

# Some Exact Results in QCD-like and Chiral Gauge Theories

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# Introduction

# Can we solve QCD?

- When we first learn about quarks, we get told we can never see them
  - Internet Scam?

Dear friend,

I am Andre Ouedraogo, a banker by profession from Burkina Faso in West Africa and currently holding the post of Director Auditing and Accounting unit of the bank. It's my urgent need for a foreign partner that made me to contact you for this business. I have the opportunity of transferring the left over funds (\$11.5 million) of one of my bank clients who died along with his entire family on 31 July 2000 in a plane crash. You can confirm the genuineness of the deceased death by clicking on this website.

<http://news.bbc.co.uk/1/hi/world/europe/859479.stm>

I need a foreign partner who will support me because i can not claim this money alone without a foreign partner since the deceased client (the owner of the fund) was a foreigner.

This fund (\$11.5 million) will be shared between us in the ratio of 60/40. I agreed that 40% of this money will be for you as a respect to the provision of a foreign account while 60% will be for me and I want to assure you that this transaction is absolutely legal and risk free since i work in this bank and i have all the necessary information that might be needed. Before we proceed, i would like to know your ability to handle this over there in your country.

Please tell me more about the political/economic stability/monetary policy of your country. I need to know all these because i don't want to have problem with the Government of your country.

Kindly update me with the following information because i want to know you more before we proceed on this transaction. Hope you will understand the importance of this request.

1. Your full name.....
2. Your age/sex .....
3. your occupation .....
4. Your residential address .....
5. Your nationality .....
6. Your private phone number .....
7. Your fax number .....

I will be waiting for your response.

Thanks for your understanding.

Have a great day.

Yours.

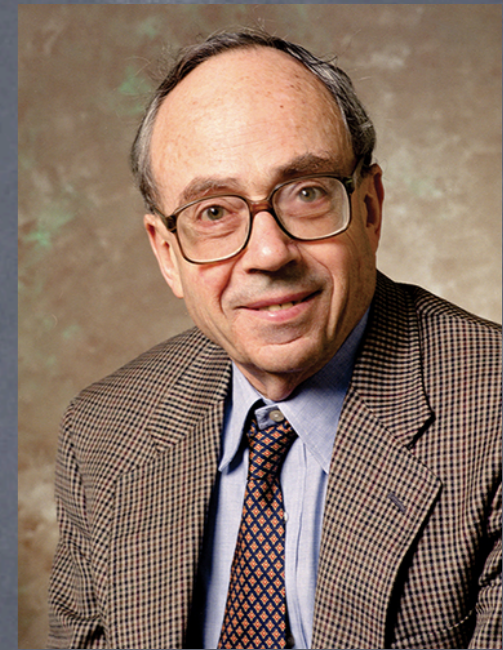
Andre Ouedraogo

# Can we solve QCD?

- When we first learn about quarks, we get told we can never see them
  - Internet Scam?
  - **Confinement!**
  - $\beta < 0$  and asymptotic freedom
  - only suggestive, doesn't prove confinement
- **Another puzzle:** proton and pion are made of same quarks
  - why pion  $\approx$  massless  $\ll$  proton?
- very mysterious!

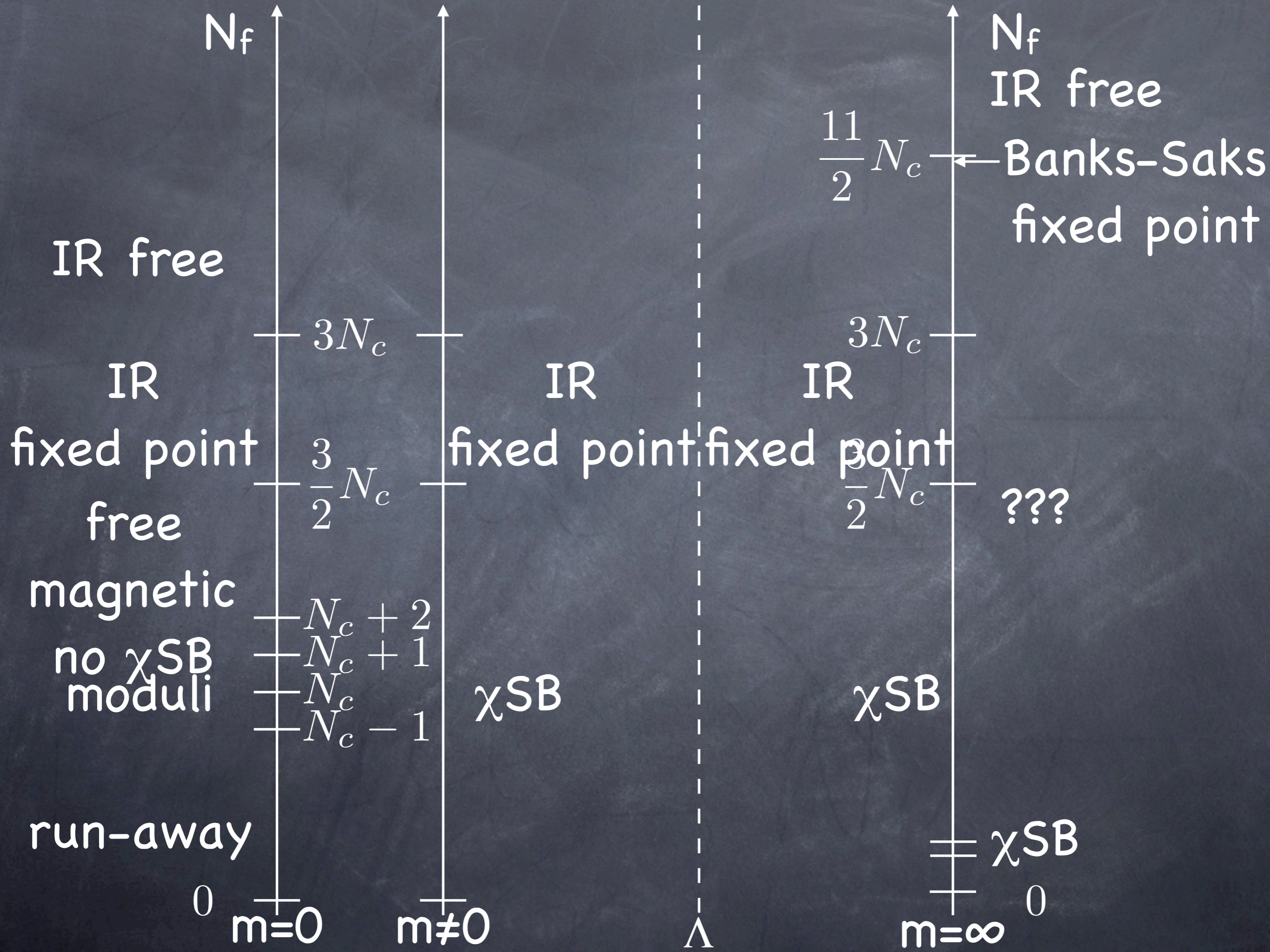
# Feeling better

- 👁 Qualitative picture makes us feel better
- 👁 **Confinement**
  - 👁 dual Meißner effect (Mandelstam)
  - 👁 monopole condensation
  - 👁 quarks confined by electric flux tube
- 👁 **Chiral symmetry breaking** (Nambu)
  - 👁 massless QCD invariant under  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$
  - 👁 pion = Nambu-Goldstone boson = massless
- 👁 but still **not derived from QCD!**



# Feeling even better but not there yet

- Progress in understanding QCD
- Confinement** (Seiberg-Witten)
  - N=2 SYM has Coulomb branch  $u = \text{Tr} \Phi^2$
  - singularities = massless monopole/dyon
  - N=1 perturbation  $W = \mu u - (u - \Lambda^2) M^+ M^-$
  - $M^+ = M^- = \sqrt{\mu} \neq 0$ : monopole condensation!
  - can further perturb to N=0 with  $m_\lambda \neq 0$
- Chiral symmetry breaking**
  - N=2 doesn't have  $\chi S$   $W = \sqrt{2} \tilde{Q}_i \Phi Q^i$
  - N=1 (Seiberg) has too unusual phases





# Main result

- We start with N=1 QCD à la Seiberg
- Most schemes to bring N=1 to N=0 do not have sufficient predictive power
- Anomaly mediation (AMSB) is uniquely suited thanks to its UV insensitivity
- Even with infinitesimal AMSB, the theory collapses to expectations in non-SUSY QCD
- N=1 SUSY + AMSB: great tool to study non-SUSY gauge theories in 4D
- Now we can “derive”  $\chi_{SB}$  from QCD!

