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# Holographic Pseudo Entropy

It from Qubit
Simons Collaboration

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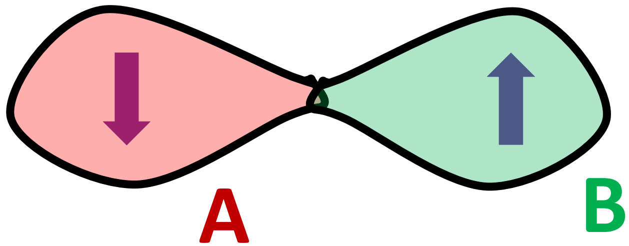
# ① Introduction




## Quantum Entanglement (QE)

Two parts (subsystems) A and B in a total system are quantum mechanically correlated.

e.g. Bell state : $\left|\Psi_{Bell}\right\rangle = \frac{1}{\sqrt{2}}\left[\left|\uparrow\right\rangle_A \otimes \left|\downarrow\right\rangle_B + \left|\downarrow\right\rangle_A \otimes \left|\uparrow\right\rangle_B\right]$

**Pure States**: Non-zero QE $\Leftrightarrow \left|\Psi\right\rangle_{AB} \neq \left|\Psi_1\right\rangle_A \otimes \left|\Psi_2\right\rangle_B$ .

**Direct Product**

**Mixed States**: Non-zero QE $\Leftrightarrow \rho_{AB} \neq \sum_i p_i\, \rho_A^{(i)} \otimes \rho_B^{(i)}$ .

**Separable**

The best (or only) measure of quantum entanglement for pure states is known to be **entanglement entropy (EE)**.

Divide a quantum system into two subsystems A and B:

$$H_{tot} = H_A \otimes H_B \quad .$$

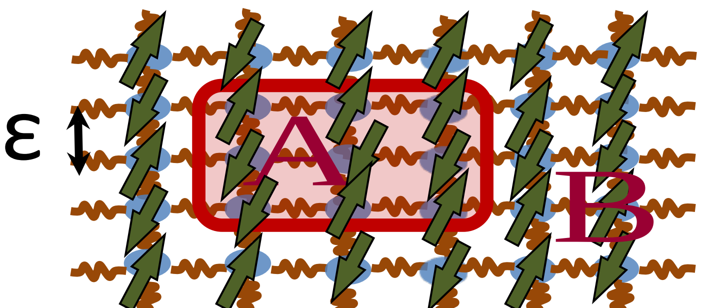
Define the reduced density matrix by $\rho_A = \mathrm{Tr}_B |\Psi\rangle\langle\Psi|$ .

The entanglement entropy $S_A$ is defined by

$$S_A = -\mathrm{Tr}_A\, \rho_A \log \rho_A \, .$$

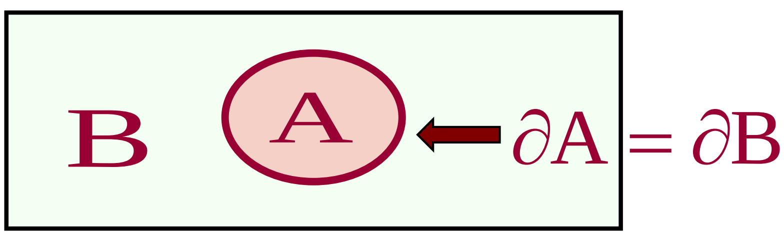
(von-Neumann entropy)

**Quantum Many-body Systems**




**Continuum Limit ε→0**

**Quantum Field Theories (QFTs)**



# Holographic Entanglement Entropy for Static Spacetimes

[Ryu-TT 06; derived by Lewkowycz-Maldacena 13]

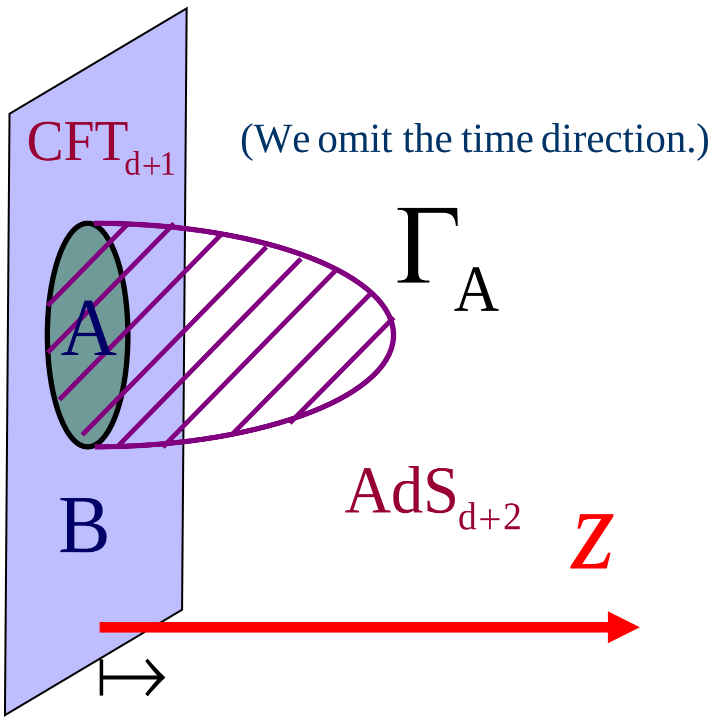
For static asymptotically AdS spacetimes:

$$S_A = \underset{\substack{\partial\Gamma_A = \partial A \\ \Gamma_A \approx A}}{\mathrm{Min}} \left[ \frac{\mathrm{Area}(\Gamma_A)}{4G_N} \right]$$

$\Gamma_A$ is the minimal area surface (codim.=2) on the time slice such that

$$\partial A = \partial\gamma_A \ \text{ and } \ A \sim \gamma_A \ .$$

homologous




$z > \varepsilon$ (UV cut off)

$$ds^2 = R^2 \cdot \frac{dz^2 - dt^2 + \sum_{i=1}^{d} dx_i^2}{z^2}$$

# Covariant Holographic Entanglement Entropy

[Hubeny-Rangamani-TT 07, derived by Dong-Lewkowycz-Rangamani-TT 16]

A generic Lorentzian asymptotic AdS spacetime is dual to a time dependent state $|\Psi(t)\rangle$ in the dual CFT.

