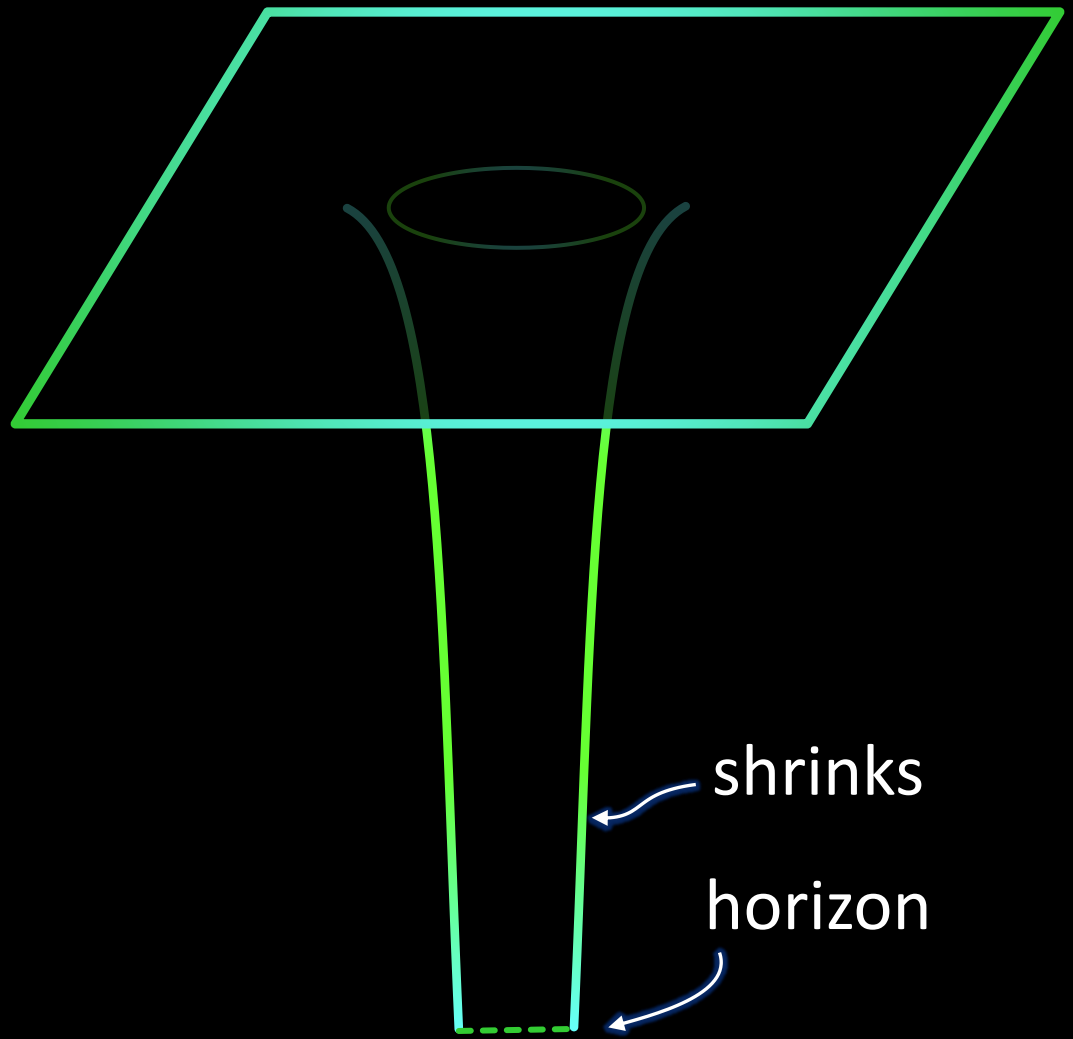
negative energy

M>Q Reissner-Nordstrom

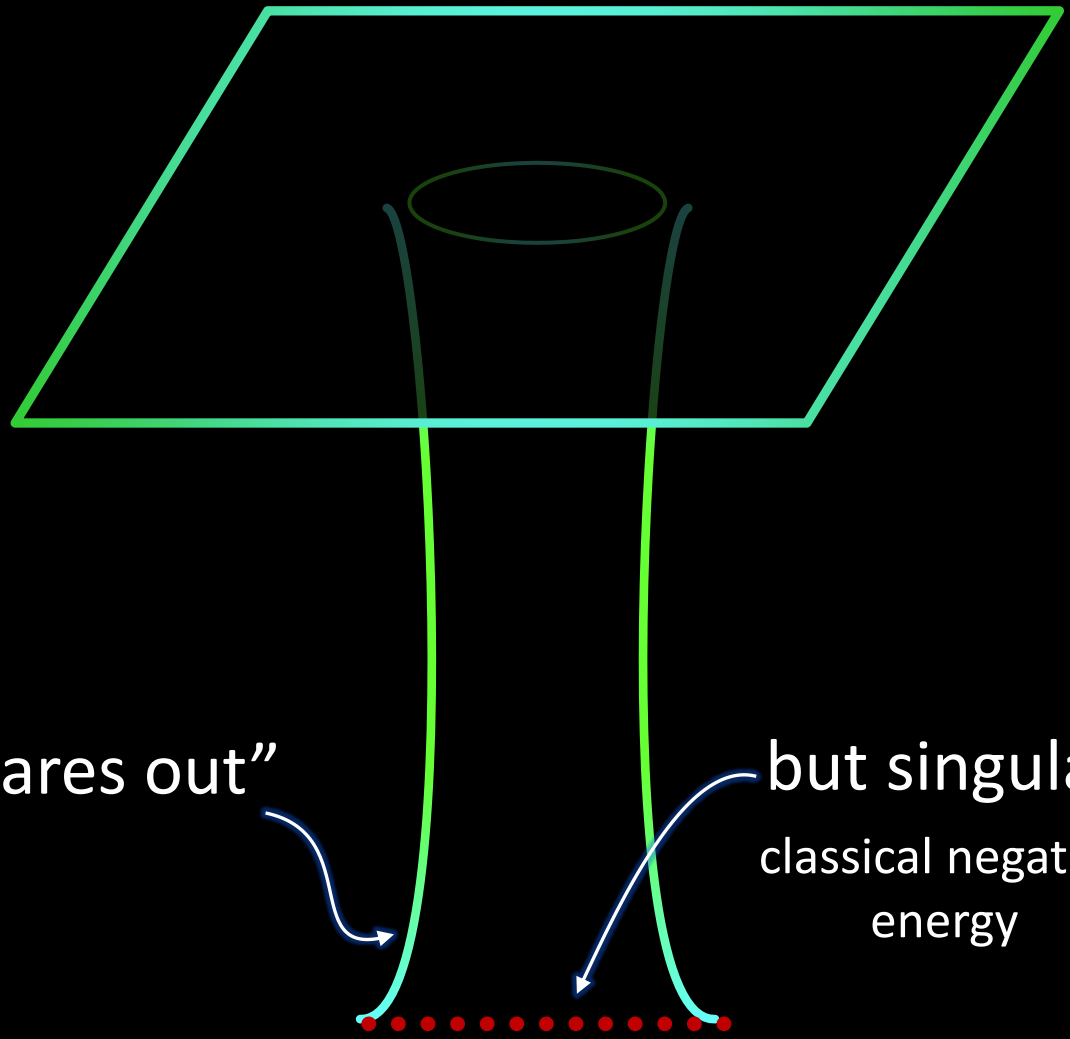


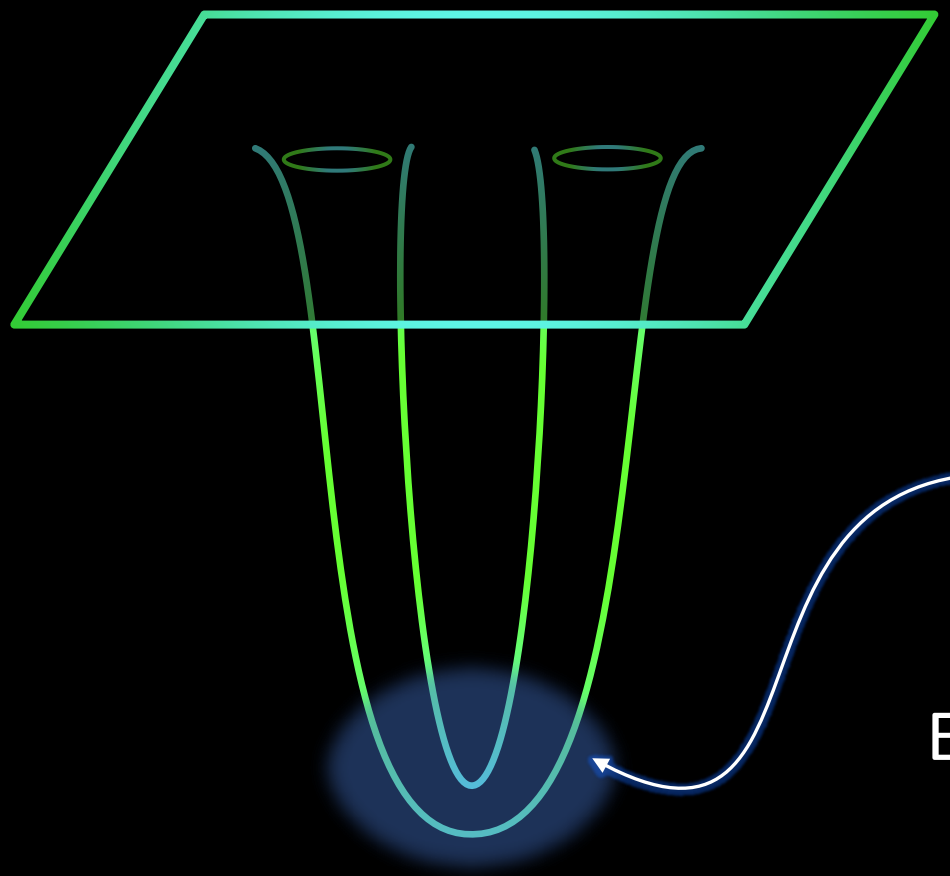
if lower energy is required,
then, how about M<Q?

M>Q Reissner-Nordstrom