# Chiral Symmetry Breaking

- Yoichiro Nambu (1960)
- 2008 Nobel Prize in Physics
- “For the discovery of the mechanism of spontaneous broken symmetry in subatomic physics”
- long-range low-energy behavior determined by symmetries  
= universality class



# Outline

- ① Introduction
- ① Anomaly Mediated Supersymmetry Breaking
- ①  $SU(N_c)$  QCD
- ① confinement and  $\chi$ SB in  $SO(N_c)$  QCD
- ① chiral  $SU(N_c)$  with anti-symmetric tensor
- ① chiral  $SU(N_c)$  with symmetric tensor
- ① Conclusion

# Anomaly-Mediated Supersymmetry Breaking

Randall, Sundrum (1998)

Giudice, Luty, HM, Rattazzi (1998)

# Our Needs

- We'd like to connect N=1 SUSY results by Seiberg to non-SUSY gauge theories
  - decouple gauginos and squarks!
  - SUSY breaking  $m_\lambda$  and  $m_{\tilde{Q}}$
- But we need to deal with composites such as mesons and baryons
- In particular, we need to know signs of their mass-squared to understand symmetry breaking patterns and universality classes

# Gaugino Mass Perturbation

- Series of papers by N. Evans, S. Hsu, M. Schwetz + S. Selipsky (1995)

- Gaugino mass:

$$\int d^2\theta S W_\alpha W^\alpha \quad S = \frac{8\pi^2}{g^2} + i\vartheta + \theta^2 \frac{16\pi^2}{g^2} m_\lambda$$

- non-perturbative superpotential is a function of  $\Lambda^{3N_c - N_f} = \mu e^{-S} = \mu e^{-8\pi^2/g^2} e^{-i\vartheta} \left( 1 - \theta^2 \frac{16\pi^2}{g^2} m_\lambda \right)$

- but Kähler potential arbitrary function of S

$$M^{ij} = \tilde{Q}^i Q^j \quad \int d^4\theta S^* S M_{ij}^* M^{ij}$$

- we don't even know the sign of meson mass<sup>2</sup>

- avoid problem with large enough quark mass

- can't discuss chiral symmetry!

$$W = m_Q \tilde{Q} Q$$

$$m_Q^2 > m_\lambda^2$$

# external symmetry

external symmetries constrain how symmetry breaking parameters enter Lagrangian

e.g. quark mass in chiral Lagrangian

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{2} \text{Tr} G_{\mu\nu} G^{\mu\nu} + \overline{Q}_L i \not{D} Q_L + \overline{Q}_R i \not{D} Q_R - \overline{Q}_L M Q_R - \overline{Q}_R M^\dagger Q_L$$

quark mass  $M$  "transforms" as  $(N_f, N_f^*)$  under  $SU(N_f)_L \times SU(N_f)_R$

Chiral Lagrangian a function of  $U = \frac{1}{f_\pi \mu^2} Q_L \overline{Q}_R$

$$\mathcal{L}_\chi = \frac{f_\pi^2}{4} \text{Tr} \partial_\mu U^\dagger \partial^\mu U + f_\pi \mu^2 \text{Tr} M^\dagger U + \text{c.c.}$$

switch from UV to IR description, but we know how symmetry breaking works

# background spacetime

- Good example of external symmetry is general coordinate invariance
- consider QCD in a background metric

$$\mathcal{L}_{\text{QCD}} = \sqrt{-g} \left( -\frac{1}{2} g^{\mu\rho} g^{\nu\sigma} \text{Tr} G_{\mu\nu} G_{\rho\sigma} + \bar{Q} e_a^\mu \gamma^a i D_\mu Q \right)$$

- switch from UV to IR description
- we know how chiral Lagrangian couples to the background spacetime!

$$\mathcal{L}_\chi = \sqrt{-g} \left( \frac{f_\pi^2}{4} g^{\mu\nu} \text{Tr} \partial_\mu U^\dagger \partial_\nu U + f_\pi \mu^2 \text{Tr} M^\dagger U + c.c. \right)$$



# Sequestering

supergravity

$$m_{3/2} = e^{K/2} |W|$$

SUSY breaking

$$V = e^K \left( |F|^2 - 3 |W|^2 \right)$$

no interaction

gauge theory

$$m = \frac{W}{M_{Pl}^2}$$

# Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$
$$\mathcal{E} = 1 + \theta^2 m$$

- all effects of SUSY breaking encoded in  $\mathcal{E}$
- it can be removed by conformal transformation  $\phi \rightarrow \phi/\mathcal{E}$  if no mass params

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \lambda \phi^3$$
$$\rightarrow \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \lambda \phi^3$$

# Weyl compensator

- "superspacetime background"

$$\int d^4\theta \mathcal{E}^* \mathcal{E} K - \int d^2\theta \mathcal{E}^3 W$$
$$\mathcal{E} = 1 + \theta^2 m \quad \phi \rightarrow \phi/\mathcal{E}$$

- dimensionful parameters receive SUSY breaking

$$\mathcal{L} = \int d^4\theta \mathcal{E}^* \mathcal{E} \phi^* \phi - \int d^2\theta \mathcal{E}^3 \left( \frac{1}{2} M \phi^2 + \lambda \phi^3 \right)$$

$$\rightarrow \mathcal{L} = \int d^4\theta \phi^* \phi - \int d^2\theta \left( \frac{1}{2} M \mathcal{E} \phi^2 + \lambda \phi^3 \right)$$

$$V_{\text{AMSB}} = -m \left( \phi \frac{\partial W}{\partial \phi} - 3W \right)$$

# Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$Z\left(\frac{\mu}{M}\right) \rightarrow \mathcal{Z}\left(\frac{\mu}{M\mathcal{E}}\right) = Z\left(1 + \gamma\frac{1}{2}\ln\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \frac{1}{2}\dot{\gamma}\frac{1}{4}\ln^2\frac{\mu^2}{M^*\mathcal{E}^*M\mathcal{E}} + \dots\right)$$

$$\int d^4\theta \mathcal{Z}\phi^*\phi = Z\left(F^*F + \gamma\frac{1}{2}(m^*\phi^*F + m\phi F^*) + \frac{1}{\dot{\gamma}}m^*m\phi^*\phi\right)$$

$$V = -\frac{1}{4}\dot{\gamma}_i m^* m \phi_i^* \phi_i - \frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m \lambda_{ijk} \phi_i \phi_j \phi_k + c.c.$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

# Superconformal Anomaly

- Cutoff scale also acquires SUSY breaking

$$\frac{1}{g^2} \left( \frac{\mu}{M} \right) \rightarrow \frac{1}{g^2} \left( \frac{\mu}{M\mathcal{E}} \right) = \frac{1}{g^2} - \theta^2 \frac{\beta(g^2)}{g^4} m$$

$$\int d^2\theta \frac{1}{g^2} \left( \frac{\mu}{M\mathcal{E}} \right) W_\alpha W^\alpha \supset -\frac{\beta(g^2)}{4g^4} m \lambda\lambda$$

$$m_\lambda = -\frac{\beta(g^2)}{2g^2} m$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

# AMSB Summary

- Tree-level piece on dimensionful parameters

$$V_{\text{AMSB}} = -m \left( \phi \frac{\partial W}{\partial \phi} - 3W \right)$$

- loop-level piece from running

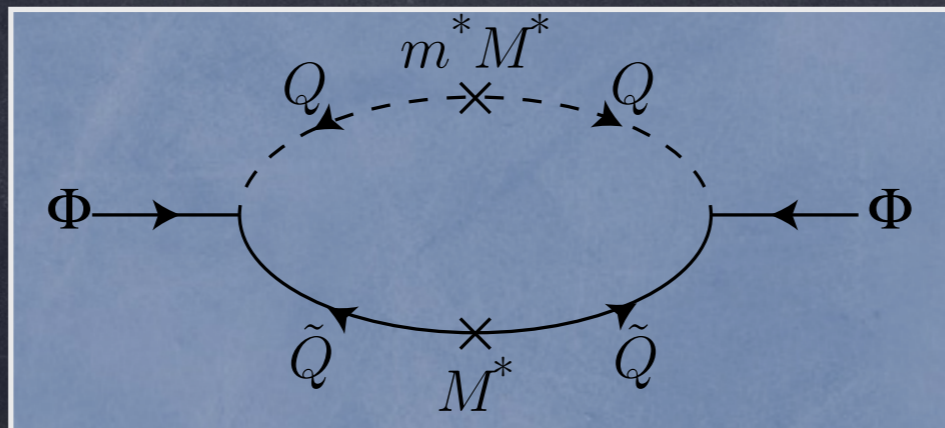
$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

- determined only by physics at the energy scale of interest
- UV insensitivity!