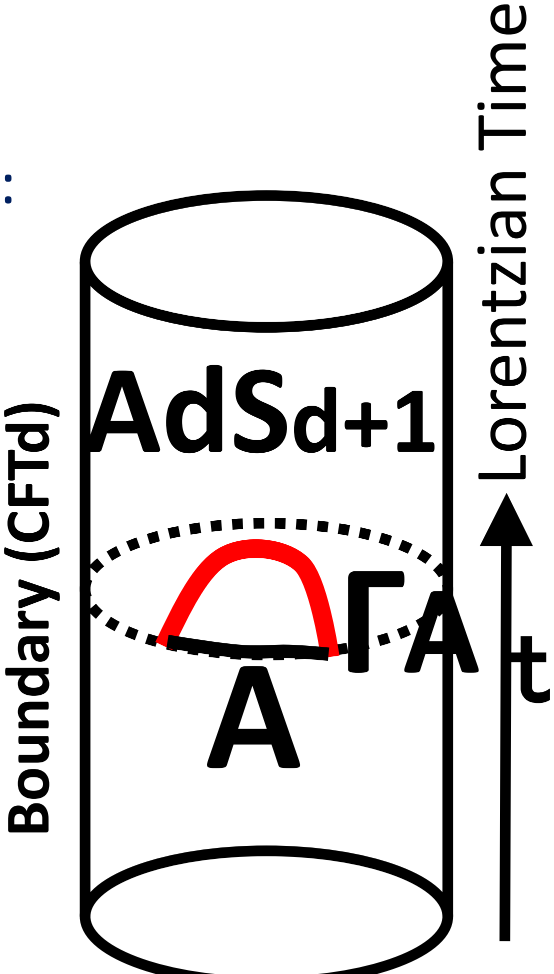
The entanglement entropy gets time-dependent:

$$\rho_A(t) = \mathrm{Tr}_B[|\Psi(t)\rangle\langle\Psi(t)|] \Rightarrow S_A(t).$$

This is computed by the holographic formula:

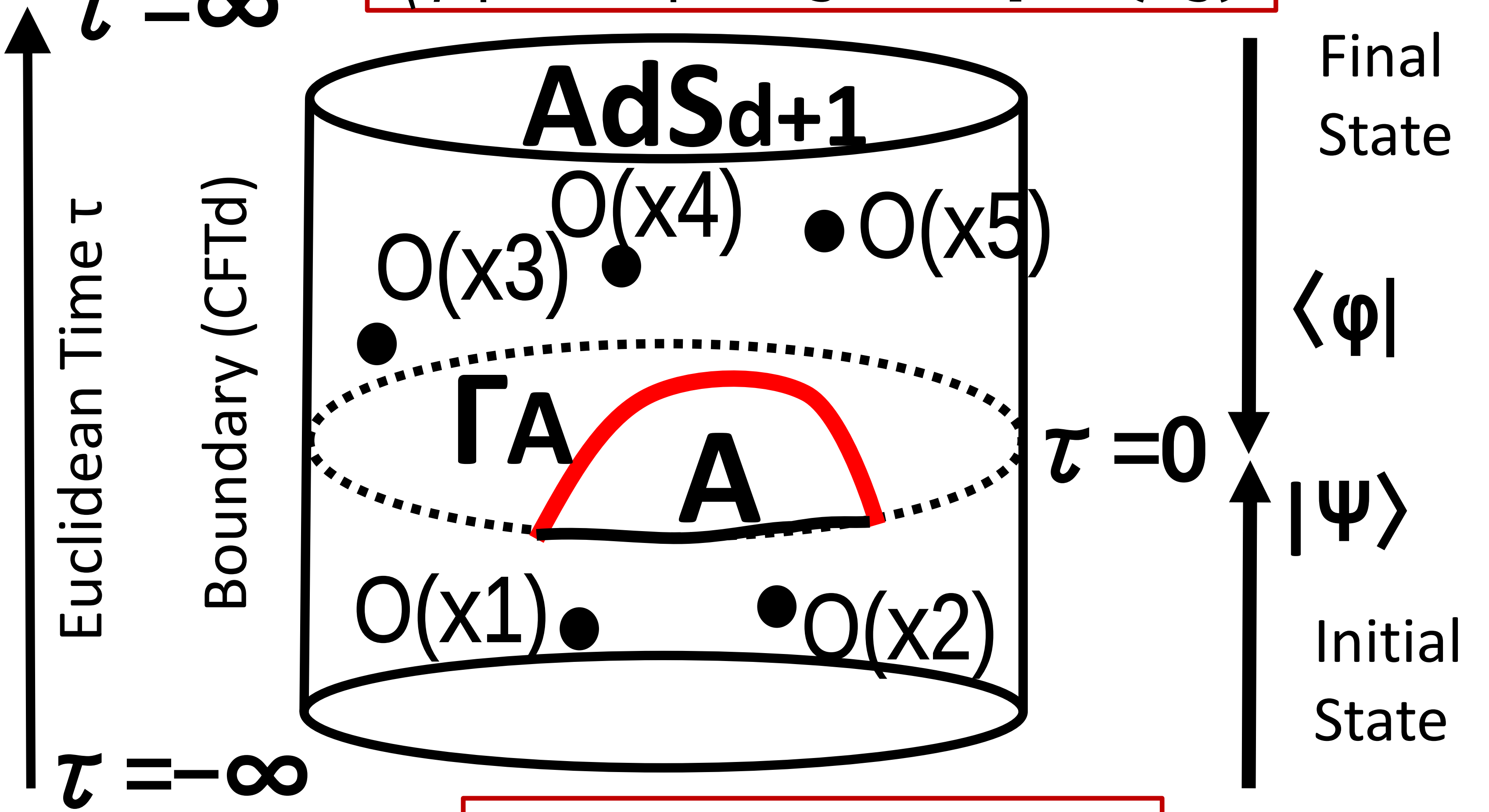
$$S_A(t) = \mathrm{Min}_{\Gamma_A}\mathrm{Ext}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right]$$

$$\partial A = \partial\gamma_A \text{ and } A \sim \gamma_A \ .$$



**Question:** **How about the area of minimal surface in *Euclidean time dependent* asymptotically AdS spaces ?**

$$\langle\varphi| = \langle 0|O(x_3)O(x_4)O(x_5)$$




$$|\psi\rangle = O(x_1)O(x_2)|0\rangle$$

We argue that the area of minimal surface calculates the following EE-like quantity:

$$S\left(\tau_A^{\psi|\varphi}\right) = -\mathrm{Tr}\left[\tau_A^{\psi|\varphi}\log\tau_A^{\psi|\varphi}\right].$$

We call this quantity *pseudo (entanglement) entropy*.

Here , the reduced transition matrix $\tau_A^{\psi|\varphi}$ is defined as

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right],$$

from the transition matrix:

$$\tau^{\psi|\varphi} = \frac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}.$$

Note that in general this transition matrix is not Hermitian.

[cf. "conditional entropy of post-selected states" Salek-Schbert-Wiesner 2013]

In this talk, we would like to study properties of pseudo entropy in quantum many-body systems, CFTs and AdS/CFT.

Note: In quantum information theory, the transition matrices arise when we consider *post-selection*.

$$\frac{\langle\varphi|O_A|\psi\rangle}{\langle\varphi|\psi\rangle} = \mathrm{Tr}[O_A \tau_A^{\psi|\varphi}]$$

Initial State

Final state after post-selection

This quantity can be complex valued in general and is called **weak value**.

[Aharanov-Albert-Vaidman 1988,...]

# Contents

# ② Basics of Pseudo (Renyi) Entropy

## (2-1) Definition of Pseudo (Renyi) Entropy

Consider two quantum states $|\psi\rangle$ and $|\varphi\rangle$ , and define the transition matrix: $\tau^{\psi|\varphi} = \dfrac{|\psi\rangle\langle\varphi|}{\langle\varphi|\psi\rangle}$.

We decompose the Hilbert space as $H_{tot} = H_A \otimes H_B$ and introduce the reduced transition matrix:

$$\tau_A^{\psi|\varphi} = \mathrm{Tr}_B\left[\tau^{\psi|\varphi}\right]$$

The pseudo n-th Renyi entropy is defined by

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].$$

The n=1 limit defined the (von-Neumann) pseudo entropy:

$$S\left(\tau_A^{\psi|\varphi}\right)=S^{(n=1)}\left(\tau_A^{\psi|\varphi}\right)$$

**Note 1**: Since $\tau_A^{\psi|\varphi}$ is not a quantum state i.e. Hermitian and positive semi-definite, the pseudo (Renyi) entropy is complex valued in general.