M<Q Reissner-Nordstrom





Perturb M>Q Reissner-Nordstrom w/

negative-energy quantum matter

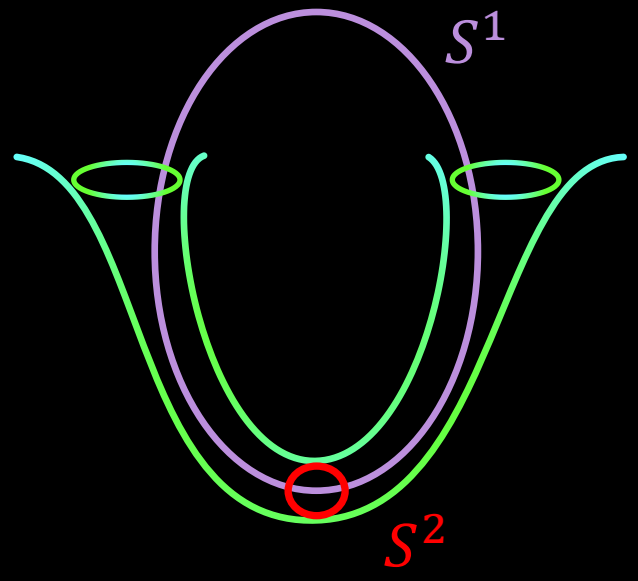
ER bridge (“almost traversable” wormhole)

⟶ traversable wormhole

Need Quantum

Negative Casimir energy

Where/how?