# UV insensitivity

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2}m_{3/2}, \quad m_i^2 = -\frac{\dot{\gamma}_i}{4}m_{3/2}^2, \quad A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k)m_{3/2}$$

- Surprising result: depends only on physics at the energy scale of interest
- No matter how complicated the UV physics is, they all disappear from low-energy soft SUSY breaking
- e.g., decouple a massive matter field:
  - Changes the beta function
  - one-loop threshold correction precisely account for the change in gaugino mass



# UV insensitivity cont.

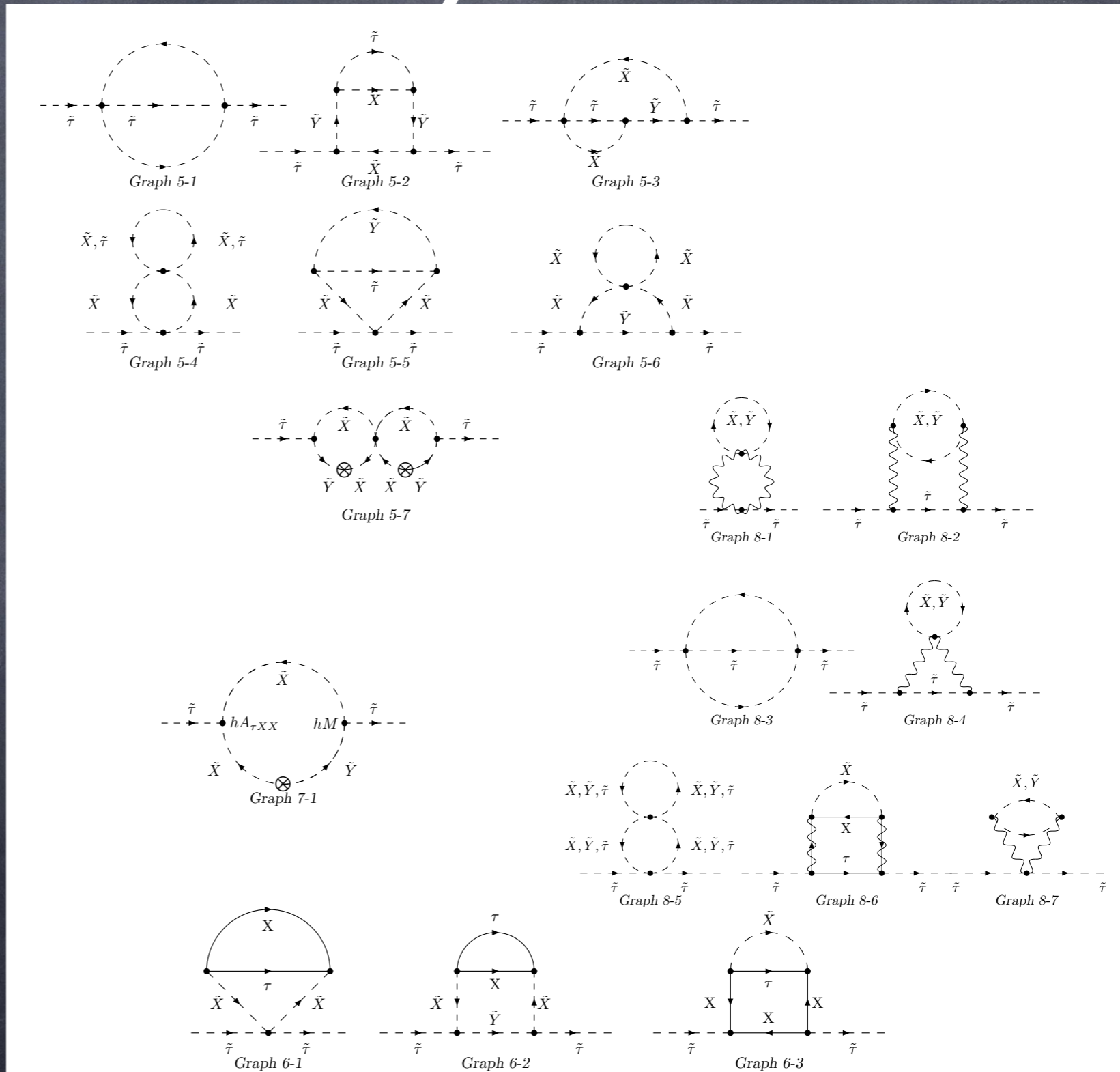
- decouple a massive matter field
- two-loop threshold correction precisely account for the change in the anomalous dimension and hence the scalar mass

$$M_i = -\frac{\beta_i(g^2)}{2g_i^2} m_{3/2}$$

$$m_i^2 = -\frac{\dot{\gamma}_i}{4} m_{3/2}^2$$

$$A_{ijk} = -\frac{1}{2}(\gamma_i + \gamma_j + \gamma_k) m_{3/2}$$

Boyda, HM, Pierce 2001







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Volume III Supersymmetry

# THE QUANTUM THEORY OF FIELDS

STEVEN WEINBERG

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*Preface*

xix

Riley for finding countless books and articles, and to Jan Duffy for many helps. I am grateful to Maureen Storey of Cambridge University Press for working to ready this book for publication, and especially to my editor, Rufus Neal, for his continued friendly good advice.

STEVEN WEINBERG

Austin, Texas  
May, 1999

# $SU(N_c)$ QCD

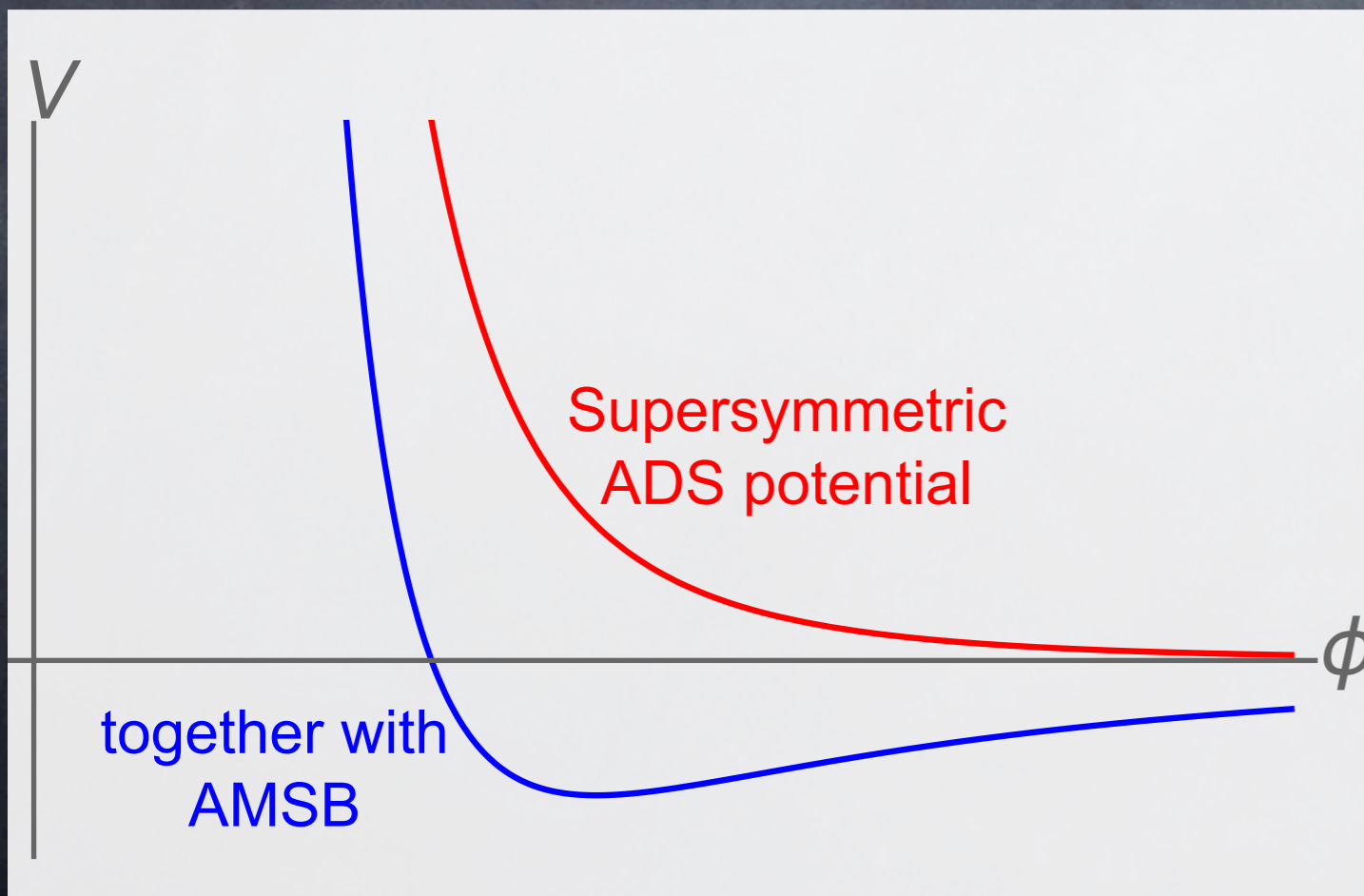
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$$N_f < N_c$$