**Note 2**: When A=total system (B=empty), we obtain

$$\left(\tau^{\psi|\varphi}\right)^n = \tau^{\psi|\varphi} \Rightarrow \mathrm{Tr}\left[\left(\tau^{\psi|\varphi}\right)^n\right]=1.$$

Thus, $S^{(n)}\left(\tau^{\psi|\varphi}\right) = 0.$

## (2-2) Basic Properties of Pseudo Entropy

**[1]** If either $|\psi\rangle$ or $|\varphi\rangle$ has no entanglement (i.e. direct product state) , then

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0.$$

→Connection to quantum entanglement !

**[2]** $$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \left[S^{(n)}\left(\tau_A^{\varphi|\psi}\right)\right]^{\dagger}.$$

**[3]** $$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = S^{(n)}\left(\tau_B^{\psi|\varphi}\right).$$

→"$S_A$=$S_B$"

**[4]** If $|\psi\rangle$=$|\varphi\rangle$, then $S^{(n)}\left(\tau_A^{\psi|\varphi}\right)$ = Renyi entropy.

# ③ Pseudo Entropy in Qubit Systems

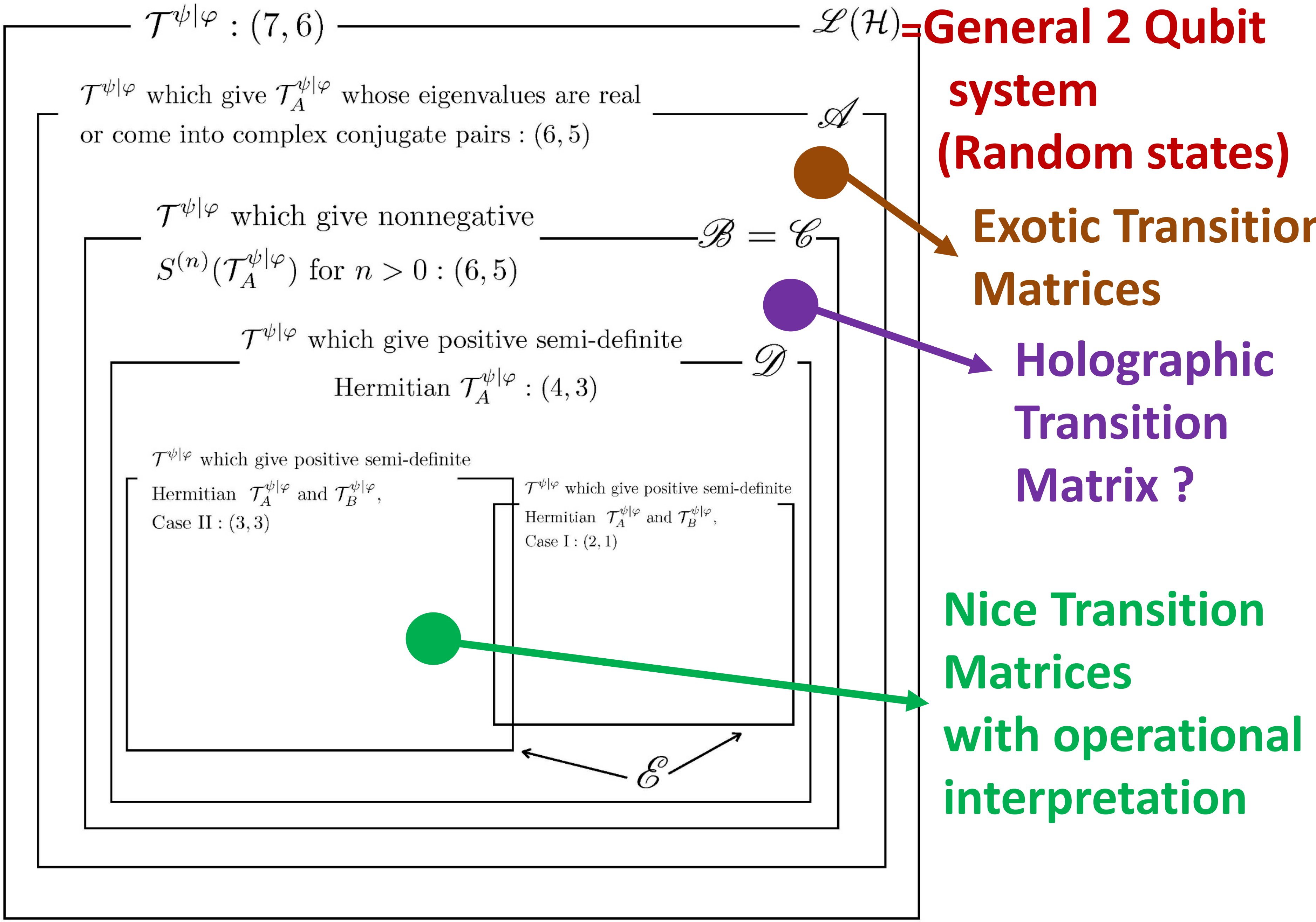
## (3-1) Classification in 2 Qubit Systems

The two pure states in 2 qubit systems are parameterized by 7 parameters:

$$|\psi\rangle \propto |00\rangle + a|11\rangle$$

$$|\varphi\rangle \propto |00\rangle + be^{-i\theta}|11\rangle + ce^{-i\xi}|01\rangle + de^{-i\eta}|10\rangle$$

$$\tau_A^{\psi|\varphi} = \frac{1}{1+abe^{i\theta}}\left(|0\rangle\langle 0| + ace^{i\xi}|1\rangle\langle 0| + de^{i\eta}|0\rangle\langle 1|\right.$$



$\mathcal{T}^{\psi|\varphi}:(7,6)$
$\mathscr{L}(\mathcal{H})$=General 2 Qubit system (Random states)
$\mathcal{T}^{\psi|\varphi}$ which give $\mathcal{T}_A^{\psi|\varphi}$ whose eigenvalues are real or come into complex conjugate pairs : (6, 5)
$\mathscr{A}$
$\mathcal{T}^{\psi|\varphi}$ which give nonnegative $S^{(n)}(\mathcal{T}_A^{\psi|\varphi})$ for $n>0$ : (6, 5)
$\mathscr{B}=\mathscr{C}$
Exotic Transition Matrices
$\mathcal{T}^{\psi|\varphi}$ which give positive semi-definite Hermitian $\mathcal{T}_A^{\psi|\varphi}$ : (4, 3)
$\mathscr{D}$
Holographic Transition Matrix ?
$\mathcal{T}^{\psi|\varphi}$ which give positive semi-definite Hermitian $\mathcal{T}_A^{\psi|\varphi}$ and $\mathcal{T}_B^{\psi|\varphi}$, Case II : (3, 3)
$\mathcal{T}^{\psi|\varphi}$ which give positive semi-definite Hermitian $\mathcal{T}_A^{\psi|\varphi}$ and $\mathcal{T}_B^{\psi|\varphi}$, Case I : (2, 1)
$\mathscr{E}$
Nice Transition Matrices with operational interpretation

## An Example of Exotic Transition Matrix

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + e^{i\theta}|11\rangle)$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$$

$$\tau_A^{\psi|\varphi} = \frac{1}{1+e^{i\theta}}\left(|0\rangle\langle 0| + e^{i\theta}|1\rangle\langle 1|\right).$$

→ Complex conjugate pair of Eigenvalues

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\left[\frac{\cos\frac{n\theta}{2}}{2^{n-1}\cos^n\frac{\theta}{2}}\right]$$

→Only special values of θ can give positive values pseudo entropy.