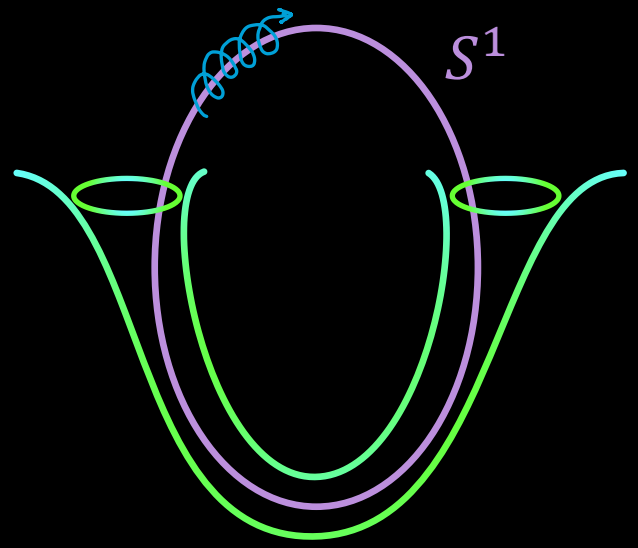


$$S^1 \times S^2 - \{\infty\}$$

Casimir in S^2 won't do

Casimir in S^1 does it

Negative Casimir energy from massless (conformal) effective 1+1 fields along S^1



Electrons in magnetic RN

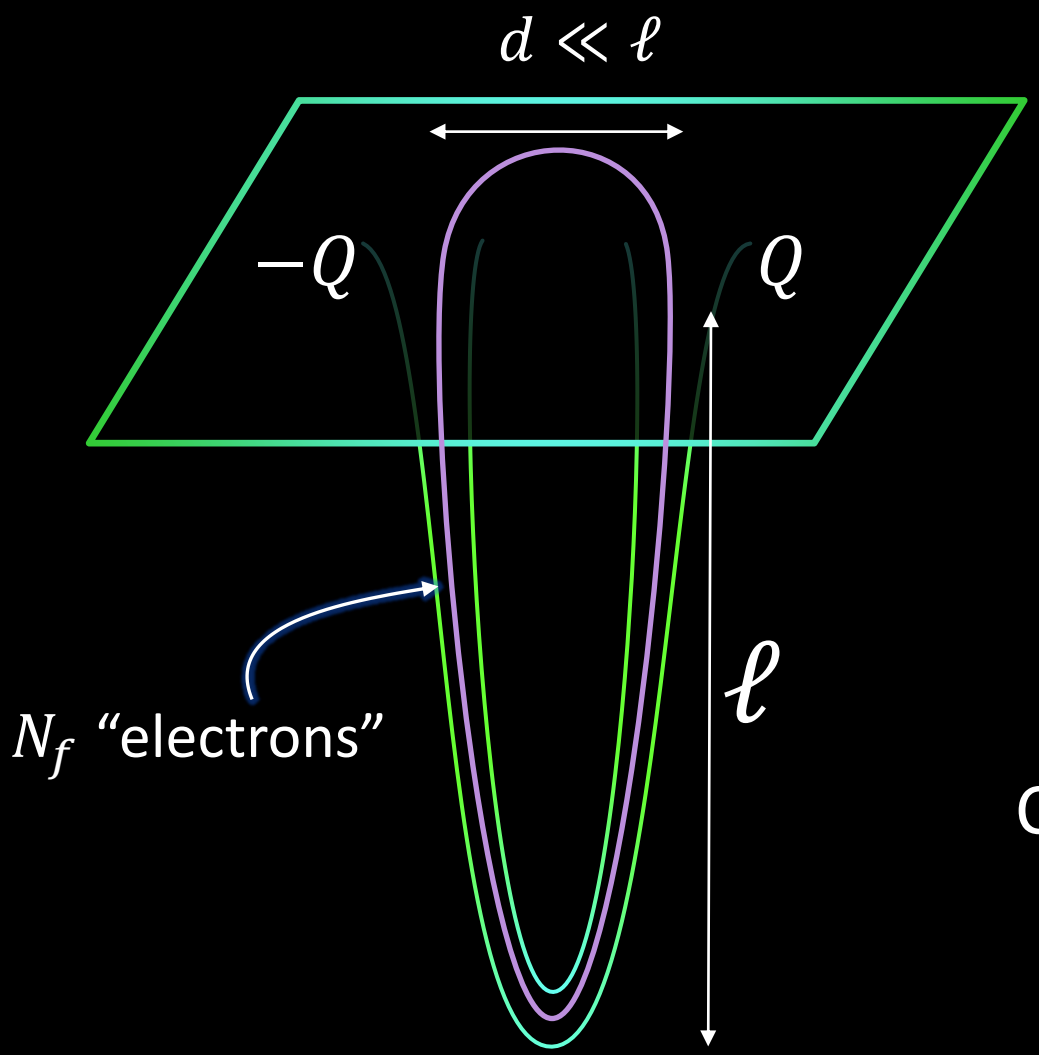
Landau ground state level

Maldacena+Milekhin+Popov 2018

“Dark CFT” from 5D bulk

Maldacena+Milekhin 2020

Energetics



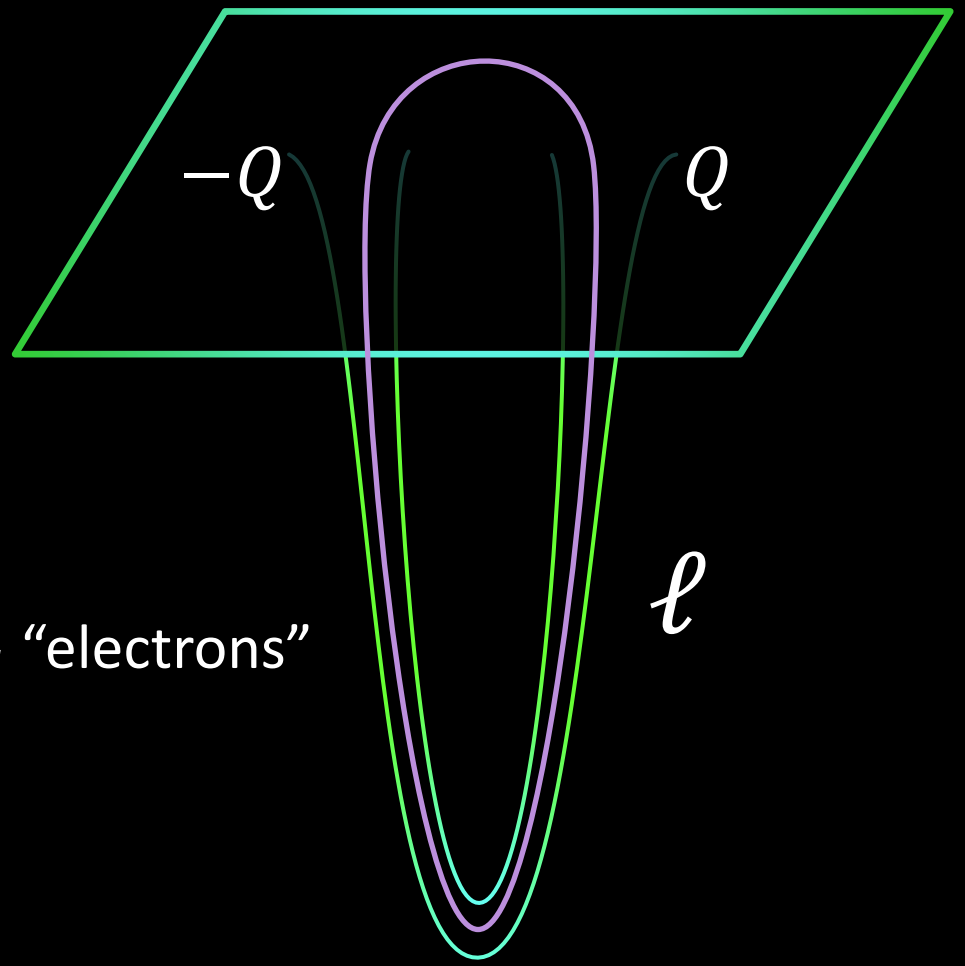
Energy "above" extremality

$$E = \frac{\pi^{3/2} Q^3 l_P}{\ell^2} - \frac{N_f Q}{8\ell}$$

Classical, from RN
redshift → small

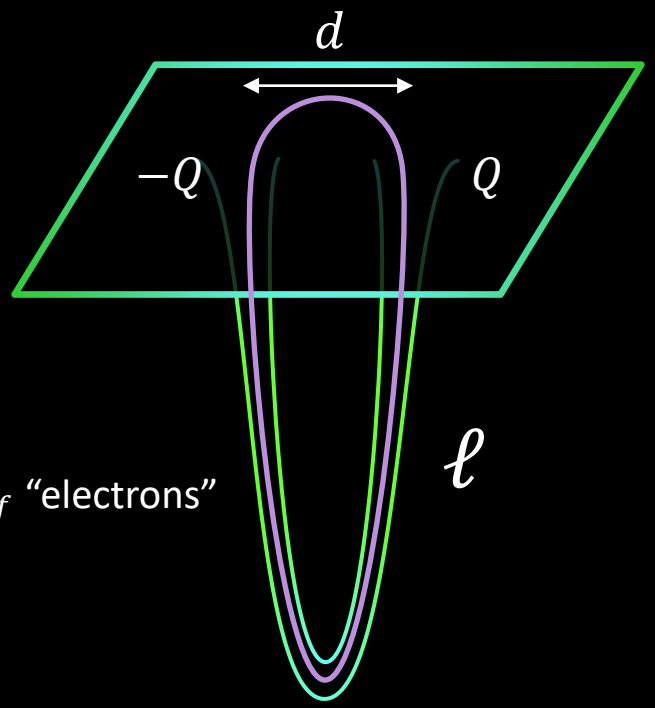