run-away superpotential for  $M^{ij} = \tilde{Q}^i Q^j$

$$W = (N_c - N_f) \left( \frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{1/(N_c - N_f)} \quad M^{ij} = \delta^{ij} \phi$$

$$V = \left| 2N_f \frac{1}{\phi} \left( \frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{1/(N_c - N_f)} \right|^2 - (3N_c - N_f)m \left( \frac{\Lambda^{3N_c - N_f}}{\phi^{2N_f}} \right)^{1/(N_c - N_f)} + c.c.$$



$$M_{ij} = \Lambda^2 \left( \frac{4N_f(N_c + N_f)}{3N_c - N_f} \frac{\Lambda}{m} \right)^{(N_c - N_f)/N_c} \delta_{ij}$$

$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

$\chi$ SB! Proving Nambu  
mesino loop  $\rightarrow$  WZW term

$N_f=1$  special  
no NGB, gapped

$$N_f = N_c$$

- “Quantum Modified Moduli Space”

$$\det M - \tilde{B}B = \Lambda^{2N_c} \quad B = \det Q, \tilde{B} = \det \tilde{Q}$$

- non-trivial 't Hooft anomaly matching implies

Kähler potential is regular for  $M, B$

- go to canonical normalization for  $M, B$

$$W = X \left( \lambda \frac{\det M}{\Lambda^{N_c-2}} - \kappa \tilde{B}B - \Lambda^2 \right)$$

$$M^{ij} = \lambda^{-1/N_c} \Lambda \delta^{ij}, \quad B = \tilde{B} = 0,$$

$$X = \lambda^{-2/N_c} m, \quad V = -N_c \lambda^{-2/N_c} m^2 \Lambda^2$$

- Naive dimensional analysis:  $\lambda \approx (4\pi)^{N_c/2}, \quad \kappa \approx 4\pi$

- $SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$

- $\chi$ SB! Proving Nambu for  $SU(3)$  with  $u, d, s$

- massless pions, massive baryons!

$$N_f = N_c + 1$$

“Confinement without  $\chi$ SB”

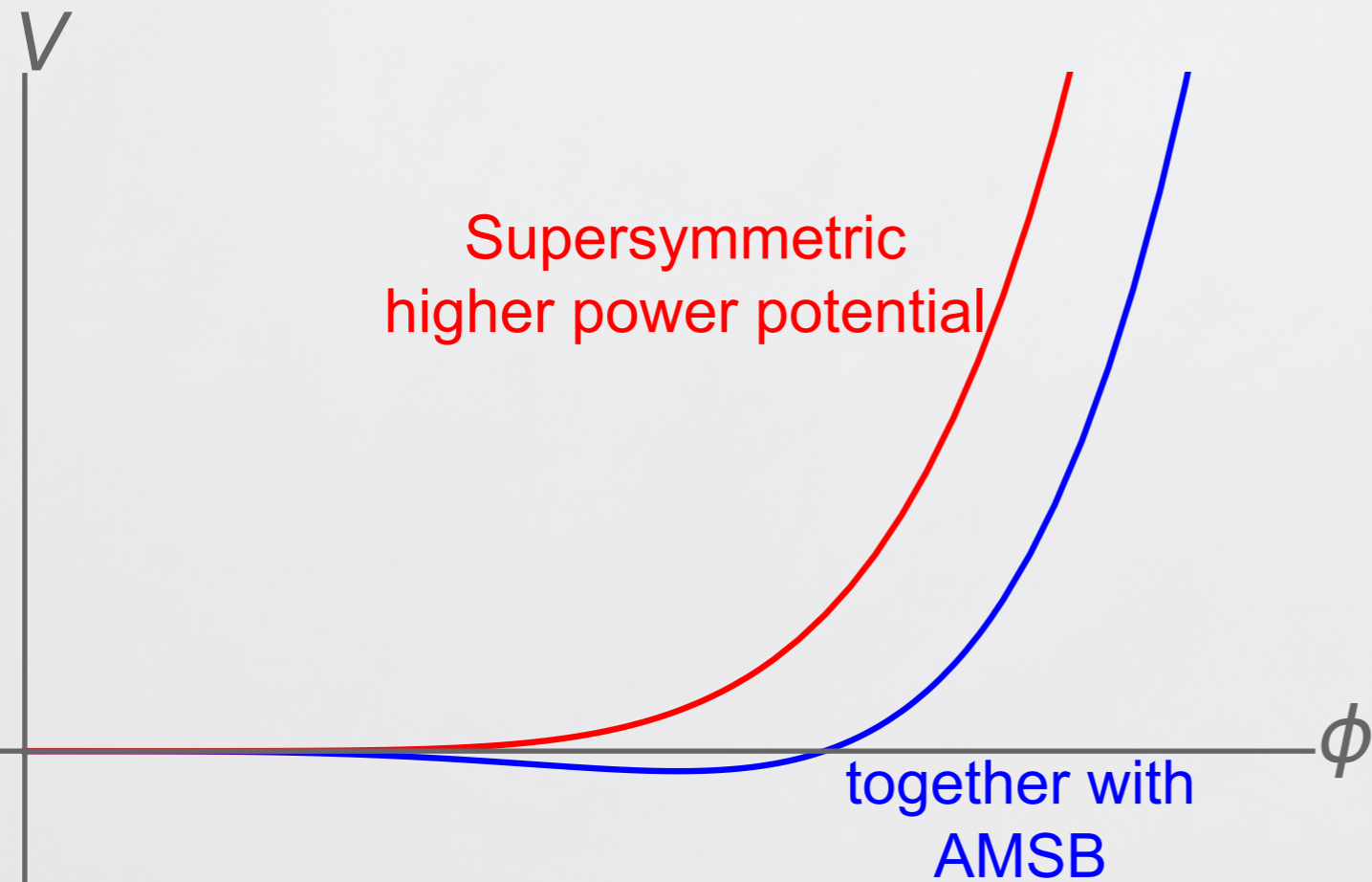
$$W = \frac{\det M - \tilde{B} M B}{\Lambda^{2N_c - 1}}$$

$$W = \lambda \frac{\det M}{\Lambda^{N_f - 3}} - \kappa \tilde{B} M B$$

$$B_i = \epsilon_{ij_1 \dots j_{N_c}} Q^{j_1} \dots Q^{j_{N_c}}$$

$$V = N_f \lambda^2 \frac{|\phi|^{2N_f - 2}}{\Lambda^{2N_f - 6}} - \lambda(N_f - 3)m\phi^{N_f} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left( \frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f - N_c)/(2N_c - N_f)} \ll \Lambda$$



$$SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V$$

- $\chi$ SB!
  - massless pions
  - massive baryons
- $$m_B = \kappa f_\pi \approx 4\pi f_\pi$$

$$N_c + 2 \leq N_f \leq 3N_c/2$$

- “free magnetic phase”
- $SU(N_c)$  becomes strong, binds baryons, which break up into dual quarks

$$B^{i_1, \dots, i_{N_c}} = Q_{\alpha_1}^{i_1} \dots Q_{\alpha_{N_c}}^{i_{N_c}} \epsilon^{\alpha_1 \dots \alpha_{N_c}} = \epsilon^{i_1 \dots i_{N_c} i_{N_c+1} \dots i_{N_f}} b_{i_{N_c+1} \dots i_{N_f}}$$

$$b_{i_{N_c+1} \dots i_{N_f}} = q_{i_{N_c+1}}^{\beta_1} \dots q_{i_{N_f}}^{\beta_{N_f - N_c}} \epsilon^{\beta_1 \dots \beta_{N_f - N_c}}$$