## (3-2) Pseudo Entropy as Entanglement Distillation

Let us focus on the class E i.e. $\tau_A^{\psi|\varphi}$ And $\tau_B^{\psi|\varphi}$ are Hermitian and semi-positive definite.

Remarkably, in this case we can show a quantum information theoretical interpretation of pseudo entropy:

**Claim** **Pseudo Entropy** $S\left(\tau_A^{\psi|\varphi}\right)$

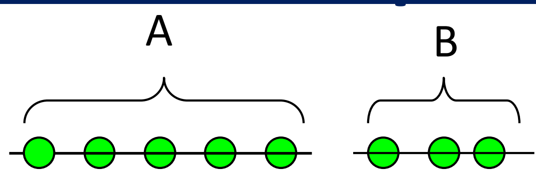
**= # of Distillable Bell Pairs**

**as an intermediate states**

**of post-selection** $|\psi\rangle \to |\varphi\rangle$.

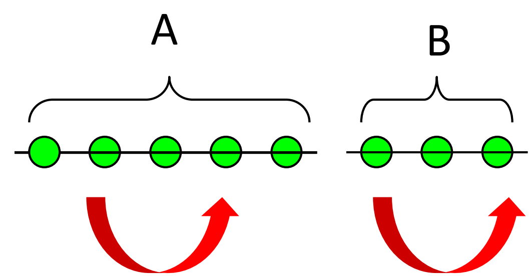
More precisely, we take asymptotic limit M→∞.

# Operational Interpretation of EE from LOCC

**Setup** 


$$\Rightarrow H_{tot} = H_A \otimes H_B$$

**LO** (=Local Operations)




Projection measurements and unitary trfs. which act either A or B only.

**CC** (=Classical Communications between A and B)

**⇒These operations are combined and called LOCC**.

A basic example of LOCC: quantum teleportation

$$(|\Psi\rangle_{AB}\langle\Psi|)^{\otimes M}$$


**Entangled in a very complicated way**

**N Bell pairs**

$$(|\Psi\rangle_{AB}\langle\Psi|)^{\otimes M} \Rightarrow (|\text{Bell}\rangle\langle\text{Bell}|)^{\otimes N}$$

Well-known fact in QI:

$$\boxed{S(\rho_A) = \lim_{M\to\infty} \frac{N}{M}}$$

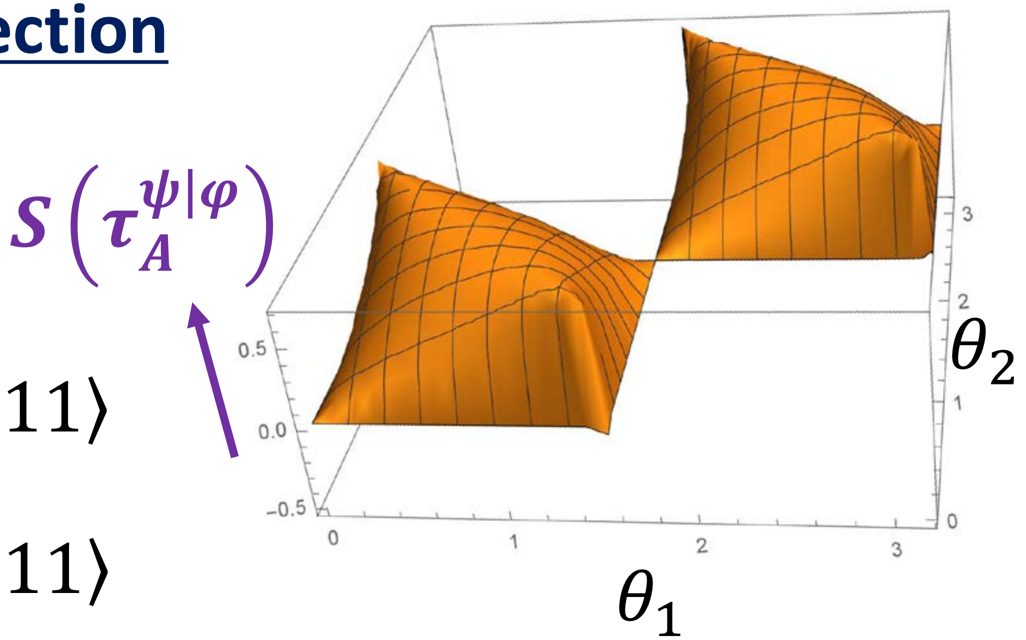
$$\rho_A \equiv \mathrm{Tr}_B[|\Psi\rangle_{AB}\langle\Psi|]$$

# Distillation from Post-selection

In class D, we can write

$$|\psi\rangle = \cos\theta_1|00\rangle + \sin\theta_1|11\rangle$$

$$|\varphi\rangle = \cos\theta_2|00\rangle + \sin\theta_2|11\rangle$$




$$\tau_A^{\psi|\varphi} = \frac{\cos\theta_1\cos\theta_2|0\rangle\langle 0| + \sin\theta_1\sin\theta_2|1\rangle\langle 1|}{\cos(\theta_1 - \theta_2)}$$

$$S\left(\tau_A^{\psi|\varphi}\right) =$$

$$-\frac{\cos\theta_1\cos\theta_2}{\cos(\theta_1-\theta_2)}\cdot\log\frac{\cos\theta_1\cos\theta_2}{\cos(\theta_1-\theta_2)} - \frac{\sin\theta_1\sin\theta_2}{\sin(\theta_1-\theta_2)}\cdot\log\frac{\sin\theta_1\sin\theta_2}{\sin(\theta_1-\theta_2)}$$

$$(|\psi\rangle)^{\otimes M} = (\cos\theta_1|00\rangle + \sin\theta_1|11\rangle)^{\otimes M}$$
$$= \sum_{k=0}^{M}(\cos\theta_1)^{M-k}\,(\sin\theta_1)^{k}\sum_{a=1}^{{}_M C_k}|P(k),a\rangle|P(k),a\rangle$$

$$k = 0: \quad |P(0),1\rangle = |00\cdots0\rangle$$
$$k = 1: \quad |P(1),1\rangle = |10\cdots0\rangle, |P(1),2\rangle = |01\cdots0\rangle, \quad \cdots$$

Projection to maximally entangled states with **Log[MCk]** entropy:

**MCk=M!/(M-k)!k!**

$$\Pi_k = \sum_{a=1}^{{}_M C_k}|P(k),a\rangle\langle P(k),a|$$

probability: $p_k = \langle\varphi|\Pi_k|\psi\rangle/\langle\varphi|\psi\rangle = \frac{(c_1c_2)^{M-k}\,(s_1s_2)^k}{(c_1c_2+s_1s_2)^M}\cdot {}_M C_k$