Quantum
Casimir in S^1

Energetics



Energy "above" extremality

$$E_{binding} = -\frac{N_f Q}{16\ell}$$

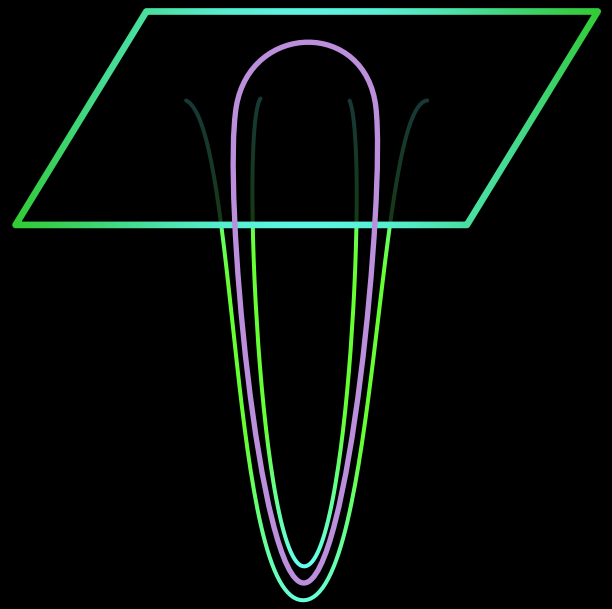


$$E_{binding} = -\frac{N_f Q}{16\ell}$$

Remarks

- Small binding energy: *fragile* wormhole
- May allow for $d \lesssim \ell$
- Can only make single-universe wormholes
- Attraction between mouths?
  - merger time $d^{3/2}$ long enough to cross
  - may balance w/ fluxtube, strings, rotation…

Variations on the theme of wormholes



More versatile constructions:

- Casimir energy from cosmic string zero-modes on S^1 *Fu+Grado-White+Marolf 2018*