- “magnetic” IR-free  $SU(N_f - N_c)$  gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \rightarrow \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$

- $\lambda$  dimensionless, only loop-level AMSB

$$N_c + 2 \leq N_f \leq 3N_c/2$$

• "magnetic" IR-free  $SU(N_f - N_c)$  gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \rightarrow \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$

•  $\lambda$  dimensionless, only loop-level AMSB

$$V = \lambda^2 |q_i \tilde{q}_j|^2 + \lambda^2 |M^{ij} q_i|^2 + \lambda^2 |M^{ij} \tilde{q}_j|^2 \\ + m_M^2 |M^{ij}|^2 + m_q^2 (|q_i|^2 + |\tilde{q}_j|^2) + A\lambda M^{ij} q_i \tilde{q}_j + c.c.$$

$$M = v \left( \begin{array}{c|c} 1_{(N_f - N_c) \times (N_f - N_c)} & 0_{(N_f - N_c) \times N_c} \\ \hline 0_{N_c \times (N_f - N_c)} & 0_{N_c \times N_c} \end{array} \right)$$

$$q = \tilde{q} = v \left( \begin{array}{c} 1_{(N_f - N_c) \times (N_f - N_c)} \\ 0_{N_c \times (N_f - N_c)} \end{array} \right)$$

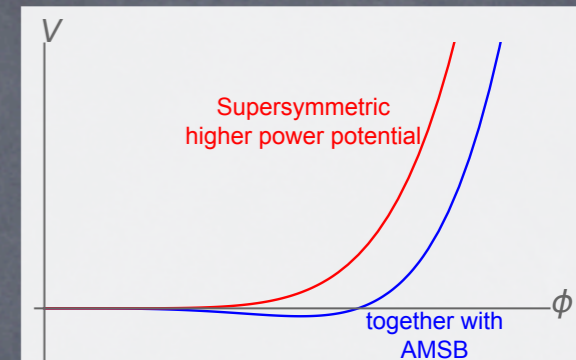
• breaks  $SU(N_f)_L \times SU(N_f)_R \times U(1)_B$   
to  $SU(N_f - N_c) \times SU(N_c)_L \times SU(N_c)_R$ ?

• vacuum energy loop-suppressed:  $V \approx - \left( \frac{\lambda^2}{16\pi^2} \right)^4 m^4$

$$N_c + 2 \leq N_f \leq 3N_c/2$$

- “magnetic” IR-free  $SU(N_f - N_c)$  gauge theory

$$W = \frac{1}{\mu} M^{ij} q_i \tilde{q}_j \rightarrow \lambda \tilde{M}^{ij} q_i \tilde{q}_j$$



- much lower global minimum
- go along the meson direction with rank  $M = N_f$
- integrate out dual quarks with  $M^{ij} = \phi \delta^{ij}$
- pure  $SU(N_f - N_c)$  YM forms gaugino condensate

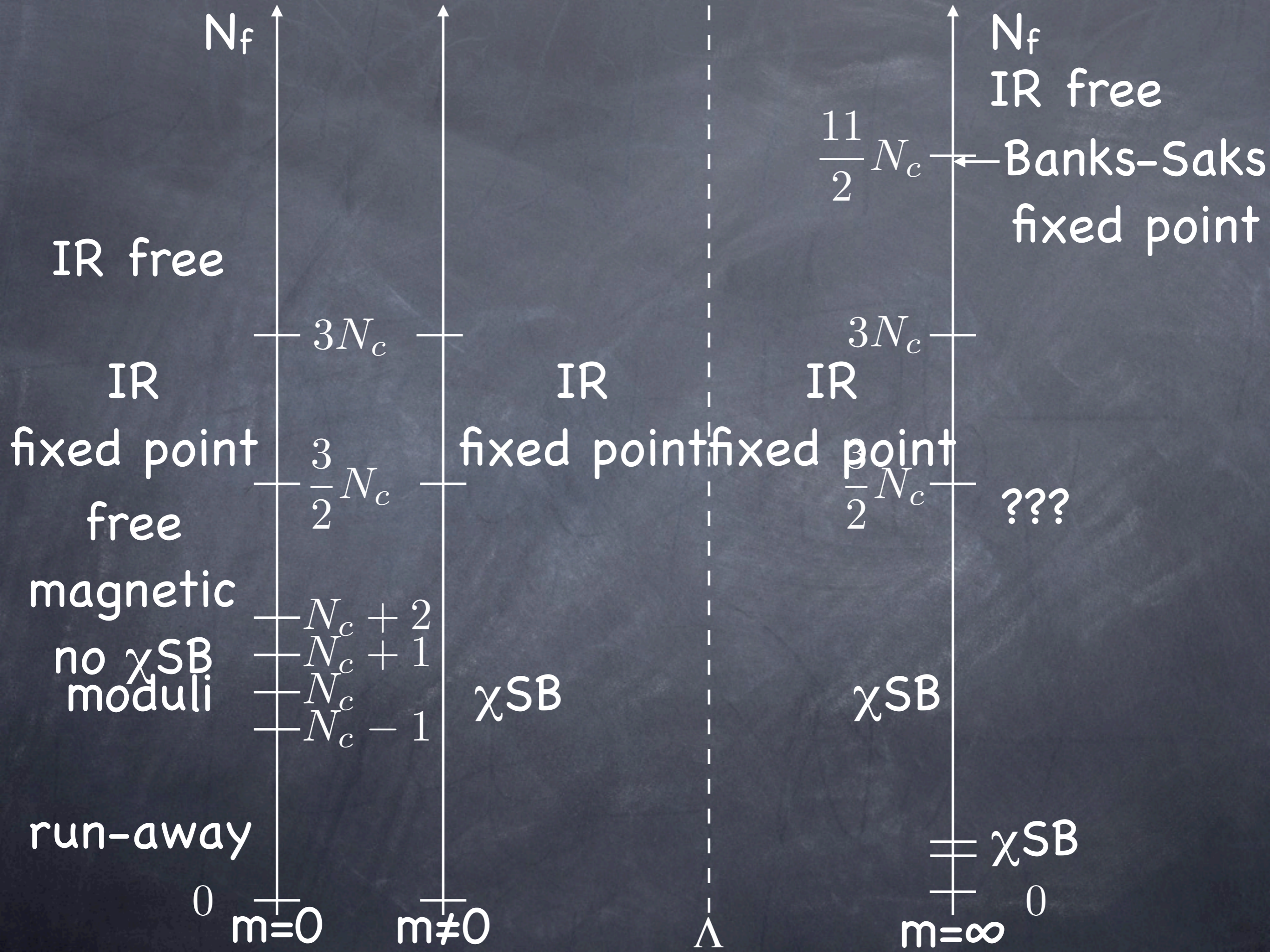
$$W = (N_f - N_c) \left( \frac{\kappa^{N_f} \det M}{\Lambda^{3N_c - 2N_f}} \right)^{1/(N_f - N_c)}$$

$$V = N_f \Lambda^4 \left| \frac{\kappa \phi}{\Lambda} \right|^{2N_c/(N_f - N_c)} - (2N_f - 3N_c) m \Lambda^3 \left( \frac{\kappa \phi}{\Lambda} \right)^{N_f/(N_f - N_c)} + c.c.$$

$$\phi = \kappa^{-1} \Lambda \left( \frac{2N_f - 3N_c}{N_c} \frac{m}{\Lambda} \right)^{(N_f - N_c)/(2N_c - N_f)} \ll \Lambda$$

$$\bullet \quad SU(N_f)_L \times SU(N_f)_R \rightarrow SU(N_f)_V \quad V \approx -\Lambda^4 \left( \frac{m}{\Lambda} \right)^{\frac{2(N_f - N_c)}{(2N_c - N_f)}}$$