**# of Distillable Bell pairs:** $\mathbf{N} = \sum_{k=0}^{M} p_k \cdot \mathrm{Log}[{}_M C_k]$

$$\approx M\cdot S\left(\tau_A^{\psi|\varphi}\right) \ !$$

# (3-3) Monotonicity in 2 Qubit systems

We can prove the following monotonicity under unitary transformation:

**Claim** Consider two states related by local unitary trf.

$$|\psi\rangle = (U_A \otimes V_B)|\varphi\rangle.$$

If $\tau_A^{\psi|\varphi}$ has non-negative eigenvalues (i.e. class B=C), then

$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) \geq S^{(n)}(\mathrm{Tr}_B[|\psi\rangle\langle\psi|]) = S^{(n)}(\mathrm{Tr}_B[|\varphi\rangle\langle\varphi|]).$$

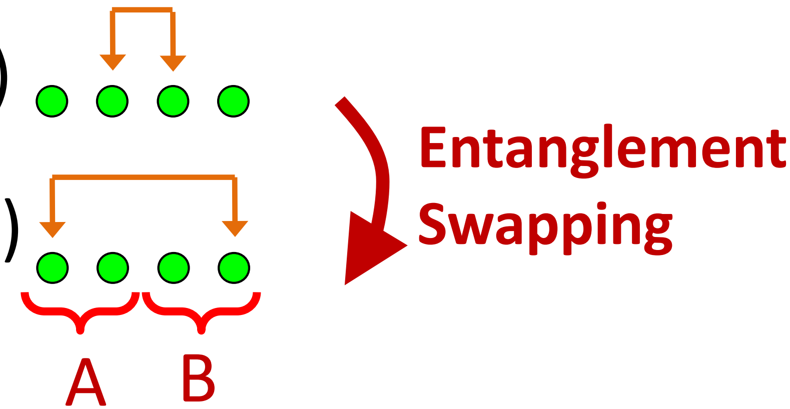
Note: However, this claim is limited to 2 qubit systems.

# Decreasing Pseudo Entropy Examples

**Ex1**

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |0110\rangle)$$

$$|\varphi\rangle = \frac{1}{\sqrt{2}}(|0000\rangle + |1001\rangle)$$




$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = 0$$

$$S^{(n)}(\mathrm{Tr}_B[|\psi\rangle\langle\psi|]) = S^{(n)}(\mathrm{Tr}_B[|\varphi\rangle\langle\varphi|]) = \log 2.$$

**Ex2** **Thermofield States in CFTs**

$$|\psi_i\rangle = \frac{1}{\sqrt{Z(\beta_1)}}\sum_n e^{-\frac{\beta_i}{2}E_n}|n\rangle|n\rangle \qquad (i = 1,2)$$

$$S\left(\frac{\beta_1 + \beta_2}{2}\right) \le \frac{1}{2}[S(\beta_1) + S(\beta_2)]$$

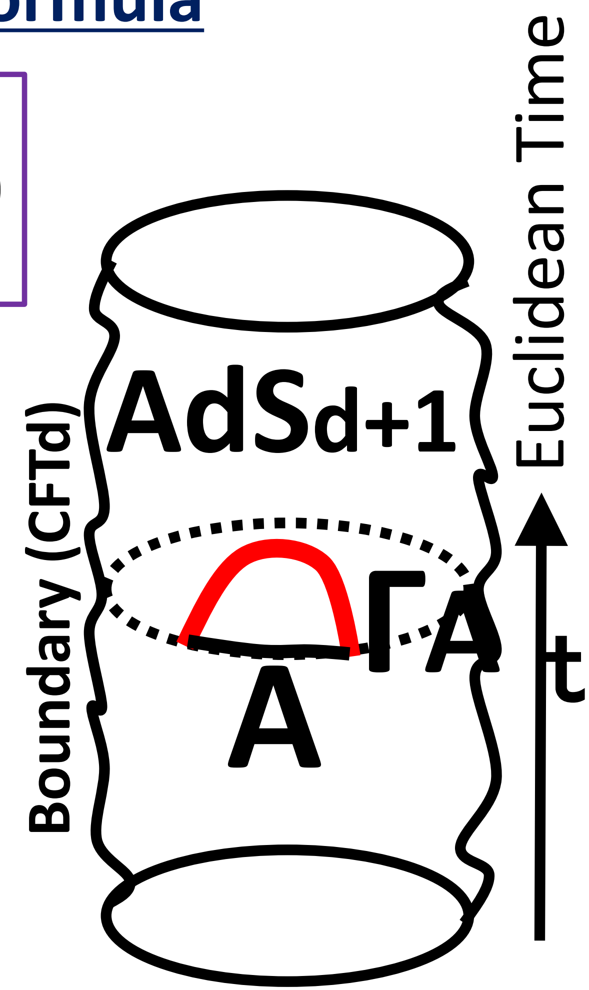
$$S\left(\tau_A^{\psi_1|\psi_2}\right) \le \frac{1}{2}[S(\mathrm{Tr}_B[|\psi_1\rangle\langle\psi_1|]) + S(\mathrm{Tr}_B[|\psi_2\rangle\langle\psi_2|])].$$

# ④ Holographic Pseudo Entropy

## (4-1) Holographic Pseudo Entropy and General Properties

### Holographic Pseudo Entropy (HPE) Formula

$$S\left(\tau_A^{\psi|\varphi}\right) = \mathrm{Min}_{\Gamma_A}\left[\frac{A(\Gamma_A)}{4G_N}\right] \geq 0$$




### Basic Properties of HPE

$(i)$ If $\rho_A$ *is pure*, $S\left(\tau_A^{\psi|\varphi}\right)$ =0.

$(ii)$ If $\psi$ *or* $\varphi$ *is not entangled*,

$S\left(\tau_A^{\psi|\varphi}\right)$ =0.

→This follows from AdS/BCFT [TT 2011]

$(iii)$ $S\left(\tau_A^{\psi|\varphi}\right) = S\left(\tau_B^{\psi|\varphi}\right)$. **"SA=SB"**

$(iv)$ $S\left(\tau_A^{\psi|\varphi}\right) + S\left(\tau_B^{\psi|\varphi}\right) \geq S\left(\tau_{AB}^{\psi|\varphi}\right)$. **"Subadditivity"**

- However, it is not clear whether the strong subadditivity holds for the holographic pseudo entropy even for a single classical geometry. (cf. Proof of SSA in Lorentzian AdS [Wall 2012] )

- Since holographic computations in general leads to

$$\mathrm{Tr}[\,(\tau_A^{\psi|\varphi})^2] \neq \mathrm{Tr}[\,\tau_A^{\psi|\varphi} \cdot (\tau_A^{\psi|\varphi})^\dagger]$$

we find that $\tau_A^{\psi|\varphi}$ in holography belongs to class C.

- So far we focus on a single classical geometry. In general, we can consider a quantum superposition of several classical gravity states $|\Psi_i\rangle = \sum_{k=1}^{M} \alpha_{ik} \, |\psi_{ik}\rangle$, where we find the linearity

$$S(\mathcal{T}_A^{\Psi_i|\Psi_j}) \simeq \frac{\langle\Psi_j|\,\frac{\hat{A}}{4G_N}\,|\Psi_i\rangle}{\langle\Psi_j|\Psi_i\rangle} = \frac{1}{\sum_k \alpha_{jk}^* \alpha_{ik}} \sum_k \alpha_{jk}^* \alpha_{ik} \frac{\mathrm{Area}(\gamma_A^{\psi_{jk}|\psi_{ik}})}{4G_N},$$

⇒We can find examples in this class where SSA is violated.