- Several $U(1)$s

Entanglement and Teleportation

Gao+Jafferis+Wall 2016
Maldacena+Qi 2018

The two “black hole mouths” are entangled

Microscopic degrees of freedom in Bell-like state

Sending a particle through a wormhole is dual to using entanglement as resource for quantum teleportation

Multiple wormholes?

Fragile!

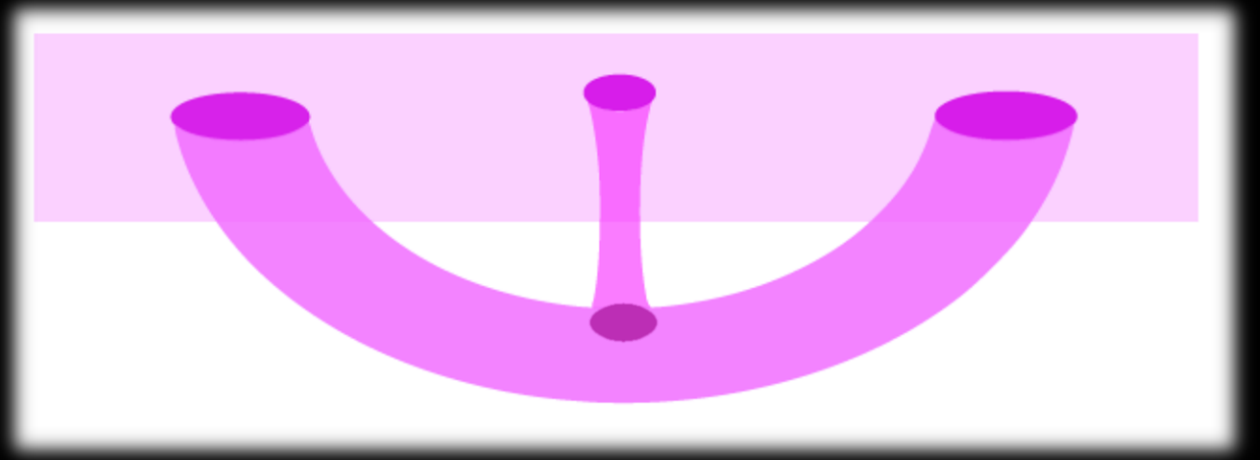
Make *small corrections*

Three-mouth wormholes

Acquire a ready-made wormhole

Insert a small black hole in it

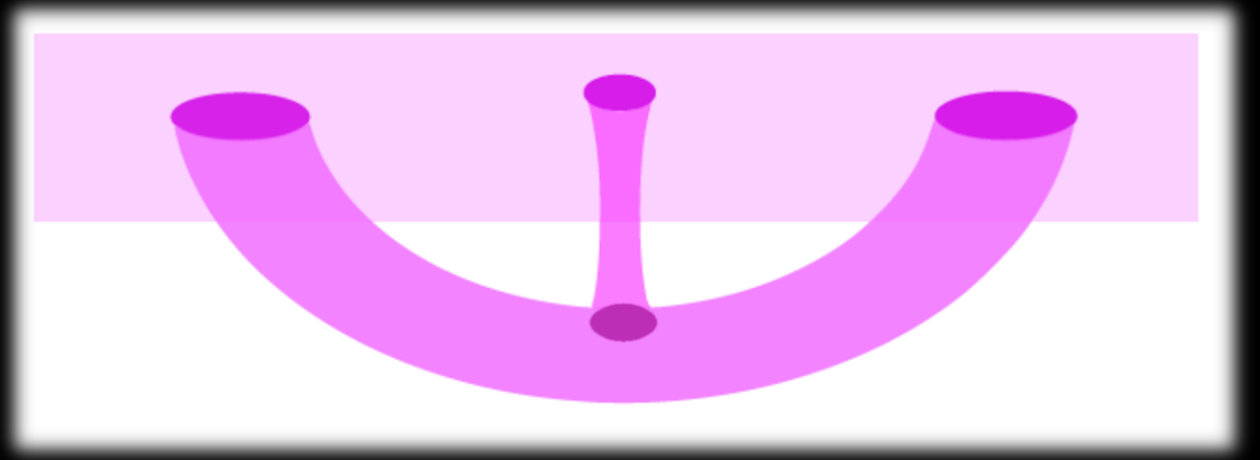
Connect to small black hole outside



Inserting the small mouth

Technically straightforward:

Perturbations w/ matched asymptotics

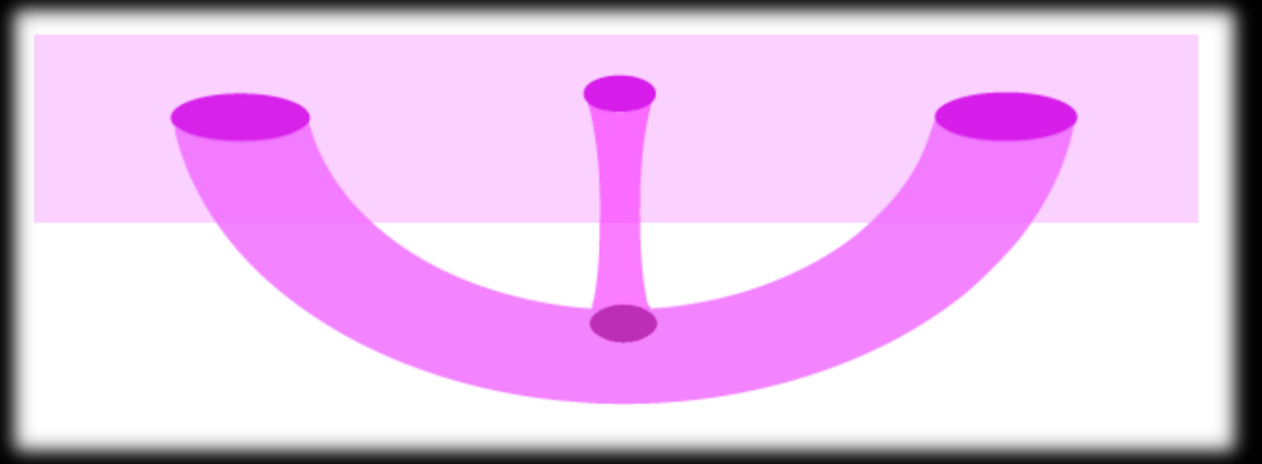


How large the third black hole?

Choking the throat?

Semiclassical black holes have $m \gg m_{Planck}$

Throat redshift helps, but how large can m be?



Backreaction calculation shows that throat remains open if

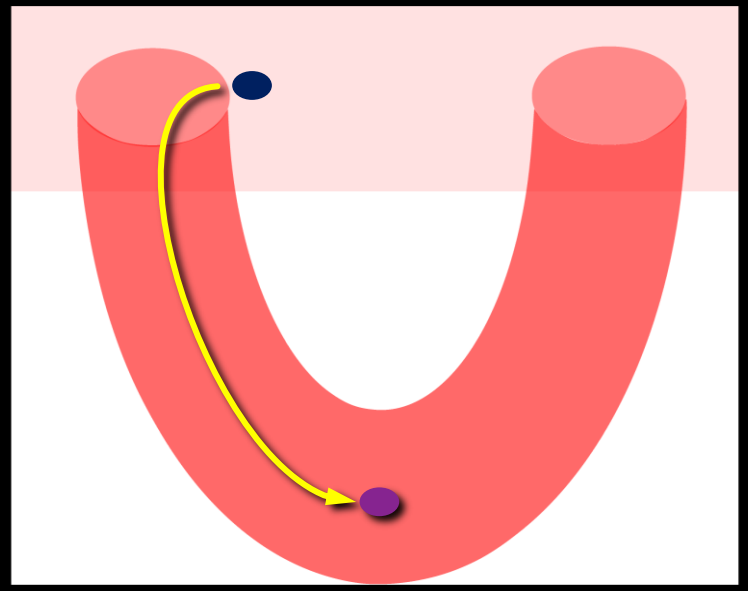
small bh mass $m < \frac{1}{8\sqrt{\pi}} N_f \, m_{Planck}$

Then $E_{bh} < |E_{binding}|$

$\Rightarrow$ if $N_f \gg 1$ then a semiclassical three-mouth wormhole is possible

(also need small enough bh radius to fit inside throat – can do it)

Lowering the mass

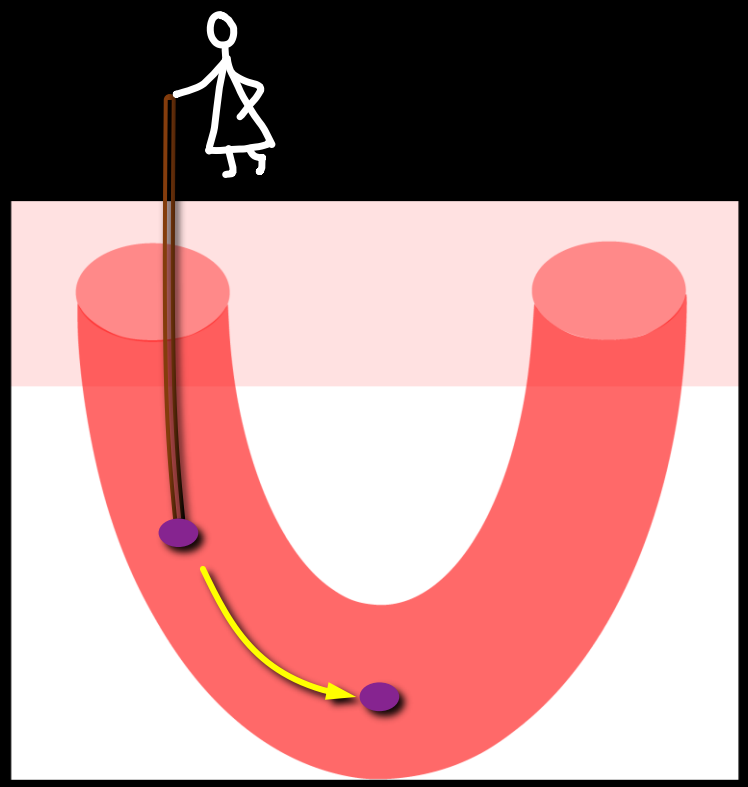


Choking the throat, now?