$$3N_c/2 < N_f < 3N_c$$

• theories flow to IR fixed point

•  $d=3R/2$

$$Z_Q(\mu) = Z_{\tilde{Q}}(\mu) = \left(\frac{\mu}{\mu'}\right)^{(3N_c - N_f)/N_f} Z_{Q, \tilde{Q}}(\mu'),$$

$$Z_M(\mu) = \left(\frac{\mu}{\mu'}\right)^{(6N_c - 4N_f)/N_f} Z_M(\mu'),$$

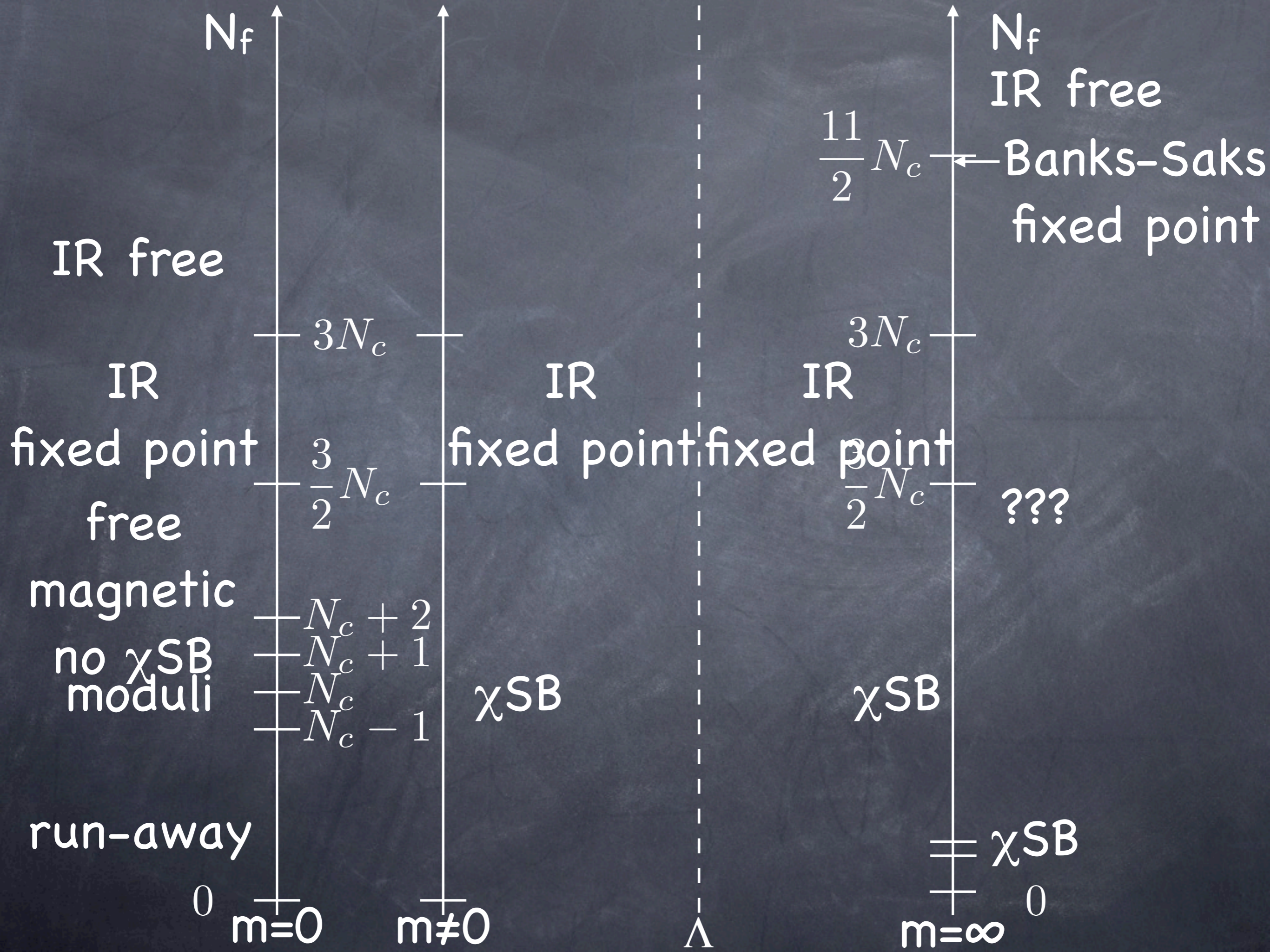
$$Z_q(\mu) = Z_{\tilde{q}}(\mu) = \left(\frac{\mu}{\mu'}\right)^{(2N_f - 3N_c)/N_f} Z_{q, \tilde{q}}(\mu')$$

• NSVZ exact beta function vanishes  $\beta = \beta'_*(g^2 - g_*^2)$

• loop-level AMSB power suppressed

$$m_\lambda(\mu) = -m\beta'_* \frac{g^2(\mu') - g_*^2}{2g^2(\mu)} \left(\frac{\mu}{\mu'}\right)^{\beta'_*}$$

• IR fixed point with "emergent SUSY" = SCFT

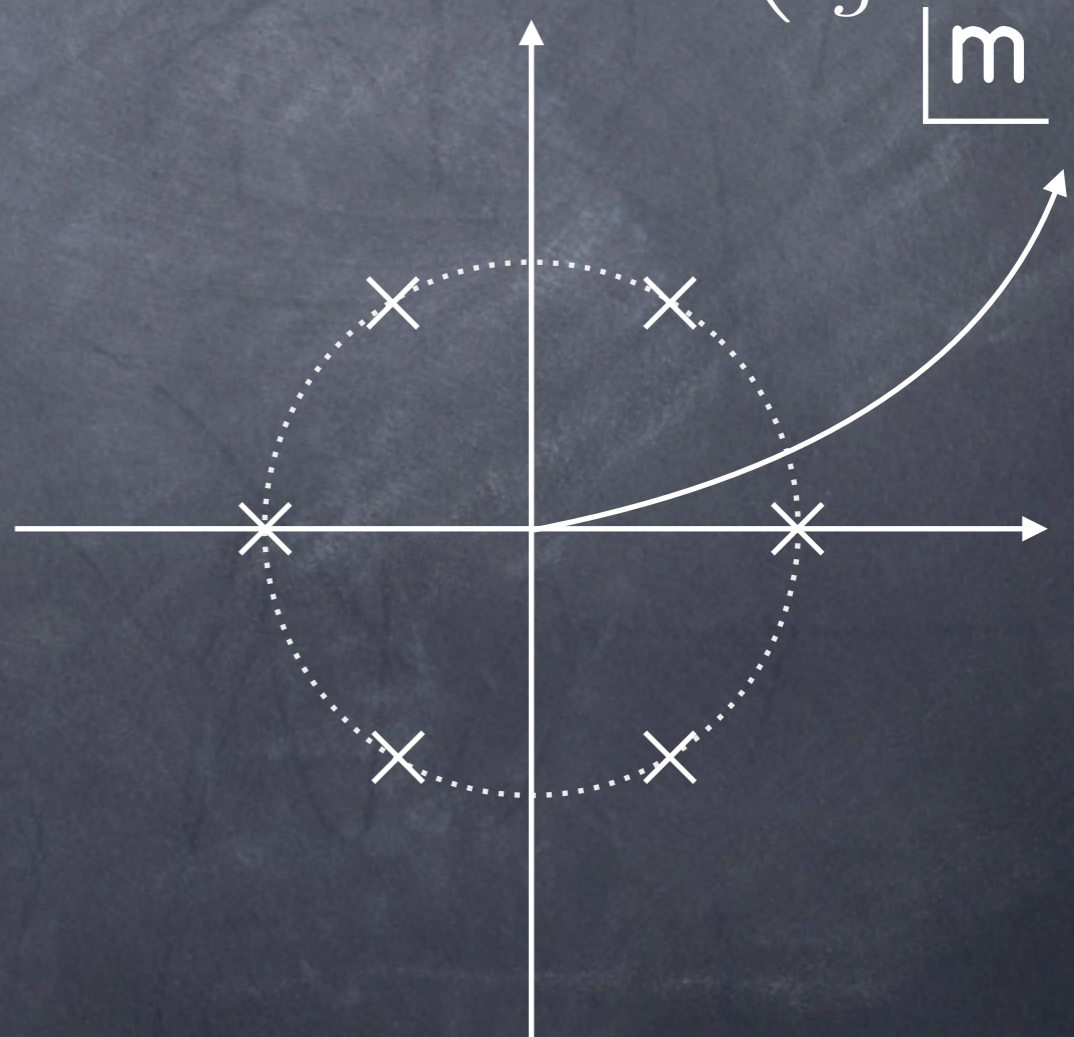


# phase transition?

- one may worry about phase transition when  $m \approx \Lambda$
- but they are both holomorphic quantities
- can't compare  $|m|$  and  $|\Lambda|$
- On the complex plane of  $m$ ,  $\Lambda$  are isolated points
- no phase transition, continuously connected
- "holomorphy argument"