# (4-2) Simple Example: Janus AdS3/CFT2

Consider a gravity dual of 2d CFT perturbed
by the exactly marginal operator O(x) such that

$$\tau < 0: \quad \phi_- \int d^2x\, O(x), \qquad \qquad \tau > 0: \quad \phi_+ \int d^2x\, O(x).$$

➡ $|\psi\rangle$ ➡ $|\varphi\rangle$

## AdS3 gravity coupled to a massless scalar

$$I = \frac{1}{16\pi G_N} \int d^3x \sqrt{g}(R - \partial_a \phi \partial^a \phi + 2),$$

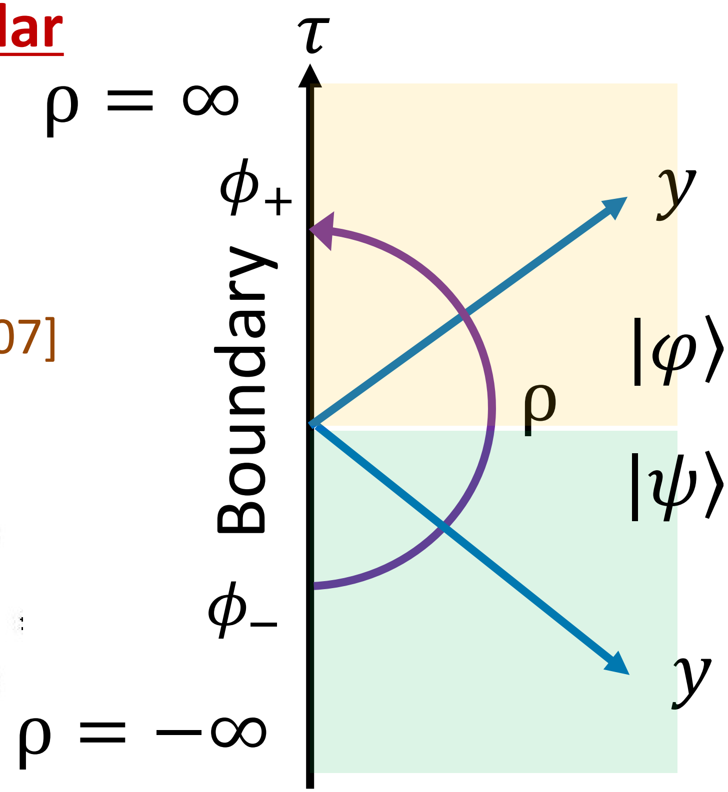
## Janus Solution

[Bak-Gutperle-Hirano 2007]

$$ds^2 = d\rho^2 + f(\rho) \frac{dx^2 + dy^2}{y^2},$$

$$\phi = \phi_0 + \frac{1}{\sqrt{2}} \log \left[ \frac{1 + \sqrt{1 - 2\gamma^2} + \sqrt{2}\gamma \tanh \rho}{1 + \sqrt{1 - 2\gamma^2} - \sqrt{2} \tanh \rho} \right],$$

$$f(\rho) = \frac{1}{2} \left( 1 + \sqrt{1 - 2\gamma^2} \cosh 2\rho \right).$$



# Holographic Pseudo Entropy

We take A=[0,l]. The minimal surface is given by

$$\tau = 0, \qquad x^2 + y^2 = l^2.$$

Thus the holographic pseud entropy reads

$$S\left(\tau_A^{\psi_1|\psi_2}\right) = \frac{c}{3} \cdot \sqrt{\frac{1+\sqrt{1-2\gamma^2}}{2}} \cdot \log\frac{l}{\delta}.$$

$$\left( g_{xx} = \epsilon^{-2} \rightarrow \delta = \sqrt{\frac{1+\sqrt{1-2\gamma^2}}{2}} \cdot \epsilon \right)$$

When γ is small, we obtain

$$S\left(\tau_A^{\psi_1|\psi_2}\right) \approx \frac{c}{3} \cdot \left(1 - \frac{\gamma^2}{4}\right) \cdot \log\frac{l}{\epsilon} + \frac{c}{12}\gamma^2.$$

Agreeing with the CFT perturbation (universal in any 2d CFTs):

$$\Delta S_A^{(n)} = \frac{J^2}{1-n}\left[\int dx^2 \int dy^2 \langle O(x)O(y)\rangle_{\Sigma_n} - n\int dx^2 \int dy^2 \langle O(x)O(y)\rangle_{\Sigma_1}\right].$$

# ⑤ Pseudo Entropy for Locally Excited States

## (5-1) Pseudo Renyi Entropy in Free CFT

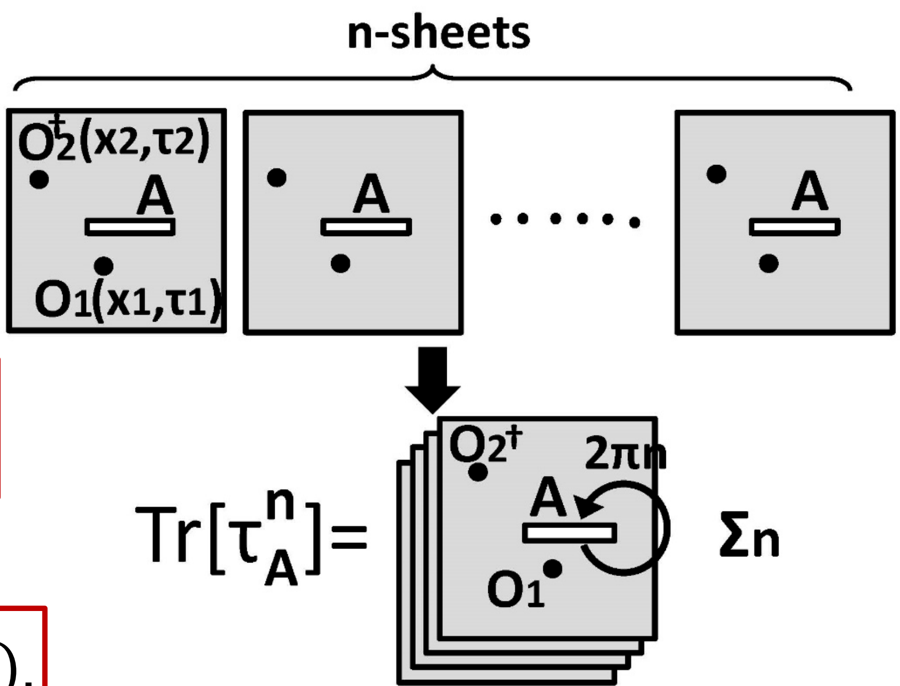
Consider the following 2 locally excited states:

$$|\psi\rangle = N_1 \cdot O(x_1, \tau_1)|0\rangle, \qquad |\varphi\rangle = N_2 \cdot O(x_2, \tau_2)|0\rangle$$

We would like to calculate their pseudo Renyi entropy.

We take the replica of complex plane $w = x + i\tau$.