Near the big mouth

$$m \gg |E_{binding}|$$ (no redshift help)

Lowering the mass



Lower it *à la Geroch*

Slowly, hanging from a string

Calculation shows it is safe for

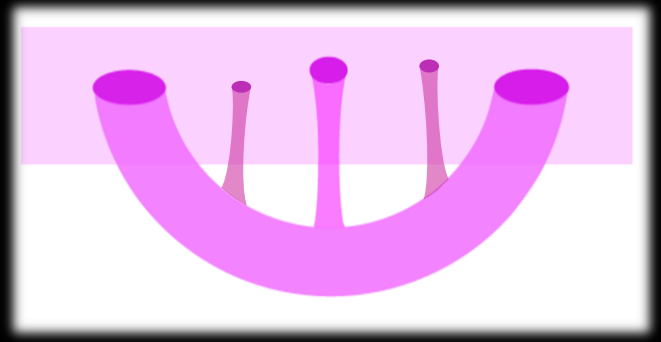
$$m \lesssim \frac{1}{8\sqrt{\pi}} N_f \, m_{Planck}$$

So

It is possible to construct three-mouth wormholes

with two big mouths/one small mouth

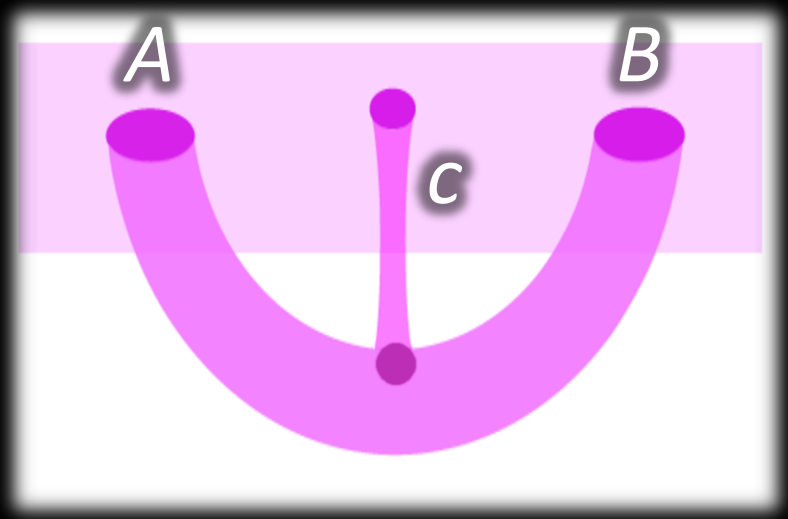
Smallsmouth strikes again



Can insert many small mouths

as long as $\sum_i E_{bh,i} < |E_{binding}|$

Signalling between mouths



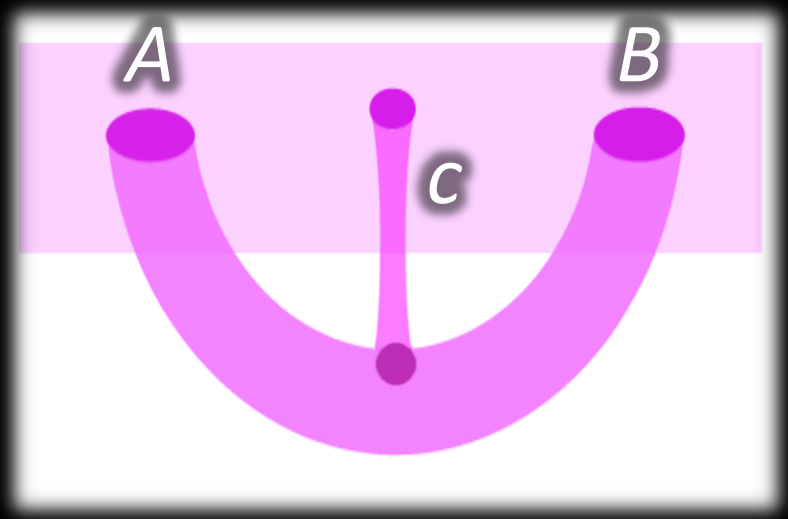
Leaky pipeline

Particle or wave from *A* to *B* may be (partially) absorbed by *c*

Absorption $\propto$ area of *c* $\propto$ dof's of *c*