$$m = \frac{W}{M_{Pl}^2},$$

$$\Lambda^{b_0} = \mu \exp - \left( \frac{8\pi^2}{g^2} + i\vartheta \right)$$



# confinement and $\chi$ SB

Csáki, Gomes, HM, Telem, in preparation

# confinement vs screening

- We've derived  $\chi$ SB in  $SU(N_c)$  QCD
  - it has no confinement
  - massless quarks in the fundamental rep can screen any color charges
  - Wilson loop is perimeter law
- $SO(N_c)$  QCD with quarks in vector rep
  - cannot screen  $Z_2$  center (e.g. spinor rep)
  - rigorous definition of confinement
  - can we see an interplay with  $\chi$ SB?



$$N_f = N_c - 2$$

for  $M_{ij} = Q_i^i Q_j^j \neq 0$  with rank  $M = N_f$ ,  $SO(N_c)$  is broken to  $SO(2)$

Coulomb branch  $u = \det M$

two singularities

$u = \det M = 0$

dyons:  $q_i^\pm$   $W = \frac{1}{\mu} M^{ij} q_i^+ q_j^-$

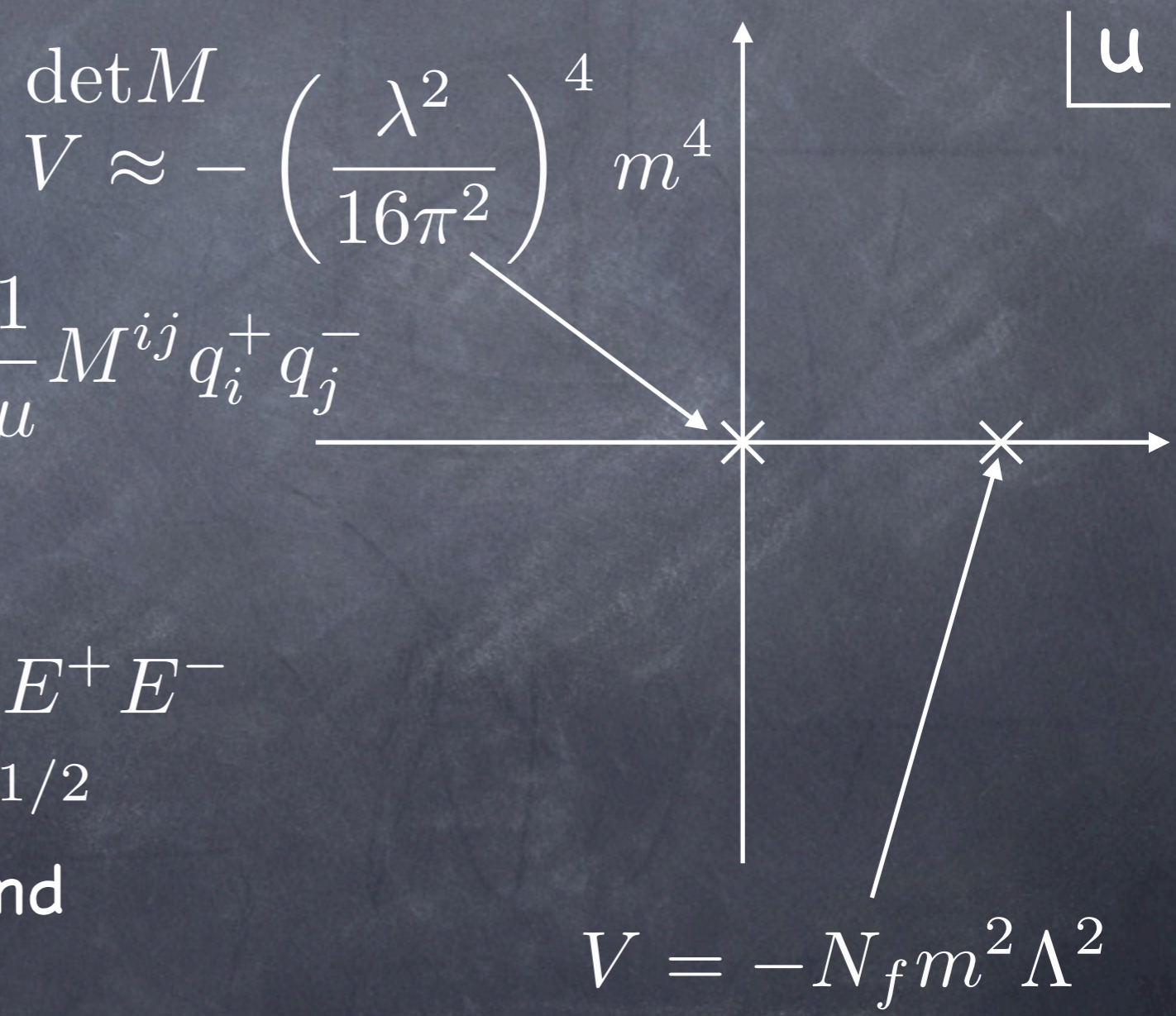
$u = \det M = \Lambda^{2N_f}$

monopoles:

$$W = (u - \Lambda^{2N_f}) E^+ E^-$$

$$|E^\pm| = (m\Lambda)^{1/2}$$

both monopoles and meson condense!





Chiral  $SU(N_c)$   
with  $A + (N_c - 4) F^*$

Csáki, HM, Telem, 2104.10171

# conjectures

- $A + (N_c - 4)F^*$ : anomaly free chiral gauge th
  - $SU(N_c - 4) \times U(1)$  global symmetry
1. massless composite fermions:  $A\bar{F}_{\{i, \bar{F}_j\}}$  satisfy 't Hooft anomaly matching conditions without breaking any global symmetries (E. Eichten, R. D. Peccei, J. Preskill, and D. Zeppenfeld (1986))
  2. condensate  $A\bar{F}_i$  breaks  $SU(N_c)_g \times SU(N_c - 4) \times U(1)$  to  $SU(N_c - 4) \times U(1) \times SU(4)_g$  with massless  $(\frac{1}{2}N_c(N_c + 1), N_c)$  fermion  $A\bar{F}_{\{i, \bar{F}_j\}}$  (S. Dimopoulos, S. Raby, L. Susskind (1980)) "tumbling"

# SUSY + AMSB, $N_c$ odd

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left( \begin{array}{c|c} J_{(N-5)} & 0 \\ \hline 0 & 0_{5 \times 5} \end{array} \right), \quad \bar{F} = \varphi \left( \begin{array}{c|c} I_{(N-5)} & 0 \\ \hline 0 & 0_{5 \times 1} \end{array} \right)$$

$$W = \left( \frac{\Lambda_N^{2N+3}}{(\text{Pf}' A \bar{F} \bar{F})(\text{Pf}' A)} \right)^{3/13}$$

Pouliot  
(1995)

$$\varphi \approx \Lambda \left( \frac{\Lambda}{m} \right)^{13/(4N-7)} \gg \Lambda$$

- $SU(N_c-4) \times U(1)$  broken to  $Sp(N_c-5) \times U(1)$
- massless fermions  $(N_c-4, N_c)$

# revised tumbling

- $A\bar{F}_i$  breaks  $SU(N_c)_g \times SU(N_c-4) \times U(1)$  to  $SU(4)_g \times SU(N_c-4) \times U(1)$
- $\bar{F}_{[i}, \bar{F}_{j]}$  breaks it further to  $SU(4)_g \times Sp(N_c-5) \times U(1)$
- $SU(4)_g$  becomes strong and  $A(4)$  and  $F^*(4^*)$  condense