$\Rightarrow \Sigma_n$




$$S^{(n)}\left(\tau_A^{\psi|\varphi}\right) = \frac{1}{1-n}\log\mathrm{Tr}\left[\left(\tau_A^{\psi|\varphi}\right)^n\right].$$

We compute the difference:

$$\Delta S_A^{(n)} = S^{(n)}\left(\tau_A^{\psi|\varphi}\right) - S^{(n)}(\mathrm{Tr}_B|0\rangle\langle 0|).$$

For simplicity, we will focus on 2nd Renyi pseudo entropy in the 2d massless free scalar CFT.

[EE in the same setup: Nozaki-Numasawa-TT, He-Numasawa-Watanabe-TT 2014]

$$\Delta S_A^{(2)} = -\log \frac{\langle O(w_1)O^\dagger(w_2)O(w_3)O^\dagger(w_4)\rangle_{\Sigma 2}}{\langle O(w_1)O^\dagger(w_2)\rangle_{\Sigma 1}^2}$$

We choose $O(x,\tau) = e^{i\phi(x,\tau)/2} + e^{-i\phi(x,\tau)/2}$ (~Bell pair).

After we perform the conformal map

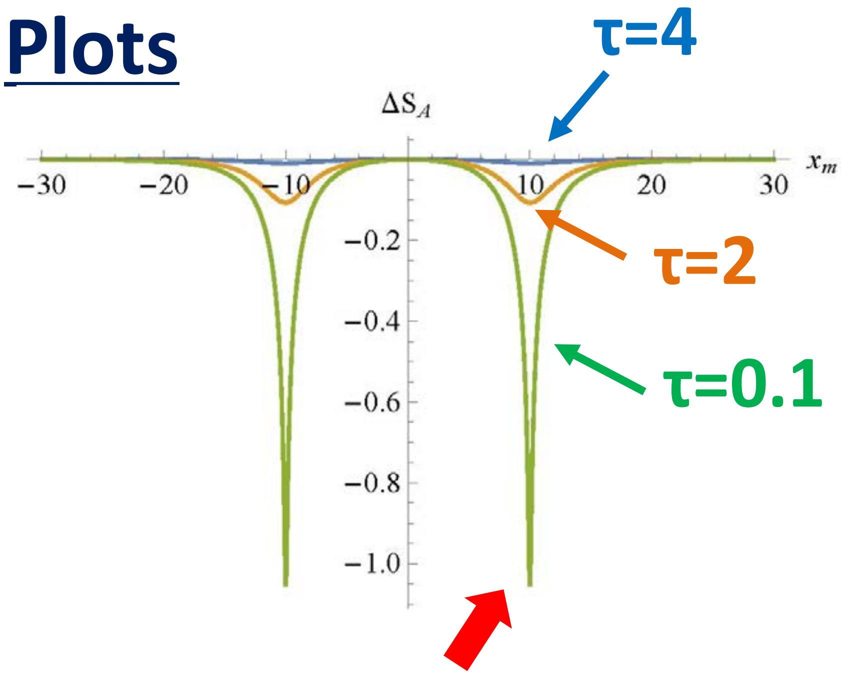
$$\frac{w-a}{w-b} = z^2$$

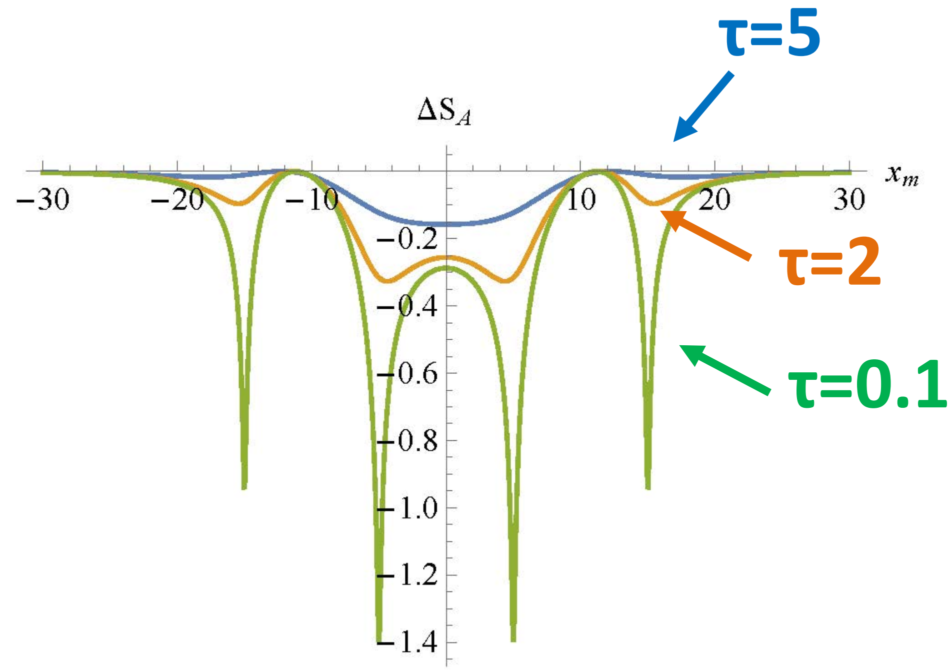
we obtain $\Delta S_A^{(2)} = \log\left(\frac{2}{1+|\eta|+|1-\eta|}\right) \leq 0.$

Cross ratio:

$$\eta = \frac{(z_1-z_2)(z_3-z_4)}{(z_1-z_3)(z_2-z_4)}.$$

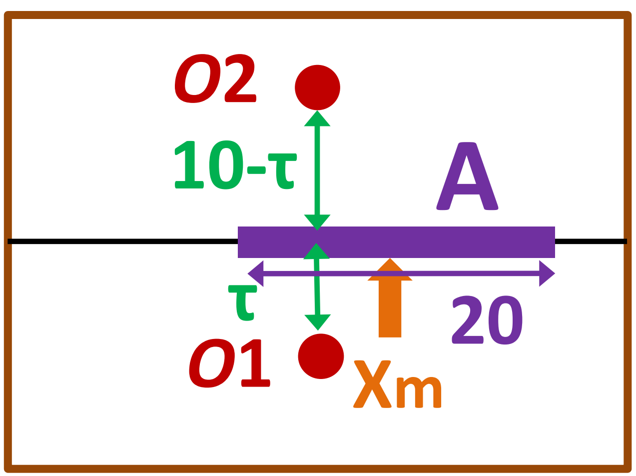
# Plots

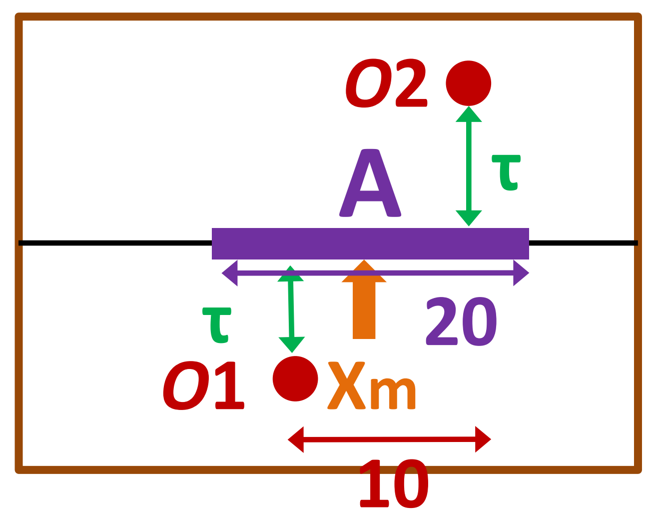



**Pseudo Entropy is reduced when ∂A gets closer to the excitation.**

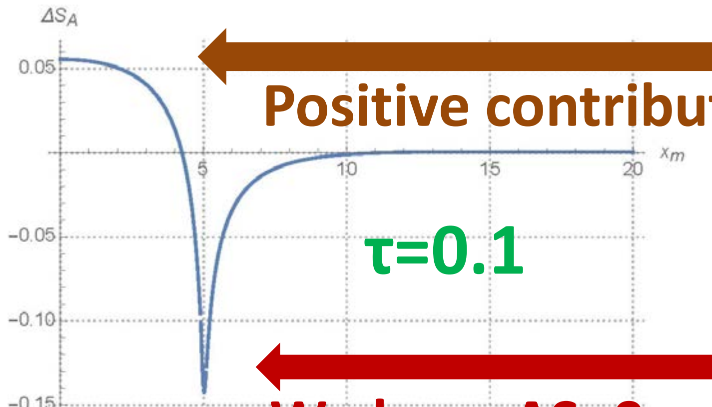
**⇒ This is due to the decreasing property under entanglement swap.**

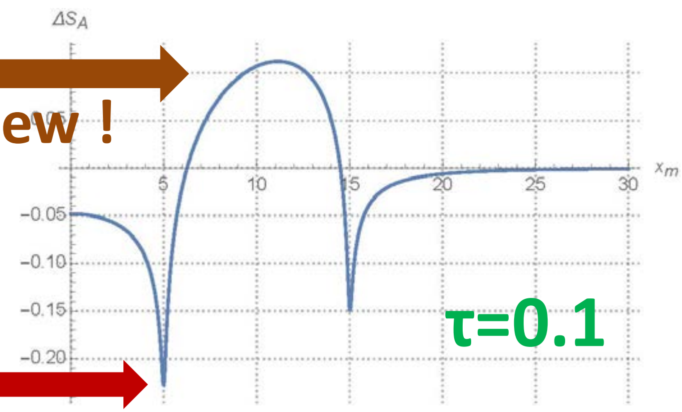


## (5-2) Holographic CFT Calculations

We can calculate the pseudo entropy for the same setup in holographic CFTs:



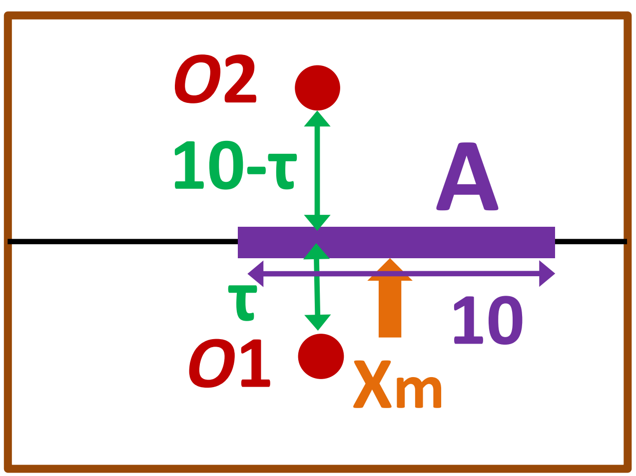

**Positive contribution is new !**

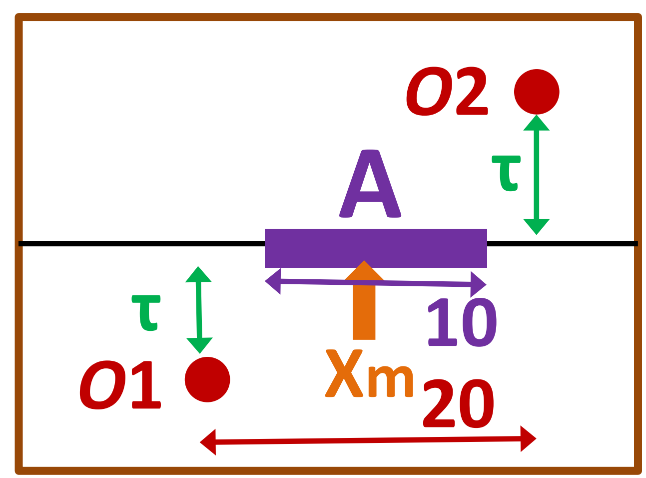
**ΔS>0**

**τ=0.1**

**τ=0.1**

**We have ΔS<0 when ∂A get closer to Os.**
**(in the same way as in free scalar.)**



# ⑥ Mixed State Generalization

Can we define pseudo entropy when the total system AB is mixed ?

Assume $\tau_{AB}^{\psi|\varphi}$ is given.

One possibility is to extend the reflected entropy. [Dutta-Faulkner 2019, Kusuki-Tamaoka 2019]

$$X_{AB} = \sum_{i,j} X_{ij} |i\rangle \langle j|$$

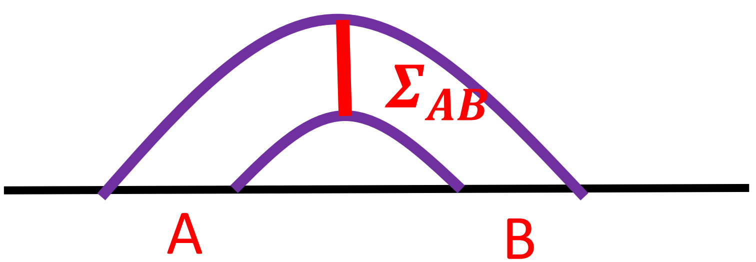
$$\Rightarrow |X\rangle_{AB\tilde{A}\tilde{B}} = \sum_{i.j} X_{ij} |i\rangle_{AB\tilde{A}\tilde{B}} |j^*\rangle_{AB\tilde{A}\tilde{B}}$$

Define pseudo reflected entropy by $S_R(\tau_{AB}^{\psi|\varphi}) = S\left(\mathrm{Tr}_{B\tilde{B}}\left[\frac{|\tau_{AB}^{\psi|\varphi}\rangle\langle(\tau_{AB}^{\psi|\varphi})^\dagger|}{\langle(\tau_{AB}^{\psi|\varphi})^\dagger|\tau_{AB}^{\psi|\varphi}\rangle}\right]\right).$

This coincides with the twice of entanglement wedge cross section:

$$\boldsymbol{S_R(\tau_{AB}^{\psi|\varphi}) = 2 \cdot \frac{A(\Sigma_{AB})}{4G_N}}.$$

[cf. EW=EoP: Umemoto-TT 2017
EW=Odd entropy: Tamaoka 2018
EW∝Negativity: Kudler-Flam-Ryu 2019]



# ⑦ Conclusions

## Main claim of this talk

**Pseudo Entropy = Area of Minimal surface in Euclidean asymptotically AdS with time-dependence**

- Pseudo Entropy is in general complex valued.
  ⇒ Why is holographic pseudo entropy non-negative ?
  [ Euclidean path-integrals are positive valued !]

- Pseudo Entropy for `non-exotic states' measures the amount of quantum entanglement in the intermediate states.
  [How about general states ? What is the meaning of complex values?]

- Pseudo entropy tends to decrease in CFT computations due to entanglement swapping. However, we find also a positive contribution only for holographic CFTs, as opposed to free scalar CFT.

Thank you very much !