Angular dependence $\leftrightarrow SU(2)$ R-charge of qubits

Signalling between mouths



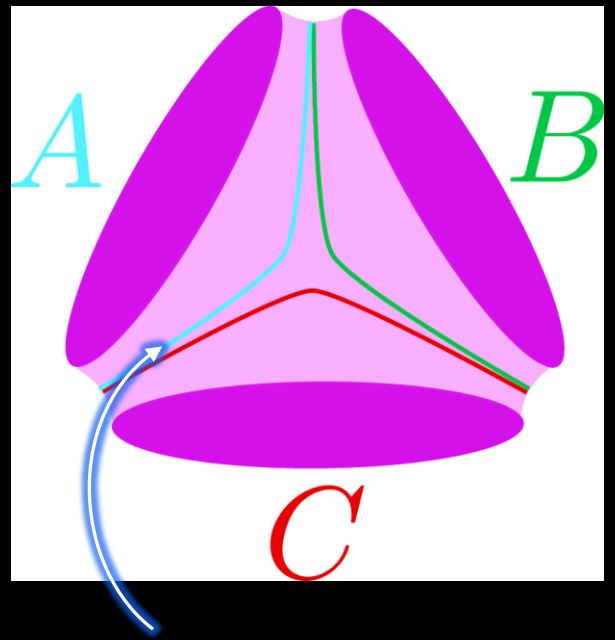
Time delay

Message from *A* to *B* delayed (Shapiro) by mass *c*

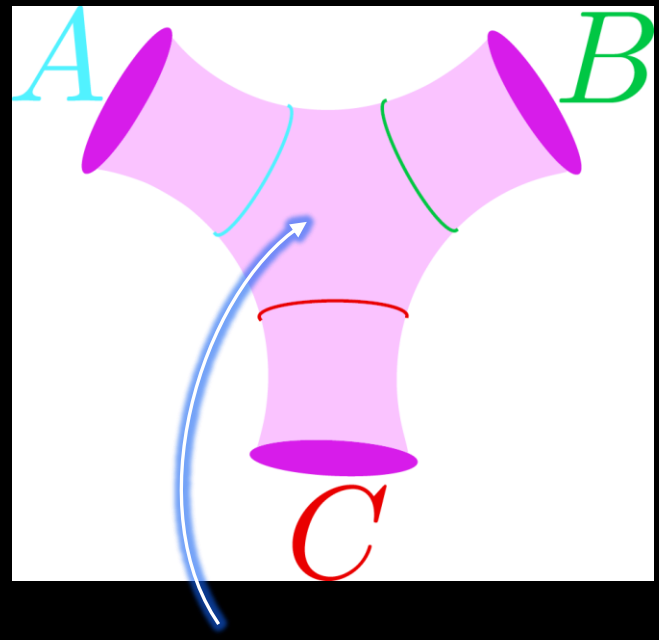
Increased complexity of decoding the message

Multi-partite entanglement

Marolf+Maxfield+Peach+Ross 2015

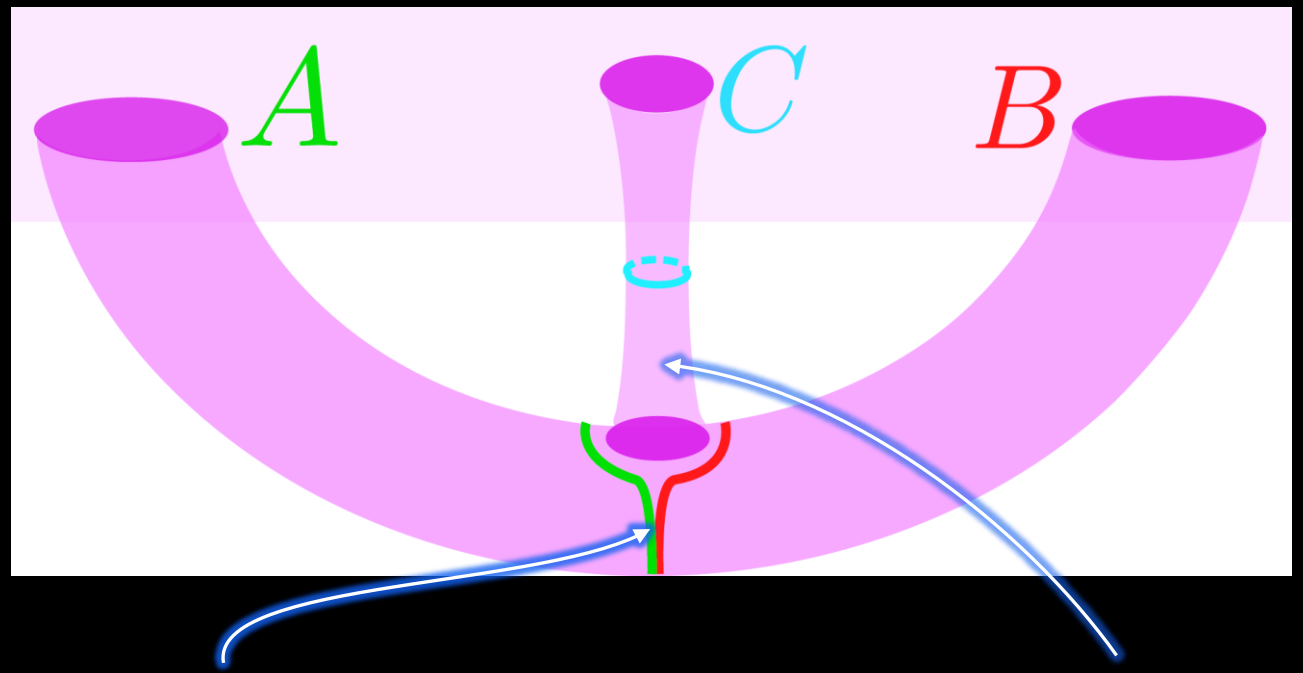


Mostly bipartite entanglement



Tripartite entanglement

Multi-partite entanglement



Conclusions

- Intra-universe traversable wormholes are physically allowed

Maldacena+Milekhin+Popov
Fu+Grado-White+Marolf

- Multi-mouth traversable wormholes also allowed – two big mouths + small mouths
- Multi-wormholes reveal new multi-partite holographic entanglement structures