# SUSY + AMSB, $N_c$ even

- non-perturbative run-away superpotential

$$A = \frac{\varphi}{\sqrt{2}} \left( \begin{array}{c|c} J_{(N_c-4)} & 0 \\ \hline 0 & 0_{4 \times 4} \end{array} \right), \quad \bar{F} = \varphi \left( \begin{array}{c} I_{(N_c-4)} \\ 0 \end{array} \right)$$

$$W = \left( \frac{\Lambda^{2N+3}}{(\text{Pf} A \bar{F} \bar{F})(\text{Pf} A)} \right)^{1/3}$$

Pouliot  
(1995)

$$\varphi \approx \Lambda \left( \frac{\Lambda}{m} \right)^{3/2N_c}$$

- $SU(N_c-4) \times U(1)$  broken to  $Sp(N_c-4)$
- no massless fermions

# revised tumbling

- $A\bar{F}_i$  breaks  $SU(N_c)_g \times SU(N_c-4) \times U(1)$  to  $SU(4)_g \times SU(N_c-4) \times U(1)$
- $\bar{F}_{[i}, \bar{F}_{j]}$  breaks it further to  $SU(4)_g \times Sp(N_c-4)$
- $SU(4)_g$  becomes strong and  $A(4)$  and  $F^*(4^*)$  condense

Chiral  $SU(N_c)$   
with  $S + (N_c+4) F^*$

Csáki, HM, Telem, 2105.03444

# conjectures

- $S + (N_c+4)F^*$ : anomaly free chiral gauge th
  - $SU(N_c+4) \times U(1)$  global symmetry
1. massless composite fermions:  $S\bar{F}_{[i}, \bar{F}_{j]}$  satisfy 't Hooft anomaly matching conditions without breaking any global symmetries (E. Eichten, R. D. Peccei, J. Preskill, and D. Zeppenfeld (1986))
  2. condensate  $S\bar{F}_i$  breaks  $SU(N_c)_g \times SU(N_c+4) \times U(1)$  to  $SU(N_c) \times SU(4)$  with massless  $(N_c^*, 4)$  fermion (S. Dimopoulos, S. Raby, L. Susskind (1980))  
"tumbling"



# SUSY + AMSB

- dual Spin(8) with  $N_c+4$   $q^i$  and one spinor  $p$

$$M_{ij} = S \bar{F}_{\{i} \bar{F}_{j\}}, \quad U = \det S$$

$$W = \frac{1}{\mu_1^2} M_{ij} q^i q^j + \frac{1}{\mu_2^{N_c-1}} U p p$$

Pouliot  
Strassler  
(1995)

- rank  $M=N_c+4$ ,  $U \neq 0$ , integrate out  $q^i$  and  $p$

$$W_{\text{dyn}} = (\tilde{\Lambda}_L^{18})^{1/6} = \left( \frac{\det \tilde{M} \tilde{U}}{\tilde{\Lambda}^{N-17}} \right)^{1/6}$$

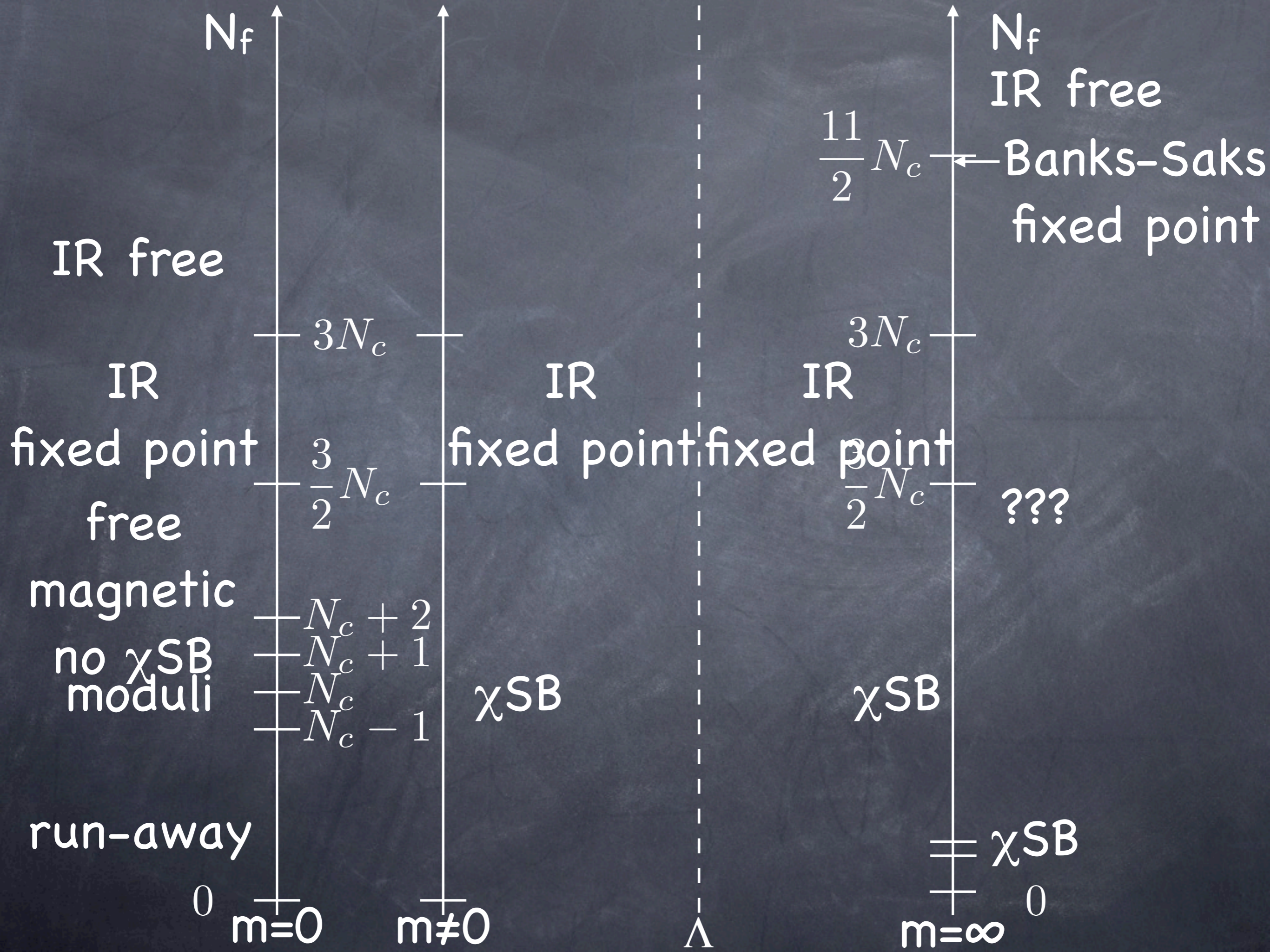
$$\mathcal{L}_{\text{tree}} = m \frac{N-17}{6} W_{\text{dyn}} + c.c.$$

$$\tilde{M}_{ij} \approx \delta_{ij} m \left( \frac{\tilde{\Lambda}}{m} \right)^{\frac{N-17}{N-11}}, \quad \tilde{U} \approx m \left( \frac{\tilde{\Lambda}}{m} \right)^{\frac{N-17}{N-11}}, \quad V \approx -m^4 \left( \frac{\tilde{\Lambda}}{m} \right)^{\frac{2(N-17)}{N-11}}$$

- $SU(N_c+4) \times U(1)$  broken to  $SO(N_c)$
- no massless fermions
- SCFT if  $N \leq 16$  + R-charge by a-minimization

# revised tumbling

- $SS$  in the  $\boxplus$  channel breaks  $SU(N_c)_g \times SU(N_c+4) \times U(1)$  to  $SO(N_c)_g \times SU(N_c+4) \times U(1)$
- $\bar{F}_{\{i, \bar{F}_j\}}$  is singlet under  $SO(N_c)_g$  and condenses, breaks it further to  $SO(N_c+4)$



# Conclusion

- $N=1$  SUSY + AMSB: a great tool to study non-SUSY gauge theories
- $SU(N_c)$  QCD:
  - $\chi$ SB ( $N_f \leq 3N_c/2$ ), CFT ( $3N_c/2 < N_f < 3N_c$ )
  - derived  $\chi$ SB from QCD for the first time!
- $SO(N_c)$  QCD:
  - $\chi$ SB and confinement by monopole condensation
- chiral gauge theories:
  - different from past conjectures
  - seems to minimize massless fermions